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DISCUSSION PRÉPARÉE / VORBEREITETE DISKUSSION / PREPARED DISCUSSION

Special Problems of Tall Buildings (Shear Walls, Stability of Columns, Effect of Thermal Gradients, Construction Problems)

Problèmes spéciaux aux bâtiments de grande hauteur (murs de contreventement, stabilité élastique des poteaux, effets de gradients thermiques, problèmes constructifs)

Spezielle Probleme bei Hochhäusern (Schubwände, Stabilität der Stützen, thermische Einflüsse, konstruktive Probleme)

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The authors' subject is one of great interest in view of the increasing world population. In this area of design there is a need for more information to define physiological and psychological response limits. Although some information is available on tolerable structural movements based on frequency of vibration and displacement, these should be extended in scope to cover a wider range of frequencies and displacements. There should be further development of response criteria based in terms of acceleration with regard to discomfort for the human body is most sensitive to this aspect of motion. It is recognized that criteria for comfort can be difficult to establish since personal reactions to vibration vary among individuals.

There is a need for more sophisticated wind measuring devices to obtain wind gust readings within fractions of a second. This information can be valuable in determining dynamic responses of structures. Further statistic and probabilistic studies should be undertaken in order to develop a more rational design criteria for earthquake and wind loads.

The importance of energy absorption and ductility in multistory frames is well recognized by all engineers. This, however, may not be easily achieved in reinforced concrete or prestressed concrete building frames unless more emphasis is given to the detailing of primary joints between columns, beams, girders and shear elements. This is now especially important in view of the potential increase in the use of precast units for the construction of high rise buildings. It may be more advantageous to precast such primary joints between columns, beams and girders and locate the splicing points between these precast units in an area midway between the primary joints in order to optimize the quality of construction and ultimate behavior of the critical column-beam-girder intersections. For instance, Figure 1 shows a method of high rise building construction utilizing precast column-beam elements in combination with pretensioned precast concrete floor slabs. The interaction of the columns and beams at the factory-produced joints will furnish the necessary energy absorption and ductility required in the resistance of lateral dynamic forces. The bending moments due to lateral forces are usually minimal at the splice points shown and these locations will be required primarily to transmit shear under the action of lateral forces. These connections, therefore, can be simply fabricated by means of grouted sleeves joining the column reinforcing bars and cast in situ splices for the beams. This prefabrication concept provides considerable economic benefit through the advantage gained by factory work replacing a large part of in situ work in the multistory structure.

Minimum depth in structural framing systems is important for multistory buildings not only from the point of view of cost savings but also with regard to the increasingly strict height limitations now being introduced in recent zoning and building set-back regulations. Figure 2 shows the cross section of a 33-story apartment building recently constructed under strictly regulated height limitations. The basic structural concept utilizes a 3-1/2 inch thick precast prestressed concrete slab soffit in composite action with a 2-1/2 inch thick cast in situ reinforced concrete topping making a total overall depth of 6 inches in a clear span of 26 feet. The maximum overall slab span from center line of beam to center line of wall support is 30 feet 7 inches. This thin slab was built to achieve these spans through the combined use of prestressing steel and lightweight aggregate concrete. It was found that when local pumice lightweight aggregate was prestressed its stiffness increased to the extent that it developed only two thirds the deflection for dead and live load of that experienced by similar construction using regular weight blue basalt aggregate concrete. This appeared to be an unusual phenomenon inasmuch as standard reinforced concrete using this same lightweight aggregate would develop about double the deflection experienced by similar construction with regular weight aggregate. The minimum thickness of these slabs enabled the builder to construct an extra story height within the restricted building height envelop as regulated by local building ordinances.

Weight is a critical factor in the construction of multistory buildings. If lightweight aggregate concrete is used in place of standard weight concrete, the reduction in total dead weight of the building means a reduction in earthquake response forces, reduction in the reinforcing required for each basic element such as floor slabs, beams, walls, columns and finally a reduction in the size of the footings and the requirements for foundation piling. However, the apparent initial disadvantage in the use of lightweight concrete because of its normally increased unit cost per cubic yard of material has discouraged many designers from investigating it further. In the building project shown in Figure 2, a structural cost comparison was made on alternate designs using various lightweight aggregate concrete combinations versus all standard weight concrete construction and it was proven that although there was a premium of $5.00 per cubic yard on the lightweight aggregate concrete, the use of this material throughout the entire building would result in a savings of $113,100. Figure 3 is a summary of this comparative cost analysis.

Other factors that can influence multistory construction cost are methods of framing with relation to the utilities that must be accommodated. One of the framing methods that has often been successfully employed in the past few years is the interrupted beam system whereby alternate spans of the beams are cantilevered off the columns and stopped short of the midspan, thus allowing a break for the passage of air conditioning ducts. This is an effective way to reduce the floor to floor height in multistory buildings and such reduction of height means a reduction in each run of stairs, elevator shafts, plumbing stacks, vertical duct work, area of exterior perimeter walls, overturning moment, etc.

The most desirable multistory building design solution requires a complete integration of numerous factors balancing structural methods and speed of construction with accommodation of electrical, mechanical and other utilities to produce the most functional, economical and aesthetic end product. The writer agrees with the authors that, "Structural design is still almost as much of an art as a science ...".

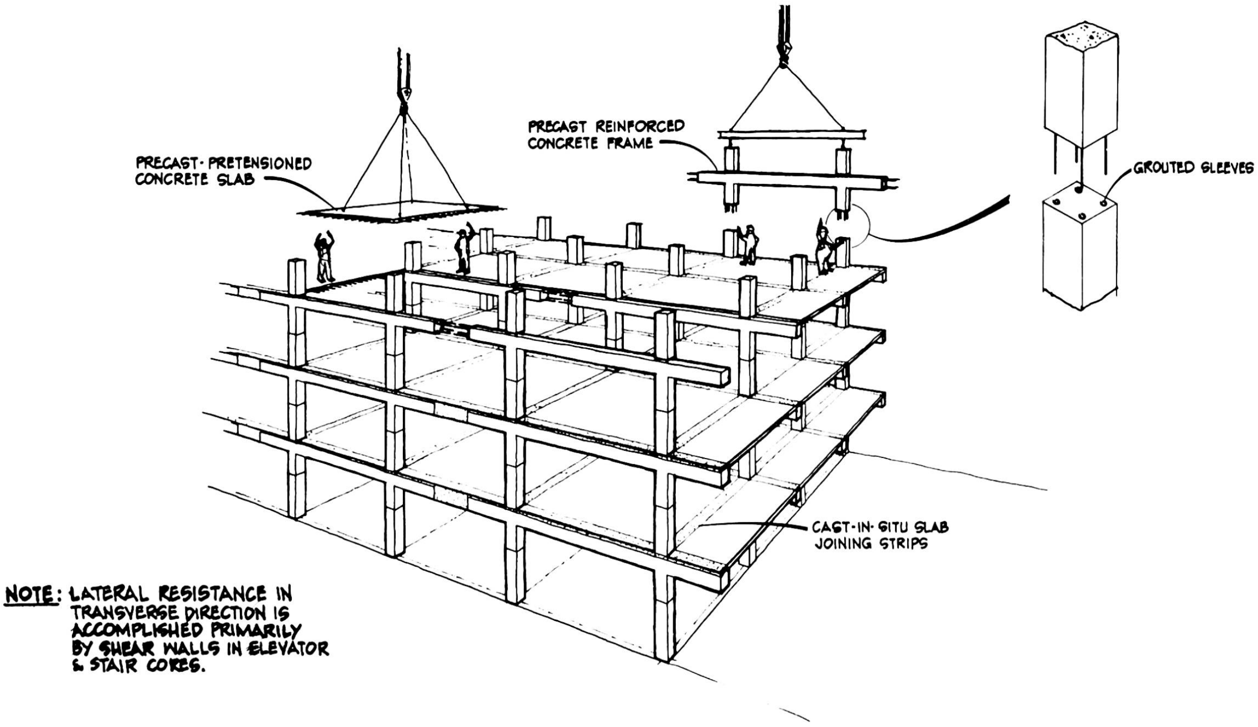
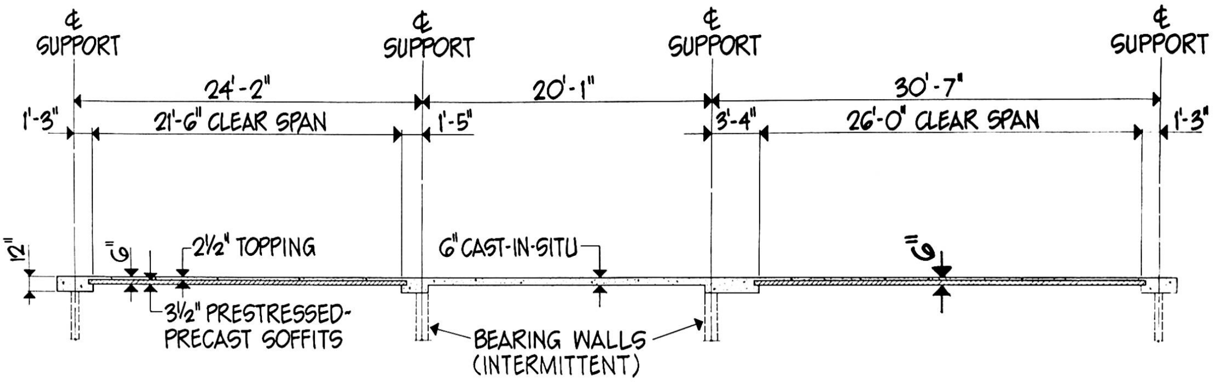

PRECAST-PRETENSIONED CONCRETE SLAB
PRECAST REINFORCED CONCRETE FRAME
GROUTED SLEEVES
CAST-IN-SITU SLAB JOINING STRIPS
NOTE: LATERAL RESISTANCE IN TRANSVERSE DIRECTION IS ACCOMPLISHED PRIMARILY BY SHEAR WALLS IN ELEVATOR & STAIR CORES.

FIGURE 1



TYPICAL APARTMENT FLOOR CROSS SECTION

FIGURE 2

COMPARATIVE COST ESTIMATE WITH VARIOUS COMBINATIONS OF AGGREGATE TYPES BASED ON TYPICAL BAY EXTRAPOLATION

Schemes	Reinforcing Costs	Concrete Costs	Pile Costs	Pile Cap Deductions	Totals
SCHEME I Blue Basalt Concrete throughout	$481,700	$264,400	$590,000	$ -	$1,336,100
SCHEME II Lightweight Concrete Topping and Beams - Blue Basalt Concrete Planks, Walls and Columns	434,100	296,400	574,000	800	1,303,700
SCHEME III Lightweight Concrete Topping, Beams and Planks - Blue Basalt Concrete Walls and Columns	387,200	299,400	552,000	1,900	1,236,700
SCHEME IV Lightweight Concrete throughout	380,300	315,700	530,000	3,000	1,223,000

TOTAL DIFFERENCE - SCHEME IV TO SCHEME I $113,100

Based on the Following Unit Costs:

Lightweight concrete @ $25.00/cu. yd.*
Regular weight concrete @ $20.00/cu. yd.*

A-15 reinforcing steel @ $.16/lb.**
A-432 reinforcing steel @ $.17/lb.**

200T piles @ $10.00/ft. (prestressed concrete)

* Material only
** In-place cost

FIGURE 3

SUMMARY

The importance of energy absorption and ductility in multistory building frames is well recognized by all engineers and the most important area of consideration is the detailing of primary joints between columns, beams, girders and shear wall elements. For factory fabrication there appears to be some advantage in precasting joints between primary members and locating splicing points at positions of minimum moments under lateral forces. In view of increasing building height restrictions, some advantages can be gained in minimizing the depth of structural framing systems. The use of lightweight concrete for weight reduction in multistory buildings is especially desirable in seismic areas.

RÉSUMÉ

L'importance de l'absorption de l'énergie et de la ténacité dans les portiques à étages multiples est bien connue par tous les ingénieurs. L'intérêt primordial est porté vers les joints principaux entre colonnes, poutres, traverses et éléments de mur de cisaillement. Pour la fabrication industrielle, il apparaît avantageux de préfabriquer les joints entre éléments primaires, et de placer des raccords aux endroits de moment minimum sous charge latérale. Vu les restrictions de plus en plus sévères de la hauteur des bâtiments, il peut être avantageux de minimizer la hauteur du système de portiques. L'emploi de béton léger réduisant le poids dans les structures élevées est particulièrement recommandé dans les zones sismiques.

ZUSAMMENFASSUNG

Frühzeitig ist die Wichtigkeit der Energieabsonderung sowie die Biegbarkeit in Stockwerkrahmen von allen Ingenieuren erkannt worden; der wichtigste Teil der Betrachtung ist der der Hauptknoten zwischen Stützen, Unterzug (Träger), Hauptträger und Scheiben. Für die Herstellung scheint es vorteilhaft, die Verbindungen zwischen Hauptelementen vorzufertigen und Montagestösse dort anzubringen, wo das Moment infolge seitlicher Kräfte minimal bleibt. Aus der Sicht der wachsenden Beschränkung von Gebäudehöhe können einige Vorteile durch die Minimalisierung der Rahmentiefe gewonnen werden. Die Anwendung leichten Betons zur Gewichtsabminderung in vielstöckigen Gebäuden ist besonders in erdbebengefährdeten Gebieten erwünscht.

New Practices in Concrete Buildings

Développements nouveaux relatifs aux bâtiments de grande hauteur en béton

Neue Entwicklungen bei Beton-Hochhäusern

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INTRODUCTION

The authors are to be commended for their concise presentation of the practices in concrete high-rise buildings. Their description is applicable where only wind forces act laterally against the buildings. In areas of high seismicity, both the design and construction is markedly different.

Professors N. M. Newmark and W. J. Hall are presenting, at this Congress, an extensive paper on the Dynamic Behavior of Reinforced Concrete Buildings. It is the philosophy expressed in their paper which led the Structural Engineers Association of California, (SEAOC) to initiate new procedures in the design of high-rise buildings.

Damaging earthquakes seem to indicate that, in order to safeguard life and property, the building frames have to resist lateral forces brought about by the earthquakes, and, furthermore, such building frames to be able to absorb energies without failures. The implementation of these criteria, however, is not a simple matter. Attempts to design within the elastic range would create buildings beyond economic feasibilities.

Forty years of evolution in producing a realistic design procedure has resulted in the latest SEAOC requirements which, for high-rise structures particularly, demand a ductile, energy absorbing rigid frame, with the following behavior:

a. Minor earthquake: No damage to the building.

b. Moderate earthquake: No structural damage, although minor damage to enclosing materials may be expected.

c. Major earthquake: Considerable non-structural damage, but the reinforced concrete frame absorbs the excess energy through ductile yielding and formation of elasto-plastic hinges.

EARTHQUAKE INTENSITIES. While no location may be guaranteed against earthquakes, past histories have been used for the production of a map of seismic probability.

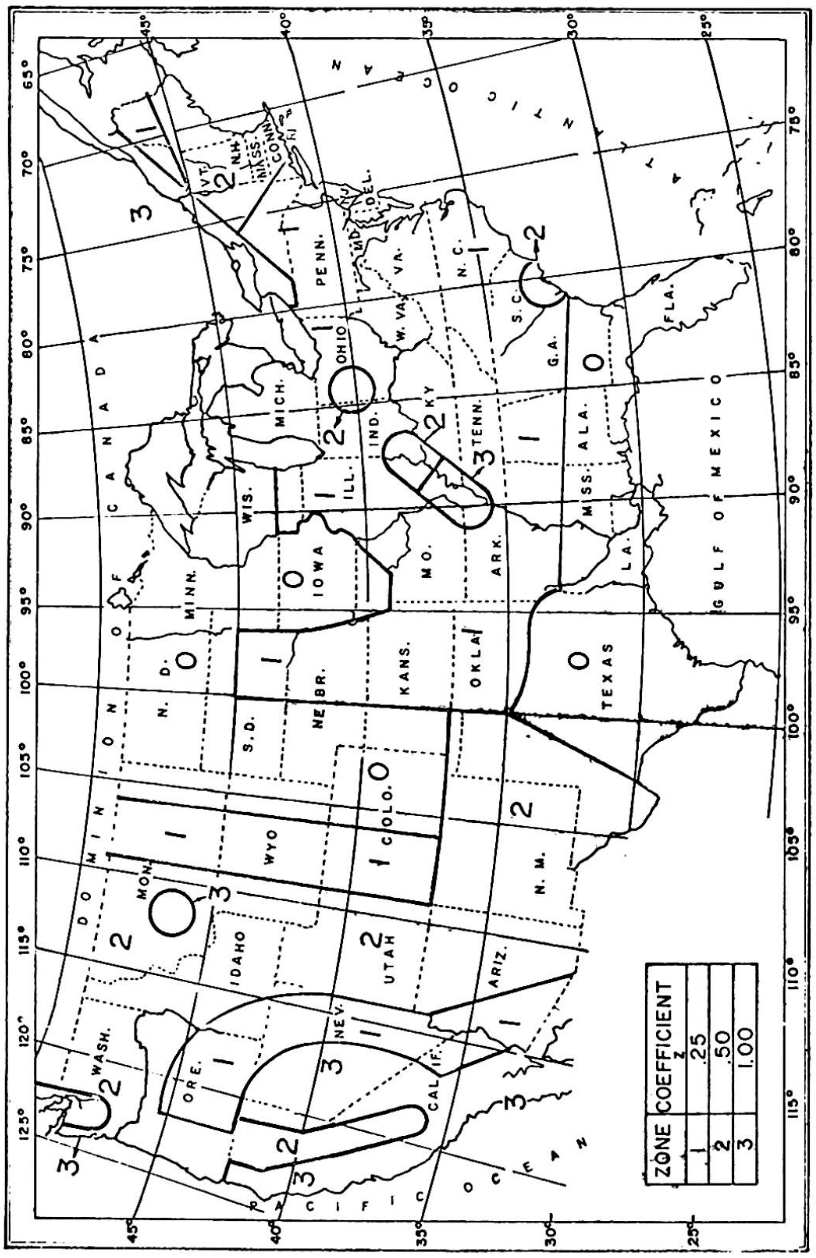


FIG. I.
EARTHQUAKE ZONES AND COEFFICIENTS "Z"

This map is now in process of revision (e.g.: Substantial earthquakes were registered in Denver, Colorado, a zero intensity location on the map.) In the calculation of seismic lateral forces a coefficient "Z" to be employed. The value "Z" varies according to the probability zone, such as:

Zone 0: "Z"= 0; Zone 1: "Z"= 0.25; Zone 2: "Z"= 0.50;
Zone 3: "Z"= 1.00.

FORCE FACTOR "K". Dependent on the framing system employed, a coefficient "K" is to be used:

HORIZONTAL FORCE FACTOR "K" FOR BUILDINGS OR OTHER STRUCTURES(1)

TYPE OR ARRANGEMENTS OF RESISTING ELEMENTS	VALUE OF K(2)
All building framing systems except as hereinafter classified.	1.00
Buildings with a box system as defined in Section 2313(b).	1.33
Buildings with a dual bracing system consisting of a ductile moment resisting space frame and shear walls designed in accordance with the following criteria: 1. The frames and shear walls shall resist the total lateral force in accordance with their relative rigidities considering the interaction of the shear walls and frames. 2. The shear walls acting independently of the ductile moment resisting space frame shall resist the total required lateral force. 3. The ductile moment resisting space frame shall have the capacity to resist not less than 25 per cent of the required lateral force.	0.80
Buildings with a ductile moment resisting space frame designed in accordance with the following criteria: The ductile moment resisting space frame shall have the capacity to resist the total required lateral force.	0.67
Elevated tanks plus full contents, on four or more cross-braced legs and not supported by a building.(3), (4), (5).	3.00
Structures other than buildings and other than those set forth in Table 23-D.	2.00

(1) Where prescribed wind loads produce higher stresses, these loads shall be used in lieu of the loads resulting from earthquake forces.

(2) The coefficients determined here are for use in the State of California and in other areas of similar earthquake activity. For areas of different activity, the coefficient may be modified by the building official upon advice of seismologists and structural engineers specializing in aseismic design.

(3) The minimum value of KC shall be 0.12 and the maximum value of KC need not exceed 0.25.

(4) For overturning, the factor J as set forth in Section 2313(h) shall be 1.00.

(5) The torsional requirements of Section 2313(g) shall apply.

High-rise buildings in excess of 160 ft. ($\pm$ 49 m.) can only be of ductile system, with a "K" factor of 0.80 or 0.67. There is a penalty for use of shear walls, as stated above; however, shear walls may be necessary to restrict drift either against seismic or wind forces; or they are difficult to eliminate due to architectural layouts. (Enclosure walls around stairs and elevators. Solid cast-in-place stairs act as trusses similar to shear walls.)

The minimum seismic base shear, in the direction of each of the main axes is to be:

$$V = ZKCW, \text{ where}$$

W= total dead load, (KIPS) $W = \sum_{i=1}^{n} w_i$

$C = \dfrac{0.05}{\sqrt[3]{T}}$; (C= 0.10 for one and two-story buildings.)

$T = \dfrac{0.05h_n}{\sqrt{D}}$; (T= 0.10 x number of stories above the base for rigid frame high-rises.)

= fundamental period of vibration in seconds in the direction under consideration. Properly substantiated data is also acceptable. (Such as computer calculations.)

h_n= total height of building above base. (ft.)

D= dimension of building in the direction of applied forces. (ft.)

In order to account for the higher modes of vibration and for whipping forces, the base shear "V" is to be distributed as follows:

F top level= $0.004V \left(\dfrac{h_n}{D_s}\right)^2 \leq 015V$, ($F_t = 0$ if $\left(\dfrac{h_n}{D_s}\right) \leq 3$)

F_x at level x= $\dfrac{(V-F_t)\, w_x h_x}{\sum_{i=1}^{n} w_i h_i}$, where

D_s= plan dimension of the vertical lateral force resisting system(ft)

wi, wx= that portion of "W" which is located at or is assigned to level "i" or "x" respectively.

TORSIONAL MOMENTS. As a rule, high-rise structures should be designed for a minimum torsional eccentricity of 5% of maximum building dimension.

OVERTURNING. Every high-rise building to resist the overturning effect caused either by wind or earthquake. For the latter there is a modified moment:

$M_{o.t.} = J\,(F_t\, h_n + \sum_{i=1}^{n} F_i h_i)$ where

$J = \dfrac{0.6}{\sqrt[3]{T}} \leq 1.$

SPECIAL PROVISIONS. Ductile frame buildings of 160 ft. or higher are subject to several restrictive provisions. The most important ones are enumerated as follows:

1. The main ductile moment resisting frame has to be cast-in-place monolithic reinforced concrete. Other members may be precast, prestressed, composite, etc.

2. The Ultimate Strength Design Method (USD) is specified for ductile frames.

3. The following load factors are to be used:

$U = 1.40\ (D + L + E)$ where D= Dead Load, L= Live Load, E= Earthquake Force

$U = .90\ D \pm 1.25E$

$= \text{(suggested)}\ .90\ D \pm 1.35E$

4. Under no conditions should plastic hinges be formed in columns but only in beams preferably near the columns.

5. In order to insure ductility both columns and beams at end near the joints shall be of confined concrete. Concrete may be confined by closely spaced ties, or stirrup-ties.

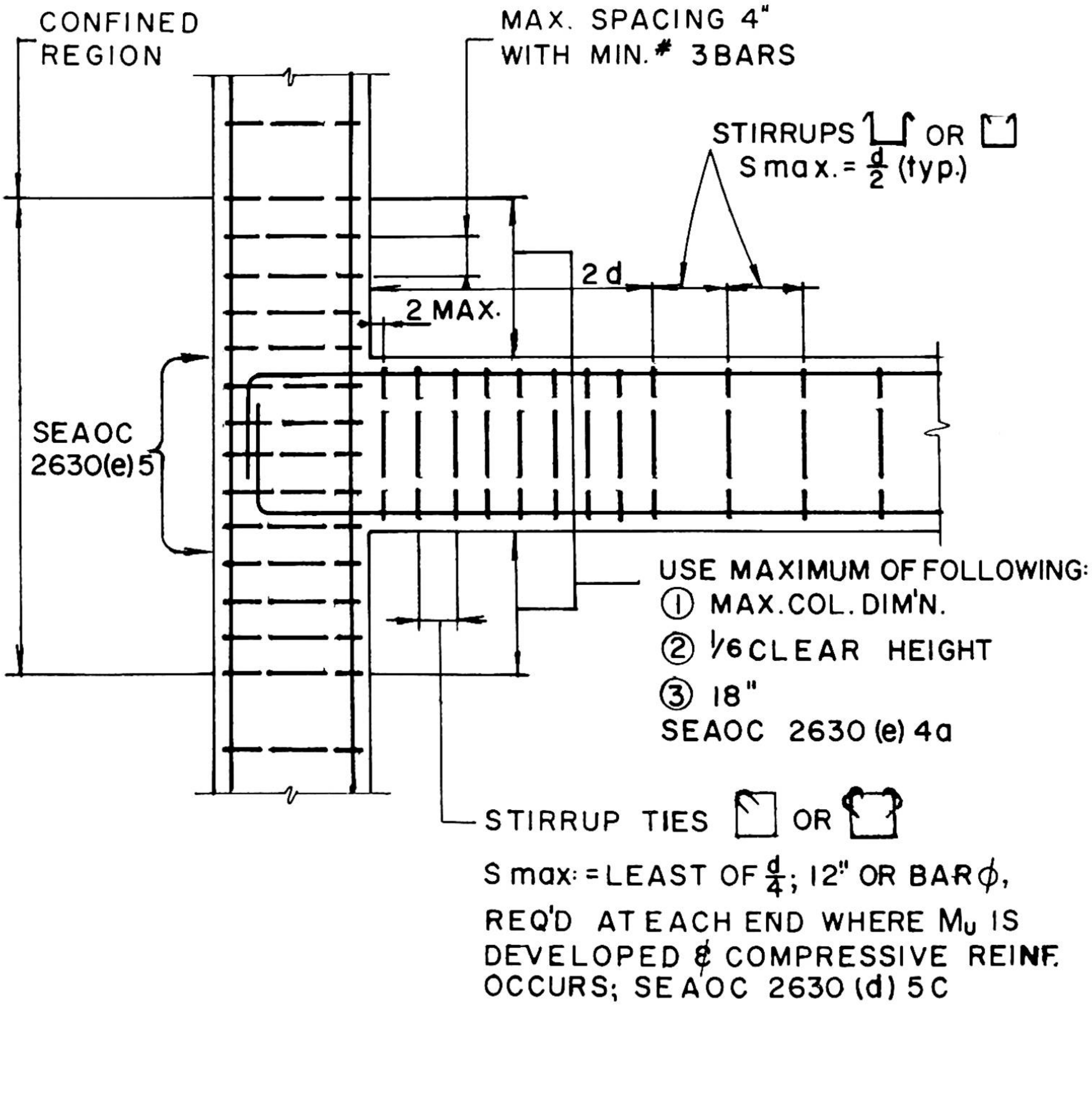
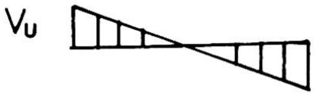


FIG. 2

COLUMN - BEAM JOINT REINFORCEMENT

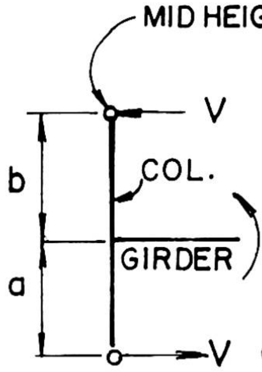
WEB REINFORCEMENT MIN. # 3 BARS IN ACCORDANCE WITH ACI, CHAPTER 17 --- SEAOC 2630 (d) 5

M_U^A & M_U^B = ULTIMATE MOMENT CAPACITY OF MEMBER OF OPPOSITE SENSE.



$$V_U \geq \frac{M_U^A + M_U^B}{L} + 1.4\,V(D+L) \text{ --SEAOC 2630(d)5a(30-5)}$$

MID HEIGHT OF STORY AS DETERMINED BY RATIONAL ANALYSIS.



$M_U = V(a+b)$

OR $V = \dfrac{M_U}{a+b}$

$V_U = T - V$ ------------------------- SEAOC 2630 (C) 5

DESIGN IN ACCORDANCE WITH ACI 318 CHAPTER 17, SPECIAL TRANSVERSE REINFORCEMENT REQ'T MAY GOVERN... SEAOC 2630 (C) 4

FOR COLUMNS WITH GIRDER FRAMING ON ALL FOUR SIDES ONE HALF OF THE SPECIAL TRANSVERSE REINFORCEMENT IS REQUIRED.

FIG. 3

GIRDER - COLUMN JOINT ANALYSIS

$V_u = \dfrac{M_u^T + M_u^B}{h}$ — — — — — — — — — — — SEAOC 2630 C (6) 30 - 9

OR $= \dfrac{M_u^B + \frac{1}{2} M_b}{h}$ — — — — — — — — — — — SEAOC 2630 C (6) 30 - 8

(OMIT FACTOR $\frac{1}{2}$ FOR ONLY ONE COLUMN IN TOP CONNECTION.

$V_u = V_c = A_v f_y \ \frac{d}{s}$ — — — — — — — — — — — SEAOC 2630 (e) 6 30 - 7

WHERE $V_c = v_c bd$ — — — — — — — — — — — ACI 318 CHAPTER 17

EXCEPT FOR $\frac{P}{A_g} \leq 0.12 f'_c$, $V_c = 0$.

A_v = TOTAL AREA OF TRANSVERSE REINF. IN TENSION WITHIN DISTANCE BETWEEN TIES

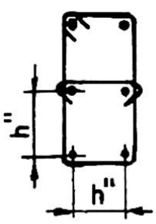
FOR SPIRALS USE $2/3\ A_v$ — — — — — — SEAOC 2630 (e) 6

FOR SPIRALS:

MIN. VOLUME SPIRAL $P_s = 0.45 \left(\frac{A_g}{A_c} - 1\right) \frac{f'_c}{f_y}$ - ACI 913 (b) (9-1)
BUT NOT LESS THAN REQUIRED BY — — SEAOC 2630 e (6)

FOR HOOPS:

MIN.: $2\ P_s$



SUPPLEMENTARY CROSS TIES; NOT TO EXCEED 25% OF TOTAL TIE VOLUME & SHALL HAVE STANDARD HOOKS AND ENGAGE EXTERIOR HOOP & VERTICAL BAR.

$h'' \leq \dfrac{2A''_{sh}\ f''_y h}{p'' a f''_y}$ — — — — — — — — — — SEAOC 2630 (e) 6 30 - 6

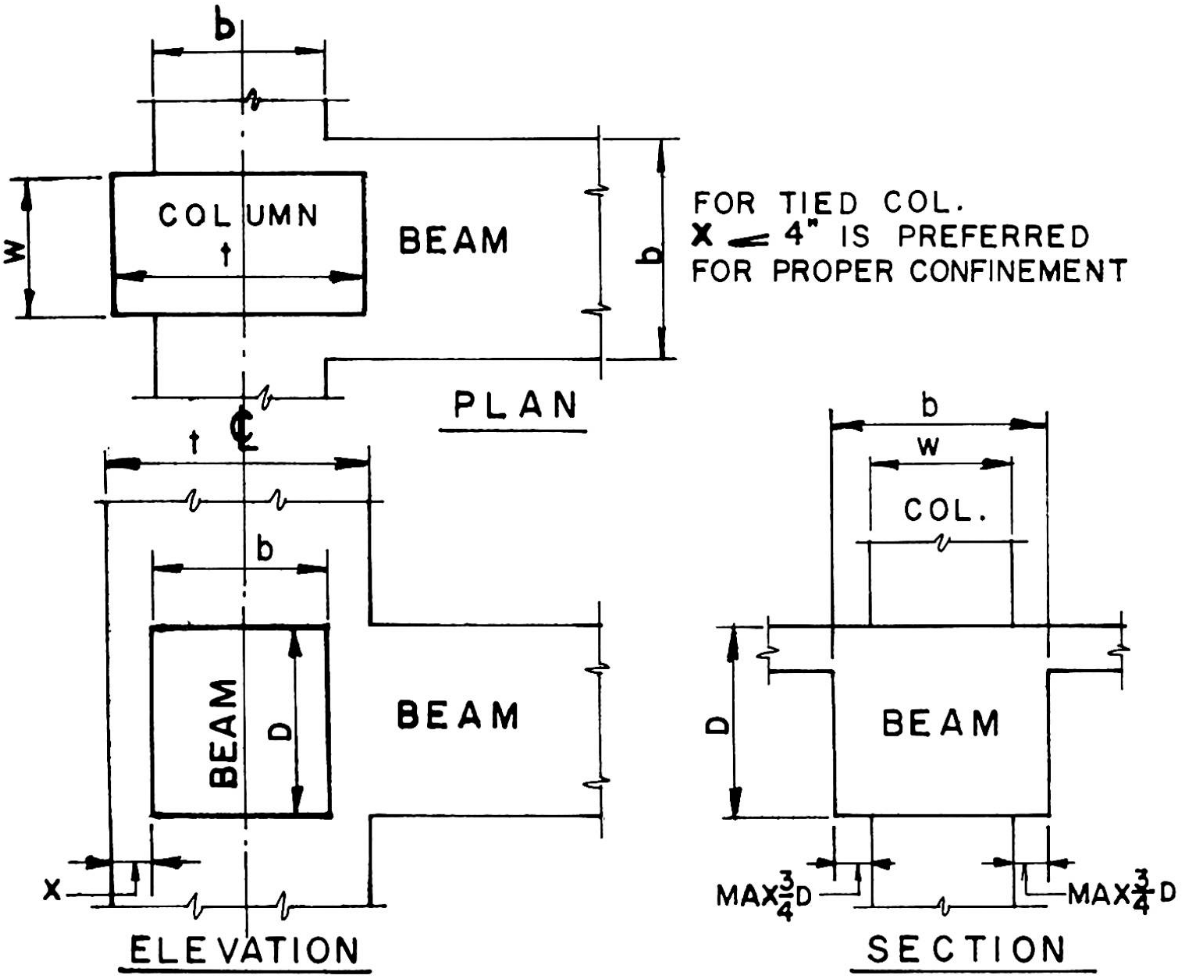
IF $h'' \leq$ MAX. COLUMN DIMENSION, PROVIDE NECESSARY OVERLAPPING HOOPS — SEAOC 2630 (e) 4 C

FIG. 4

ANALYSIS FOR COLUMN TRANSVERSE REINFORCEMENT

6. Only intermediate grade reinforcing steel (A-15, 40,000 psi yield point) shall be used in flexural members. (This restriction will be lifted as soon as the ductility requirements of high strength steel will be guaranteed.) Higher strength steel (A-432, 60,000 psi yield point) may be used in columns.

7. Compact sections are prescribed both for columns and beams.



BEAMS: $\frac{b}{D} \geq 0.4$

OR 10" $\leq$ b $\leq$ W + MAX.($\frac{3}{4}$D) EACH SIDE OF COLUMN -- SEAOC 2630(d)1

b $\geq \frac{3}{4}$t OR $\frac{3}{4}$ W (RECOMMENDED) ------------- SEAOC 2630(e)40

COLUMN: $\frac{W}{t} \geq 0.4$

W OR t OR ϕ OF ROUND COL. $\geq$ 12" ----------- SEAOC 2630(e)1

DESIGN: FOR $\frac{P}{Ag} \geq 0.12 f'_c$ WHERE Ag = (W) (t)

M_u OF COL. WITH P $\geq$ M_u OF BEAMS. FOR $\frac{P}{Ag} < 0.12 f'_c$ COLUMN SHALL CONFORM AS FLEXURAL MEMBER ----- SEAOC 2630(C)7

FIG. 5

SEAOC CODE LIMITATIONS ON FRAME DIMENSIONS.

8. The requirements under (7.) eliminate, until further studies are conducted, the use of flat slabs and flat plates as systems of unproven ductility.

9. In order to avoid brittle failures, not only minimum but also maximum reinforcements are prescribed.

10. Extreme attention is to be given to splices and anchorages.

11. Shear walls must be provided with strong boundary members designed to carry all vertical loads attributed to the shear walls including overturning axial forces. Thus, in case of failure of a shear wall during an earthquake, the boundary members would take over and insure safety against collapse.

CONCLUSIONS

The recently adopted Code provisions of the SEAOC permit now the construction of reinforced concrete high-rise buildings. (Previously only buildings less than 160 ft. in height could be built of reinforced concrete.) The writer has designed a 21-story medical building and a 26-story office building using the ductile frame reinforced concrete principles as described in this discussion.

SUMMARY

Reinforced concrete high-rise buildings in seismic areas have to be designed and constructed differently from the customary types described in the author's presentation. This discussion attempts to describe the present state of art for earthquake resisting high-rise buildings in the Western States of the United States.

RÉSUMÉ

Les bâtiments élevés en béton armé situés dans des zones sismiques doivent être projetés et construits différemment des types habituels décrits par l'auteur. La discussion essaye de faire le point sur les méthodes actuelles employées dans l'Ouest des Etats-Unis pour des bâtiments élevés résistants aux secousses sismiques.

ZUSAMMENFASSUNG

Hohe Stahlbetongebäude in Erdbebengebieten müssen anders als die üblichen, in des Verfassers Darstellung beschriebenen Typen entworfen und durchgeführt werden. Dieser Beitrag versucht, in den derzeitigen Stand der Bauweise erdbebensicherer, hoher Gebäude in den Weststaaten der USA Einblick zu geben.

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Design of Tall Buildings of Lightweight Superstructure

Projection de bâtiments élevés de construction légère

Entwurf hoher Gebäude im Leichtbau

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Introduction

Generally the floors in tall buildings are repetitive due to the shape of the structures. Table 1 gives an analysis of the estimated weights of the various components within typical floors of several buildings in London.1, 2, 3.

These buildings are approximately 35 storeys high with the exception of Moor House, which is 19 storeys high. It can be seen that the dead weight of the structure is 50 - 60% of the total weight and is thus by far the largest single item. Possible savings in weight on cladding, finishes and partitions are likely to be small in comparison with savings in dead weight of structure.

The average weight of structure of a typical floor including walls and columns is approximately 140 lbs/sq.ft. for a reinforced concrete frame 35 storeys high. The floor slabs vary in weight from 50 lbs/sq.ft. to 110 lbs/sq.ft. for spans up to 30 ft. with superloads of 80 lbs/sq.ft. including demountable partitions.

Those structures with light floors generally have a greater weight of walls and columns which yields a remarkably uniform average weight of structure. For lower structures of 20 storeys with spans up to 20 ft. the overall weight is approximately 120 lbs/sq.ft. and the slab weighs 85 lbs/sq.ft.

Table 1 - Weights of Tall Buildings.

	STAG PLACE		MILLBANK		DRAPERS GDNS		EUSTON CENTRE		MOOR HOUSE	
	weight lbs/ft^2	%	weight lbs/ft^2	%	weight lbs/ft^2	%	weight lbs/ft^2	%	weight lbs/ft^2	%
FLOOR SLAB	75	27.3	76	30.0	105	39.0	101	41.0	85	37.2
WALLS	60	21.8	48	18.7	37	13.8	17	6.8	22	9.6
COLUMNS	13	4.7	21	8.2	6	2.3	3	1.2	9	4.2
EXTERNAL CLADDING	20	7.3	10	3.9	14	5.4	9	3.6	10	4.2
FINISHES FLOORS CEILINGS	27	9.8	23	9.0	27	10.0	34	13.7	27	11.8
PARTITIONS	20	7.3	27	10.6	20	7.5	34	13.7	25	11.0
SUPERLOADS	60	21.8	50	19.6	60	22.0	50	20.0	50	22.0
TOTAL	275	100.0	255	100.0	269	100.0	248	100.0	228	100.0

Methods of reducing the dead weight of superstructure

Reduction in dead weight may be accomplished by the following:-

1. Use of high strength materials, i.e. high grade concrete, high tensile reinforcement or prestressed and/or precast concrete. These invariably cost more than average strength materials in common use, but reduction in weight and size may compensate.

2. The use of deeper structural sections of reduced thickness, i.e. ribbed and waffle slabs or open web joists. The deeper section increases the strength with very little increase in the weight. Increased fabrication costs are normally involved.

3. Use of lightweight materials of comparable strength to conventional materials i.e. lightweight concrete, plastics and aluminium. These usually cost more than their equivalent volume of conventional material, but the saving in weight may enable these costs to be recouped.

4. Reducing the floor spans, thus reducing the thickness of the floor. This technique is obviously limited as present day requirements are for open floor areas without supports.

5. Using the stiffness of the structural frame to withstand the horizontal loads without increasing the size of the members as determined by consideration of the vertical loads i.e. accommodating the stresses due to horizontal loads within the permitted 25% overstress (U.K. Standards).

6. The use of lightweight fire protection to structural steelwork in lieu of solid protection.

7. Special design techniques i.e. suspended structure where hangers may be used in lieu of columns and whole building loads ultimately supported on the core walls.

Factors to be balanced against savings in dead weight

Dead weight savings on structure are always desirable but must be reconciled with the other functions and also letting of the building. The importance of the latter is sometimes lost on engineers concerned primarily with structural design, but is vitally important to the client. The following factors should be balanced against the reduction in weight:-

a. Site Cost.
b. Building Cost.
c. Area of space available for letting.
d. Amenity values which may increase the prospect of letting.
e. Serviceability of the building.
f. Speed of construction.
g. Sound insulation and vibration.

TYPES OF STRUCTURE SUITABLE FOR LIGHTWEIGHT CONSTRUCTION AND EXAMPLES

a. Flat Plate Construction. In this type of construction the floors are designed as solid plates which act with columns to form a multi-storey rigid frame. The height for which this type of building is suitable is limited by the stresses within the plate floor and the deflections of the frame horizontally. Buildings up to 20 storeys can be constructed in this way, but the thickness of floors and the quantities of reinforcement required tend to make flat plate frame construction uneconomic above this limit.

The design imposes certain restrictions and advantages namely:- the external columns should be preferably inset from the face of the building; floor openings adjacent to the columns should be restricted; lightweight cladding should be used; the building should be preferably at least three bays wide to develop adequate lateral stiffness; the bay sizes should be approximately square. The compensations are that the elevators and staircases may be placed in any position; the shape of the building is not restricted; the construction is extremely simple, no shear walls are necessary, and it provides a flat soffite to the floors which may be plastered direct without false ceilings. It also reduces floor thickness to a minimum.

The trend today is to construct these buildings with lightweight concrete with a density of approximately 100 lbs/cu.ft. which reduces floor and column loadings, resulting in more economic design. In the U.K. buildings are often restricted in height and cubic content and therefore this form of construction, which takes up as little floor depth as possible, is often essential to obtain the maximum number of floors and therefore lettable area.

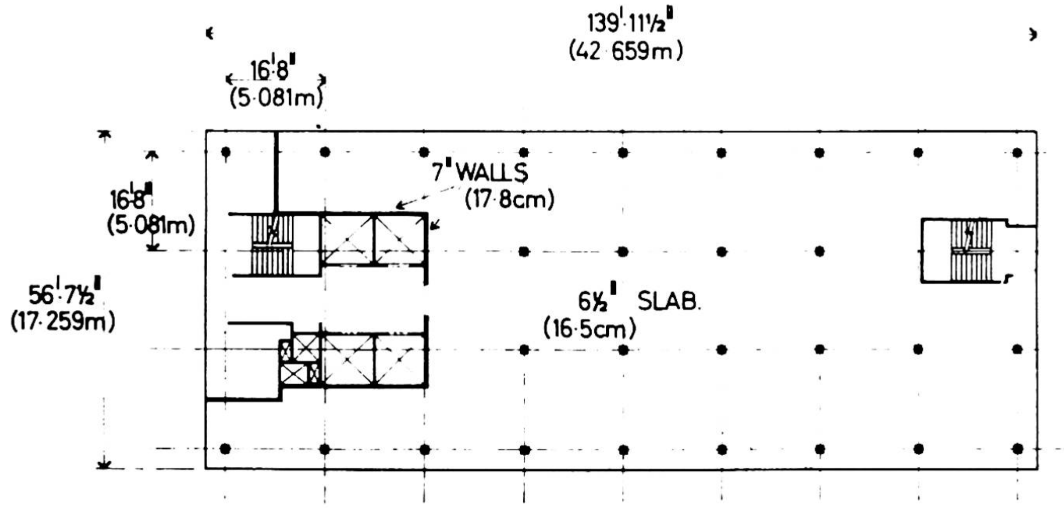


Fig. 1. MOOR HOUSE – TYPICAL FLOOR PLAN

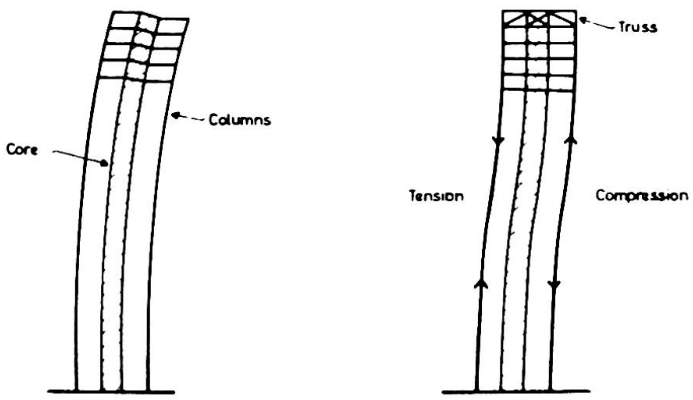


Fig. 2 FREE CANTILEVER Fig. 3 TIED CANTILEVER COLUMN – CORE INTERACTION

Figure 1 shows the floor construction of Moor House, London, 228 ft. high, and illustrates the principles outlined above. The floors are of normal gravel aggregate concrete and are only 6½ ins. thick. No column heads are provided.

b. <u>Central Core Construction, with External Edges of Floor supported on either columns or hangers</u>. The utilisation of the central core to withstand all lateral loads is becoming standard technique in buildings constructed in the U.K. up to 450 ft.high. Above this height the cores are rarely large enough to limit the lateral deflection of the building without increasing the thickness of the walls and columns as designed for vertical loading. The core supports a high proportion of the vertical loads of the

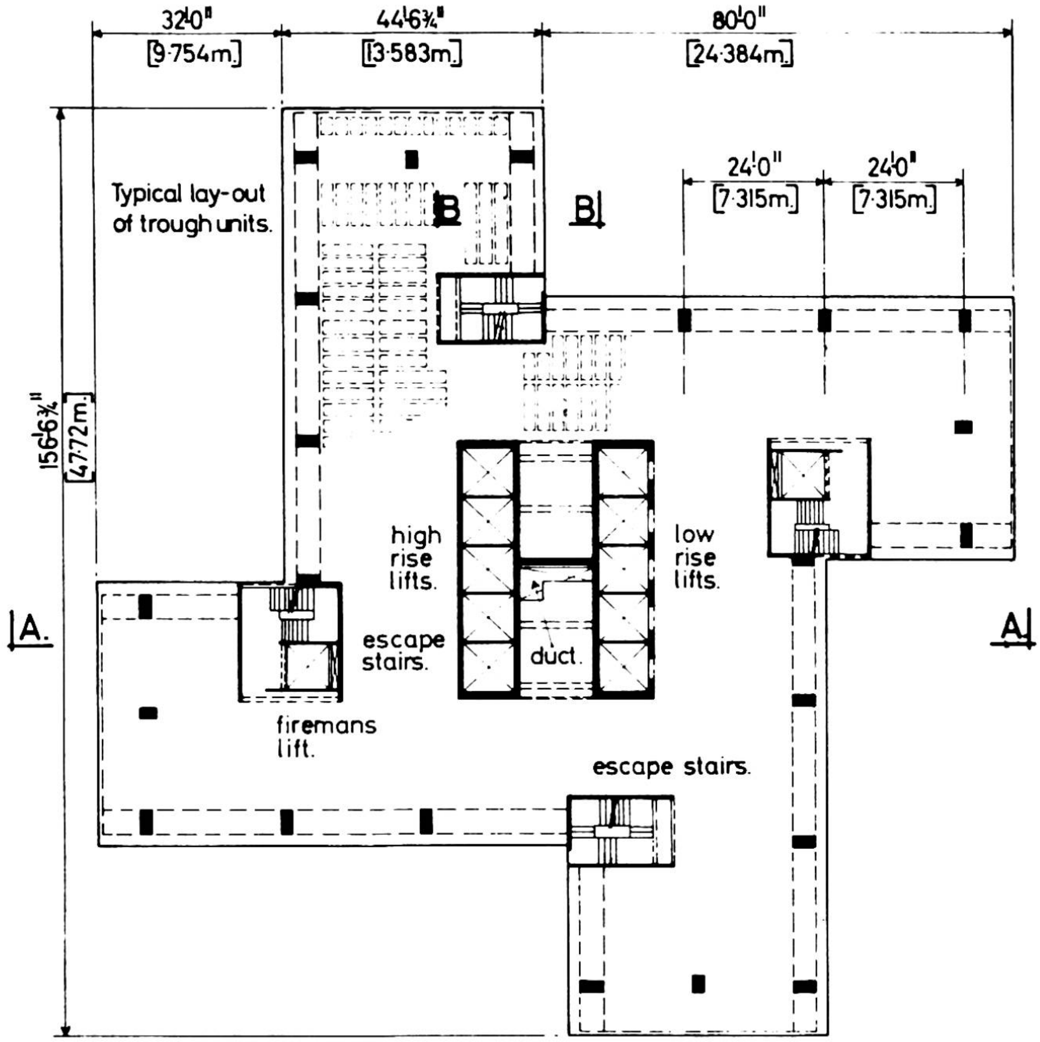
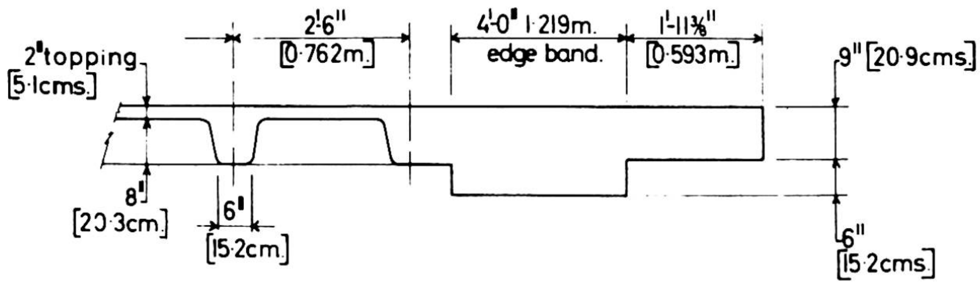


Fig. 4. – EUSTON CENTRE
TYPICAL FLOOR PLAN.



SECTION B-B.

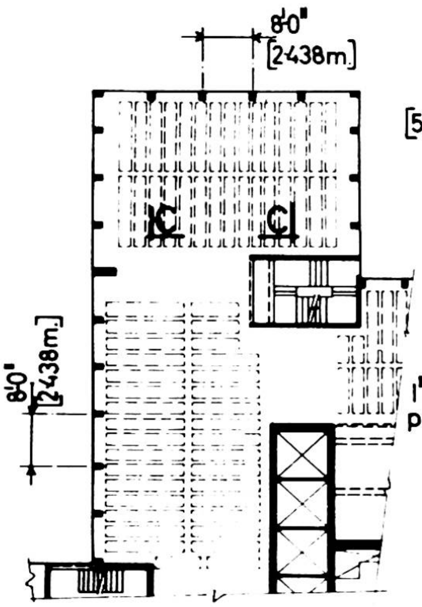
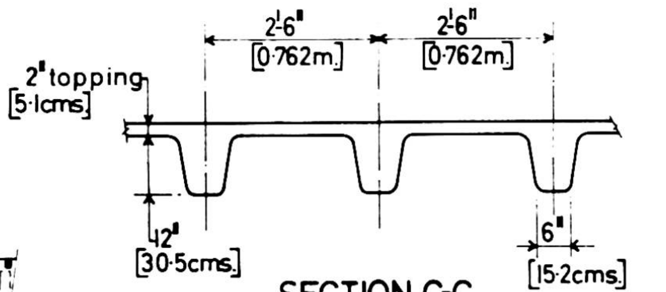


Fig.5. ALTERNATIVE FLOOR LAYOUT.



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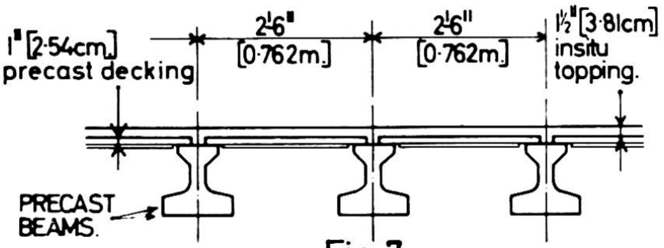

Fig.7.

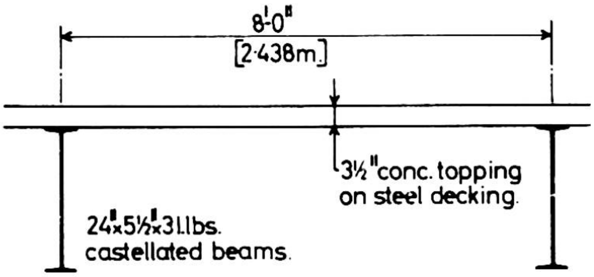

Fig.6.

Table 2 - Comparisons of alternative floor construction for Euston Centre

Case No.	Layout Fig.No.	Section Fig.No.	Ribbed Areas Wt. lbs/sq.ft.	Weight lbs/sq.ft.	% Weight	% Cost
1.	1	B - B	52	92	100	100
2.	1	-	38	64	70	107
3.	5	C - C	65	87	95	107
4.	5	C - C	48	64	69	108
5.	5	7	56	82	89	123
6.	5	6	-	42	46	125 *

* Excluding fire protection.

building thus fulfilling three functions i.e. vertical support, lateral support and service enclosures. The columns or hangers support vertical loads only, so that they may be designed to a minimum cross section and occupy as little floor area as possible.

The 35 storey Euston Tower, which is approximately 428 ft. high illustrates the principle of central core design with the external edges of the building supported on high strength (6,500 lbs/sq.in.) concrete columns. The core area and structure is limited so that 85% of the overall building area is usable.

Alternative floor constructions considered are shown on Figures 4,5,6 & 7 and Table 2.

Case 1 was in fact used and constructed using plywood formers and table forms. It provided a reasonably light structure with a minimum of reinforcement (8.5 lbs per sq.ft. including walls and columns) and a strong insitu structure to distribute lateral loads. The floor depth for the main floor areas was only 10 inches.

Case 6 using steel decking and beams was the lightest form of construction but was unacceptable due to the depth of floor construction and high cost.

Cases 2 & 4 using lightweight concrete were attractive but produced shear problems and required greater floor thickness than Case 1.

Case 3. The inclusion of structural mullions would have entailed large transfer girders at second floor level, which would have been expensive and were undesirable architecturally.

Case 5 using composite construction was more expensive than insitu construction with an increased floor thickness. Also it did not provide as rigid a structure as insitu construction and would have required a transfer girder at second floor level.

Suspended Structures

In this type of design the edges of the floors are supported by steel hangers, which are connected to cantilever trusses at the top of the building. These cantilever trusses are supported by a large central core. This arrangement produces the minimum area of external columns and provides additional dead load in the core to prevent tensile stresses being developed due to wind or lateral loading. The advantages of the method are that the contractor has the plant rooms available at an early stage in the contract, the floor space is uninterrupted by columns and there is a clear space at ground floor level which may be used for storage and access. It also allows a flexible ground floor layout. Due to the method of erecting the building from the top downwards, it is usual to build the structural floor of steel deck and castellated or lattice beams to avoid the need for formwork. Floor depths are greater than flat slab construction and building costs are usually slightly higher than an equivalent simple structure with columns.

Two examples of this form of construction are the Commercial Union Building and 20-23 Fenchurch Street, London. The difficulties of brittle fracture of the high strength steel trusses found in the latter seem to have been overcome in the P. & O. Building by using smaller made-up sections, which can be normalised instead of heavy welded Universal columns.

c. Central Core + Frame Action for Buildings over 400 ft. It is not normally economic to take all the lateral forces on the core as a vertical cantilever over 400 ft. tall, unless the core is very large; even if this is done, some account must be taken of secondary stresses induced in the remainder of the structure by consideration of the deflection of the structure. One variation is to allow the floor and columns to act integrally with the core to increase its stiffness. This solution is only practicable if the arrangement of the structural framing allows the stresses to be absorbed economically and is generally used in steel framed buildings. An alternative for reinforced concrete flat slab framed buildings is to provide trusses or beams at roof level to which the external columns are connected as shown in Figs. 2 & 3. This allows transfer of some of the tensile stresses which would otherwise develop in the core, to the external columns and therefore utilises more fully the total depth of the building. One example is Moorfields, London, 444 ft. total height and 36 storeys high, where the overall size of the floors were 66 ft. 9 in. x 202 ft. 9 in. and the core width only 21 ft.[4]

d. Hull Core Structures, where the external frame forms a hollow space tube and acts in conjunction with a central core.

No buildings, to our knowledge, have been erected in the U.K. which fall into this category, but several have been constructed in the U.S.A. One example is the World Trade Centre, New York, which is 1,350 ft. high. The principle is to use the external cladding not only to carry vertical loads but also to resist horizontal loads as a perforated box. It has the advantage that the internal floors can be constructed to give uninterrupted spans.

The external hull can be constructed in a variety of forms, either as a series of close centre mullions connected by beams at floor levels to form a series of inter-connected, very stiff portal frames, or, as a diagonal open lattice frame. The latter is particularly economical structurally since the forces within the cladding are mostly axial and result in high efficiency.

If the external cladding is constructed in steel as part of the curtain walling, it will be light in weight. If the floors are also constructed in steel joists and deck the resulting structure will be light in weight and capable of spanning 40 ft. clear without much difficulty.

This type of construction is particularly suitable in the United States where large floor areas are required and building heights are much greater than those permitted by the Planning Authorities in the U.K.

A variation on the hull core structure for buildings up to 400 ft. high is to omit the central core and use only the external wall frame to resist wind forces. The advantage is that flat slabs may be used within the building without the encumbrance of internal concrete walls, which would slow up progress of construction on site. e.g. The 42 storey DeWitt Apartments.[5].

DESIGN OF COMPONENTS

a. Floors

The two most favoured forms of floor construction are ribbed slabs, either precast or in situ, constructed as part of a floor of uniform thickness or a steel deck floor with concrete topping supported on steel joists. The former is most used in the U.K. as it is more economical for spans up to 30 ft. Generally storey heights are approximately 10 ft. 6 in. as against 12 ft. for steel deck and beam construction. In a 35 storey block another five floors can be built within the same building envelope using a ribbed slab instead of a steel deck and beam floor.

The relative costs of the two forms of construction vary according to the country in which they are built, since the ratio of labour and material costs vary considerably. The cost per sq.ft. of floor area of the steel in a steel framed structure as built in the U.S.A. or Canada would vary from 30/- to 50/- based on U.K. costs. Cost per sq. ft. of equivalent concrete building would be 20/- to 35/-.

A ribbed floor in normal gravel aggregate concrete 10 ins. thick weighs approximately 50 lbs/sq.ft. whereas a steel deck with concrete topping weighs only 35 lbs/sq.ft. If lightweight concrete were used in a ribbed floor the weights would be almost identical. The average weight of a floor incorporating ribs is much higher than might be expected because heavier solid strips have to be provided at the edges to support the ends of ribs and transfer loads back to the columns. It would seem unlikely that the dead weight of floor construction could ever be reduced much below 40 lbs/sq.ft. since a minimum thickness of floor would be required to provide mass to damp vibration and prevent undue sound transmission through the floors.

For long spans, prestressed concrete double T beams or I beams used at 2 ft. 6 in. to 3 ft. centres with precast planks provide a rapid method of erection. So far, the use of precast elements in tall buildings has not proved as successful in speeding up erection times as could be hoped. This is largely due to the labour required in propping, making and pouring insitu portions between precast elements and making joints, and also because the core areas often determine the speed of erection. As floors act as horizontal diaphragms to transfer lateral loads back to the core, it is essential that they have rigidity and any precast scheme must be carefully detailed to provide this. Shear heads should be avoided by the use of shear reinforcement either in the form of channels, collars or flat plates providing mechanical support, or diagonally inclined "snake" reinforcement in rings round columns.

b. Columns

To avoid the introduction of heavy beams or strips spanning between columns at the edges of the building, load bearing mullions at close centres may be used which do not project as far into the building as columns at greater centres and therefore do not break up the building area.

In order to reduce floor spans, columns may be inset a small distance to enable vertical service ducts to pass between column and cladding as shown in the Euston Centre. In this case the columns are designed in high strength concrete but even so are comparatively large in the lower storeys. At least one building (Drapers Gardens, London) has been constructed with solid steel columns (billets) to achieve the lightest weight and smallest amount of floor space occupied by columns. However, the increase in cost due to the billets is considerable and these should be carefully balanced against the increase in income due to the difference in lettable areas between concrete and steel columns.

c. Vertical Service Cores

These should be simplified as much as possible to enable them to be formed by slip-form or rapidly demountable large formwork panels; complicated core sections will slow down progress in the building as a whole. Small internal variations are most economically built in blockwork. The upper parts of the service core can be cast in lightweight concrete and in the lower parts the use of high strength concrete is essential to limit the wall thicknesses. Fire escape staircase enclosures can also contribute to the lateral strength of the building provided that this can be transmitted to the foundations. Unfortunately architectural requirements often prevent their being taken down to ground floor level.

d. Cladding

The lightest form of cladding is glass curtain walling amounting to within 3% - 4% of the total dead weight. The back-up wall to the curtain walling should be constructed in a light-weight, fire resistant material or wood-wool, rather than concrete or brickwork which has a greater density. Curtain walling has a further advantage in that it occupies a minimum thickness of wall, increasing the amount of floor area available for letting. This assumes that the exterior face of the building is fixed by the building line.

FACTORS AGGRAVATED BY LIGHTWEIGHT CONSTRUCTION

a. Thermal Movement

There is a faster build up of heat in exposed lightweight materials on the external face of the building which produces differential movement between the core and external columns. This can be overcome by making special provision in the structure to allow movement to take place, possibly by pin joints in the upper floors, or by inserting trusses to redistribute stresses between external columns. Internal partitions must be designed to allow a certain amount of distortion to take place in the frame. One method of minimising differential movement is to provide insulation to the external faces of columns to prevent such a rapid build-up. Careful detailing is required for glazing which fits between structural members of this type.

Thermal movement within the floor structure is usually easily accommodated and in fact, the use of lightweight concrete does help to reduce this.

If columns are set on the periphery they will be fully or partially exposed and therefore subject to temperature movement, which is considerable in buildings over 400 ft. high. The columns on one face of the building will expand or contract at a much greater rate than its core or columns on the other face. This will tend to crack the partition walls and possibly the structure, if excessive. Measurements taken on tall buildings give a differential movement of between .34 in. - 1.12 in. on structures varying between 200 ft. and 450 ft.; the amount of movement varying according to the height of the building and degree of exposure of the columns. The greater the degree of exposure, the greater the differential. Some cracking where the partitions join the external columns has been noted on existing buildings, although no structural damage has been recorded, probably because the ratio of depth:span did not exceed L/200. The reasonable limit for temperature movement seems to be approximately $\frac{3}{4}$ in. up or down from the horizontal position, assuming a clear floor span of 35 ft.[5]

b. Shrinkage

Lightweight aggregate concrete has a higher shrinkage rate than conventional gravel aggregate concrete and therefore adequate tensile reinforcement must always be included to control cracking.

c. Deflection

Generally lightweight concrete structures give rise to greater deflections than conventional structures due to their lower modulus of elasticity. There has been considerable research into the properties of lightweight concrete. This suggests that initial fears that the span:depth ratio would have to be adjusted to allow for the lower modulus are unfounded, provided that the stress in the reinforcement is not increased above 27,000 lbs/sq.in. The reduction of dead weight on the structure may give rise to tensile forces within the core and unacceptable horizontal deflections. In most practicable types of structure the height:width ratio is sufficient to avoid these difficulties. Reduction in weight also serves to decrease the damping effect of the building in its response to gusting of wind, although generally the likelihood of dangerous oscillations is improbable for conventional buildings up to 600 ft. unless very slender and with $ND<25$. (Where N is the natural frequency and D a typical cross section dimension.)

d. Sound Insulation

Lightweight structures and partitions allow greater sound transmission, which although acceptable in offices, would not be so in apartments. For this reason apartments are often constructed with solid plate floors to avoid too high a noise level. In offices, false ceilings help to reduce the level of airborne sound and the insertion of glass quilt under floors will reduce the transmission of structure-borne sound. In the case of plant rooms an acceptable solution seems to be to provide a thick concrete raft which rests on a layer of insulation material which will not transmit most of the troublesome frequencies of vibration from the plant above. If further insulation is required, wood-wool slabs may be suspended from the ceiling underneath.

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SUMMARY

The paper considers various designs suitable for lightweight superstructure, typical weights of differing construction and factors affecting the design of structural components.

RÉSUMÉ

Ce document envisage différentes conceptions convenant à des super-structures extrèmement légères, les poids types de constructions différentes et les facteurs affectant le dessin des componants structurels.

ZUSAMMENFASSUNG

Das Referat befaßt sich mit verschiedenen Entwurfen, die für leichte Aufbauten geeignet sind, sowie mit charakteristischen Gewichten verschiedener Konstruktionen und Faktoren, die den Entwurf von Bauteilen beeinflussen.

Influence Lines for Shear around Columns in Flat Plates

Les lignes d'influence d'efforts tranchants autour des colonnes aux dalles plates

Einflußlinien für Schub im Stützenbereich von Flachdecken

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Introduction

Shear stresses near columns in flat plate structures are caused by the column reaction. This reaction can be subdivided into a force, V_v, acting perpendicular to the plate and into a moment, M, whose vector is parallel to the plate. Only a portion of this moment, M, is transmitted to the plate by shear stresses. The remainder is transferred by bending stresses (Fig. 1).

The stress concentrations resulting from the above reactions often govern the design, i.e., they determine the required plate thickness and column periphery. It is the purpose of this paper to contribute to the evaluation of these stress concentrations.

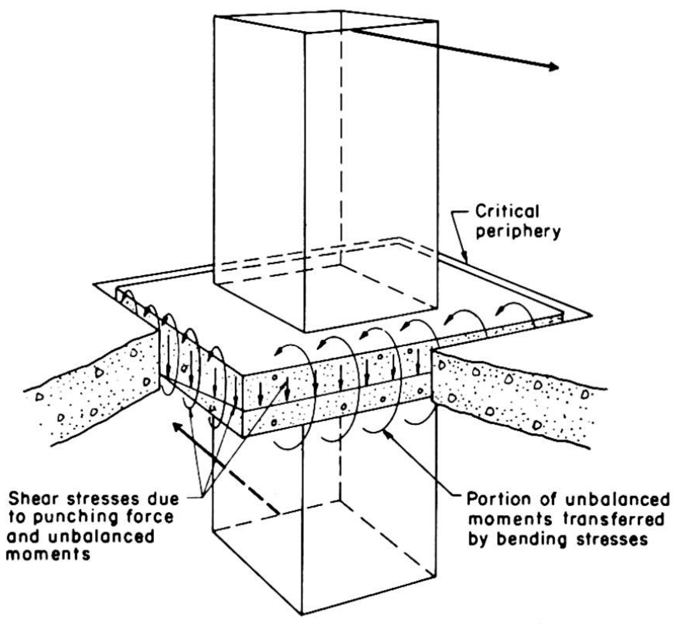

FIG. I TRANSFER OF UNBALANCED MOMENT

The moment transfer in the vicinity of columns has been investigated in extensive test programs [1 through 6]. While these tests resulted in design methods developed on a hypothetical basis [6 through 9], they did not reveal precisely which portion of the unbalanced moment is transferred by bending stresses and which portion by shear. The design methods commonly used in the USA [8, 9, 12] assume the moment transfer by shear to take place in accordance with the equation

$$v = \frac{V_v}{A_c} + \frac{KM}{J_c}e \qquad (1)$$

V_v = Total punching shear force

M = Total unbalanced moment

K = Percentage of unbalanced moment transferred by shear

A_c = Area of failure plane

J_c = Polar moment of inertia of failure plane

e = Distance from shear centroid to point on failure plane

Experimentally determined values for K, A_c and J_c vary and are available in tabulated form [9, Table 8-6]. The following is an approach to determine these values analytically and to evaluate the resulting shear stresses by means of influence lines.

An Analytical Method to Determine K

The deflection function of a simply-supported single-span plate strip, subject to a concentrated moment, is known [10].

$$w = \frac{M\,a}{2D\pi^2}\sum \frac{1}{n^2}\cos\frac{n\pi u}{a}\left(1+\frac{n\pi y}{a}\right)e^{-\frac{n\pi y}{a}}\sin\frac{n\pi x}{a} \tag{2}$$

Visualizing the plate supported by flexible columns at its center and applying the concentrated moment by one of these columns (Fig. 2), one can write a similar deflection function

$$w = M\frac{L}{D\pi^2}\sum_{n=1,2,3}\frac{1}{n^2}\cos\frac{n\pi}{2}\left(1+\frac{n\pi y}{2L}\right)e^{-\frac{n\pi y}{2L}}\sin\frac{n\pi(L+x)}{2L} \tag{3}$$

The boundary conditions at the remote columns are satisfied by this equation only partially. This, however, does practically not affect the stress configuration in the vicinity of the column at which the unbalanced moment is applied.

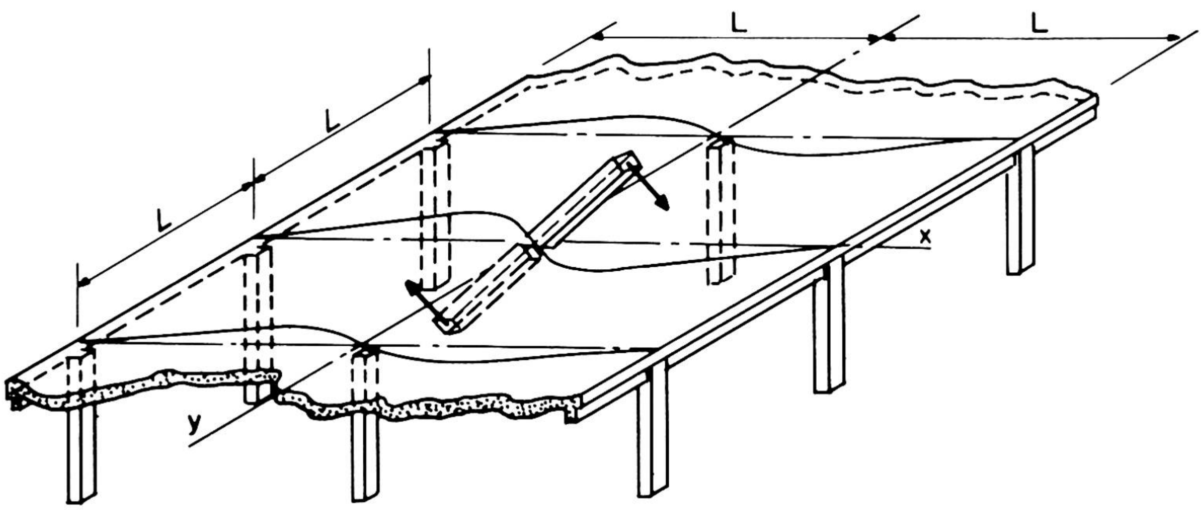

FIG. 2 CONCENTRATED MOMENT APPLIED TO FLATE PLATE

The series represented by Equation 3 converges slowly and, hence, is of limited practical use. It can be summed up, however, by means of transcendental functions [11], similar to the deflection function of a simply-supported plate strip [10]. We first determine the derivatives of the deflection function, w, with respect to the x and the y axis in closed form, and then the expressions for all bending and twisting moments and for the shear forces. It should be noted that simple expressions for the latter ones can be obtained best by determining $\partial(\Delta w)/\partial x$ and $\partial(\Delta w)/\partial y$, respectively.

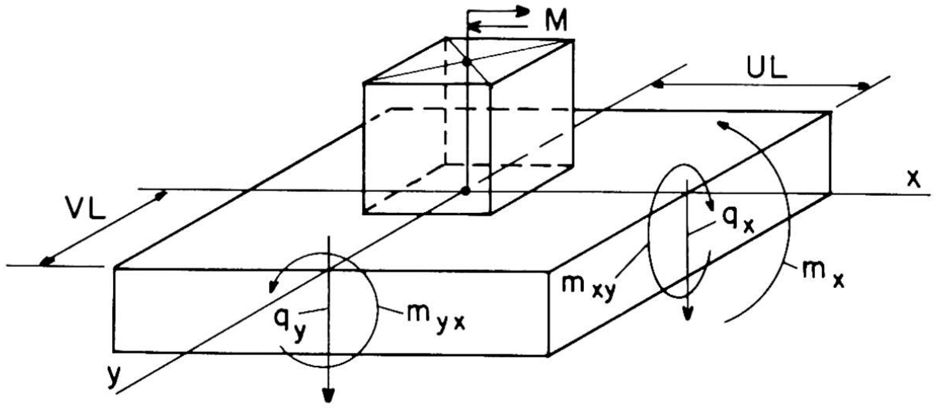

FIG. 3 SHEARS AND MOMENTS AROUND COLUMN

$$\frac{\partial w}{\partial x} = \frac{-M}{8D\pi}\left\{\log\left[2\cosh\frac{\pi y}{2L} + 2\cos\frac{\pi x}{2L}\right] + \log\left[2\cosh\frac{\pi y}{2L} - 2\cos\frac{\pi x}{2L}\right] - \frac{\frac{\pi y}{2L}\sinh\frac{\pi y}{2L}}{\cosh\frac{\pi y}{2L} + \cos\frac{\pi x}{2L}} - \frac{\frac{\pi y}{2L}\sinh\frac{\pi y}{2L}}{\cosh\frac{\pi y}{2L} - \cos\frac{\pi x}{2L}}\right\} \tag{4}$$

$$\frac{\partial w}{\partial y} = \frac{M}{16D}\left(\frac{y}{L}\right)\sin\frac{\pi x}{2L}\left\{\frac{1}{\cosh\frac{\pi y}{2L} + \cos\frac{\pi x}{2L}} - \frac{1}{\cosh\frac{\pi y}{2L} - \cos\frac{\pi x}{2L}}\right\} \tag{5}$$

$$m_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right)$$

$$= \frac{M}{16L}\left\{(1+\nu)\left[\frac{-\sin\frac{\pi x}{2L}}{\cosh\frac{\pi y}{2L} + \cos\frac{\pi x}{2L}} + \frac{\sin\frac{\pi x}{2L}}{\cosh\frac{\pi y}{2L} - \cos\frac{\pi x}{2L}}\right] + (1-\nu)\left[\frac{-\frac{\pi y}{2L}\sinh\frac{\pi y}{2L}\sin\frac{\pi x}{2L}}{\left(\cosh\frac{\pi y}{2L} + \cos\frac{\pi x}{2L}\right)^2} + \frac{\frac{\pi y}{2L}\sinh\frac{\pi y}{2L}\sin\frac{\pi x}{2L}}{\left(\cosh\frac{\pi y}{2L} - \cos\frac{\pi x}{2L}\right)^2}\right]\right\} \tag{6}$$

$$m_{yx} = m_{xy} = -D(1-\nu)\frac{\partial^2 w}{\partial x \partial y}$$

$$= \frac{M\pi y}{32L^2}(1-\nu)\left\{-\frac{\cosh\frac{\pi y}{2L}\cos\frac{\pi x}{2L}+1}{\left(\cosh\frac{\pi y}{2L}+\cos\frac{\pi x}{2L}\right)^2}+\frac{\cosh\frac{\pi y}{2L}\cos\frac{\pi x}{2L}-1}{\left(\cosh\frac{\pi y}{2L}-\cos\frac{\pi x}{2L}\right)^2}\right\} \tag{7}$$

m_y is similar to m_x, except that the terms associated with $(1-\nu)$ are of opposite sign.

$$q_x = \frac{\partial m_x}{\partial x}+\frac{\partial m_{xy}}{\partial y} = -D\frac{\partial(\Delta w)}{\partial x}$$

$$= \frac{M\pi}{16L^2}\left\{-\frac{\cosh\frac{\pi y}{2L}\cos\frac{\pi x}{2L}+1}{\left(\cosh\frac{\pi y}{2L}+\cos\frac{\pi x}{2L}\right)^2}+\frac{\cosh\frac{\pi y}{2L}\cos\frac{\pi x}{2L}-1}{\left(\cosh\frac{\pi y}{2L}-\cos\frac{\pi x}{2L}\right)^2}\right\} \tag{8}$$

$$q_y = \frac{\partial m_y}{\partial y}+\frac{\partial m_{xy}}{\partial x} = -D\frac{\partial(\Delta w)}{\partial y}$$

$$= \frac{M\pi}{16L^2}\left\{\frac{\sinh\frac{\pi y}{2L}\sin\frac{\pi x}{2L}}{\left(\cosh\frac{\pi y}{2L}+\cos\frac{\pi x}{2L}\right)^2}-\frac{\sinh\frac{\pi y}{2L}\sin\frac{\pi x}{2L}}{\left(\cosh\frac{\pi y}{2L}-\cos\frac{\pi x}{2L}\right)^2}\right\} \tag{9}$$

To determine the value K of Eq. 1 from the above expressions, one can define it in two ways. If K is defined as that portion of the unbalanced moment M, which is not transferred between columns and slab by pure bending stresses, then K becomes

$$K = \frac{\int_0^{UL} m_{yx}dx - \int_0^{VL} q_x UL\,dy - \int_0^{UL} q_y x\,dx}{\int_0^{VL} m_x dy + \int_0^{UL} m_{yx}dx - \int_0^{VL} q_x UL\,dy - \int_0^{UL} q_y x\,dx}$$

$$= 1 - \frac{4}{M}\int_0^{VL} m_x dy \tag{10}$$

The values U and V in this expression define the assumed failure plane (Fig. 1) at which the stress configuration is to be determined (Fig. 3). The integrations can be performed numerically or, with certain approximations, in closed form as follows.

We are primarily interested in the stress configuration near the columns, where the terms which have $(\cosh\pi y/2L + \cos\pi x/2L)$ in the denominator are very small compared to the remainder of the equation. They may, therefore, be neglected. Furthermore, setting

$$\sin\frac{\pi x}{2L} = \frac{\pi x}{2L}, \text{ and } \cos\frac{\pi x}{2L} = 1 - 1/2\left(\frac{\pi x}{2L}\right)^2 \tag{11}$$

$$\text{and } \cosh\frac{\pi y}{2L} = 1 + 1/2\left(\frac{\pi y}{2L}\right)^2, \text{ and } \sinh\frac{\pi y}{2L} = \frac{\pi y}{2L} \tag{12}$$

the expression for m_x in Eq. 6 simplifies to

$$m_x = \frac{M}{4\pi L}\left\{(1 + \nu)\frac{x/L}{(x/L)^2 + (y/L)^2} + (1 - \nu)\frac{2(y/L)^2 x/L}{[(x/L)^2 + (y/L)^2]^2}\right\} \tag{13}$$

This equation can be integrated to

$$\int m_x dy = \frac{M}{2\pi}\left\{\arctan\frac{y/L}{x/L} - \left[\frac{(1 - \nu)}{2}\right]\frac{(y/L)(x/L)}{(x/L)^2 + (y/L)^2}\right\} \tag{14}$$

so that a closed solution for K as a function of the critical periphery (Fig. 3) becomes

$$K = 1 - \frac{2}{\pi}\left\{\arctan\frac{V}{U} - \left[\frac{(1 - \nu)}{2}\right]\frac{UV}{U^2 + V^2}\right\} \tag{15}$$

The above definition of K is based solely on the transfer of bending moments. There is, of course, also the possibility of expressing K in terms of the shear stresses, q_x, directly. This approach is even more justified since we are interested primarily in the maximum shear stresses along the critical periphery.

Making similar approximations as outlined before, the expression for q_x of Eq. 8 becomes

$$q_x = \frac{M}{4\pi L^2}\left\{\frac{\left[2(y/L)^2 - 2(x/L)^2\right] - \left[\frac{\pi^2}{4}(y/L)^2(x/L)^2\right]}{[(x/L)^2 + (y/L)^2]^2}\right\} \tag{16}$$

Remembering from Eq. 1 how the shear stress, v, due to the unbalanced moment, M, was defined, the definition of K then becomes

$$K = \left(\frac{-q_x}{M}\right)\left(\frac{J_c}{de}\right) \tag{17}$$

where d = structural depth of the plate. While J_c and e are determined by the failure plane chosen, the shear force, q_x, as given by Eq. 16, varies, of course, along this periphery and does not suffice to define K. The required additional condition comes from the fact that the slab is built integrally with the column. This results in zero twisting moment, m_{yx}, along the column face, so that $q_x = \partial m_x/\partial x$. On the other hand, the term $\partial^3 w/\partial x^3$ is almost constant with respect to y in the vicinity of the column. It can, therefore, be assumed that the actual distribution of q_x along the column face is uniform and that it is justified to assume an average value, q_x = constant, to prevail along the assumed failure plane along the y-axis in the vicinity of the column. To find this average value, $\overline{q_x}$, one must integrate Eq. (16) as follows.

$$\overline{q_x} = \frac{1}{VL}\int_0^{VL} q_x dy$$

$$= M\frac{\pi}{32L^2}\left\{-\frac{x/L}{V}\arctan\frac{V}{x/L} + \frac{(x/L)^2 - (4/\pi)^2}{(x/L)^2 + V^2}\right\} \tag{18}$$

To determine K from Eq. (17), one may use the definition of J_c from Reference 9, Eq. 8-24, so that

$$\frac{J_c}{de} = L^2\left\{\frac{4}{3}U^2 + 1/3\left(\frac{d}{L}\right)^2 + 4UV\right\} \tag{19}$$

in which the structural depth, d, of the plate may be assumed as L/40. This term is rather insignificant so that any other reasonable assumption will yield similar results. The resulting K-value for $q_x = \overline{q_x}$ can thus be expressed as a function of the critical periphery, i.e., in terms of U and V (Fig. 3)

$$K = \frac{\pi}{32}\left\{\left[\frac{4}{3}U^2 + 1/3\left(\frac{1}{40}\right)^2 + 4UV\right]\left[\frac{U}{V}\arctan\frac{V}{U} - \frac{U^2 - (4/\pi)^2}{U^2 + V^2}\right]\right\} \tag{20}$$

Another simplification suggests itself by neglecting terms in Eq. 20 which are small compared to the remainder of the equation. Hence, with

$$R = \frac{J_c}{L^2 de} \tag{21}$$

a simple expression for K results, which is within 1/2 percent identical with Eq. 20:

$$K = \frac{R}{2\pi(U^2 + V^2)} \tag{22}$$

This shows that the resulting averaged maximum shear stress, $v = \overline{q_x}/d$, due to an unbalanced moment, M, is inversely proportional to the square of the distance from the center of the column to the corner of the critical periphery:

$$v = \frac{M}{2\pi d L^2 (U^2 + V^2)} \tag{23}$$

Influence Lines for Maximum Shear Stress

As revealed by Eq. 1, the influence line for maximum shear stress is a combination of the influence lines for the column reaction, V_v, and for the unbalanced slab moment, M. Fig. 4 shows these influence lines for a typical flat plate structure extending over three spans (slab thickness: 8 in.; columns 18x18 in.; story height: 10 ft.; spans: 20 ft.; bay widths: 20 ft.). For a structure with longer spans, the ordinates of the moment influence line would, of course, be bigger, whereas the ones for V_v would remain about as shown.

In order to combine these influence lines, we multiply the ordinates of the one for M by the factor

$$Q = K\frac{A_c}{J_c}\, e \tag{24}$$

and add them to the ordinates of the one for V_v, as shown in Fig. 4c. The force, S, obtained by putting a load on the ordinates, η_s, yields the maximum shear stress as $v = S/A_c$.

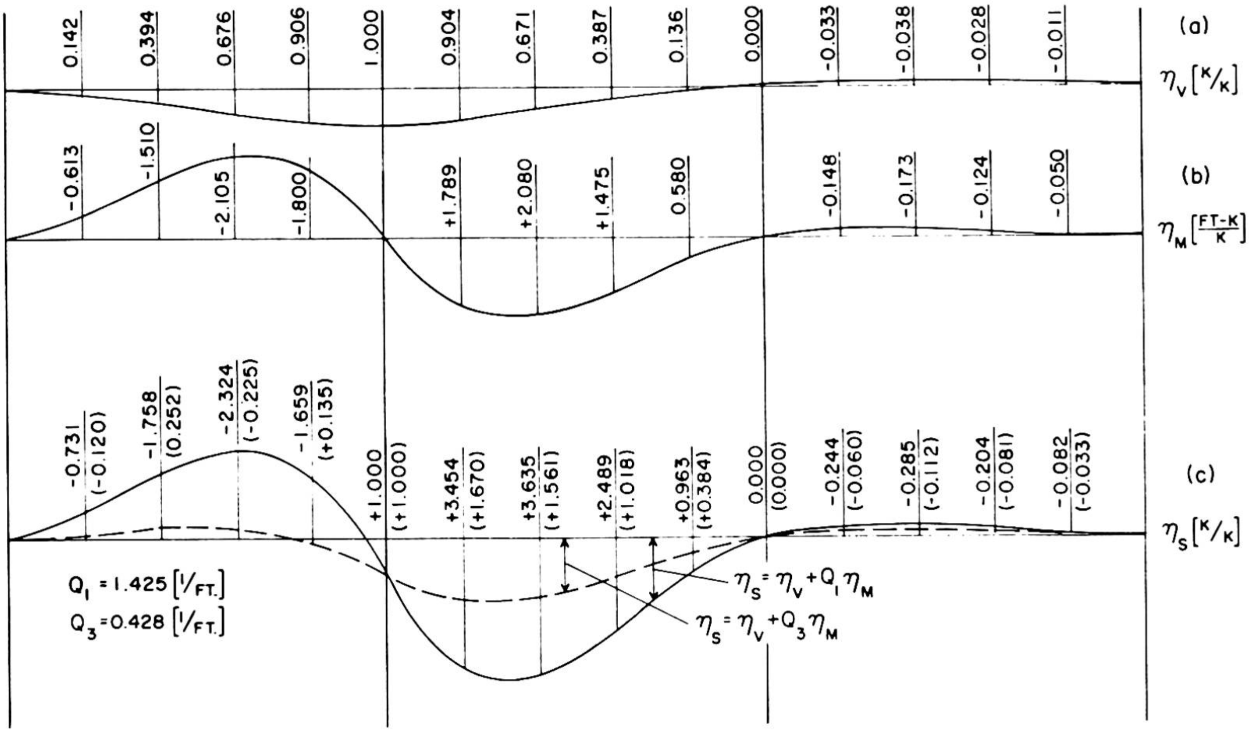

FIG. 4 INFLUENCE LINES FOR MAXIMUM SLAB SHEAR AT COLUMNS

Tables 1 through 3 tabulate K-values and the corresponding Q-factors as a function of the shape of the critical periphery, which is expressed by its coordinates, U and V. Table 1 is set up for K = 1.0. The corresponding Q-factors pertain to designs in accordance with common practice in the USA [8, 12]. Table 2 uses K-values based on the moment transfer, as defined by Eq. 15. Table 3 uses K-values based on maximum shear stress, as defined by Eqs. 20 or 22.

These Q-values in Tables 1 through 3 differ considerably, and so do the shapes of the resulting influence lines for shear. The lower portion of Fig. 4, for example, compares the influence lines based on the Q-factors from Table 1 (solid line) and from Table 3 (dashed line). The chosen coordinates of the critical periphery are U = 0.045 and V = 0.089, which comply with common practice [8, 12] for the column and slab dimensions stated above. The K-values pertaining to these Q-factors are $K_1 = 1.00$ (for $Q_1 = 1.425$) and $K_3 = 0.301$ (for $Q_3 = 0.428$). It should be noted that these influence lines resemble the ones for kern-moments in columns of continuous frames. The width of the "shear kern," measured from the centroid of the critical periphery, amounts to

	V \ U	0.025	0.050	0.075	0.100
K_1	0.025				
	0.050	K_1 = constant			
	0.075	= 1.000			
	0.100				
Q_1	0.025	2.823	1.756	1.315	1.062
	0.050	2.482	1.477	1.101	0.894
	0.075	2.341	1.348	0.993	0.803
	0.100	2.264	1.274	0.928	0.747

Table 1. K and Q - Values without Plate Theory

	V \ U	0.025	0.050	0.075	0.100
K_2	0.025	0.620	0.800	0.867	0.900
	0.050	0.391	0.620	0.736	0.800
	0.075	0.277	0.485	0.620	0.705
	0.100	0.213	0.391	0.525	0.620
Q_2	0.025	1.750	1.406	1.140	0.956
	0.050	0.971	0.915	0.810	0.716
	0.075	0.649	0.654	0.615	0.567
	0.100	0.482	0.499	0.487	0.463

Table 2. K and Q Values Based on Moment Transfer

	V \ U	0.025	0.050	0.075	0.100
K_3	0.025	0.451	0.435	0.387	0.353
	0.050	0.308	0.431	0.445	0.427
	0.075	0.218	0.364	0.428	0.444
	0.100	0.166	0.300	0.385	0.427
Q_3	0.025	1.273	0.764	0.509	0.374
	0.050	0.764	0.637	0.490	0.382
	0.075	0.510	0.490	0.425	0.357
	0.100	0.375	0.383	0.358	0.319

Table 3. K and Q Values Based on Maximum Shear

$$k = 1/Q = J_c/KA_c e \tag{25}$$

Evaluation of Influence Lines

The variation of the influence lines (Fig. 4c) shows that the shape of the critical periphery and the theoretical assumptions of moment transfer affect the shear stresses around a column in two ways: First, there is a direct effect due to the magnitude of the factors associated with V_v and M, i.e., the magnitude of the variables K, J_c, A_c, and e. The other effect results from the positioning of the live load as determined by the positive and negative regions of the influence lines.

In designing multi-span frames, we are used to positioning the live load either on the two spans at both sides of a column (Case A) or on just one span, i.e., to the left or to the right of a column (Case B). The first arrangement, Case A, results in a maximum punching force, V_v, whereas the latter one, Case B, results in a maximum unbalanced moment, M. We will investigate these two loading conditions with respect to the maximum shear stress which they produce. Furthermore, we will see what effect partial loading of the span has, i.e., loading up to the zero-point of the influence lines (Case C).

The influence lines of Fig. 4c represent two extremes. The solid line ($K_1 = 1.00$; $Q_1 = 1.425$) puts the maximum emphasis on the unbalanced moment, whereas the dashed line assumes a big portion of the unbalanced moment being transferred by bending ($K_3 = 0.301$; $Q_3 = 0.428$). Shear stresses obtained from these influence lines cannot be compared directly because the ones for $K_1 = 1.00$ may be reduced by the provision of flexural reinforcement [8]. It is for this reason that the resulting shear stresses are compared separately in Tables 4 and 5. In other words, these tables are meant to show the significance of the live load positioning only.

In comparing Cases A through C, the dead-to-live load ratio is important. We assume a feasible range of live loads varying from 50 lb./sq.ft. to 100 lb./sq.ft. Considering the slab thickness given, other factors affecting the dead-to-live load ratio are the type of concrete (lightweight or normal weight) and, due to load factors, the design method used (Ultimate Strength Method or Working Stress Method [12]).

K_1 = 1.000 Q_1 = 1.425	Ultimate Strength Normal weight		Lightweight		Working Stress Normal weight		Lightweight	
Live Load:	50	100	50	100	50	100	50	100
Case A	112	159	95	142	69	96	58	84
Case B	171	278	154	261	102	162	91	150
Case C	173	280	155	263	103	163	92	152

Table 4. Shear Stresses (psi) as Function of Live Load Positioning

K_3 = 0.301 Q_3 = 0.428	Ultimate Strength Normal weight		Lightweight		Working Stress Normal weight		Lightweight	
Live Load:	50	100	50	100	50	100	50	100
Case A	124	172	104	152	77	104	64	91
Case B	124	173	104	154	78	105	64	92
Case C	130	185	110	165	80	111	67	98

Table 5. Shear Stresses (psi) as Function of Live Load Positioning

The values of Tables 4 and 5 were computed in compliance with standard practice [8, 12] by means of a computer program [13]. They show that it is always the live load position for maximum positive span moments (Case B) which causes maximum shear stresses at the columns. They, furthermore, show that the increase in shear stress due to extending the live load to the zero-point of the influence lines (Case C) is insignificant. It should be mentioned that some building codes [12] call for only 75 percent of the live load to be applied in pattern loading, whereas 100 percent of the live load must be placed on all spans. Under this condition it is possible that the positioning of Case A governs, especially when the spans are short and, therefore, the η_M are small compared to the η_V (Fig. 4). The loading of spans which are not adjacent to the column is insignificant due to the restraining effect of the remote columns.

It should be noted that the experimentally determined K-values [9, Table 8-6] correspond well with the K-values of Table 3, if the critical periphery is at least d/2 away from the face of the column. This is reasonable since the theoretical assumption of a concentrated moment is justified only in view of the theorem of St. Venant, i. e., at some distance away from the point of application. Numerical refinements are, of course, always possible by using series expansions of the applied moment and of the boundary reactions.

The K-values and Q-factors depend on the shape of the critical periphery and, thereby, on the column size. In addition to this primary effect, the column size affects, of course, the shape of the influence lines for V and M. Visualizing the latter one as the deflection curve due to a unit moment applied at the joint, one could expect the ordinates, η_M, to decrease with increasing column stiffness. In reality, however, these ordinates increase, because of the smaller joint rotation. They would diminish to zero, of course, if the columns had no stiffness at all. Doubling the column stiffness in the example, for which influence lines are shown in Fig. 4, would result in an increase of the ordinates, η_M, by about 30 percent.

Summarizing the above, it can be stated that all of these factors, the assumed shape of the critical periphery, the stiffness ratio between slabs and columns, the slab spans, and the theoretical assumptions of moment transfer have an effect on the shape of the influence lines. As far as the critical live load positioning is concerned, however, Case B (Tables 4 and 5) will usually be the governing one.

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SUMMARY

An analytical method is developed to determine the stress configuration in flat plates subject to column moments. The results are presented in closed form for values, K, as a function of the shape of the critical periphery. These values agree well with test results. Influence lines for maximum shear stress are drawn for the bounds of the feasible range of K-values. Their evaluation shows that live load on alternate spans usually governs. The effect of partial loading of spans, however, is insignificant.

RÉSUMÉ

On présente une méthode analytique pour déterminer les efforts aux dalles plates produits par les moments aux colonnes. Les résultats sont présentés en formules fermées définissant les valeurs K, qui sont des fonctions du profil de la périphérie critique. Les valeurs s'accordent avec les résultats expérimentaux. Les lignes d'influence d'efforts tranchants ont été tirées pour des valeurs extrêmes K à portée de service et évaluées pour des conditions différentes. L'évaluation fait preuve du fait, qu'une charge utile aux portées alternes est décisive, tandis qu'une charge partiale des portées est d'insignifiance.

ZUSAMMENFASSUNG

Der Aufsatz schlägt eine analytische Methode vor, um die Spannungen in Flachdecken zu ermitteln, welche durch Stützenkopfmomente hervorgerufen werden. Die Ergebnisse sind in geschlossenen Formeln für'K'in Abhängigkeit der Bruchform dargestellt, und sie stimmen gut mit vorhandenen Versuchsergebnissen überein. Mit den Extremen der K-Werte im brauchbaren Bereich sind Einflusslinien gezeichnet und für verschiedene Bedingungen ausgewertet. Sie zeigen, dass abwechselnd feldweise Belastung ausschlaggebend ist, dass aber teilweise Feldbelastung die Ergebnisse kaum beeinflusst.

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Load Distribution in Multi-Storey Shear Wall Structures

Répartition des charges dans des constructions à étages multiples avec murs de cisaillement

Lastverteilung in mehrstöckigen Scheibentragwerken

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1. Introduction

Recent developments in multi-storey buildings for residential purposes have led to the extensive use of shear-walls, or cross-walls, for the basic structural system. Whilst the walls are designed primarily to resist both vertical and horizontal loads, they can in addition, by careful planning, be utilised fully for the non-structural functions of dividing and enclosing space, whilst simultaneously providing fire resistance and acoustic insulation between dwellings. By this means, efficient structural designs can be achieved. The floor slabs, which keep storey and overall heights to a minimum, act as deep beams transferring the wind loads to the vertical wall elements. The regular system of walls and slabs lends itself to industrialised building techniques, using either in-situ or precast construction.

A typical example of such slab-type structures is shown in Fig.1. Planning requirements tend to evolve parallel assemblies of walls, coupled by floor slabs or lintel beams, in conjunction with box-type assemblies surrounding lift shafts and stair wells.

The majority of studies of coupled shear-wall structures has been devoted to the problem of plane walls subjected to standard systems of loads in their own plane. In the case of a complete building, the results are strictly accurate only if the structure consists of parallel systems of identical wall assemblies, so that any lateral load is divided equally amongst them, it being assumed that all walls deflect equally due to the very high in-plane stiffness of the floor slabs.

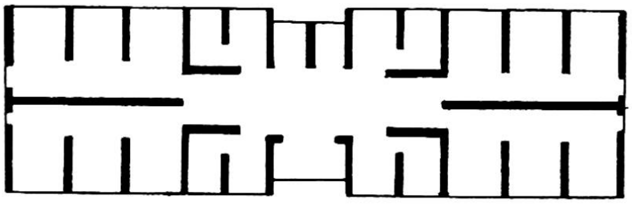


FIG. 1

If the structure consists of different forms of load bearing elements, or if, as is often the case, walls are curtailed at first floor level to provide an open concourse, considerable redistribution of load may take place. It may then become important to examine in some detail the distribution of load between the walls throughout the height of the building.

For shear-wall structures, suggestions have been made that lateral loads should be distributed amongst the walls in proportion to their stiffnesses[1]. If any coupling occurs, this process will give rise to significant errors, as may readily be seen by comparing the modes of deformation of a coupled wall and a cantilever box structure subjected to uniformly distributed loads[2]. The former bends with a reversal of curvature in the upper levels, whilst the latter bends in single curvature. In order to constrain the two to deflect equally, tensile linking forces are required in the upper levels, and compressive forces in the lower regions.

Two different methods have been developed for the analysis of coupled shear walls, the frame analogy and the continuous connection techniques. The first replaces the deep wall by a line column at the centroid, the finite depth being incorporated by the use of rigid arms to link the ends of the connecting beams to the column. The analysis is then carried out using standard frame-analysis techniques. In this case, an increase in the number of storeys leads to an increase in the degree of statical indeterminacy, and a corresponding increase in the number of equations to be solved. The second method replaces the discrete system of connecting beams by a continuous medium of the same stiffness. By assuming that the connecting beams deform with a fixed point of contraflexure, but do not deform axially, the behaviour of the system may be expressed by a single second-order differential equation, enabling a general closed solution to the problem to the obtained. This method has the advantage that the accuracy of the solution increases with an increasing number of storeys, with no extra labour involved. Because of its analytical nature, the method is less versatile in that it is more difficult to deal with variations in geometrical and stiffness parameters, although, because of the essential uniformity of the shear wall structure, this is not a serious limitation. The effects of variable thickness can easily be incorporated[3], and changes in width may be included in the general solution[4]. The accuracy of the technique has been demonstrated by numerous experimental investigations[3,4]. The discontinuities which may be present at ground floor level, where shear walls may be discontinued, may be treated by incorporating their influence in the lower boundary condition[5].

The aim of this paper is to present a method of analysis of complete multi-storey apartment-style concrete buildings, whose load-bearing structure consists essentially of parallel systems of shear wall and box elements, subjected to any system of lateral loads. The method is based on the continuous connection technique, and numerical results are presented for a typical representative structure.

2. Method of Analysis

Consider the coupled shear wall shown in Fig.2 subjected to a point load Pi applied at any height xi. The individual connecting beams of stiffness EI_p are replaced by an equivalent continuous medium or set of laminas of stiffness EI_p/h per unit height. It is assumed that the connecting beams do not deform axially so that both walls deflect equally, and that the point of contraflexure occurs at the mid-span position in all beams. If the laminas are assumed 'cut' at their mid-points, the only force acting at the cut section is a shear force of intensity q per unit height. On considering the deformations of the cut laminas, the compatibility condition may be set up to give no resultant relative deformation at the cut, and, when used in conjunction with the moment-curvature relationships for the walls, this leads to the establishment of the following governing differential equation,

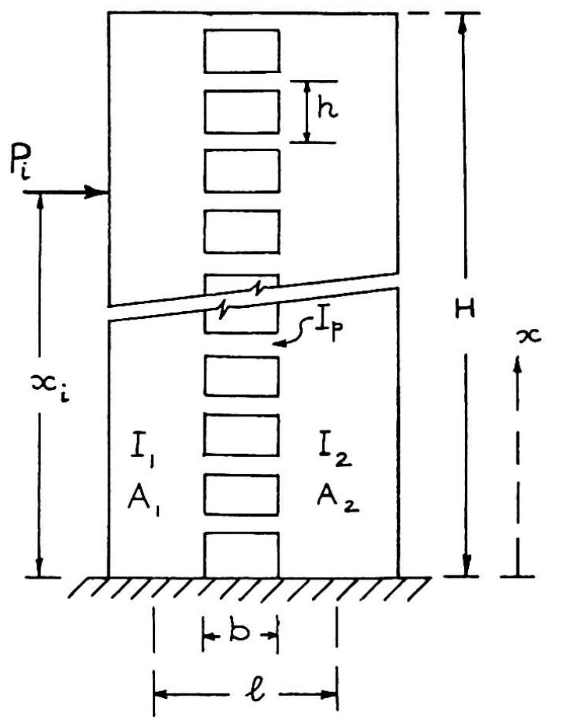


FIG. 2

$$\frac{d^2q}{dx^2} - \mu^2 q = -\alpha^2 Pi \langle x - xi \rangle^0 \qquad (1)$$

where $$\alpha^2 = \frac{12\ell I_p}{b^3 hI}$$

and $$\mu^2 = \frac{\alpha^2}{\ell}\left(\ell^2 + \frac{IA}{A_1A_2}\right)$$

For convenience in writing a single solution for the entire structure, a system of Macaulay's brackets has been employed in equation (1). These are defined in the usual manner as, When $x < xi$, $\langle x - xi \rangle^n = 0$, $\langle x - xi \rangle^o = 0$

When $x > xi$, $\langle x - xi \rangle^n = (x - xi)^n$, $\langle x - xi \rangle^o = 1$

The detailed derivation of equation (1) is not given, since similar equations have been derived in detail in earlier papers for different load conditions[3,4]. The appropriate boundary conditions for the fixed and free ends, expressed in terms of the shear force intensity 'q', may be shown to be, respectively,

$$\text{At } x = 0, \qquad q = 0$$

$$\text{At } x = H, \qquad \frac{dq}{dx} = 0 \tag{2}$$

The solution of equation (1), subject to the boundary conditions (2), may be shown to be,

$$q = \frac{\alpha^2 pi}{\mu^2} \cdot \frac{1}{\cosh \mu H} \Big\{ \cosh \mu H \,.\, \cosh \langle \mu (x - xi) \rangle^1 - \cosh \mu (H - x) - \sinh \mu (H - xi) \,.\, \sinh \mu x \Big\} \tag{3}$$

At any level, the axial force in the wall, tensile or compressive, is given by,

$$N = \int_x^H q \, dx$$

$$= \frac{\alpha^2 pi}{\mu^3} \frac{1}{\cosh \mu H} \Big\{ \sinh \mu (H - xi) \,.\, \cosh \mu x - \sinh \mu (H - x) - \cosh \mu H \,.\, \sinh \langle \mu (x - xi) \rangle^1 - \langle \mu (xi - x) \rangle^1 \Big\} \tag{4}$$

The corresponding bending moments in the walls are,

$$M_1 = \Big\{ Pi \langle x - xi \rangle^1 - T\ell \Big\} \frac{I_1}{I}$$

$$M_2 = \Big\{ Pi \langle x - xi \rangle^1 - T\ell \Big\} \frac{I_2}{I} \tag{5}$$

since the moment carried by each wall is proportional to its stiffness, by virtue of the assumption that both walls deflect equally.

The moment-curvature relationship is,

$$EI \frac{d^2 y}{dx^2} = Pi \langle x - xi \rangle^1 - T\ell \tag{6}$$

Integration of equation (6), and inclusion of the appropriate boundary conditions at top and bottom, yields the deflection at any level,

$$y = \frac{Pi}{EI}\left\{\frac{1}{6}\left(1 - \frac{\alpha^2 \ell}{\mu^2}\right)\left(3xi\,.\,x^2 - x^3 + \langle x - xi\rangle^3\right)\right.$$
$$+ \frac{\alpha^2 \ell}{\mu^4}\left[\left(x - \langle x - xi\rangle^1\right) + \frac{1}{\mu \cosh \mu H}\left(\sinh \mu (H - x)\right.\right.$$
$$- \sinh \mu H + \sinh \mu (H - xi)\ \ (1 - \cosh \mu x)$$
$$\left.\left.\left. + \cosh \mu H \ \sinh \langle \mu (x - xi)\rangle^1\right)\right]\right\} \qquad (7)$$

The single equation (7) yields the complete relationship between a unit load at any height xi and the deflection at any height x, enabling a complete set of influence coefficients fij (deflection at xi due to a unit load at xj) to be evaluated readily.

It may readily be shown that the same solution holds for a symmetrical shear wall containing two bands of openings, provided the parameters α and μ are defined slightly differently[3,4].

Analysis of Complete Structure

Suppose that the structure consists of a number of parallel wall assemblies. For a coupled-wall element, the load-deflection relationship is given by equation (7). For a box-type element, the corresponding load-deflection relationship may be shown to be,

$$y = \frac{Pi}{6EI}\left\{3xi^2 x - xi^3 + \langle xi - x\rangle^3\right\} \qquad (8)$$

In either case, for the k^{th} wall element, the load-deflection relationship may be expressed in matrix form as,

$$\underset{\sim}{y}_{ik} = \underset{\sim}{fij}_k \ \underset{\sim}{Pi}_k \qquad (9)$$

where $\underset{\sim}{y_i}$ and $\underset{\sim}{Pi}$ are column vectors of deflections and applied loads at any chosen set of levels xi, and $\underset{\sim}{f_{ij}}$ is a square matrix of influence coefficients evaluated from equations (7) or (8). A similar relationship can be derived for each wall assembly. Fortunately, in buildings of this nature, the number of different forms of wall elements is quite small, so that only a very limited number of matrices $\underset{\sim}{fij}$ will have to be evaluated.

For any applied load system whose resultant passes through the centre of rotation, all deflections will be the same at any given level, if it is assumed that the floor slabs are infinitely stiff in their own plane. Hence,

$$\underset{\sim}{fij}_k \ \underset{\sim}{Pi}_k = \underset{\sim}{y_i} = \text{constant for all 'k'} \qquad (10)$$

For equilibrium, the total applied load Pi at each storey level must be equal to the sum of the loads on the wall assemblies at that level. That is,

$$Pi = \sum_k \underset{\sim}{Pi}_k \qquad (11)$$

summing over all wall assemblies.

Hence, from equations (10) and (11), the complete load distribution throughout the structure may be obtained. From equation (10) the applied loads on any wall assembly 'k' may be expressed in terms of the load Pi_1 on any particular wall '1', say,

$$\underset{\sim}{Pi}_k = \underset{\sim}{fij}_k^{-1} \; \underset{\sim}{fij}_1 \; \underset{\sim}{Pi}_1 \tag{12}$$

Hence, from equation (11), the loads on wall '1' become,

$$\underset{\sim}{Pi}_1 = \left\{ \underset{\sim}{I} + \sum_{k = 2,3,\ldots n} \underset{\sim}{fij}_k^{-1} \; \underset{\sim}{fij}_1 \right\}^{-1} \underset{\sim}{Pi} \tag{13}$$

and the loads on all other walls follow from equation (12).

The deflections and stresses in individual wall assemblies may then be evaluated by the continuous connection solution, using equations (3), (4), (5) and (7).

Suppose, for example, that the structure consists of four, six and one element of types 'a', 'b' and 'c' respectively. Then, from equation (9),

$$\underset{\sim}{yi} = \underset{\sim}{fij}_a \; \underset{\sim}{Pi}_a = \underset{\sim}{fij}_b \; \underset{\sim}{Pi}_b = \underset{\sim}{fij}_c \; \underset{\sim}{Pi}_c \tag{14}$$

For equilibrium, $$\underset{\sim}{Pi} = 4\underset{\sim}{Pi}_a + 6\underset{\sim}{Pi}_b + \underset{\sim}{Pi}_c \tag{15}$$

or using equation (14)

$$\underset{\sim}{Pi} = \left\{ 4\underset{\sim}{I} + 6\underset{\sim}{fij}_b^{-1} \; \underset{\sim}{fij}_a + \underset{\sim}{fij}_c^{-1} \; \underset{\sim}{fij}_a \right\} Pi_a \tag{16}$$

The loads on the wall assemblies of type 'a' become,

$$\underset{\sim}{Pi}_a = \left\{ 4\underset{\sim}{I} + 6\underset{\sim}{fij}_b^{-1} \; \underset{\sim}{fij}_a + \underset{\sim}{fij}_c^{-1} \; \underset{\sim}{fij}_a \right\}^{-1} \underset{\sim}{Pi} \tag{17}$$

and the loads on the other elements follow from equation (14).

If the resultant of the applied loads does not pass through the centre of rotation, but acts at a distance z' from it, a torsional moment Ti of magnitude $Pi\,z'$ is applied to the building. Owing to the high in-plane stiffness of the floor slabs, they will undergo a rigid body rotation through an angle θi, say, such that the total displacement of the k^{th} wall assembly is equal to $yi + \theta i z_k$, where z_k is the distance of the wall element from the centre of rotation.

If the twisting moments on the wall elements are neglected, the second condition of rotational equilibrium becomes, for the entire system,

$$\underset{\sim}{Ti} = \sum_k \left(\underset{\sim}{Pi}_k \, z_k \right) \tag{18}$$

The second condition of equilibrium enables the additional unknown displacement $\underset{\sim}{\theta}_i$ to be evaluated and a complete solution obtained.

By virtue of the assumption of a rigid body rotation of all floor slabs, the rotations of all wall elements may be obtained and the twisting moments evaluated in terms of the rotations Θi from standard strength of materials relationships. If the twisting movements on individual wall elements are included, therefore, the second condition of rotational equilibrium (18) must be amended to,

$$\underset{\sim}{Ti} = \sum_k (\underset{\sim}{pi}_k \; 3_k) + \sum_k \underset{\sim}{Ti}_k \tag{19}$$

where Ti_k is the twisting movement on the k^{th} wall element at level xi. These twisting movements may then be expressed in terms of the rotations, and the complete solution obtained as before.

Lack of space does not permit the torsional condition to be treated in greater detail.

3. Numerical Example

In order to illustrate the results, a representative shear-wall structure of the form shown in Fig.3 (in plan) is considered. The structure consists of a central core (wall 3), heavy flank walls (walls 1), and four pairs of interior cross-walls (walls 2). It is assumed that the building is 20 storeys high, with a storey height of 8 ft.9in., the floor slabs being assumed to be 6 in. thick. For the purpose of this example, it is assumed that the building is subjected to a wind loading of intensity 10 lb/ft^2 in a direction parallel to the cross-walls.

The effective width, or stiffness, of the floor slabs may be determined by a subsidiary calculation, using such numerical techniques as the finite element or finite difference method; in the present instance, the latter was employed, using curves prepared by Qadeer and Stafford Smith[6]. In this case, the effective widths of slab connecting walls 1 and 2 are taken to be 10 ft. and 13.2 ft. respectively.

A computer program has been written to perform the entire calculations necessary for a complete analysis of the building. Because of the regularity of such structures, the program requires a relatively small amount of input data. The output consists of the influence coefficients for each type of wall unit, the loads on the various walls at each level, the bending moments, axial forces, and shear forces on all walls at each level, the stresses at the extreme fibres of each wall, and the deflection. In order to check the computation, the deflection of each wall is calculated separately.

In the present case, the influence coefficients and forces were computed at each storey level. The computed deflections for the different wall units agreed with each other to six significant figures. The results obtained are shown in Figs.4, which show (a) the percentage of the total applied load carried by each type of wall unit throughout the height of the building, and (b) the deflected form.

The results indicate that for structures of this nature, considerable variations in the load distribution can occur throughout the height of the building.

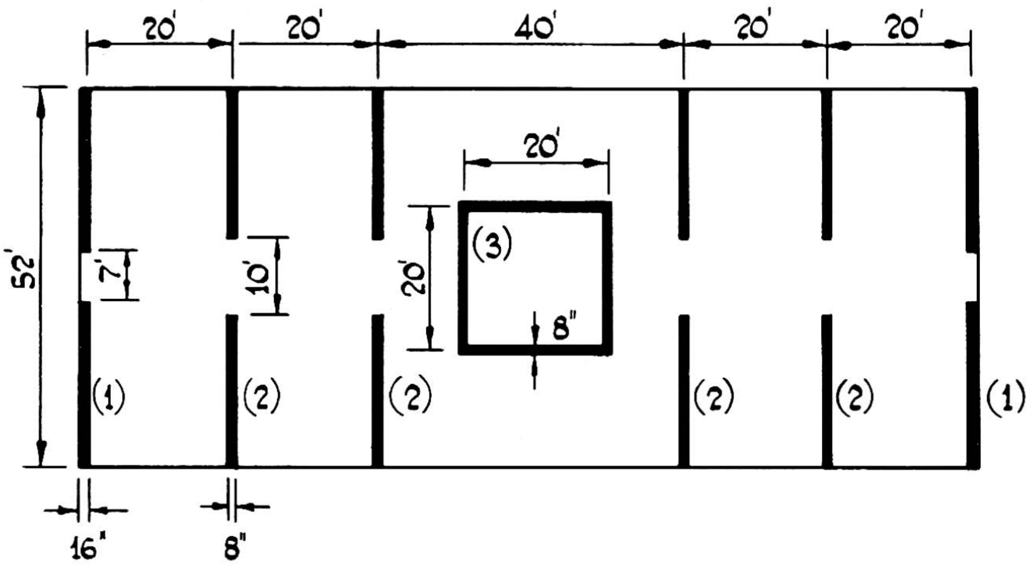
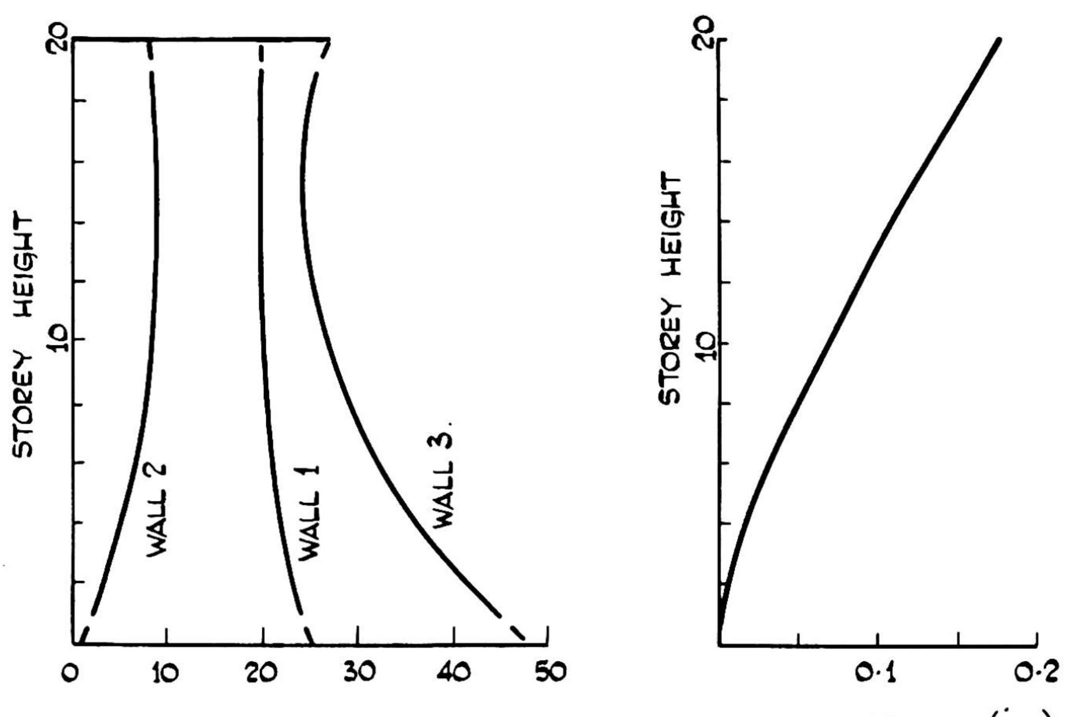


Fig. 3.



(a) LOAD DISTRIBUTION (b) DEFLECTED FORM.

Fig. 4.

4. Conclusions

A method has been presented for the analysis of multi-storey structures which consist essentially of parallel assemblies of coupled shear-walls. The continuous connection technique is used to determine the load-deformation characteristics of individual wall assemblies, and the complete structure is analysed using compatibility and equilibrium conditions. Any applied load system can be dealt with, and the technique can be extended to include torsion of the structure.

The analysis deals with matrices of small order, and can readily be programmed for digital computation. In order to reduce the amount of computation, the influence coefficients need not be evaluated for every storey level, since rapid variations in load distribution do not occur in regular structures of the form considered. Having regard to the form of structure and applied load, the designer can assess the number of reference levels which may be required in the analysis to give a solution which is sufficiently accurate for practical purposes. A closer spacing in the reference levels may be adopted for regions where the load distribution is changing most rapidly.

In the paper, the general solution for a coupled shear wall structure subjected to a joint load at any height is, as far as the authors are aware, presented for the first time. By simple integration, the general solution for any load form may be derived from equation (3).

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Notation The following symbols are used in this paper:

x	Height
y	Horizontal deflection
θ	Rotation
ζ	Distance from centre of rotation
H	Total Height
h	Storey Height
b	Clear distance between walls
ℓ	Distance between centroidal axis of two walls
I_p	Moment of inertia of connecting beam
I_1, I_2	Moment of inertia of walls 1 and 2
I	$I_1 + I_2$
$A_1 A_2$	Cross-sectional areas of walls 1 and 2
A	$A_1 + A_2$
Pi	Applied load at level xi
Ti	Applied twisting moment at level xi
q	Shear force intensity in substitute connecting medium
$M_1 M_2$	Bending moments in walls 1 and 2
N	Axial force in wall
fij	Deflection at level xi due to unit load at xj
k	Suffix denoting k^{th} wall assembly

SUMMARY

A method is presented for the analysis of the distribution of load amongst the shear walls of a multi-storey apartment-style building. The method is based on the continuous connection technique. Numerical results are presented for a typical twenty-storey structure.

RÉSUMÉ

L'article présente une méthode pour l'analyse de la distribution des charges parmi les murs de cisaillement dans une maison d'appartement de beaucoup d'étages. La méthode est basée sur la technique de liaison continue. Des résultats numériques sont donnés pour un bâtiment typique de 20 étages.

ZUSAMMENFASSUNG

Dargestellt ist ein Verfahren zur Berechnung der Lastverteilung in Scheiben eines mehrstöckigen Gebäudes. Das Verfahren baut auf der durchlaufenden Gewebetechnik auf. Numerische Ergebnisse werden für ein typisches zwanzigstöckiges Haus gegeben.

Study of the Distribution of Wind Loads Between Stiffening Elements and Framing of Multi-Storey Buildings

Etude de la distribution des charges du vent entre des éléments de la rigidité et des portiques des carcasses des immeubles géants

Untersuchung über die Verteilung der Windlasten zwischen Versteifungen und Stockwerkrahmen von Hochhäusern

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Modern multi-storey buildings consist, as a rule, of framing, vertical diametric walls, adding transverse rigidity to the building, floors and light railing structures attached to the framework. In order to design such buildings for horizontal loads it is necessary, in general, to determine the forces arising in the structures of the framework and its deformations. The latter is required to calculate the strength of railing structures, their connections with each other and with the framework. (When designing the framework, the influence of railing structures is, commonly, not taken into account).

The present paper gives some results of the investigation of the first, most difficult part of the problem of designing framed structures with due account of the spatial behaviour of the structures.

Horizontal loads in framed buildings are taken up by vertical diametric walls and frames. Floors joined in rigid disks, serve as bracing distributing the load between the load-bearing members of the building and equalizing their transverse displacements. In very high buildings with diametric walls closely arranged, disks of floors also exert a direct affect on longitudinal deformations of load-bearing members due to the resistance of floors to torsion at unequal angles of rotation of cross sections of diametric walls and frames.

When diametric walls are placed symmetrically in the plan, the three dimensional problem is reduced to a two dimensional one, viz. bending of each load-bearing member from an appropriate portion of the horizontal load, the size of which is to be determined; with asymmetric location the problem is divided in two: bending and torsion, the latter being set up by torsional moments distributed along the height of the building arising from non-coincidence of the resultants of horizontal loads with the line connecting the centres of torsion of horizontal sections of buildings.

Consider briefly the torsion.

Rigid horizontal disks formed by floors do not permit the framework to deform at torsion otherwise than by rotations of all sections round the vertical axis, and so V. Z. Vlasov's theory of thin-wall systems with closed undeformed contour of the cross section may be used for design. In fact, longitudinal forces arising in the members of the framework due to torsion may be neglected too, and making use of Bredt's postulate, shear forces in the planes of frames and diametric walls on the level of each floor can be determined on the basis of the diagram of torsional moments, which is easily drawn. These forces will give rise to local bending of posts (and rafters) of frames, and in continuous vertical diametric walls shear stresses will occur due to their effects.

When designing for bending, vertical diametric walls and frames of the framework, for simplification, may be grouped into two independent blocks with summed up parameters of stiffness, viz. flexural parameters (EJ_1 and EJ_2), characterizing the resistance of elements of blocks to sectional rotations, and shear parameters (GF_1 and GF_2) showing their resistance to shear-distortions / 2 / .

The present problem comes to determining forces $q(x)$ and $m(x)$ arising at the contact between two design blocks.

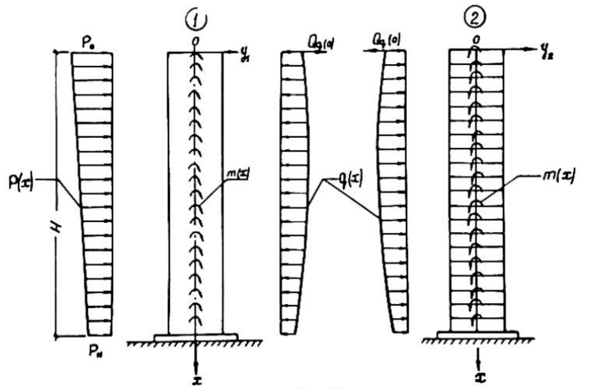

Рис. 1
(Fig 1)

Fig. I illustrates the design scheme corresponding to the three-dimensional behaviour of structures described above with a single foundation slab under the building.

If in the design scheme the block with index 2 is the framing, then $q(x)$ is the fraction of the horizontal load resisted by the framing. Concurrently with this load there occur generally distributed bending moments $m(x)$ due to the difference in the angles of rotation of sections and resistance shown by the floors to these displacements.

Assuming the beginning of the coordinates on the upper end of the building where, as it is clear beforehand, there are no concentrated imposed bending moments, interconnected deformations of the blocks may be described by the following system of equations:

$$\begin{cases} EJ_1 y_1'' - \frac{EJ_1}{GF_1} M_q'' + M_q + \frac{EJ_1}{GF_1} M_m'' - M_m = \frac{EJ_1}{GF_1} |M_p|'' - |M_p|, \\ EJ_2 y_2'' + \frac{EJ_2}{GF_2} M_q'' - M_q - \frac{EJ_2}{GF_2} M_m'' + M_m = 0, \\ (y_1 - y_2) C = M_q'', \\ -\frac{M_m''}{\varepsilon} = \frac{1}{EJ_2} (M_q - M_m) - \frac{1}{EJ_1} (-|M_p| - M_q + M_m) \end{cases} \qquad (I)$$

In Equations (1):

M_q, M_m, M_p are the bending moments from unknown contact forces $q(x)$ and $m(x)$ and outer load $p(x)$ respectively;

C [kg/cm²] is the rigidity of floors measured by the value of the linear force on contact $\bar{q}$ [kg/cm] causing reciprical transverse displacement of the blocks by a linear unit [cm] in the direction of the force;

ε [kg cm/cm] is the rigidity of floors thought of as longitudinal bracing measured by the value of a linear torsional moment $\bar{m}$ [kg cm/cm] causing torsion of the floors by an angle equal to unity.

It should be noted that we find $M_q(x)$ and $M_m(x)$, but not directly $q(x)$ and $m(x)$. The loads are easily determined by the formulae:

$$q(x) = M_q''(x),$$
$$m(x) = M_m'(x) \tag{2}$$

Solution of the system of equations (1) gives the following resolution differential equations:

$$M_m^{VI} - \left[\varepsilon\left(\frac{1}{EJ_1} + \frac{1}{EJ_2}\right) + C\left(\frac{1}{GF_1} + \frac{1}{GF_2}\right)\right] M_m^{IV} + C\left(\frac{1}{EJ_1} + \frac{1}{EJ_2}\right) M_m'' =$$

$$= -\frac{\varepsilon}{EJ_1}\left|M_p\right|^{IV} + C\varepsilon\left(\frac{1}{EJ_1 \cdot GF_2} - \frac{1}{EJ_2 \cdot GF_1}\right)\left|M_p\right|''; \tag{3}$$

$$M_q = -\frac{M_m''}{\varepsilon\left(\frac{1}{EJ_1} + \frac{1}{EJ_2}\right)} + M_m - \frac{EJ_2}{\sum EJ}\left|M_p\right|$$

Boundary conditions added to Equations (3) to find the constants of integration are:

$$x=0 \qquad M_m=0, \quad M_q=0, \quad M_q'=0;$$

$$x=H \qquad M_m'=0, \quad M_q'=\frac{GF_2}{\sum GF}|M_P|', \quad M_q''=0.$$

The sign convention is as follows: the bending moment rotating the blocks counter clockwise and shear force of the same direction are thought of as positive. A negative sign of $M_q(x)$ **and** $M_m(x)$ **will indicate that the direction of** $q(x)$ **and** $m(x)$ **on the scheme was assumed improperly.**

When designing buildings for wind load, the latter is as a rule defined by expression $P(x)=P_0-\alpha x$ **, where** $\alpha=\frac{P_0-P_H}{H}$ **(the diagram is of a trapezium form). Floors are often taken as absolutely rigid in the plane of discs (** $c=\infty$ **) and flexible out-of-plane (** $\mathcal{E}=0$ **). Then** $m(x)=0$ **, the appropriate fraction of wind load resisted by the framing and forces may be determined by the formulae:**

$$M_q(x)=A\left\{\frac{B}{K}Sh\,Kx-\frac{1}{K^2}\left[(Ch\,Kx-1)+\alpha x\right]\right\}+\frac{EJ_2}{\sum EJ}\left(\frac{P_0x^2}{2}-\frac{\alpha x^3}{6}\right);$$

$$Q_q(x)=A\left[B\,Ch\,Kx-\frac{1}{K}\left(P_0Sh\,Kx+\frac{\alpha}{K}\right)\right]+\frac{EJ_2}{\sum EJ}\left(P_0x-\frac{\alpha x^2}{2}\right);$$

$$q(x)=A\left(KB\,Sh\,Kx-P_0Ch\,Kx\right)+\frac{EJ_2}{\sum EJ}\left(P_0-\alpha x\right);$$

$$A=\frac{EJ_2}{\sum EJ}-\frac{GF_2}{\sum GF}; \tag{4}$$

$$B = \frac{1}{Ch\,kH}\left[\left(\frac{Sh\,kH}{K} - H\right)P_0 + \left(\frac{H^2}{2} + \frac{1}{K^2}\right)\alpha\right];$$

$$K^2 = \frac{\frac{1}{EJ_1} + \frac{1}{EJ_2}}{\frac{1}{G\cdot F_1} + \frac{1}{G\cdot F_2}}.$$

The equation for the axis of a bent building is:

$$(\Sigma EJ)y = \left(\frac{EJ_2}{G\cdot F_2} - \frac{EJ_1}{G\cdot F_1}\right)\left[M_q(H) - M_q(x)\right] + \frac{EJ_1}{G\cdot F_1}\left[|M_p(H)| - |M_p(x)|\right] - x\left(\frac{P_0 H^3}{6} - \frac{\alpha H^4}{24}\right) + \frac{P_0 x^4}{24} - \frac{\alpha x^5}{120} + \frac{P_0 H^4}{8} - \frac{\alpha H^5}{30}. \quad (5)$$

The limit values of angles of distortion of railing structures and inner partition walls which characterize their strength and deformability, provided they are fastened rigidly to the cross frames of the framework, must not be less than the values obtained by the formulae:

$$\gamma = m_\gamma \frac{Q_q(max)}{G F_2}; \quad (6)$$

where m_γ is the safety factor stipulated by the Building Code of a particular country. The zone of the largest distortions, where $Q_q(x)$ reaches its maximum is located at one-third of the building height counting from the ground base.

To study the nature of the distribution of horizontal loads between the elements of the framework, Central Research Institute for Building Structures of Gosstroy of the USSR has carried out experiments on a large model of 30-storey buildings of an original design which were then built in a slightly changed version in Moscow / 3 /. The buildings under consideration have a broken configuration in the plan (30° angle), they consist of a rigid core, two additional vertical diametric walls on the ends of the building and framework which is enclosed between the diametric walls and the core and which bears almost all the useful load. The framework rests upon a solid foundation slab reinforced in the plane of posts of frames by longitudinal and transverse ribs. Fig. 2 shows the plan and cross-section of the building, figures in brackets relate to the model.

The model was made of organic glass following the similarity conditions with the scale 1/50 (Fig. 3). The vertical load on floors was produced by small shots, and the horizontal load, the diagram of which is shown in Fig. 4, was set up through a distribution arrangement by loads hung on elastic threads pulled over pulleys.

Longitudinal deformations of the material were measured by wire strain gauges and displacements of the model were measured by indicators of 0.01 to 0.001 mm sensitivity.

The measurements have shown that transverse displacements of the model frames, core and diametric walls were practically the same. This testifies to great rigidity of discs formed by the floors despite a significant space between the diametric walls and core.

The influence of torque rigidity of the floors on longitudinal deformations of posts of frames and diametric walls has appeared to be also negligible as it should have been expected.

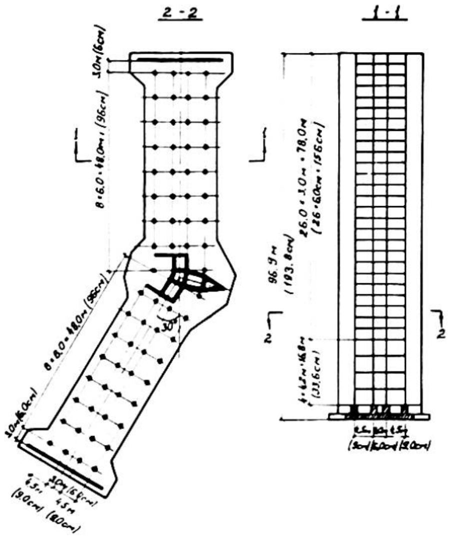

Рис 2
(Fig. 2)

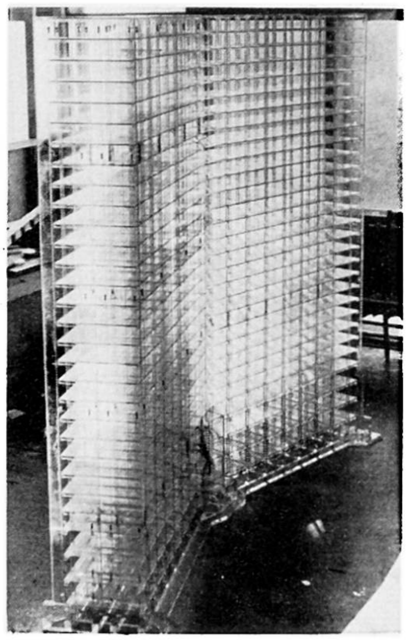

Fig. 3

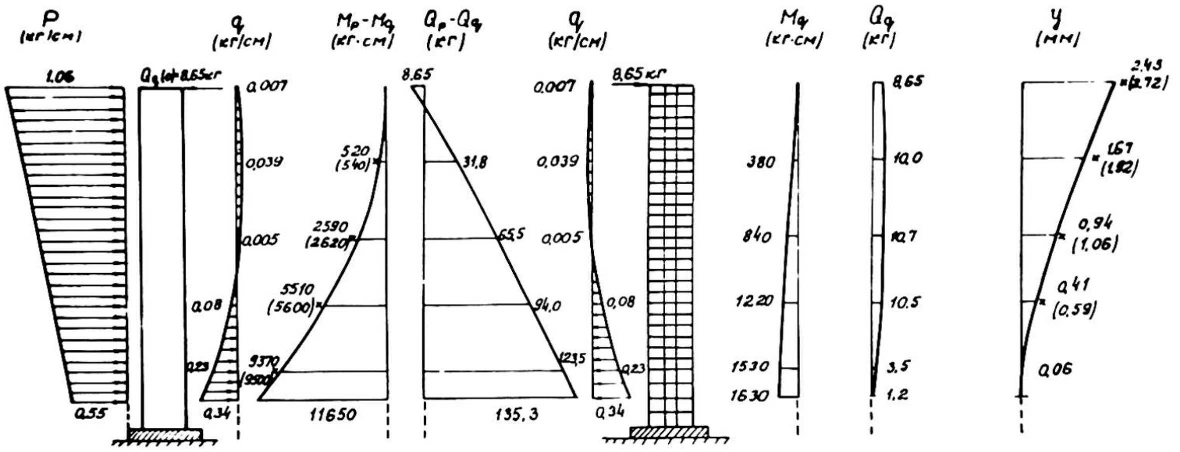

Рис. 4
(Fig 4)

The results of the design of the model under the supposition of embedment of the framework in the plane of the floor over the second floor by formulae (4), the diagrams of loads acting on framing and core, of bending moments and shear forces are given in Fig. 4. On the diagram of bending moments small crosses show the data obtained from the experiment by gauges attached to the core of the building and processed appropriately.

The data of these gauges were most stable, as well as of the indicators measuring lateral displacements; the results of measurements are also illustrated in Fig. 4 in the same way.

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SUMMARY

The theoretical investigations and model test results enable to conclude the following. When designing multi-storey buildings it should be born in mind that wind loads are distributed between the stiffening elements and the frames of the framework and overload the latter. The degree of the distribution of wind loads depends on the rigid characteristics of the framing and diametric walls and may be determined with a sufficient degree of precision by the approximate design method discussed in the paper.

RÉSUMÉ

Lors des calculés des immeubles géants avec des carcasses et des panneaux il faut prendre en considération les charges du vent distribuées entre des éléments de la rigidité et des portiques des carcasses ce que fait venir la sur charge les derniers. Le degré de la distribution des charges du vent dépend du rapport des caractéristiques de la rigidité des éléments de la carcasse et ce degré est déterminé avec la precision parfaite à l'aide de la méthode du calcul exposé au rapport.

ZUSAMMENFASSUNG

Man kann aus den theoretischen Untersuchungen und Versuchsergebnissen folgende Schlussfolgerungen ziehen: Bei der Berechnung der Fachwerkgebäude mit Bauplatten ist zu berücksichtigen, dass die Windbelastung eine Überbelastung des Fachwerkrahmens hervorruft und zwischen den Elementen der Steifen und des Fachwerkrahmens verteilt ist. Der Verteilungsgrad der Windbelastung hängt von dem Verhältnis der Steifigkeit der Fachwerkelemente ab und kann laut Bericht nach der Methode der Näherungsrechnung ziemlich genau bestimmt werden.

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Effects of Column Temperature, Creep and Shrinkage in Tall Structures

Effets de la température, du fluage et du retrait dans les colonnes des structures élancées

Temperatur-, Schwind- und Kriecheinflüsse in Stützen hoher Bauwerke

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In recent years a large number of multi-story apartment and office buildings have been built in reinforced concrete. While in the low rise buildings the effects of temperature creep and shrinkage in the columns did not substantially control the stress or design of the structure, these otherwise secondary effects may become primary and must be considered in the analysis, design and detailing of the high-rise structure. The effects of temperature, creep and shrinkage in high-rise buildings are not only structural, but also architectural in that the exterior window wall details as well as the interior partition details must be designed to incorporate relative movements caused by these factors. A brief discussion of the philosophy for planning and design procedures of high-rise buildings subjected to these effects follows:

1. Temperature Effects. Exposed columns when subjected to seasonal temperature variations change their length relative to the interior columns which remain unchanged in a controlled environment. Furthermore, if the exterior columns have difference in size and are subjected to different average temperature due to the location of glass lines, there will be relative displacement between these adjacent columns when exposed to seasonal changes.

The philosophy of design of structure with exposed columns involves one of the two basic concepts: (a) To use an effective method of analysis and design and to develop details to accommodate large expected relative movements, or (b) To plan a building for a controlled temperature movement.

When for architectural or other reasons it is not possible to limit the relative movements between the exterior column and the interior column to a reasonable value, the structural system should be analyzed by a simple and effective method such as proposed by the authors in papers published in the ACI Journal (1, 2, 3). If the analysis indicates that the stresses are acceptable and can be designed for, then only the partition details should be developed to accommodate the expected maximum distortions. However, if the initial analysis indicates extremely high stresses in the upper floors it may be advisable to hinge the floor system at the exterior columns as was done in the 38-story Brunswick Building in Chicago.

If the exterior columns are unequal in size as was used for the One Shell Plaza Building in Houston, the analysis may indicate that the glass line should be controlled in a manner that the average temperature of all the columns is approximately the same.

2. Effects of Creep and Shrinkage. With increasing height of buildings, the importance of time dependent shortening of columns and shear walls becomes more critical due to the cummulative nature of such shortening. It is known that columns with varying percentage of reinforcement and varying volume-to-surface ratio will have different creep and shrinkage strains. Increasing the percentage of reinforcement and the volume-to-surface ratio reduces strains due to creep and shrinkage. In very tall structures where a large heavy reinforced column may be adjacent to a lightly reinforcing shear wall a differential inelastic shortening causes moments in the horizontal members and also a load redistribution from the shear wall to the column which has less creep and shrinkage.

Although a large amount of research information is available on shrinkage and creep, it is not directly applicable to column of high-rise buildings but are applicable to flexural elements only. In the construction of a high-rise building columns are loaded in as many increments as there are stories above the level under consideration. Such incremental loading over a long period of time makes a considerable difference in the magnitude of creep and consequently in the differential movement and load redistribution between adjacent columns.

The significance of incremental loading was first questioned during the design of the 52-story, 715' (218m) high One Shell Plaza Building in Houston, built entirely with high strength (6,000 psi & 4,500 psi) lightweight concrete. Theoretical work to predict incremental creep was then jointly undertaken by the authors, the results of which have been submitted to ACI for publication. The Portland Cement Association at the suggestion of the senior author, undertook a series of tests with incremental loading conducted under direct supervision of Dr. Eivind Hognestad and Mr. D. Pfeifer. These results clearly pointed out the difference between the incremental loading in a column and the full load applied to a beam. The test results confirmed the authors theoretical findings indicating that the overall time vs. strain characteristics due to incremental loading surprisingly resembles the theoretical linear curve made on the basis of elastic shortening at each incremental loading. The classical creep characteristic is almost non-existent.

This linear type of creep characteristic can be translated into an "equivalent creep modulus" and can then be used to determine load redistribution between adjacent columns or columns and shear walls. Such an analysis can be made by the use of the iterative method developed by the first author (4).

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SUMMARY

It is concluded on the basis of previous discussion that even though further research is necessary, sufficient information is now available for the design of ultra high-rise buildings in reinforced concrete to take into consideration the temperature creep and shrinkage effects both for normal weight and lightweight concrete.

RÉSUMÉ

A la suite de discussions précédentes, et malgré la nécessité de pousser les recherches, on dispose de suffisamment d'informations à l'heure actuelle pour tenir compte des effets de température, de fluage et de retrait dans les colonnes en béton armé (normal ou extraléger), dans le dimensionnement de structures très élevées en béton armé.

ZUSAMMENFASSUNG

Auf Grund vorangegangener Diskussionen und trotz der Einsicht, dass weitere Studien unerlässlich sind, kann man behaupten, dass im Moment genug Informationsmaterial zur Verfügung steht, um Temperatur-, Kriech- und Schwindeinflüsse in den Stützen extrem hoher Stahlbetonbauten, sowohl mit Normal- als auch mit Leichtbeton, bei der Bemessung zu berücksichtigen.

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Time-Dependent Performance of Reinforced Concrete Columns — Field Investigation of a 70-Story Building

Performance de colonnes en béton armé en fonction du temps — Essais sur nature d'un bâtiment de 70 étages

Zeitabhängiges Verhalten von Stahlbetonsäulen — Felduntersuchung eines 70-stöckigen Gebäudes

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INTRODUCTION

Reinforced concrete buildings in the United States are being constructed to heights greater than 600 ft. (183 m). Within a story, columns shorten only a fraction of an inch. When these shortenings are added over the full height of the structure, the cumulative deformation may be several inches. Both elastic and time-dependent axial deformations of columns may therefore be important design considerations.

To determine the actual behavior of columns in tall buildings, a 70-story structure was instrumented. After columns were cast, installations were made for measuring vertical strains. These field measurements are supplemented by laboratory tests on non-reinforced specimens made with concrete taken from batches used in the building. This report describes the structure, instrumentation, laboratory tests, and initial field measurements.

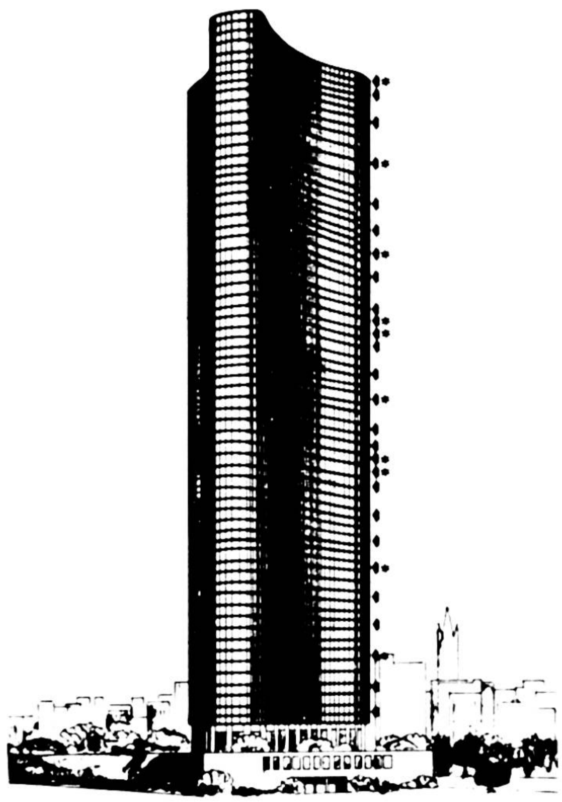

Fig. 1 - Elevation of building

DESCRIPTION OF STRUCTURE

A sketch of the structure is shown in Fig. 1. Total height above grade is 645 ft. (197 m). In plan, the building has three wings shaped as shown in Fig. 2. Each wing is 65 ft. (20 m) wide and extends 117 ft. (36 m) from the center of the building.

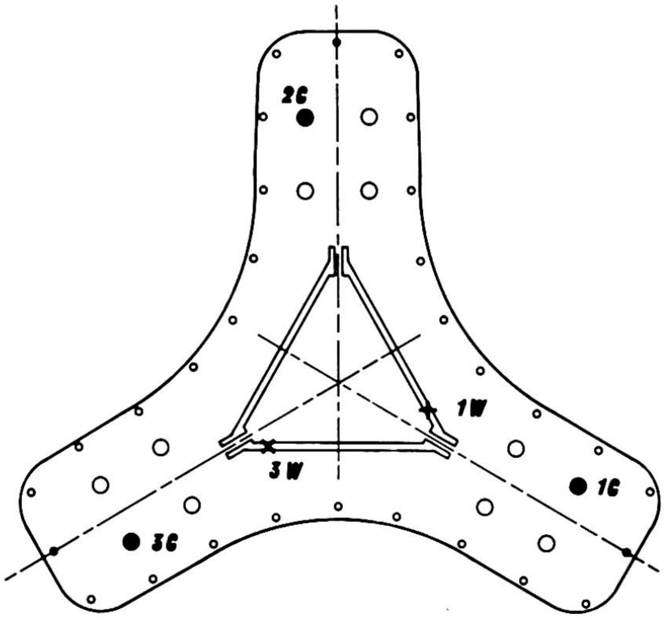

Fig. 2 - Floor Plan

The floors are flat plates 8 in. (20 cm) thick made of lightweight aggregate reinforced with high strength deformed bars. Diameter, design strength of concrete at 28 days, and design yield stress of reinforcement for the interior columns are given in Table 1. Normal weight concrete is used in the columns.

Construction of the first story of the building was started in June, 1966. The 70th story was completed in December, 1967. Weather permitting, the building was cast at the rate of one floor every three working days.

TABLE 1 -- PROPERTIES OF INTERIOR COLUMNS

Story No.	Column Diameter		Concrete Design Strength		Yield Stress of Reinforcement	
	in.	cm	psi	kg/cm^2	ksi	kg/mm^2
1	40	102	7500	528	75	53
11	40	102	7500	528	75	53
12	40	102	7500	528	60	42
16	40	102	7500	528	60	42
17	40	102	6000	422	60	42
29	40	102	6000	422	60	42
30	36	91	6000	422	60	42
34	36	91	6000	422	60	42
35	36	91	5000	352	60	42
43	36	91	5000	352	60	42
44	30	76	5000	352	60	42
58	30	76	5000	352	60	42
59	30	76	3500	246	60	42
68	30	76	3500	246	60	42

FIELD MEASUREMENTS

A Whittemore mechanical strain gage[1] with a gage length of 20 in. (51 cm), is used to measure vertical shortening of the columns and core walls. With an initial gage length of 20 in., the 0.0001 in. (0.0025 mm) dial gage indicates five millionths strain per dial division. Gage installations were made after the columns were cast and forms were removed. Reference discs and a portion of the Whittemore gage are shown in Fig. 3. In taking readings, the Whittemore dial gage is first set to a fixed reading over a gage length determined by a standard bar made of Invar steel. Air temperature and surface temperature of the concrete are recorded using a thermocouple connected to a potentiometer.

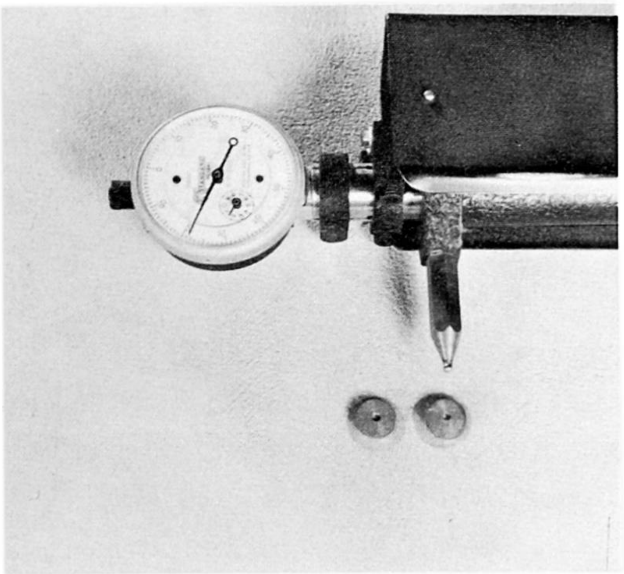

FIG. 3 - Whittemore Strain Gage

Three columns and two locations on the core wall of selected stories are instrumented. These installations are identified as 1C, 1W, 2C, 3C and 3W on Fig. 2. About every third story, Column 1C and core wall location 1W are instrumented with one gage line. Specified stories where these single gage lines are installed are indicated by arrows in Fig. 1. Asterisks in Fig. 1 indicate stories in which all three columns, each with three gage lines, and both core wall locations are instrumented. Stories above and below floors where column sizes change are monitored at installations 1C, 2C, and 3C, each with three gage lines. These gage lines are spaced at about 90 degree intervals around half the column perimeter. The more completely instrumented stories are spaced about every ten floors.

Relative humidity in Column 1C is measured with a Monfore humidity probe[2] at floors 10, 19, 43, 58 and 61. Hollow tubes are cast into these columns to permit measuring humdity at the center of the column, midway between the center and surface of the column, and about 2 in. (5cm) from the surface of the column. Temperature within the humidity wells can also be measured. Surface temperature and interior column temperature agree within ±3F (± 1.7 C).

LABORATORY TESTS

Samples of each of the four concrete mixes used in casting columns and core walls of the building were obtained and brought to the laboratory for tests. For each mix, sixteen 6 x 12-in. (15 x 30 cm) cylinders were cast at the building site. The cylinders were transported to the laboratory the day following casting. At the laboratory, the cylinders were stripped from the molds and stored at 100 percent relative humidity and 73 F (23 C) until ready for test.

Compressive strength and modulus of elasticity are measured 28, 90, and 180 days, and also one and two years after casting. At 28, 90, and 180 days, one cylinder is placed under sustained constant load to measure time-dependent shortening. The applied load produces a cylinder stress equal to 25 percent of the nominal concrete design strength. With each sustained load test, a companion cylinder having no applied load is used to measure drying shrinkage. The cylinders used in this time-dependent study are stored at 50 percent relative humidity and 73 F (23 C) once the test is started.

The coefficient of thermal expansion was measured for each of the four concrete mixes. To do this, one cylinder from each mix was taken from moist storage and sealed in copper foil. Length of these moist concrete specimens was measured at temperatures of 40, 73, and 100 F (4, 23, and 38 C).

PRESENTATION OF DATA

Because of the limited length of this paper permitted by IABSE, only data related to columns in the 1st and 30th stories will be presented and discussed. Concrete used in casting these columns are designated as A and B, respectively. Measured compressive strength and modulus of elasticity versus age for these concretes are shown in Table 2.

TABLE 2 -- MEASURED PROPERTIES OF CONCRETE

Age of Concrete	Compressive Strength, *				Modulus of Elasticity, *			
	Mix A		Mix B		Mix A		Mix B	
	psi	kg/cm²	psi	kg/cm²	10^6 psi	10^6 kg/cm²	10^6 psi	10^6 kg/cm²
28-day Design	7500	528	6000	422	5.00	0.35	4.46	0.31
28 days	7940	558	7810	549	4.80	0.34	5.01	0.35
90 days	9250	650	9530	670	5.78	0.41	4.84	0.34
180 days	9910	697	9790	688	5.84	0.41	6.16	0.43
1 year	10300	724	10210	718	6.59	0.46	6.29	0.44

* Average of two cylinders

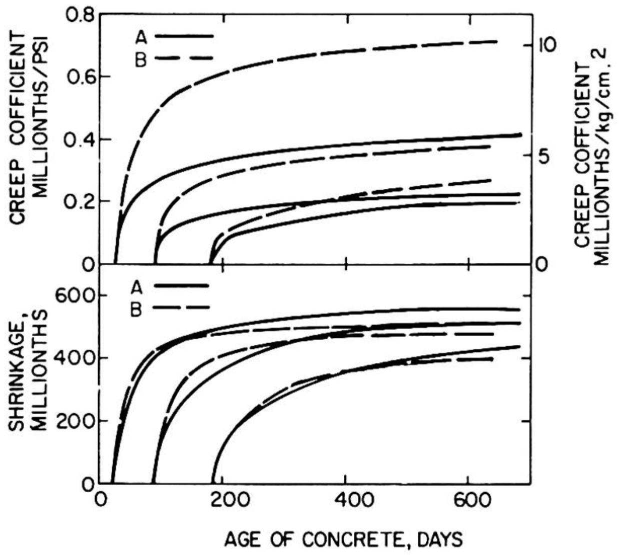

Fig. 4 - Creep and Shrinkage of Concrete

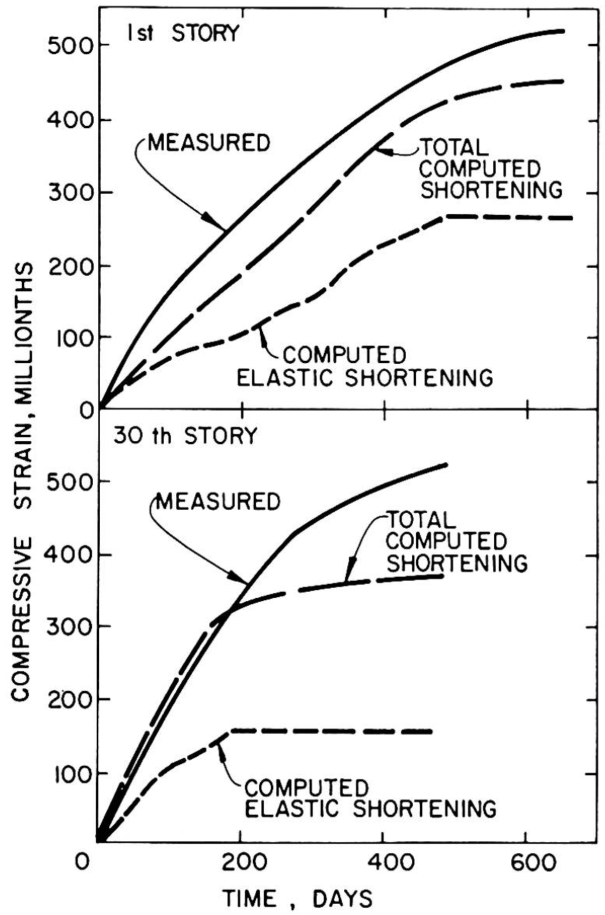

Fig. 5 - Deformation of Columns

Creep and drying shrinkage characteristics of the two field-obtained concrete mixes are shown in Fig. 4. These data are from laboratory tests of the 6x12-in. (15x30 cm) plain concrete cylinders. Cylinders were removed from moist curing and placed under test at 28, 90, and 180 days after casting.

The temperature coefficient measured in the laboratory tests was about 5 millionths per degree Fahrenheit (9 milliionths per degree Centigrade). This value applies over the temperature range of 40F to 100F (4C to 38C).

Measured deformations of the reinforced columns in the 1st and 30th stories are shown in Fig. 5. The curves are the average strains for the 3 columns in each story. Strain readings are adjusted to a column temperature of 73F (23C) using the measured temperature coefficient. Temperature of the columns varied between 40F and 80F (4C and 27C) during this field investigation.

Elastic shortening was computed using the steel percentage and the measured modulus of elasticity of concrete since this value changed with time. This shortening is due to the weight of the structure alone; i. e., columns and weight of a tributary floor area used in design. Increments of load applied to a column were about 37 kips (17 t) per story. Actual time of construction was used in computing column loads. Column reinforcement was 5.5 percent in the 1st story and 3.8 percent in the 30th story.

At the end of construction, computed elastic shortening was about 260 and 160 millionths for the 1st and 30th story columns, respectively. Total measured strain was about 510 millionths in the 1st story columns at 600 days and 500 millionths in the 30th story columns at 400 days.

Relative humidity in the interior of the columns was greater than 90 percent when the column was 18 months old. Most readings at the two interior locations showed a relative humidity greater than 95 percent. Relative humidity of concrete near the surface of the column was difficult to measure accurately due to influence from the atmosphere.

TENTATIVE ANALYSIS

Time-dependent column shortening is affected by volume/surface ratio and the amount of reinforcement. By taking these two factors into account, deformation of columns in a building may be predicted from laboratory tests on small plain concrete cylinders.[3]

Hanson and Mattock[4] have shown that the amount and rate of creep and drying shrinkage of concrete decreases as the volume/surface ratio increases. Tests were made on plain concrete specimens that had volume/surface ratios from 1 to 6 in. (2.5 to 15 cm). Data from these tests were extrapolated to the volume/surface ratios of 10 and 9 in. (25.4 and 22.8 cm) for the 1st and 30th story columns. Thus, size-effects between time-dependent shortening of the laboratory test cylinders and that of the columns were determined.

Pfeifer[5] conducted tests on specimens with equal volume/surface ratio but with reinforcement varying from 0 to 8.4 percent. The tests demonstrated that time-dependent deformation decreases with increasing amounts of reinforcement. Using data from Pfeifer's tests, creep and drying shrinkage of plain concrete cylinders were related to creep and drying shrinkage of specimens with 3.8 and 5.5 percent reinforcement by interpolation. These reinforcement percentages correspond to the amounts in the 30th and 1st story columns.

Using the laboratory test data shown in Fig. 4, creep and drying shrinkage strains for concretes A and B were determined for the same time interval and stress conditions as the field measurements. These strains were modified by factors to account for volume/surface ratio and amount of reinforcement for the 1st and 30th story columns.

By this method, total computed shortenings agree satisfactorily with measured shortenings to the end of construction. Only dead load of the structure was used in computing total shortening. The differences shown in Fig. 5 between measured and computed total shortening after the end of construction may be due to live load effects.

In this tentative analysis it is assumed that effects of volume/surface ratio and amount of reinforcement may be treated separately. It is also assumed that the factors determined for these two effects can be applied to any concrete mix. The extensive test data obtained during this long-time study should permit evaluation and improvement of this analysis.

CONCLUDING REMARKS

The data described and presented in this report are examples of the initial results of a long-term study. It is intended that these measurements of the behavior of the building will be continued for ten years after construction began. Such information should lead to methods of closely predicting the performance of actual structures.

ACKNOWLEDGMENTS

These tests are being conducted with the cooperation of William Schmidt and Associates, Structural Engineers; Schipporeit/Heinrich, Architects; Harnett-Shaw and Associates/ Fluor Properties, Inc., Developers; Crane Construction Co., Inc., General Contractors; and Lake Point Tower Ltd., Owners. The field and laboratory program is being directed by personnel of the Concrete Products Research Section and the Structural Research Section of the Portland Cement Association. Particular credit is due Bernard J. Doepp and Eugene A. Valko, Technicians.

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SUMMARY

A 70-story reinforced concrete building has been instrumented to obtain measurements of time-dependent deformation of columns and core walls. Standard laboratory tests are being conducted on cylinders of concrete taken from batches used in casting the structure. Measurements of column shortening at two levels in the building and of characteristics of the concretes used are presented. It is intended that measurements will continue for a total of ten years after construction began. These tests should provide a correlation between laboratory test data and the performance of prototype buildings.

RÉSUMÉ

Un bâtiment en béton précontraint de 70 étages a été équipé d'instruments pour obtenir des mesures des déformations en fonction du temps sur les colonnes et les murs du noyau. Des tests de laboratoire standards sont faits sur des cylindres de béton pris des mêmes mélanges. Les mesures des raccourcissements des colonnes à deux niveaux du bâtiment et des caractéristiques des bétons utilisés sont présentés. On a l'intention de continuer les mesures pendant 10 ans depuis le début de la construction. Ces tests devraient permettre une comparaison entre les données déterminées au laboratoire et la performance de bâtiments-type.

ZUSAMMENFASSUNG

Ein siebzigstöckiges Stahlbetongebäude wurde mit Instrumenten dergestalt ausgerüstet, dass Messungen über zeitabhängige Verformungen von Säulen und Kernwänden angestellt werden konnten. Standardversuche wurden an Betonzylindern aus im Bauwerk gebrauchten Teilen durchgeführt. Es werden die Messungen der Säulenkürzung auf zwei Höhen des Gebäudes und die Betoncharakteristiken angegeben. Es ist beabsichtigt, die Messungen über zehn Jahre nach Baubeginn durchzuführen. Diese Prüfungen sollten eine Beziehung zwischen Laborversuchen und der Verformung am Versuchsgebäude ergeben.

Basic Design Considerations for the Moscow 533 Metre T.V. Tower

Considérations principales pour le projet de la tour de télévision à Moscou, 533 m haute

Grundsätzliche Berechnungsprinzipien für die Projektierung des 533 m hohen Fernsehturmes in Moskau

N.V. NIKITIN
USSR

Construction of a new 533 m TV tower was terminated at the end of 1967 (Fig.1).

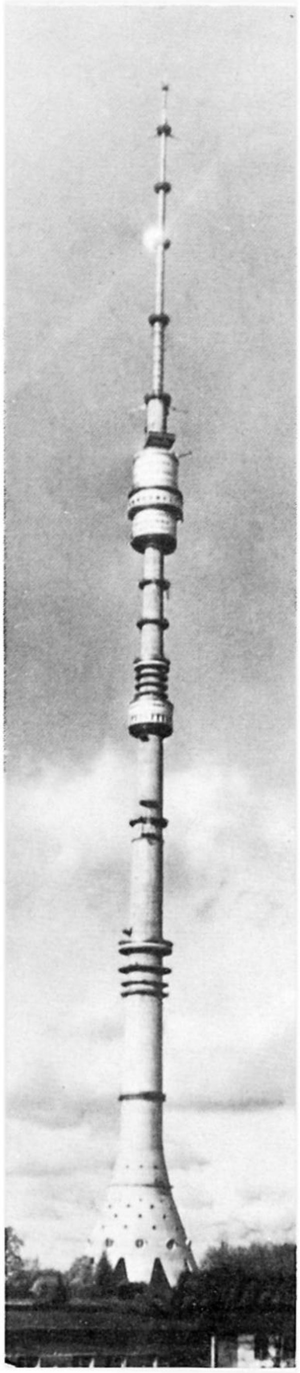
Up to the level of 385 m the tower is a cone prestressed reinforced concrete tube with a sharp break in its generating line at the height of 63 m. The lower part of the cone rests on foundation containing ten separate supports which form ten open arches 17 m high each.

Beginning from the height level 385 m to 533 m (148 m) a steel telescopic pipe shaft is situated which carries radio aerials as well as TV antennas.

Within the cone and the shaft of the tower 48 floors of various premises are situated, floors N°, 5, 6, 7 are occupied by the radio transmitting station. Partially the radio transmitters are located in the upper part on the level of 348 and 353 m. Simultaneous transmission of 5 television programs is secured (one of them is in technicolour) and 6 radio programs on ultrashort wave lengths. The height of the antenna guarantees secure reception of TV and radio broadcasting programs at a distance of 120-150 km.

Three floors at the height levels 328, 331 and 334 m are taken by the restaurant. Simultaneously the restaurant can serve 288 persons. The tables are placed on the circular revolving floor.

At the height levels of 147, 269, 337 and 341 m the observation towers are located. Three of them are closed and the upper one is open (Fig.2).



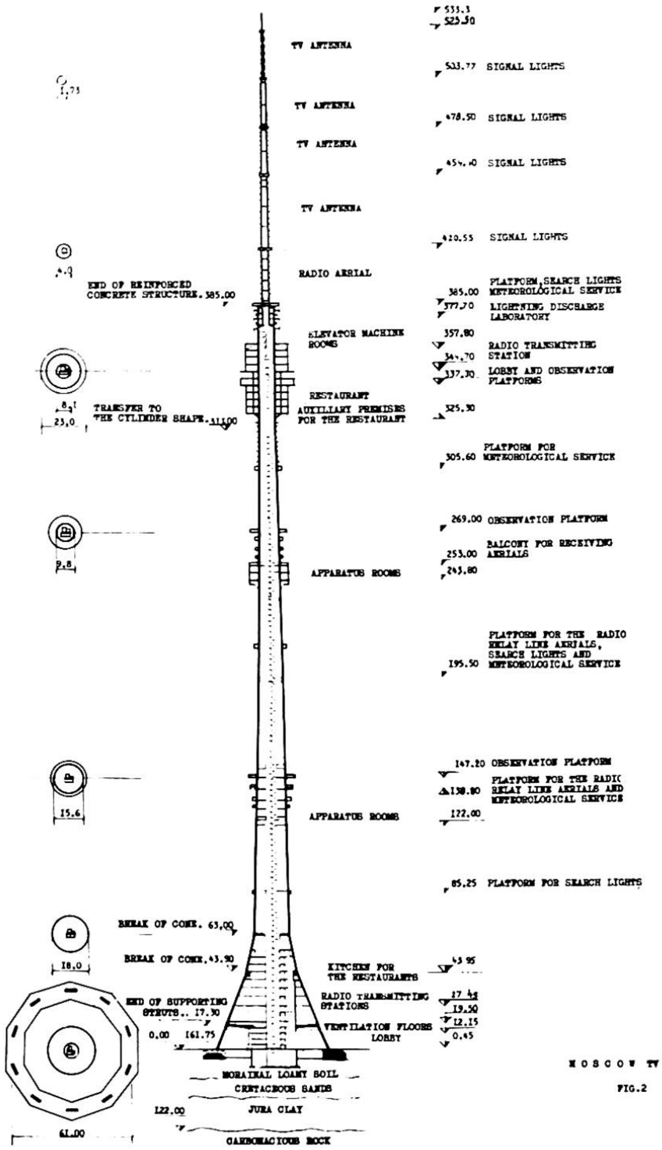
In the shaft there are 4 elevators. One of them serves the restaurant. The rise and descent takes 150 sec. at a speed of 7 m/sec.

In the shaft proper and around it on the platforms numerous technical services are located such as a receiving station for subsequent retranslation of the TV programs, meteorological apparatus, signal lights, laboratory for lightning discharges and radio relay line antennas. The shaft of the tower on top of this contains all kinds of communication, wiring for radio, TV transmissions, electric conduits, water supply pipes, sewerage and telephone.

The architectural outlook of the tower considerably is determined by the technical requirements. The lower part has large openings and being left free reveals the structural design. On this elevation the cross-section of the shell is represented. The concrete surface of the tower is left unpainted in order to better reveal the type of material used. The light aluminium constructions contrast well with rough concrete shell. The interior decoration contains modern materials: aluminium, glass, plastics.

The Basis and Foundation

On the building site the subsoil conditions are the following from top to bottom: 10-12 m of very dense loamy soil of the glacial period containing pebbles and bolders, water is at a depth of 5-6 m, further down 12-15 m more ancient deposite in form of fine dusty sands and sandy soil. Further down old dense clay and only at a depth of 40 m - rock. Under the above-mentioned circumstances it was decided to rest the tower on the upper loams with the minimum depth of foundation in order to leave as much as possible the layer of good soils between the lower part of the foundation and relatively weak lower sandy soils saturated with water.



MOSCOW TV TOWER

FIG.2

The foundation of the tower has the form of 10-sided polygonal ring slab with an average diametre - 60 m, width 9.5m and the thickness 3 - 5.5 m.

When testing the soils by the method of loading the punch with the area of 600 cm^2 in the pits we have received the following values of the modules of deformation: morainal soil 800-900 kg/cm^2, underlying sand soils, loamy soils 300-400 kg per sq cm and the lower jura deposits - 300 kg per sq cm. The sedimentations of the foundations were determined at a usual supposition that the stresses under the foundation are being distributed as in a elastic homogeneous half shere 5 - 6 cm, what at subsequent observations was found to be true. Such sedimentation has no importance as far as the construction is concerned. The statical design of the foundation was carried out assuming the design scheme to be a ring continuous 10 span beam with a hinged support with a given load as the reaction of lower soils with the flat distribution of the load.

As a principal scheme for designing such a beam it is convenient to adopt a 10-sided polygon with the hinges in the middle of each side (Fig.3).

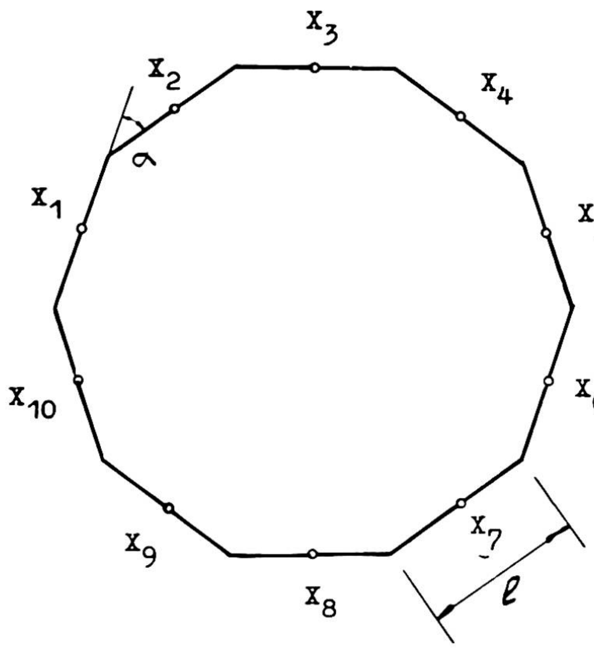


FIG.3

The hinges allow free rotation around the axis of the rod, that is they exclude the torsion moments but do not allow any shear or rotation around the axis perpendicular to the axis of the rod. In other words the hinge would allow the bending moments and the shearing stresses to appear. Under this assumption the design is reduced to solution of a system of five-member equations.

$$-x_1-2\cos\alpha\, x_2+(4+2\cos^2\alpha+f)x_3-2\cos\alpha\, x_4-x_5+$$
$$+a_{3p}+a_{35}+a_{3m} = 0$$

where x are unknown torsion moments in the hinges

$$f = \frac{6\sin^2\alpha\, EJ}{GJ_k}$$

where E and G are modules of elasticity and shear of the concrete; J and J_k - bending and torsion moments of inertia; $a_{35}a_{3p}a_{3m}$ - the load members from the vertical loads, from the junction loads and from the uniformly distributed torsion moments.

The total weight of the tower is 31400 tons, the foundation 14500 tons soil on the foundation 5500 tons. All these loads produce stresses under the foundation equal to 2.64 kg/cm^2. The side stresses from wind loads are equal to 0.42 kg/cm^2.

When designing the foundation the necessary measures were taken to increase its durability and safety. The concrete used had according to our standards strength N^o 400, the main and cross reinforcement was increased. The protecting layer was increased to 10 cm. Along the perimetre of the foundation the reinforcement was prestressed and was formed by 10 strends according to the number of sides of the foundation polygon. Every strend has 104 wire cables containing each 24 wires 5mm in diametre. This reinforcement was prestressed by means of jacks each one developing the stress of 57 tons in the cable. Thus, in the foundation a prestressing force was created by compression equal to 5900 tons. This prestressing permitted to reduce the tension in concrete to such a value under which one does not have to expect appearance of cracks.

Supports, the Cone and Shaft

In the cone there are 10 round openings 4 m in diametre each as well as a considerable number of other smaller ones. Over the openings special reinforced concrete awnings are provided to protect from possible fall of icicles.

At the height level 63 m a very powerful diaphragme was introduced, further on up the shaft of the tower is a cone shell with the slant of the generating line equal to 2% compared with a vertical line. The diametre of the shaft is being reduced from 18 m at a height level 63 m to 8 m at the level 311 m, further up the shaft has a cylindrical form. The thickness of the shell of the shaft within the entire cone part is constant and equal to 40 cm. In the cylindrical part it is 35 cm.

The aerial consists of 5 cylindrical sections having the following diametres: 4.0 m; 3.0 m; 2.6 m; 1.72 m; 0.72 m with the lengthes from 19 to 36 m. The sixth section 8 m long has a square cross section 16x16 cm. The sections of the aerial have the thickness from 30 to 12 mm. Inside the aerial are located the feeders from the radio transmitting sets and a lift designed for one person. The last stop of the lift is at a height of 470 m. At the sections where the cross section of the aerial is changing the ring platforms are located. To these balconies special suspended platforms are affixed. By means of these suspended platforms the possibility to reach any external point of the aerial surface is secured. The aerial is protected from corrosion by means of galvanized zinc layer or plastic materials.

The total weight of the aerial is 360 tons.

The Strends for the Prestressing of the Structure

In order to increase the rigidity and to avoid appearance of cracks the shaft of the tower was specially prestressed to create compression in the concrete in the vertical direction. This was provided by means of a series of strends prestressed parallel to the inner surface of the shaft. Altogether there were 150 strends stressed. Thirty strends are fixed at a height of 63 m and 120 strends at a height of 43 m. At the upper part these strends are gradually fixed at 7 different horizons beginning from the height of 195 m and to the upper part only 60 strends are reaching. These strends are embedded into the ring cantilevers. The strend having a diametre of 38 mm consists of 259 wires 1.8 mm in diametre each. The wires have high quality zinc coating.

Each strend is prestressed with a force equal to 72 tons. The total stress of these strends in the lower part of the shaft reaches 10800 tons.

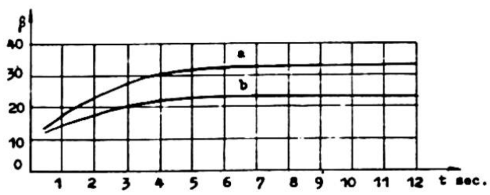
After the final stressing the strends were brought close to the walls of the shaft and affixed at the intervals of 7 m to the wall of the shaft. This is important since the affixed strends work as reinforcement and the strends not affixed to the walls keep their normal stress unchanged. This fact helps to gain around 10% in the safety factor.

Wind loads

When designing this tower the wind pressure was taken into consideration according to the usual norms for similar structures increased by 8%. The following wind velocities were considered:

Height above the ground level	10	20	40	100	350 and higher
Velocity in m/sec	24.7	28.7	33.1	36.6	42.7
m	0.35	0.35	0.32	0.21	0.10

Besides the statical wind load which corresponds to the above given velocities a frontal dynamic wind pressure is taken into consideration. The dynamic wind pressure is being evaluated through the coefficient β added to the statical pressure. By means of this coefficient the distribution of the dynamic part of the wind pressure along the height is calculated. The type of change of load pressure in terms of time is evaluated through a dynamic coefficient which is a function of the period of oscillation of the structure and its material. Having two different materials such as: reinforced concrete shaft and steel aerial the dynamic co-



a- for metal and timber structures
b- for reinforced concrete structures

FIG.4

efficient is determined by means of interpolation-proportional to the quantity of the potential energy stored during the oscillations of the steel and reinforced concrete parts of the structure (Fig.4ab).

$$Q_m = \int \frac{M^2}{2EJ_c} dx$$

Aerodynamic coefficient for the cylinder part is assumed to be equal to 0.6, for all sorts of protruding parts - 1.00. At that the wind load was assumed acting on all protruding parts along the entire perimetre.

Besides longitudinal oscillations in the direction of air pressure the resonance transversal oscillations were also considered which occur through periodical cessations of wind vortexes.

The critical velocity of air which leads to a resonance of the vortex cessation and proper oscillations of the structure can be determined by the formula:

$$V = \frac{5D}{t}$$

where D is the diametre of the tower

t - period of oscillation

The amplitude of the transversal aerodynamic force is determined by the formula

$$F = \frac{V^2 D}{80}$$

When determining the full resonance amplitude the damping of oscillations on account of friction is taken into consideration. The logarithmic decrement is assumed: for steel structures 0.10 and for the reinforced concrete structures 0.30. The mixed structure is being calculated through the interpolation similar to the design of frontal oscillations.

In connection with the very large period of fluctuation of transversal oscillation for the tower the latter ones were found to be considerably less than frontal oscillations although the resonance was considered to appear along the entire height of the tower.

The Dynamic Design

When making the dynamic design the tower was modeled in the form of a cantilever rod with the 24 concentrated loads (masses) and 24 elastic hinges in the same points where the concentrated loads were applied. By means of consecutive approach method the forms of free oscillations were determined together with the amplitude of acceleration at the top level of the tower which was equal to the acceleration of the gravity force further called unity oscillations.

In this case the inertia forces at the time of maximum deviations are determined by the formula:

$$T = G \frac{Y}{f} = G\lambda$$

where G is concentrated mass; Y - deviation of the mass;
f - deflection of the upper point of the rod.

Having assumed in the first approximation the diagram of deflections in form of a parabola the inertia forces T are determined and on the basis of these diagrams we determine the diagram of deflections. By means of the following procedure we gradually approach the diagram of deflections. After the fourth attempt we have come to a satisfactory precision.

After the unit form of oscillation has been determined the period of fluctuations can be determined by the formula:

$$t = \sqrt{\frac{4\pi^2 f}{g}} = 0.2\sqrt{f}$$

where f is in cm.

When determining the second harmonic the basic curve was reduced to orthogonal position with the first one.

$$Y_1^{I} = Y_0^{I} - a\lambda^{II}$$

where

$$a = \frac{\sum T^{I} Y_0^{II}}{\sum T_1^{II} \lambda^{I}}$$

When determining the third harmonic the original curve was reduced to the orthogonal position with the two first ones

$$Y_1^{III} = Y_0^{III} - a_1\lambda^{I} - a_2\lambda^{II}$$

where

$$a_1 = \frac{\sum T^I\ Y_o^{III}}{\sum T^I\ \lambda^I}\ ; \quad a_2 = \frac{\sum T^{II}\ Y_o^{III}}{\sum T^{II}\ \lambda^{II}}$$

The amplitude of oscillations and the accompanying stresses can be determined by the formula:

$$K^n = \eta \beta\ k^n$$

where K^n is deviations and stresses according to harmonic n-power, k^n - the same deviations and stresses in the singular harmonic; β - the mentioned above dynamic coefficient; η - the influence coefficient

$$\eta^n = \frac{\sum P^n\ \lambda^n}{\sum T\ \lambda^n}$$

where P^n - the load applied at the point n; λ^n - relative deviation at the singular harmonic n.

Summing up of all the amplitudes of all harmonics is done by means of the square root:

$$K = \sqrt{(K^I)^2 + (K^{II})^2 + (K^{III})^2}$$

The Static Design

The static design was carried out on the basis of deformed scheme, i.e., all bending moments from the vertical loads were taken into consideration, bending moments which appear on account of deflection of the shaft. The deflection of the shaft taken into consideration gives us increase of the bending moments up to 10-15%.

Since the design was carried out on the basis of the deformed scheme separate design for stability was not performed.

A special design for the season fluctuations of the temperature was performed. It was assumed that the foundation has permanent temperature but the cone and supports can be heated up to + 30°(C) and cooled up to -30°(C). This design led us to the necessity to make the supports of the tower flexible in the radial direction.

Strength, Stability and Crack Resistance Design

The strength design of any usual structure is being carried out by means of comparison of stresses caused by the designing loads having considerably small probability together with the limit stresses which the structure can receive. In this case a reduced stress compared to the nominal strength of materials of the structure is taken into consideration.

For the tower such an approach was found to be nonconvincing. There is a definite assurance as far as the size of the normal stress is concerned, stress which is caused to 95% by the weight of the entire structure. The quality of building materials were under a very thorough observation and there is no doubt in total reliability of the concrete and reinforcement's strength. Still doubtful is the correct choice of the designing wind load as well as the correct determination of breaking stress in the ring section. In connection with this the following condition of the rigidity of the shaft was adopted: the breaking bending moment in each section was determined under the given permanent normal force, must be twice the size of the bending moment caused by the wind load. The breaking bending moment in the section of the shaft of the tower is determined under the assumption that in the part of the ring section the concrete is stressed and these stresses reached the prismatic strength; and in the reinforcement they reached the ultimate compression strength in concrete (0.2%) $R^1a = 4000$ kg/cm^2.

In the tensile zone the concrete resistance was not taken into consideration and the reinforcement was considered to have the yield point at $R_a = 4600$ kg/cm^2. This conventional yield limit somewhat exceeds the reject minimum (4000 kg/cm^2) and approaches the average statical. It is assumed that the following distribution of stresses will take place along the cross section (Fig.5). In the compressed zone (φ) the stresses in concrete and in the reinforcement are uniform (the orthogonal diagram); in the tensile zone (ψ) as well. Between the compressed and tensile zones it is assumed that a neutral zone is located in which neither concrete nor reinforcement are stressed.

It is assumed that the strends in the tensile zone have a stress limit that is $R_H = 19200$ kg/cm^2; and in the compressed zone

$$\sigma_c^1 = 19200 - 4000 \frac{1.5 \times 10^6}{2.0 \times 10^6} = 6200 \text{ kg/cm}^2$$

and in the neutral zone the stress in strends is assumed to be equal to original minus the losses $\sigma_c = 11050$ kg/cm^2.

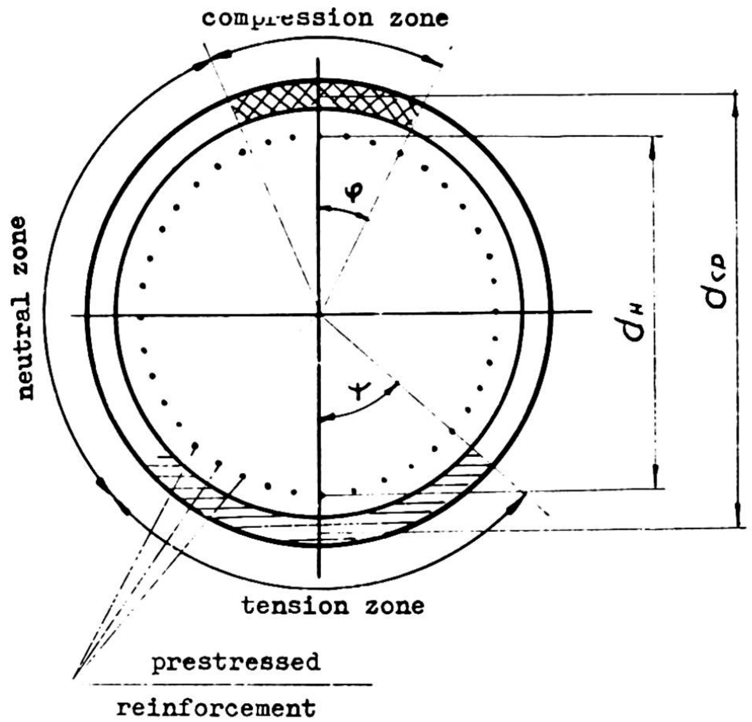


FIG.5

The size of the tensile zone is selected in such a way that it must satisfy the following conditions: $S_\delta = 0.8\, S_0$, where S_δ - the statical moment of compressed zone calculated in relation to the centre of gravity of tensile zone, S_0 - the statical moment of the entire area of concrete situated above the centre of gravity of the tensile zone, related to this centre.

From this condition we receive an equation to determine the size of the tensile zone depending on the size of the compressed zone

$$\varphi\left(\frac{\sin\varphi}{\varphi}+\frac{\sin\psi}{\psi}\right) = 0.8\left(\frac{\sin\theta}{\theta}+\frac{\sin\psi}{\psi}\right)$$

This equation is invariant to the reinforcement of the normal force/diametre.

Solving this equation we can compile the following table which gives us the values of the compressed and tensile zones:

φ	<1.28	1.29	1.31	1.33	1.35	1.37	1.39	1.41	1.43
ψ	1.55	1.46	1.37	1.28	1.19	1.10	1.00	0.89	0.77

The size of the compressed zone can be determined from the condition of equilibrium of all stresses acting in the cross section:

$$\varphi = \frac{N + d_{cp} f_a R_a \psi + d_H f_n \left[\pi \sigma_0 + \psi (R_H - \sigma_0)\right]}{d_{cp}\,\delta R_{np} + d_{cp} f_a R_a' + d_H f_H (\sigma_c - \sigma_c')}$$

After this the breaking moment

$$M_p = \tfrac{1}{2} d^2_{cp}\,\delta R_{np} \sin\varphi + \tfrac{1}{2} d_{cp} f_a (R_a \sin\psi + R_a' \sin\varphi) + \tfrac{1}{2} d^2_n f_n \times$$
$$\times \left[(R_H - \sigma_c)\sin\psi + (\sigma_c - \sigma_a')\sin\varphi\right]$$

where N is normal force in the cross section; f_a, f_n - the cross section area of unstressed and stressed reinforcement related to the unity of the perimetre. The crack resistance design was carried out in the elastic state. At the wind load equal to 0.75 of assumed there was no tensile stresses allowed in the concrete.

The Design Data and Observations

For the three forms of harmonic oscillations the periods of the fluctuations were determined at 13.1 seconds, 4.6 seconds and 2.7 seconds.

Actually the period of fluctuations was found to be 11.3 sec in the main form. This fact shows that the rigidity of the shaft was 1.35 times underestimated. When designing the fluctuations the module of deformation of concrete was also reduced; it should have been taken considering the age of concrete at the impact load equal to 390000 kg/cm^2, actually was taken 300000 kg/cm^2. Evidently all numerous elements filling the tower inside as well as external constructions participated in the total work.

As a nominal wind load it is assumed by the norms such a probable wind load which is equal to 1/7300, i.e., such a load which would be surpassed on the average during 1/7300 of the considered period of time, for instance, during 1.2 hours a year. At this wind velocity according to the design the statical deflection of the upper part of the tower will be equal to 5.8 m, the amplitude of fluctuations according to the first form 1.4 m, the second - 1.1 m and the third 0.2 m.

It can be expected that the actual deflections of the tower will be less than mentioned above since the rigidity of the entire structure was not taken into account in design.

A one-sided heating by the sun was also taken into consideration. According to design the deformation must reach 3.3 m. The small amount of observations carried so far give somewhat smaller value.

During the 24 hours the upper part of the tower moves along a very complicated closed curve with a maximum diametre of 2.5 m.

There were apprehensions that the visitors of the tower would experience large and unpleasant fluctuations of the tower. The experience showed that visitors do not feel any of those fluctuations.

A very vast program of observation of the condition of the structure was organized. The wind velocity is being measured at various heights. Deformations, oscillations of the tower, conditions of the concrete, lightning discharges, the status of the prestressed strends as well as sedimentation and deformation of the soil under the structure are also being observed and measured.

SUMMARY

In the design of the Moscow prestressed r.c. 533 m TV tower both static and dynamic effects of the wind load were taken into consideration. Three forms of harmonic oscillations were determined. The amplitudes of frontal and transversal fluctuations were also found. The design was based on the deformed scheme. The s strength computation was performed at the failure stage. Crack resistance and rigidity were determined at the elastic stage.

RÉSUMÉ

Pour le calcul de la tour de télévision à Moscou, 533 m haute et construite en béton armé précontraint, les effects statiques et dynamiques de la force du vent ont été pris en considération. Trois formes des oscillations harmoniques et les amplitudes des divergences frontales et transversales ont été déterminées. Le schéma déformé a été adopté pour les calculs. La résistance de la structure a été calculée pour l'état de destruction, la résistance à la fissuration ainsi que la rigidité pour l'état d'élasticité.

ZUSAMMENFASSUNG

Bei der Projektierung des 533 m hohen Turmes aus vorgespanntem Stahlbeton in Moskau wurden die statischen und dynamischen Wirkungen der Windbelastung in Betracht gezogen. Es wurden drei Formen von harmonischen Schwingungen festgelegt und die Amplituden der Frontal- und Querschwingungen bestimmt. Die Berechnungen wurden auf Grund eines deformierten Schemas vorgenommen. Die Widerstandsfestigkeit wurde auf dem Stadium der Zerstörung, die Rissfestigkeit und Steifigkeit auf dem Stadium der Elastizität berechnet.

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