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On the Damping of Vibrations

Amortissement des vibrations

Über die Dämpfung von Schwingungen

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It is well known that classical elasticity offers to the structural engineer many valuable results and some powerful general methods to calculate the characteristic frequencies of his buildings.

But we have to remember that, from the dynamic viewpoint, a world of perfect elasticity would be very unstable and brittle (with materials of finite strength) whilst, fortunately in practice, many resonant frequencies are not at all dangerous, thanks to damping.

Our knowledge on this subject is not as wide as it should be to answer, at least with a practically sufficient approximation, to questions like the following:

what is the maximum alternating stress that a given structure can endure in a definite interval of time?

is it possible to build with materials of higher static strength without losing something as to the capability to withstand dynamic actions?

since the damping coefficients increase with stress, to what extent does abundancy of dimensions really improve dynamic safety?

With a view to bring some contribution towards the solution of such problems, or of many others that naturally arise from them, we started in February 1967 a program of systematic research on the damping of vibrations in the frequency range 1 ÷ 100 hz which is of interest not only for civil engineering (1 ÷ 10 hz) but also for the design of machines (10 ÷ 100 hz).

This program is carried out at the Building Science Institute of Turin Polytechnic School, with the support of the Italian National Council for Research (C.N.R.).

Without any claim to have reached final conclusions, I wish to point out some results, both theoretical and experimental, that appear to be promising or that deserve at least a deeper analysis.

1) Deduction of the Equations of Small quasi-Elastic Oscillations and Discussion on the Relationship between Relaxation Time and Frequency for Beams in Bending.

From a thermodynamic view point, the simplest hypothesis that can be made about the dissipation of energy for unit time and unit volume is the following:

$$\frac{dW}{dt} = g_1 \cdot (\text{rate of change of elastomechanical energy})^2 + g_2 \cdot (\text{local gradient of velocity})^2 \qquad (1)$$

without any "a priori" assumption concerning g_1, g_2 (but, of course, for irreversibility $g_1 > 0$, $g_2 \geq 0$)

It is fundamental to observe: first, the logical simmetry connecting elastic after-work (caused by the variation of elastomechanical energy with respect to time) and internal friction (caused by the variation of kinetic energy in space); second, that to evaluate the local density of energy and its rate of change we have to take into account the <u>static stresses pre-existing to vibratory motion</u>.

If we apply hypothesis (1) to bending of a uniform beam, neglecting shear and rotatory inertia and denoting with

f the elastic deplacement

M_o the pre-existing bending moment,

the principle of conservation of energy expresses the stationary property of the form :

$$\int_0^L [EJ \left(\frac{\partial^2 f}{\partial x^2}\right)^2 + \rho A \left(\frac{\partial f}{\partial t}\right)^2 + \int_0^t g_1 M_o^2 \left(\frac{\partial^3 f}{\partial x^2 \partial t}\right)^2 dt + \int_0^t g_2 \, \rho A \left(\frac{\partial^2 f}{\partial x \partial t}\right)^2 dt] dx \qquad (2)$$

By transformation into a double integral we get a normal problem of the calculus of variations, and if we put:

$g_1 M_o^2 / \rho A = 2a \qquad EJ/ \rho A = b^2 \qquad g_2 = 2c$

the indefinite equation may be written (in the case of constant coefficients):

$$2a \frac{\partial^5 f}{\partial x^4 \partial t} + b^2 \frac{\partial^4 f}{\partial x^4} - 2c \frac{\partial^3 f}{\partial x^2 \partial t} + \frac{\partial^2 f}{\partial t^2} = 0 \qquad (3)$$

For a simply supported beam, of length L, under its own weight only, it is easy to deduce from (3) the relaxation time in the form:

$$\theta = \frac{A}{g_1 M_o^2 \nu^2 + g_2 \frac{B}{L^2}} \qquad (4)$$

or, since $M_o \equiv L^2 \equiv 1/\nu$

$$\theta = \frac{1}{\alpha + \beta \nu} \qquad (5)$$

where A,B α,β are constants.

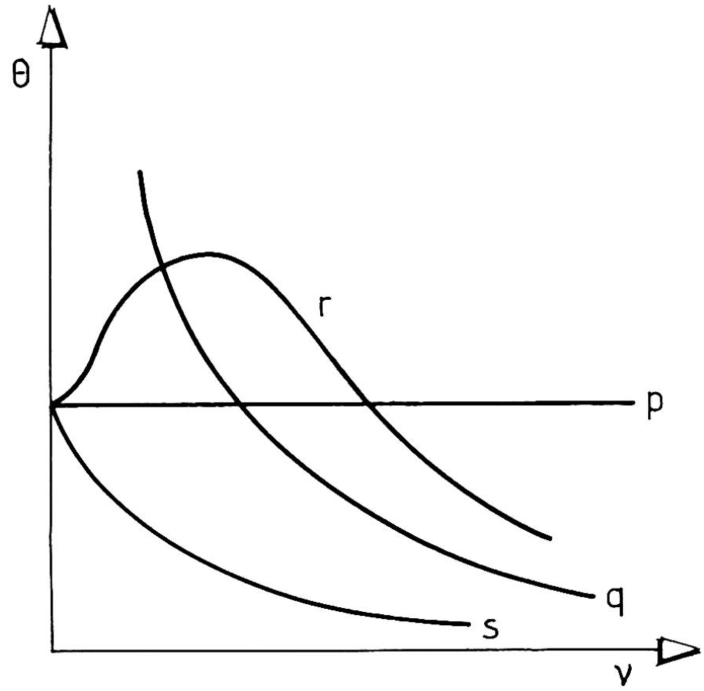

fig.1

In fig. 1 a comparison is made between different theories about the relationship $\theta=\theta\ (\nu)$:

Neglecting internal friction (g_2 = 0) and the influence of pre-existing stresses ($g_1 M_o^2$ = const) we have the curve q that would mean disaster as to antiseismic strength because by $\theta\nu^2$= const damping would be too low for low frequencies.

If we put g_2= 0, g_1 = const we get the line p (in agreement, e.g. with POZZO's observations on concrete ('))

By assuming g_1 = const, g_2 = const, we find the hyperbola s that eliminates the paradox of undamped low frequencies.

However a real curve may look like r (see fig. 2 from BO and LEPORATI's experiments on Burback tracks ('')) and this fact can be explained by considering

2) The Influence of Microstructure

As it is reasonable to suppose that energy be dissipated mainly through weak grains, large enough to have a chance of undergoing plastic deformation, we are interested in knowing the minimum grain size D_0 that may be "activated" in this sense and the fraction F(D) of mass constituted by grains of size D or more ($F(0) = 1$, $F(\infty) = 0$)

D_0 may be given by a formula as $\sigma^2 D_0 =$ const or the like, decreases with stress and increases with frequency, whereas the contrary occurs for $F(D_0)$.

Consequently in (4) g_1, g_2 that increase with $F(D_0)$ will decrease with ν and in (5), with $\frac{d\alpha}{d\nu} < 0, \frac{d\beta}{d\nu} < 0$, it will be possible to have a maximum for θ (ν).

So we come to think that damping depends chiefly on stress, especially at low frequencies when the controlling factor is the number of cycles.

In fig. 2 we see the results of experiments on small oscillations of uniform beams with the same cross section vibrating at the same frequency under different end conditions (''). The amplitude of oscillation A was between 1/25th and 1/50th of the static deflection.

Plotting the relaxation time $\theta = -\frac{1}{2} A/(\frac{dA}{dt})$ against frequency seems to demonstrate that the assumption of seismic coefficients depending only on frequency (as prescribed by several regulations) is an over-simplification too far from reality.

To study the combined effect of stress, frequency and grain-size we have performed many more experiments using I beams HE 100 B UNI (5397-64) on

3) <u>The Damping of Large Oscillations</u>

During each experiment the variation of θ with A, σ has been quite evident.

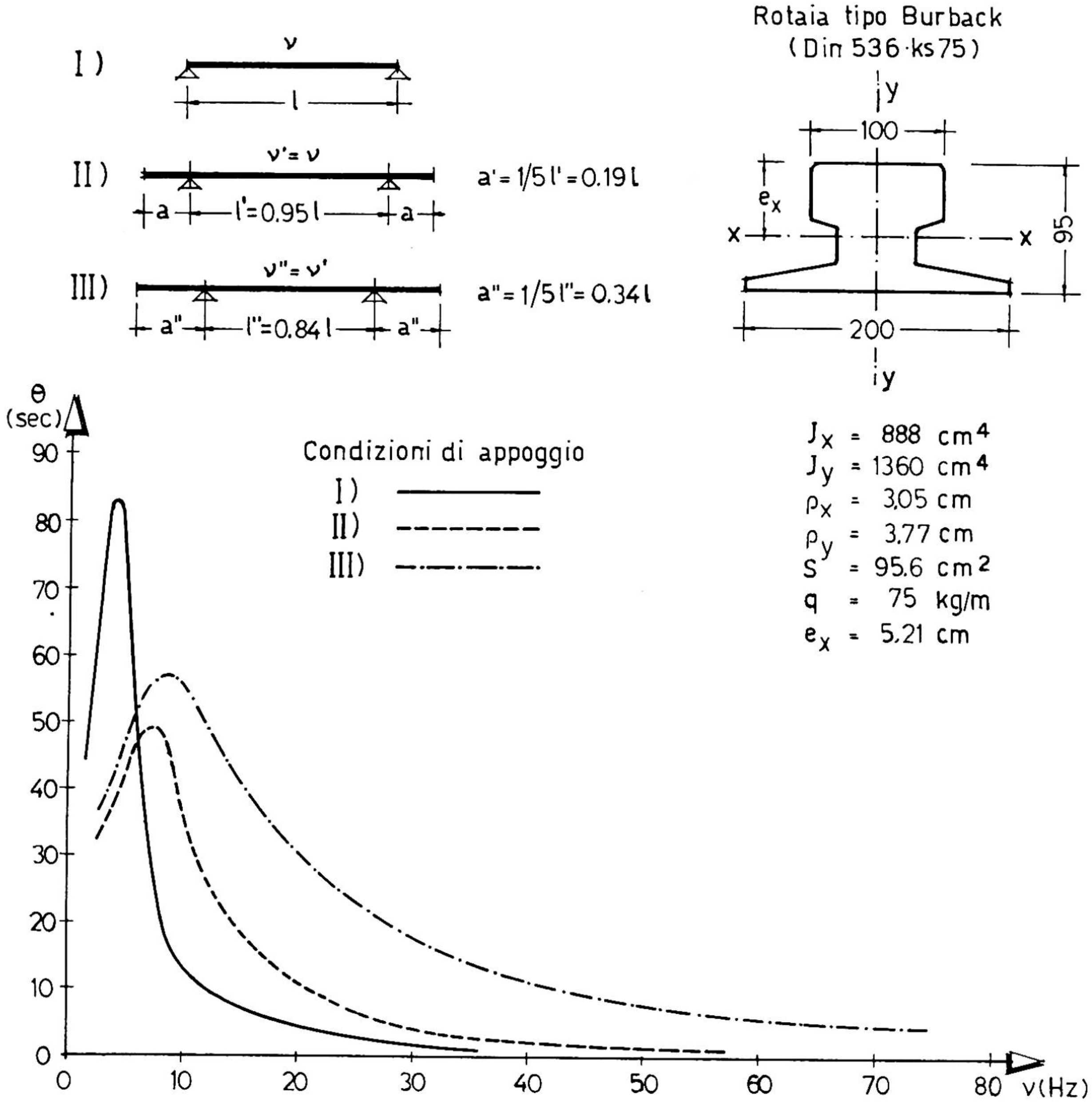
Rotaia tipo Burback
(Din 536·ks75)
I)
II)
III)
a' = 1/5 l' = 0.19 l
a'' = 1/5 l'' = 0.34 l
l' = 0.95 l
l'' = 0.84 l
100
200
95
e_x
J_x = 888 cm⁴
J_y = 1360 cm⁴
ρ_x = 3.05 cm
ρ_y = 3.77 cm
S = 95.6 cm²
q = 75 kg/m
e_x = 5.21 cm
θ (sec)
Condizioni di appoggio
ν (Hz)

fig. 2

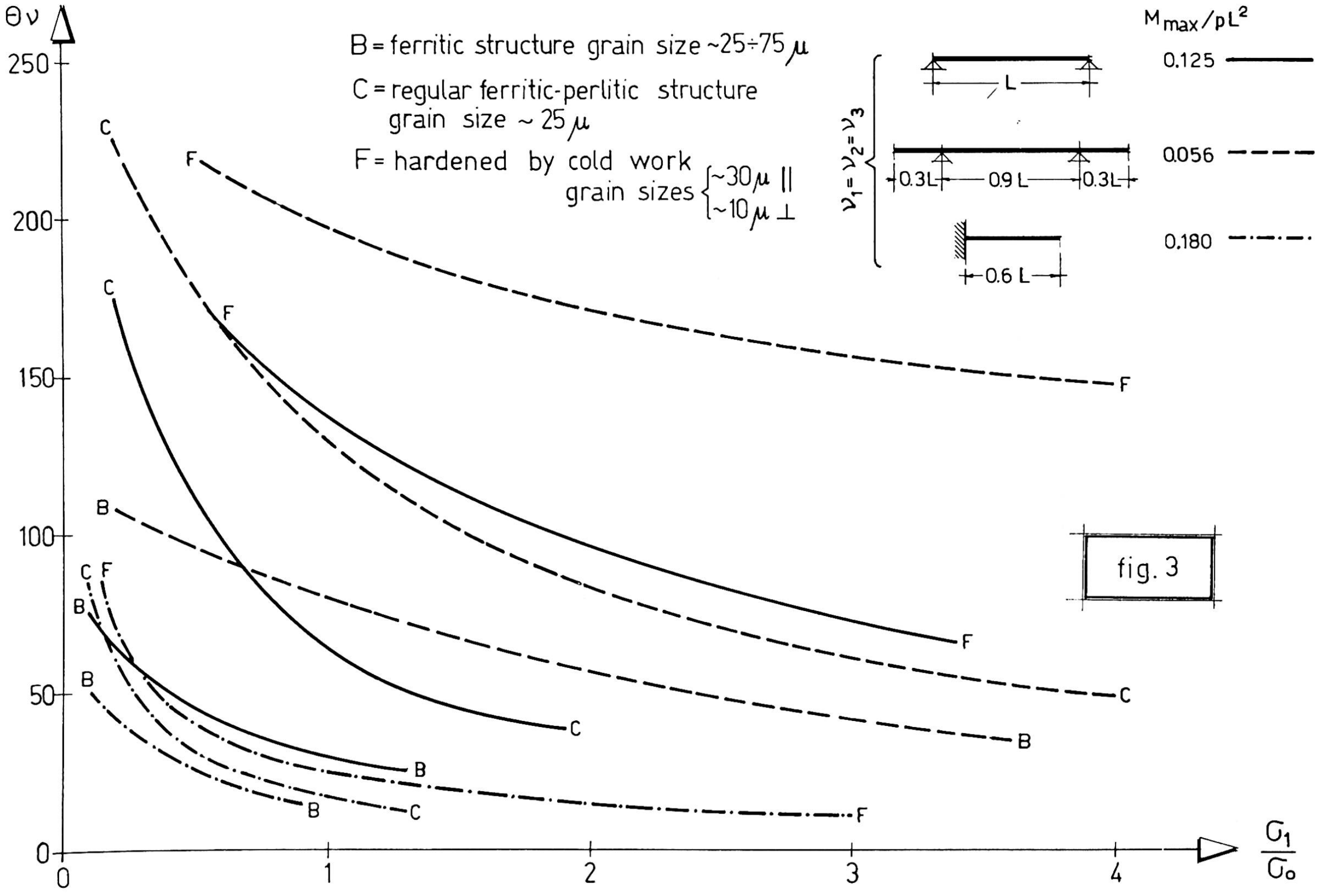
Θν
250
200
150
100
50
0
0
1
2
3
4
σ1/σ0
B = ferritic structure grain size ~25÷75 μ
C = regular ferritic-perlitic structure grain size ~ 25 μ
F = hardened by cold work grain sizes {~30μ ∥ ~10μ ⊥
ν1 = ν2 = ν3
L
0.3L
0.9 L
0.3L
0.6 L
M max/pL²
0.125
0.056
0.180
B
C
F

fig. 3

The results appear in fig. 3 where two dimensionless parameters have been introduced: $\theta\,\nu$ (a number of cycles) and

$$\frac{\sigma_1}{\sigma_0} = \frac{\text{dynamic amplitude of stress oscillation}}{\text{pre-existing static stress}}$$

$(\sigma = \sigma_0 + \sigma_1 \sin 2\pi\nu\, t)$

Here again three different geometrical conditions corresponding to nearly equal frequencies have been considered. The maximum bending moments are roughly as 1:2:3.

A micrography of every specimen has been kindly prepared and analyzed by prof. BURDESE.

We observe that

a) a hyperbolic law fits well enough with the experimental curves

$$\theta\,\nu = \frac{1}{A + B(\sigma_1/\sigma_0)} \qquad (6)$$

So far we are in agreement with (1): the rate of dissipation of energy is proportional to $\sigma_1 . \sigma_0 / \theta$ and must be equalized to the terms of the second member that necessarily comprehend a velocity factor $\sigma_1 \nu$ multiplied by a function of σ_0, σ_1 which, to a first approximation, can be expressed as $A\sigma_0 + B\sigma_1$

b) the constants of hyperbolae depend on material and end conditions:
for a given material, the structures under heavier stress damp out vibrations more rapidly; for a given geometry of structure damping is quicker in materials with coarse and larger grains.

Normalized damping tests may be useful in quality control.

Finally I wish to draw attention to the influence of dissimetry in the excitation of vibrations and emphasize the reasons that cause large variations of the damping coefficients in the general problem.

Let us consider

4) <u>The Characteristic Equation</u>

$$b^2 \xi^4 + 2 (a \xi^2 - c) \xi^2 z + z^2 = 0 \qquad (7)$$

connecting the exponents of the elementary solution $\exp (\xi x + z t)$ of eq. (3)

If we put:

$$\xi = u + i v = r e^{i\varphi} = \frac{1}{R} e^{-i\varphi}$$

$$z = - \chi + i\omega$$

and consider (7) as an equation of the 2nd degree in z ,we get, with the assumption, surely acceptable for steel, of dissipation so modest as to influence frequency but little:

$$z = \xi^2 (c - a \xi^2 \pm ib)$$

Writing:

$$c \pm ib = \gamma e^{i\epsilon} \qquad a = \lambda \gamma$$

and using the inversion to represent long waves at increasing distances from the origin, we obtain:

$$\begin{aligned} - \chi &= \frac{\gamma}{R^2} [\cos (2 \varphi+\epsilon) - \frac{\lambda}{R^2} \cos 4 \varphi] \\ \omega &= \frac{\gamma}{R^2} [\sin (2 \varphi+\epsilon) - \frac{\lambda}{R^2} \sin 4 \varphi] \end{aligned} \qquad (8)$$

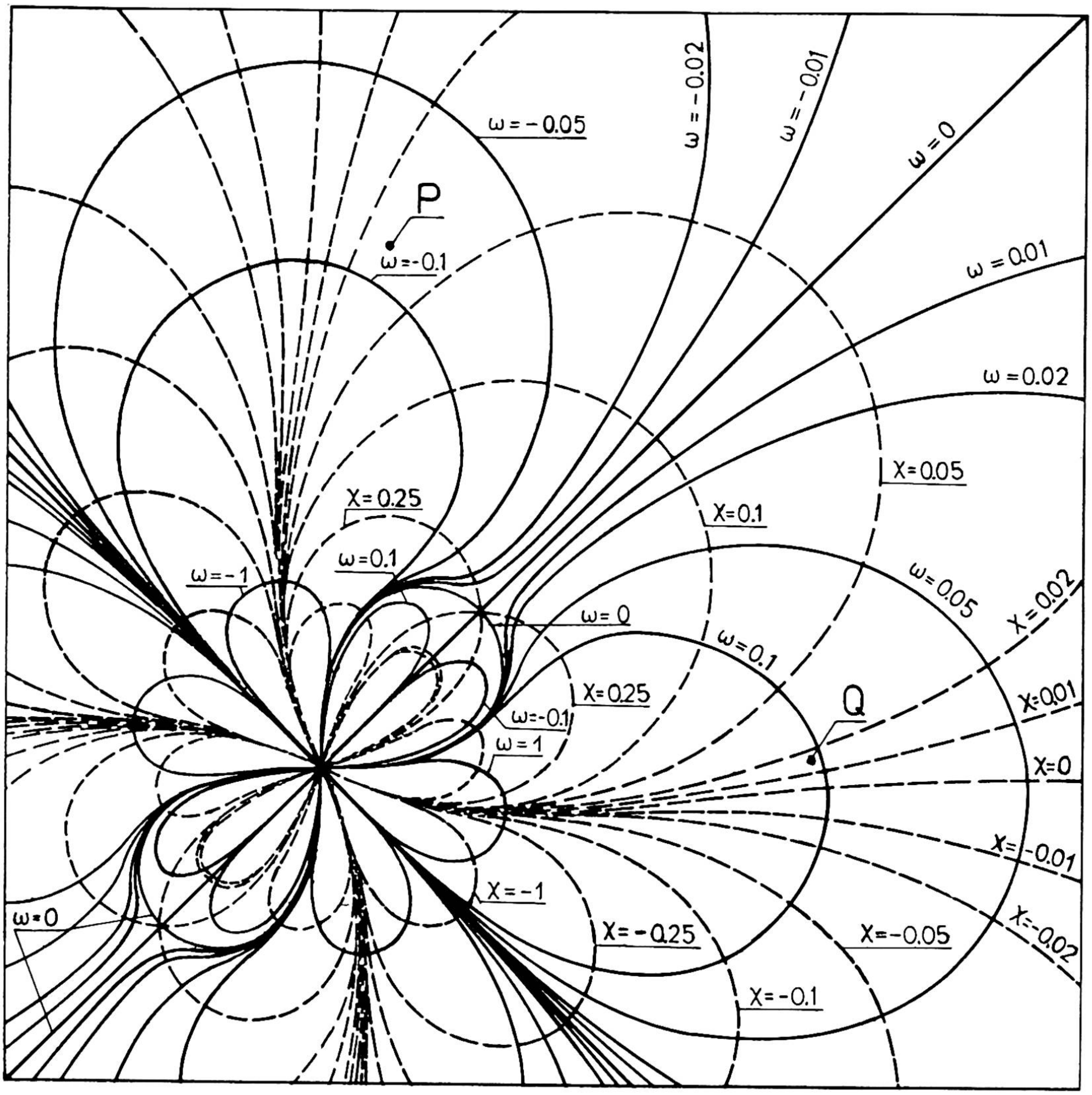

fig.4

In fig. 4 the curves of constant damping χ and their orthogonal trajectories of constant pulsation ω are traced in the particular case $\gamma = \lambda = 1, \varepsilon = \pi/2$

So, in Gauss' plane, we have an immediate representation of the correspondence between the dissimetrical attenuation of waves in space and time respectively.

It may be seen at once that near points like P, representing long slightly unsimmetrical waves, the same value of χ corresponds to very different frequencies; whilst near Q a minor change in the wave form causes different damping for equal frequencies.

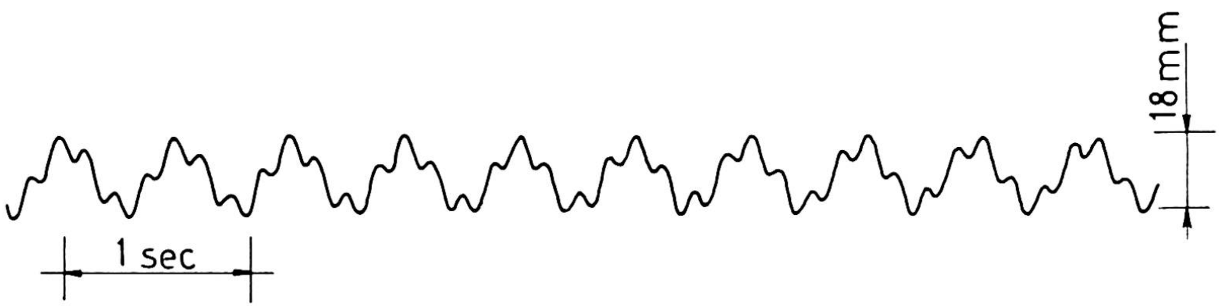

fig.5

The almost undistorted permanence of complicated wave forms, like the one of Fig. 5 observed (") during more than 5 minutes of free oscillations of a Burback track, may be perhaps better understood through these considerations rather than by associating definite damping coefficients to simple harmonics.

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SUMMARY

Elastic systems with the same frequency may have different relaxation times. Materials under heavy stresses and coarse-grained materials damp out vibrations more quickly and so show a greater reserve of dynamical strength. In propagation phaenomena the damping factor may be very sensitive to comparatively small changes in the shape of waves.

RÉSUMÉ

Des systèmes élastiques de la même fréquence peuvent avoir des temps d'amortissement très différents. Les matériaux plus sollicités et ceux à gros grains amortissent les vibrations plus rapidement et montrent ainsi une plus grande réserve de résistance dynamique. Dans les phénomènes de propagation le facteur d'amortissement peut être très sensible à des modifications relativement petites de la forme des ondes.

ZUSAMMENFASSUNG

Elastische Systeme mit gleicher Frequenz können sehr verschiedene Relaxationszeiten zeigen.
Stark belastete sowie grobkörnige Materialien dämpfen die Schwingungen schneller und zeigen eine grössere dynamische Widerstandsreserve. In Verbreitungsphänomena kann der Dämpfungsfaktor sehr empfindlich auf relativ geringe Veränderungen der Wellenform sein.