

# Influence lines for shear around columns in flat plates

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## Influence Lines for Shear around Columns in Flat Plates

Les lignes d'influence d'efforts tranchants autour des colonnes aux dalles plates

Einflußlinien für Schub im Stützenbereich von Flachdecken

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### Introduction

Shear stresses near columns in flat plate structures are caused by the column reaction. This reaction can be subdivided into a force,  $V_v$ , acting perpendicular to the plate and into a moment,  $M$ , whose vector is parallel to the plate. Only a portion of this moment,  $M$ , is transmitted to the plate by shear stresses. The remainder is transferred by bending stresses (Fig. 1).

The stress concentrations resulting from the above reactions often govern the design, i. e., they determine the required plate thickness and column periphery. It is the purpose of this paper to contribute to the evaluation of these stress concentrations.

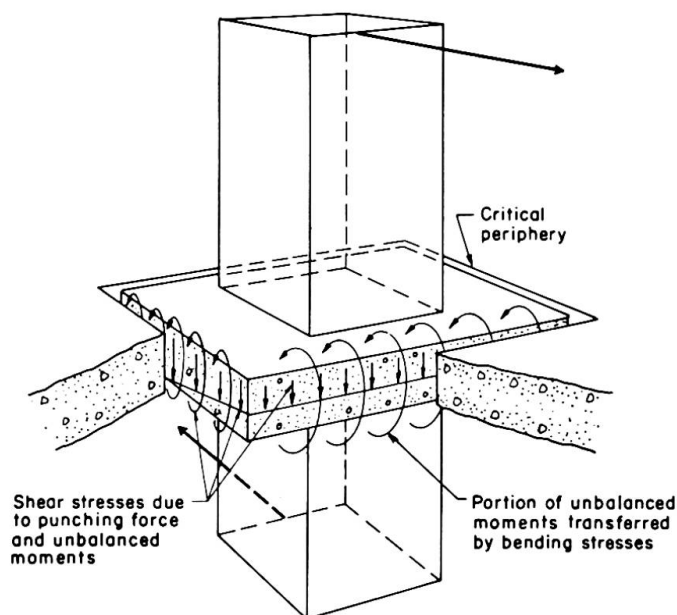


FIG. 1 TRANSFER OF UNBALANCED MOMENT

The moment transfer in the vicinity of columns has been investigated in extensive test programs [1 through 6]. While these tests resulted in design methods developed on a hypothetical basis [6 through 9], they did not reveal precisely which portion of the unbalanced moment is transferred by bending stresses and which portion by shear. The design methods commonly used in the USA [8, 9, 12] assume the moment transfer by shear to take place in accordance with the equation

$$v = \frac{V_v}{A_c} + \frac{KM}{J_c} e \quad (1)$$

$V_v$  = Total punching shear force

$M$  = Total unbalanced moment

$K$  = Percentage of unbalanced moment transferred by shear

$A_c$  = Area of failure plane

$J_c$  = Polar moment of inertia of failure plane

$e$  = Distance from shear centroid to point on failure plane

Experimentally determined values for  $K$ ,  $A_c$  and  $J_c$  vary and are available in tabulated form [9, Table 8-6]. The following is an approach to determine these values analytically and to evaluate the resulting shear stresses by means of influence lines.

#### An Analytical Method to Determine $K$

The deflection function of a simply-supported single-span plate strip, subject to a concentrated moment, is known [10].

$$w = \frac{M a}{2D\pi^2} \sum \frac{1}{n^2} \cos \frac{n\pi u}{a} \left(1 + \frac{n\pi y}{a}\right) e^{-\frac{n\pi y}{a}} \sin \frac{n\pi x}{a} \quad (2)$$

Visualizing the plate supported by flexible columns at its center and applying the concentrated moment by one of these columns (Fig. 2), one can write a similar deflection function

$$w = M \frac{L}{D\pi^2} \sum_{n=1,2,3} \frac{1}{n^2} \cos \frac{n\pi}{2} \left(1 + \frac{n\pi y}{2L}\right) e^{-\frac{n\pi y}{2L}} \sin \frac{n\pi(L+x)}{2L} \quad (3)$$

The boundary conditions at the remote columns are satisfied by this equation only partially. This, however, does practically not affect the stress configuration in the vicinity of the column at which the unbalanced moment is applied.

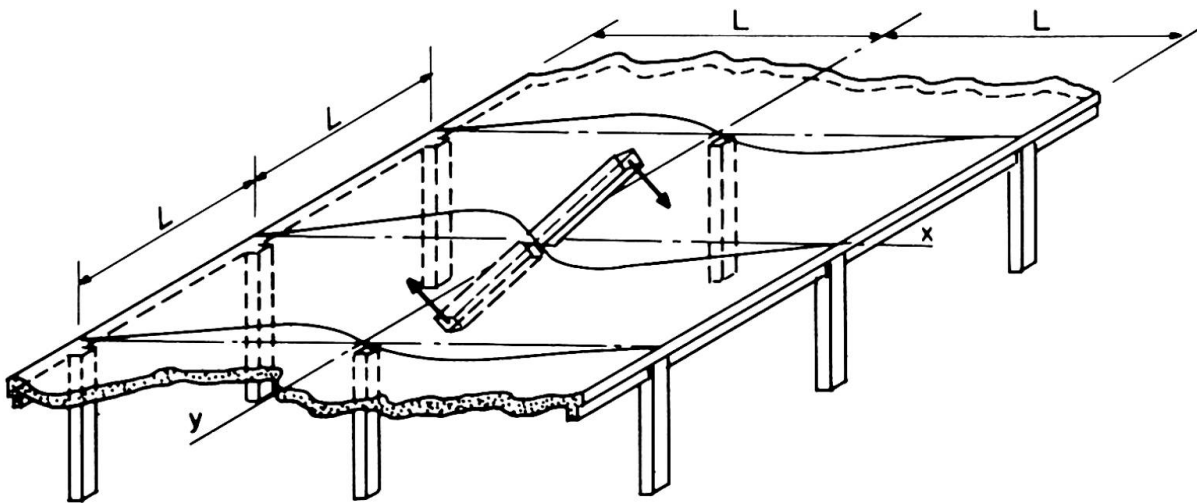


FIG. 2 CONCENTRATED MOMENT APPLIED TO FLATE PLATE

The series represented by Equation 3 converges slowly and, hence, is of limited practical use. It can be summed up, however, by means of transcendental functions [11], similar to the deflection function of a simply-supported plate strip [10]. We first determine the derivatives of the deflection function,  $w$ , with respect to the  $x$  and the  $y$  axis in closed form, and then the expressions for all bending and twisting moments and for the shear forces. It should be noted that simple expressions for the latter ones can be obtained best by determining  $\partial(\Delta w)/\partial x$  and  $\partial(\Delta w)/\partial y$ , respectively.

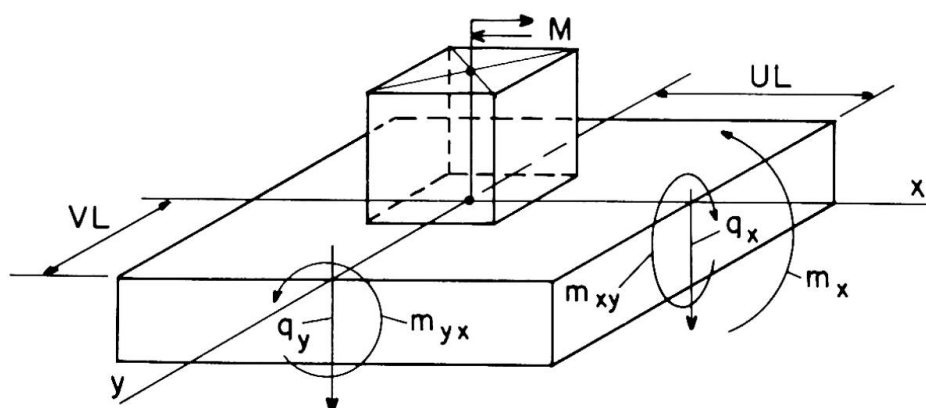


FIG. 3 SHEARS AND MOMENTS AROUND COLUMN

$$\frac{\partial w}{\partial x} = \frac{-M}{8D\pi} \left\{ \log \left[ 2 \cosh \frac{\pi y}{2L} + 2 \cos \frac{\pi x}{2L} \right] + \log \left[ 2 \cosh \frac{\pi y}{2L} - 2 \cos \frac{\pi x}{2L} \right] \right. \\ \left. - \frac{\frac{\pi y}{2L} \sinh \frac{\pi y}{2L}}{\cosh \frac{\pi y}{2L} + \cos \frac{\pi x}{2L}} - \frac{\frac{\pi y}{2L} \sinh \frac{\pi y}{2L}}{\cosh \frac{\pi y}{2L} - \cos \frac{\pi x}{2L}} \right\} \quad (4)$$

$$\frac{\partial w}{\partial y} = \frac{M}{16D} \left( \frac{y}{L} \right) \sin \frac{\pi x}{2L} \left\{ \frac{1}{\cosh \frac{\pi y}{2L} + \cos \frac{\pi x}{2L}} - \frac{1}{\cosh \frac{\pi y}{2L} - \cos \frac{\pi x}{2L}} \right\} \quad (5)$$

$$m_x = -D \left( \frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right) \\ = \frac{M}{16L} \left\{ (1 + \nu) \left[ \frac{-\sin \frac{\pi x}{2L}}{\cosh \frac{\pi y}{2L} + \cos \frac{\pi x}{2L}} + \frac{\sin \frac{\pi x}{2L}}{\cosh \frac{\pi y}{2L} - \cos \frac{\pi x}{2L}} \right] \right. \\ \left. + (1 - \nu) \left[ \frac{\frac{\pi y}{2L} \sinh \frac{\pi y}{2L} \sin \frac{\pi x}{2L}}{\left( \cosh \frac{\pi y}{2L} + \cos \frac{\pi x}{2L} \right)^2} + \frac{\frac{\pi y}{2L} \sinh \frac{\pi y}{2L} \sin \frac{\pi x}{2L}}{\left( \cosh \frac{\pi y}{2L} - \cos \frac{\pi x}{2L} \right)^2} \right] \right\} \quad (6)$$

$$\begin{aligned}
 m_{yx} = m_{xy} &= -D(1-\nu) \frac{\partial^2 w}{\partial x \partial y} \\
 &= \frac{M\pi y}{32L^2}(1-\nu) \left\{ -\frac{\cosh \frac{\pi y}{2L} \cos \frac{\pi x}{2L} + 1}{\left(\cosh \frac{\pi y}{2L} + \cos \frac{\pi x}{2L}\right)^2} + \frac{\cosh \frac{\pi y}{2L} \cos \frac{\pi x}{2L} - 1}{\left(\cosh \frac{\pi y}{2L} - \cos \frac{\pi x}{2L}\right)^2} \right\}
 \end{aligned} \quad (7)$$

$m_y$  is similar to  $m_x$ , except that the terms associated with  $(1-\nu)$  are of opposite sign.

$$\begin{aligned}
 q_x &= \frac{\partial m_x}{\partial x} + \frac{\partial m_{xy}}{\partial y} = -D \frac{\partial(\Delta w)}{\partial x} \\
 &= \frac{M\pi}{16L^2} \left\{ -\frac{\cosh \frac{\pi y}{2L} \cos \frac{\pi x}{2L} + 1}{\left(\cosh \frac{\pi y}{2L} + \cos \frac{\pi x}{2L}\right)^2} + \frac{\cosh \frac{\pi y}{2L} \cos \frac{\pi x}{2L} - 1}{\left(\cosh \frac{\pi y}{2L} - \cos \frac{\pi x}{2L}\right)^2} \right\}
 \end{aligned} \quad (8)$$

$$\begin{aligned}
 q_y &= \frac{\partial m_y}{\partial y} + \frac{\partial m_{xy}}{\partial x} = -D \frac{\partial(\Delta w)}{\partial y} \\
 &= \frac{M\pi}{16L^2} \left\{ \frac{\sinh \frac{\pi y}{2L} \sin \frac{\pi x}{2L}}{\left(\cosh \frac{\pi y}{2L} + \cos \frac{\pi x}{2L}\right)^2} - \frac{\sinh \frac{\pi y}{2L} \sin \frac{\pi x}{2L}}{\left(\cosh \frac{\pi y}{2L} - \cos \frac{\pi x}{2L}\right)^2} \right\}
 \end{aligned} \quad (9)$$

To determine the value  $K$  of Eq. 1 from the above expressions, one can define it in two ways. If  $K$  is defined as that portion of the unbalanced moment  $M$ , which is not transferred between columns and slab by pure bending stresses, then  $K$  becomes

$$\begin{aligned}
 K &= \frac{\int_0^{UL} m_{yx} dx - \int_0^{VL} q_x UL dy - \int_0^{UL} q_y x dx}{\int_0^{VL} m_x dy + \int_0^{UL} m_{yx} dx - \int_0^{VL} q_x UL dy - \int_0^{UL} q_y x dx} \\
 &= 1 - \frac{4}{M} \int_0^{VL} m_x dy
 \end{aligned} \quad (10)$$

The values  $U$  and  $V$  in this expression define the assumed failure plane (Fig. 1) at which the stress configuration is to be determined (Fig. 3). The integrations can be performed numerically or, with certain approximations, in closed form as follows.

We are primarily interested in the stress configuration near the columns, where the terms which have  $(\cosh \pi y / 2L + \cos \pi x / 2L)$  in the denominator are very small compared to the remainder of the equation. They may, therefore, be neglected. Furthermore, setting

$$\sin \frac{\pi x}{2L} = \frac{\pi x}{2L}, \text{ and } \cos \frac{\pi x}{2L} = 1 - 1/2 \left( \frac{\pi x}{2L} \right)^2 \quad (11)$$

$$\text{and } \cosh \frac{\pi y}{2L} = 1 + 1/2 \left( \frac{\pi y}{2L} \right)^2, \text{ and } \sinh \frac{\pi y}{2L} = \frac{\pi y}{2L} \quad (12)$$

the expression for  $m_x$  in Eq. 6 simplifies to

$$m_x = \frac{M}{4\pi L} \left\{ (1 + \nu) \frac{x/L}{(x/L)^2 + (y/L)^2} + (1 - \nu) \frac{2(y/L)^2 x/L}{[(x/L)^2 + (y/L)^2]^2} \right\} \quad (13)$$

This equation can be integrated to

$$\int m_x dy = \frac{M}{2\pi} \left\{ \arctan \frac{y/L}{x/L} - \left[ \frac{(1 - \nu)}{2} \right] \frac{(y/L)(x/L)}{(x/L)^2 + (y/L)^2} \right\} \quad (14)$$

so that a closed solution for  $K$  as a function of the critical periphery (Fig. 3) becomes

$$K = 1 - \frac{2}{\pi} \left\{ \arctan \frac{V}{U} - \left[ \frac{(1 - \nu)}{2} \right] \frac{UV}{U^2 + V^2} \right\} \quad (15)$$

The above definition of  $K$  is based solely on the transfer of bending moments. There is, of course, also the possibility of expressing  $K$  in terms of the shear stresses,  $q_x$ , directly. This approach is even more justified since we are interested primarily in the maximum shear stresses along the critical periphery.

Making similar approximations as outlined before, the expression for  $q_x$  of Eq. 8 becomes

$$q_x = \frac{M}{4\pi L^2} \left\{ \frac{[2(y/L)^2 - 2(x/L)^2] - \left[ \frac{\pi^2}{4} (y/L)^2 (x/L)^2 \right]}{[(x/L)^2 + (y/L)^2]^2} \right\} \quad (16)$$

Remembering from Eq. 1 how the shear stress,  $v$ , due to the unbalanced moment,  $M$ , was defined, the definition of  $K$  then becomes

$$K = \left( \frac{-q_x}{M} \right) \left( \frac{J_e}{de} \right) \quad (17)$$

where  $d$  = structural depth of the plate. While  $J_e$  and  $e$  are determined by the failure plane chosen, the shear force,  $q_x$ , as given by Eq. 16, varies, of course, along this periphery and does not suffice to define  $K$ . The required additional condition comes from the fact that the slab is built integrally with the column. This results in zero twisting moment,  $m_{yx}$ , along the column face, so that  $q_x = \partial m_x / \partial x$ . On the other hand, the term  $\partial^3 w / \partial x^3$  is almost constant with respect to  $y$  in the vicinity of the column. It can, therefore, be assumed that the actual distribution of  $q_x$  along the column face is uniform and that it is justified to assume an average value,  $\bar{q}_x = \text{constant}$ , to prevail along the assumed failure plane along the  $y$ -axis in the vicinity of the column. To find this average value,  $\bar{q}_x$ , one must integrate Eq. (16) as follows.

$$\begin{aligned} \bar{q}_x &= \frac{1}{VL} \int_0^{VL} q_x dy \\ &= M \frac{\pi}{32L^2} \left\{ -\frac{x/L}{V} \arctan \frac{V}{x/L} + \frac{(x/L)^2 - (4/\pi)^2}{(x/L)^2 + V^2} \right\} \end{aligned} \quad (18)$$

To determine  $K$  from Eq. (17), one may use the definition of  $J_e$  from Reference 9, Eq. 8-24, so that

$$\frac{J_e}{de} = L^2 \left\{ \frac{4}{3} U^2 + 1/3 \left( \frac{d}{L} \right)^2 + 4UV \right\} \quad (19)$$

in which the structural depth,  $d$ , of the plate may be assumed as  $L/40$ . This term is rather insignificant so that any other reasonable assumption will yield similar results. The resulting  $K$ -value for  $q_x = \overline{q_x}$  can thus be expressed as a function of the critical periphery, i. e., in terms of  $U$  and  $V$  (Fig. 3)

$$K = \frac{\pi}{32} \left\{ \left[ \frac{4}{3} U^2 + 1/3 \left( \frac{1}{40} \right)^2 + 4UV \right] \left[ \frac{U}{V} \arctan \frac{V}{U} - \frac{U^2 - (4/\pi)^2}{U^2 + V^2} \right] \right\} \quad (20)$$

Another simplification suggests itself by neglecting terms in Eq. 20 which are small compared to the remainder of the equation. Hence, with

$$R = \frac{J_e}{L^2 de} \quad (21)$$

a simple expression for  $K$  results, which is within 1/2 percent identical with Eq. 20:

$$K = \frac{R}{2\pi(U^2 + V^2)} \quad (22)$$

This shows that the resulting averaged maximum shear stress,  $v = \overline{q_x}/d$ , due to an unbalanced moment,  $M$ , is inversely proportional to the square of the distance from the center of the column to the corner of the critical periphery:

$$v = \frac{M}{2\pi d L^2 (U^2 + V^2)} \quad (23)$$

#### Influence Lines for Maximum Shear Stress

As revealed by Eq. 1, the influence line for maximum shear stress is a combination of the influence lines for the column reaction,  $V_v$ , and for the unbalanced slab moment,  $M$ . Fig. 4 shows these influence lines for a typical flat plate structure extending over three spans (slab thickness: 8 in.; columns 18x18 in.; story height: 10 ft.; spans: 20 ft.; bay widths: 20 ft.). For a structure with longer spans, the ordinates of the moment influence line would, of course, be bigger, whereas the ones for  $V_v$  would remain about as shown.

In order to combine these influence lines, we multiply the ordinates of the one for  $M$  by the factor

$$Q = K \frac{A_c}{J_e} e \quad (24)$$

and add them to the ordinates of the one for  $V_v$ , as shown in Fig. 4c. The force,  $S$ , obtained by putting a load on the ordinates,  $\eta_s$ , yields the maximum shear stress as  $v = S/A_c$ .

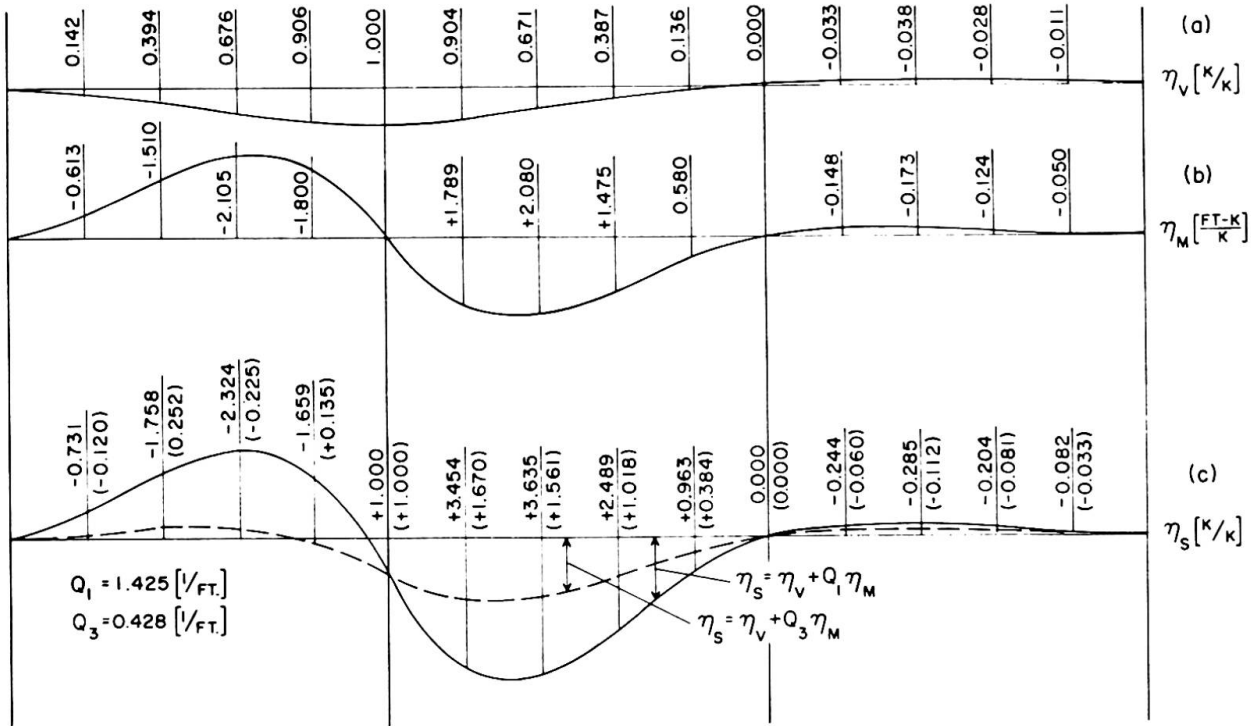


FIG. 4 INFLUENCE LINES FOR MAXIMUM SLAB SHEAR AT COLUMNS

Tables 1 through 3 tabulate K-values and the corresponding Q-factors as a function of the shape of the critical periphery, which is expressed by its coordinates, U and V. Table 1 is set up for  $K = 1.0$ . The corresponding Q-factors pertain to designs in accordance with common practice in the USA [8, 12]. Table 2 uses K-values based on the moment transfer, as defined by Eq. 15. Table 3 uses K-values based on maximum shear stress, as defined by Eqs. 20 or 22.

These Q-values in Tables 1 through 3 differ considerably, and so do the shapes of the resulting influence lines for shear. The lower portion of Fig. 4, for example, compares the influence lines based on the Q-factors from Table 1

(solid line) and from Table 3 (dashed line). The chosen coordinates of the critical periphery are  $U = 0.045$  and  $V = 0.089$ , which comply with common practice [8, 12] for the column and slab dimensions stated above. The K-values pertaining to these Q-factors are  $K_1 = 1.00$  (for  $Q_1 = 1.425$ ) and  $K_3 = 0.301$  (for  $Q_3 = 0.428$ ). It should be noted that these influence lines resemble the ones for kern-moments in columns of continuous frames. The width of the "shear kern," measured from the centroid of the critical periphery, amounts to

V \ U		0.025	0.050	0.075	0.100
$K_1$	0.025				
	0.050	$K_1 = \text{constant}$			
	0.075	$= 1.000$			
	0.100				
$Q_1$	0.025	2.823	1.756	1.315	1.062
	0.050	2.482	1.477	1.101	0.894
	0.075	2.341	1.348	0.993	0.803
	0.100	2.264	1.274	0.928	0.747

Table 1. K and Q - Values without Plate Theory



V \ U		0.025	0.050	0.075	0.100
K <sub>2</sub>	0.025	0.620	0.800	0.867	0.900
	0.050	0.391	0.620	0.736	0.800
	0.075	0.277	0.485	0.620	0.705
	0.100	0.213	0.391	0.525	0.620
Q <sub>2</sub>	0.025	1.750	1.406	1.140	0.956
	0.050	0.971	0.915	0.810	0.716
	0.075	0.649	0.654	0.615	0.567
	0.100	0.482	0.499	0.487	0.463

Table 2. K and Q Values Based on Moment Transfer

v \ U		0.025	0.050	0.075	0.100
K <sub>3</sub>	0.025	0.451	0.435	0.387	0.353
	0.050	0.308	0.431	0.445	0.427
	0.075	0.218	0.364	0.428	0.444
	0.100	0.166	0.300	0.385	0.427
Q <sub>3</sub>	0.025	1.273	0.764	0.509	0.374
	0.050	0.764	0.637	0.490	0.382
	0.075	0.510	0.490	0.425	0.357
	0.100	0.375	0.383	0.358	0.319

Table 3. K and Q Values Based on Maximum Shear

$$k = 1/Q = J_c / K A_c e \quad (25)$$

### Evaluation of Influence Lines

The variation of the influence lines (Fig. 4c) shows that the shape of the critical periphery and the theoretical assumptions of moment transfer affect the shear stresses around a column in two ways: First, there is a direct effect due to the magnitude of the factors associated with  $V_v$  and  $M$ , i. e., the magnitude of the variables  $K$ ,  $J_c$ ,  $A_c$ , and  $e$ . The other effect results from the positioning of the live load as determined by the positive and negative regions of the influence lines.

In designing multi-span frames, we are used to positioning the live load either on the two spans at both sides of a column (Case A) or on just one span, i. e., to the left or to the right of a column (Case B). The first arrangement, Case A, results in a maximum punching force,  $V_v$ , whereas the latter one, Case B, results in a maximum unbalanced moment,  $M$ . We will investigate these two loading conditions with respect to the maximum shear stress which they produce. Furthermore, we will see what effect partial loading of the span has, i. e., loading up to the zero-point of the influence lines (Case C).

The influence lines of Fig. 4c represent two extremes. The solid line ( $K_1 = 1.00$ ;  $Q_1 = 1.425$ ) puts the maximum emphasis on the unbalanced moment, whereas the dashed line assumes a big portion of the unbalanced moment being transferred by bending ( $K_3 = 0.301$ ;  $Q_3 = 0.428$ ). Shear stresses obtained from these influence lines cannot be compared directly because the ones for  $K_1 = 1.00$  may be reduced by the provision of flexural reinforcement [8]. It is for this reason that the resulting shear stresses are compared separately in Tables 4 and 5. In other words, these tables are meant to show the significance of the live load positioning only.

In comparing Cases A through C, the dead-to-live load ratio is important. We assume a feasible range of live loads varying from 50 lb./sq. ft. to 100 lb./sq. ft. Considering the slab thickness given, other factors affecting the dead-to-live load ratio are the type of concrete (lightweight or normal weight) and, due to load factors, the design method used (Ultimate Strength Method or Working Stress Method [12]).

$K_1 = 1.000$ $Q_1 = 1.425$	Ultimate Strength				Working Stress			
	Normal weight		Lightweight		Normal weight		Lightweight	
Live Load:	50	100	50	100	50	100	50	100
Case A	112	159	95	142	69	96	58	84
Case B	171	278	154	261	102	162	91	150
Case C	173	280	155	263	103	163	92	152

Table 4. Shear Stresses (psi) as Function of Live Load Positioning

$K_3 = 0.301$ $Q_3 = 0.428$	Ultimate Strength				Working Stress			
	Normal weight		Lightweight		Normal weight		Lightweight	
Live Load:	50	100	50	100	50	100	50	100
Case A	124	172	104	152	77	104	64	91
Case B	124	173	104	154	78	105	64	92
Case C	130	185	110	165	80	111	67	98

Table 5. Shear Stresses (psi) as Function of Live Load Positioning

The values of Tables 4 and 5 were computed in compliance with standard practice [8, 12] by means of a computer program [13]. They show that it is always the live load position for maximum positive span moments (Case B) which causes maximum shear stresses at the columns. They, furthermore, show that the increase in shear stress due to extending the live load to the zero-point of the influence lines (Case C) is insignificant. It should be mentioned that some building codes [12] call for only 75 percent of the live load to be applied in pattern loading, whereas 100 percent of the live load must be placed on all spans. Under this condition it is possible that the positioning of Case A governs, especially when the spans are short and, therefore, the  $\eta_M$  are small compared to the  $\eta_V$  (Fig. 4). The loading of spans which are not adjacent to the column is insignificant due to the restraining effect of the remote columns.

It should be noted that the experimentally determined K-values [9, Table 8-6] correspond well with the K-values of Table 3, if the critical periphery is at least  $d/2$  away from the face of the column. This is reasonable since the theoretical assumption of a concentrated moment is justified only in view of the theorem of St. Venant, i. e., at some distance away from the point of application. Numerical refinements are, of course, always possible by using series expansions of the applied moment and of the boundary reactions.

The K-values and Q-factors depend on the shape of the critical periphery and, thereby, on the column size. In addition to this primary effect, the column size affects, of course, the shape of the influence lines for V and M. Visualizing the latter one as the deflection curve due to a unit moment applied at the joint, one could expect the ordinates,  $\eta_M$ , to decrease with increasing

column stiffness. In reality, however, these ordinates increase, because of the smaller joint rotation. They would diminish to zero, of course, if the columns had no stiffness at all. Doubling the column stiffness in the example, for which influence lines are shown in Fig. 4, would result in an increase of the ordinates,  $\eta_M$ , by about 30 percent.

Summarizing the above, it can be stated that all of these factors, the assumed shape of the critical periphery, the stiffness ratio between slabs and columns, the slab spans, and the theoretical assumptions of moment transfer have an effect on the shape of the influence lines. As far as the critical live load positioning is concerned, however, Case B (Tables 4 and 5) will usually be the governing one.

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## SUMMARY

An analytical method is developed to determine the stress configuration in flat plates subject to column moments. The results are presented in closed form for values,  $K$ , as a function of the shape of the critical periphery. These values agree well with test results. Influence lines for maximum shear stress are drawn for the bounds of the feasible range of  $K$ -values. Their evaluation shows that live load on alternate spans usually governs. The effect of partial loading of spans, however, is insignificant.

## RÉSUMÉ

On présente une méthode analytique pour déterminer les efforts aux dalles plates produits par les moments aux colonnes. Les résultats sont présentés en formules fermées définissant les valeurs  $K$ , qui sont des fonctions du profil de la périphérie critique. Les valeurs s'accordent avec les résultats expérimentaux. Les lignes d'influence d'efforts tranchants ont été tirées pour des valeurs extrêmes  $K$  à portée de service et évaluées pour des conditions différentes. L'évaluation fait preuve du fait, qu'une charge utile aux portées alternes est décisive, tandis qu'une charge partielle des portées est d'insignifiance.

## ZUSAMMENFASSUNG

Der Aufsatz schlägt eine analytische Methode vor, um die Spannungen in Flachdecken zu ermitteln, welche durch Stützenkopfmomente hervorgerufen werden. Die Ergebnisse sind in geschlossenen Formeln für  $K$  in Abhängigkeit der Bruchform dargestellt, und sie stimmen gut mit vorhandenen Versuchsergebnissen überein. Mit den Extremen der  $K$ -Werte im brauchbaren Bereich sind Einflusslinien gezeichnet und für verschiedene Bedingungen ausgewertet. Sie zeigen, dass abwechselnd feldweise Belastung ausschlaggebend ist, dass aber teilweise Feldbelastung die Ergebnisse kaum beeinflusst.

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