Non-stationary vibrations of bridges under random moving load

Non-Stationary Vibrations of Bridges Under Random Moving Load

Vibrations non-stationnaires de ponts sous une charge en mouvement arbitraire

Nichtstationäre Brückenschwingungen unter zufälliger, beweglicher Last

LADISLAV FRÝBA
Doc. Ing.Dr.Sc.
Head Research Scientist
Research Institute of Transport
Prague, Czechoslovakia

1. Introduction

It has been assumed up to this time that the traffic loading of bridges, i.e. the static and dynamic component of the service load, is a well known function of the space and time coordinate (a deterministic process), see [1], [2]. This paper deals with the essentially opposite case supposing that the traffic loading of bridges is a random process. This new conception is in better accordance with observations because the true traffic loading is influenced by the random composition of the traffic flow, by the random initial conditions when the vehicles enter the bridge, by the irregularities of unevenness of the road surface etc.

In general the static and dynamic deflection of bridges is described by the linear differential equation

$$L[v(x,t)] = p(x,t) \quad (1)$$

where $v(x,t)$ denotes the deflection and $p(x,t)$ the load. The random variation of $p(x,t)$ is assumed not only with respect to the time coordinate t but also to the position coordinate x and in addition the load $p(x,t)$ is regarded as a nonstationary Gaussian random process of non-Markov type.

L represents a linear differential operator of the type

$$L = L_o + \mu \frac{\partial^2}{\partial t^2} + 2\mu\,\omega_b \frac{\partial}{\partial t} \quad (2)$$

where L_o is a self-adjoint linear operator in the space coordinate x, μ - mass per unit length and ω_b - circular frequency of viscous damping.

2. Probability Analysis

2.1. Normal - Mode Analysis. Elastic systems described by Eqs. (1) and (2) are with advantage solved by means of the normal-mode analysis

$$v(x,t) = \sum_{j=1}^{\infty} v_{(j)}(x)\, q_{(j)}(t) \tag{3}$$

$$p(x,t) = \sum_{j=1}^{\infty} \mu\, v_{(j)}(x)\, Q_{(j)}(t) \tag{4}$$

where $v_{(j)}(x)$ are the normal modes of vibration that are obtained with regard to the boundary conditions from the equation

$$L_o\left[v_{(j)}(x)\right] = \mu\, \omega^2_{(j)} v_{(j)}(x) \quad , \tag{5}$$

$\omega_{(j)}$ is the natural circular frequency of the system,

$$Q_{(j)}(t) = \frac{1}{V_{(j)}} \int_0^{\ell} p(x,t)\, v_{(j)}(x)\, dx \tag{6}$$

is the generalized force,

$$V_{(j)} = \int_0^{\ell} \mu\, v^2_{(j)}(x)\, dx \; , \quad \int_0^{\ell} \mu\, v_{(j)}(x) v_{(k)}(x)\, dx = 0 \text{ for } j \neq k \tag{7}$$

and $q_{(j)}(t)$ is the generalized deflection that is obtained with regard to the initial conditions from the equation

$$\ddot{q}_{(j)}(t) + 2\,\omega_b\, \dot{q}_{(j)}(t) + \omega^2_{(j)}\, q_{(j)}(t) = Q_{(j)}(t) \tag{8}$$

The solution of Eq.(8) with zero initial conditions is

$$q_{(j)}(t) = \int_0^t h_{(j)}(t-\tau) Q_{(j)}(\tau)\, d\tau = \int_{-\infty}^{\infty} h_{(j)}(\tau) Q_{(j)}(t-\tau)\, d\tau \tag{9}$$

where $h_{(j)}(t)$ denotes the impulsive function

$$h_{(j)}(t) = \begin{cases} \dfrac{1}{\omega'_{(j)}}\, e^{-\omega_b t} \sin \omega'_{(j)} t & \text{for } t \geqq 0 \\ 0 & \text{for } t < 0 \end{cases} \tag{10}$$

and $\omega'^2_{(j)} = \omega^2_{(j)} - \omega^2_b$. The limits of integration in (9) may be extended to ∞ and $-\infty$, respectively, because $Q_{(j)}(t-\tau) = 0$ for $\tau > t$ and $h_{(j)}(\tau) = 0$ for $\tau < 0$, respectively.

The functions $h_{(j)}(t)$ and $v_{(j)}(x)$ are deterministic while

$q_{(j)}(t)$, $Q_{(j)}(t)$, $v(x,t)$ and $p(x,t)$ are random ones.

2.2. Correlation Analysis. The probability analysis recquires to know the statistic characterisitcs of the input

$$p(x,t) = E\left[p(x,t)\right] + \overset{o}{p}(x,t) \qquad (11)$$

$$K_{pp}(x_1,x_2,t_1,t_2) = E\left[\overset{o}{p}(x_1,t_1)\ \overset{o}{p}(x_2,t_2)\right] \qquad (12)$$

where E represents the mean value linear operator, $\overset{o}{p}(x,t)$ - the centred value of the load and $K_{pp}(x_1,x_2,t_1,t_2)$ - the covariance of the nonstationary function $p(x,t)$.

As follows from the definition of the covariance (12) the covariance of the generalized deflection may be evaluated from (9)

$$K_{q_{(j)}q_{(k)}}(t_1,t_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_{(j)}(\tau_1)h_{(k)}(\tau_2)K_{Q_{(j)}Q_{(k)}}(t_1-\tau_1,t_2-\tau_2)\ .\ d\tau_1\, d\tau_2 \quad , \qquad (13)$$

the covariance of the deflection from (3)

$$K_{vv}(x_1,x_2,t_1,t_2) = \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} v_{(j)}(x_1)v_{(k)}(x_2)K_{q_{(j)}q_{(k)}}(t_1,t_2) \qquad (14)$$

and the covariance of the load from (4)

$$K_{pp}(x_1,x_2,t_1,t_2) = \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} \mu^2 v_{(j)}(x_1)v_{(k)}(x_2)K_{Q_{(j)}Q_{(k)}}(t_1,t_2) \qquad (15)$$

In Eqs. (13) and (15) the covariance of the generalized force is calculated from (6)

$$K_{Q_{(j)}Q_{(k)}}(t_1,t_2) = \frac{1}{V_{(j)}V_{(k)}}\int_0^{\ell}\int_0^{\ell} v_{(j)}(x_1)v_{(k)}(x_2)K_{pp}(x_1,x_2,t_1,t_2)\ .\ dx_1\, dx_2 \qquad (16)$$

2.3. Spectral Density Analysis. The spectral density of a nonstationary function is defined in [3] and for the generalized deflection the Wiener-Khinchine relations between the spectral density and the covariance are as follows

$$S_{q_{(j)}\,q_{(k)}}(\omega_1,\omega_2) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} K_{q_{(j)}\,q_{(k)}}(t_1,t_2)e^{-i(\omega_2 t_2-\omega_1 t_1)}dt_1 dt_2 \qquad (17)$$

$$K_{q_{(j)}q_{(k)}}(t_1,t_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_{q_{(j)}q_{(k)}}(\omega_1,\omega_2)e^{i(\omega_2 t_2-\omega_1 t_1)}d\omega_1\, d\omega_2 \qquad (18)$$

For the spectral density analysis it is also convenient to introduce the transfer function

$$H_{(j)}(\omega) = \int_{-\infty}^{\infty} h_{(j)}(t)\, e^{-i\omega t} dt = \frac{1}{\omega_{(j)}^2 - \omega^2 + 2i\omega_b\omega} \qquad (19)$$

as a Fourier integral transformation of $h_{(j)}(t)$ given by (10).

Then the spectral density of the generalized deflection may be evaluated as a function of the spectral density of the generalized force, see [4] :

$$S_{q_{(j)}q_{(k)}}(\omega_1,\omega_2) = \bar{H}_{(j)}(\omega_1)\, H_{(k)}(\omega_2)\, S_{Q_{(j)}Q_{(k)}}(\omega_1,\omega_2) \qquad (20)$$

where $\bar{H}_{(j)}(\omega)$ is a complex conjugate function of $H_{(j)}(\omega)$.

Here we used the spectral density $S_{Q_{(j)}Q_{(k)}}(\omega_1,\omega_2)$ of the generalized force defined similarly as in (17); this can be adapted with regard to the Eq. (16)

$$S_{Q_{(j)}Q_{(k)}}(\omega_1,\omega_2) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} K_{Q_{(j)}Q_{(k)}}(t_1,t_2)\, e^{-i(\omega_2 t_2 - \omega_1 t_1)} dt_1 dt_2 =$$

$$= \frac{1}{V_{(j)}V_{(k)}} \int_0^{\ell}\int_0^{\ell} v_{(j)}(x_1) v_{(k)}(x_2) S_{pp}(x_1,x_2,\omega_1,\omega_2) dx_1 dx_2 \qquad (21)$$

$$K_{Q_{(j)}Q_{(k)}}(t_1,t_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_{Q_{(j)}Q_{(k)}}(\omega_1,\omega_2)\, e^{i(\omega_2 t_2 - \omega_1 t_1)} d\omega_1 d\omega_2 \qquad (22)$$

The spectral density of the deflection is then with respect to (14)

$$S_{vv}(x_1,x_2,\omega_1,\omega_2) = \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} v_{(j)}(x_1) v_{(k)}(x_2) S_{q_{(j)}q_{(k)}}(\omega_1,\omega_2) \qquad (23)$$

from which the covariance of the deflection can be calculated similarly as in (18) and (14).

3. Random Moving Load

3.1. Random Moving Force. As an example we shall solve a simple beam of span ℓ loaded ba a massless force $P(t) = P + \overset{o}{P}(t)$ with constant mean value $E\left[P(t)\right] = P$ which is moving with constant velocity c along the beam. The analogous deterministic case was solved in

[2] and [4] and it represents the mean value of the present solution $E[v(x,t)]$, so that we will now investigate the stochastic case only.

The load per unit length and its mean and centred values are in our case

$$p(x,t) = \delta(x-ct)P(t) \;, \quad E[p(x,t)] = \delta(x-ct)P \;, \quad \overset{o}{p}(x,t) = \delta(x-ct)\overset{o}{P}(t) \tag{24}$$

where $\delta(x)$ represents the Dirac-delta function. The covariance of the load can be calculated from (12)

$$K_{pp}(x_1,x_2,t_1,t_2) = \delta(x_1-ct_1)\,\delta(x_2-ct_2)\,K_{PP}(t_1,t_2) \tag{25}$$

where $K_{PP}(t_1,t_2)$ is the known covariance of the load $P(t)$. We substitute (25) into (16) and then, with regard to the well known properties of the Dirac function, we obtain the covariance of the generalized force

$$K_{Q_{(j)}Q_{(k)}}(t_1,t_2) = \frac{1}{V_{(j)}V_{(k)}}\, v_{(j)}(ct_1)v_{(k)}(ct_2)K_{PP}(t_1,t_2) \tag{26}$$

Using (26) the covariances of the deflection can be calculated from (13) and (14).

As an example let us assume the covariance of the force $P(t)$ in the form

$$K_{PP}(t_1,t_2) = K_{PP}(t_2-t_1) = 2\pi S_P\,\delta(t_2-t_1) \tag{27}$$

where S_P is the constant spectral density (white noise). Then we obtain from the Eq. (13)

$$K_{q_{(j)}q_{(k)}}(t_1,t_2) = \frac{1}{V_{(j)}V_{(k)}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_{(j)}(\tau_1)h_{(k)}(\tau_2)v_{(j)}[c(t_1-\tau_1)]\,.\, v_{(k)}[c(t_2-\tau_2)]\,K_{PP}(t_1-\tau_1,t_2-\tau_2)\,d\tau_1\,d\tau_2 =$$

$$= \frac{2\pi S_P}{V_{(j)}V_{(k)}}\int_{-\infty}^{\infty} h_{(j)}(\tau_1)h_{(k)}(\tau_1+t_2-t_1)v_{(j)}[c(t_1-\tau_1)]\,.\, v_{(k)}[c(t_1-\tau_1)]\,d\tau_1 \tag{28}$$

If for simplification we neglect the cross-correlation of the generalized deflection, i.e. $K_{q_{(j)}q_{(k)}}(t_1,t_2) = 0$ for $j \neq k$, the variance of the deflection can be received from (14) :

$$\sigma_v^2(x,t) = K_{vv}(x,x,t,t) = \sum_{j=1}^{\infty} v_{(j)}^2(x) K_{q_{(j)}q_{(j)}}(t,t) =$$

$$= \sum_{j=1}^{\infty} \frac{2\pi S_P}{V_{(j)}^2} v_{(j)}^2(x) \int_{-\infty}^{\infty} h_{(j)}^2(\tau_1)\, v_{(j)}^2[c(t-\tau_1)]\, d\tau_1 \qquad (29)$$

The following expressions hold true for a simple beam of span ℓ and of bending stiffness EJ, see [4]

$$v_{(j)}(x) = \sin\frac{j\pi x}{\ell}, \quad V_{(j)} = \frac{\mu\ell}{2}, \quad \omega_{(j)}^2 = \frac{j^4\pi^4}{\ell^4}\frac{EJ}{\mu} \qquad (30)$$

Substituting (30) and (10) into (29) we obtain (note that the limits of integration may be changed as $h_{(j)}(\tau_1) = 0$ for $\tau_1 < 0$ and $v_{(j)}[c(t-\tau_1)] = 0$ for $\tau_1 > t$) :

$$\sigma_v^2(x,t) = \sum_{j=1}^{\infty} \frac{8\pi S_P}{\mu^2\ell^2\omega_{(j)}'^2} \sin^2\frac{j\pi x}{\ell} \int_0^t \left[e^{-\omega_b\tau_1}\sin\omega_{(j)}'\tau_1 \sin\frac{j\pi c}{\ell}(t-\tau_1)\right]^2 d\tau_1 =$$

$$= \sum_{j=1}^{\infty}\frac{8\pi S_P}{\mu\ell\omega_{(j)}'^2}\sin^2\frac{j\pi x}{\ell}\,\frac{1}{16}\left\{\frac{\omega_{(j)}'+j\pi c/\ell}{(\omega_{(j)}'+j\pi c/\ell)^2+\omega_b^2}\left[\sin 2j\pi ct/\ell + e^{-2\omega_b t}\cdot\right.\right.$$

$$\left.\cdot \sin 2\omega_{(j)}'t + \frac{\omega_b}{\omega_{(j)}'+j\pi c/\ell}(\cos 2j\pi ct/\ell - e^{-2\omega_b t}\cos 2\omega_{(j)}'t)\right] +$$

$$+\frac{\omega_{(j)}'-j\pi c/\ell}{(\omega_{(j)}'-j\pi c/\ell)^2+\omega_b^2}\left[-\sin 2j\pi ct/\ell + e^{-2\omega_b t}\sin 2\omega_{(j)}'t + \frac{\omega_b}{\omega_{(j)}'-j\pi c/\ell}\cdot\right.$$

$$\left.\cdot(\cos 2j\pi ct/\ell - e^{-2\omega_b t}\cos 2\omega_{(j)}'t)\right] - \frac{2\,\omega_b}{j^2\pi^2c^2/\ell^2+\omega_b^2}\left(\frac{j\pi c/\ell}{\omega_b}\sin 2j\pi ct/\ell +\right.$$

$$\left.+\cos 2j\pi ct/\ell - e^{-2\omega_b t}\right) - \frac{2\omega_b}{\omega_{(j)}^2}\left[1 - e^{-2\omega_b t}(\cos 2\omega_{(j)}'t - \frac{\omega_{(j)}'}{\omega_b}\cdot\right.$$

$$\left.\left.\cdot\sin 2\omega_{(j)}'t)\right] + \frac{2}{\omega_b}(1 - e^{-2\omega_b t})\right\} \qquad (31)$$

As the variance of the deflection is a function of the time the resulting vibration of the beam appears as a nonstationary process although we have taken into account the movement of a stationary random force.

For subcritical velocities ($c < c_{cr}$) the greatest static and dynamic effects of a moving force appear approximately in the moment when the force crosses the beam centre. Therefore the coefficient of variance, defined as $C_v(x,t) = \sigma_v(x,t)/E[v(x,t)]$, is to be calculated for $x = \ell/2$ and $t = T/2 = \ell/(2c)$. It represents then the relative dynamic increment of the deflection effected by the random moving force and it takes the following form (from (31) for j=1 approximately, see [4]) :

$$C_v(\ell/2, T/2) = C_P \cdot C_{vP} \tag{32}$$

Here C_P is an analogous coefficient of variance of the force P(t) and C_{vP} is represented graphically in Fig. 1 as a function of the parameters α and β where α is a velocity parameter and β a damping parameter, respectively :

$$\alpha = c/c_{cr} \quad ; \qquad c_{cr} = (\pi/\ell)(EJ/\mu)^{1/2} \tag{33}$$

$$\beta = \omega_b \, / \, \omega_{(1)} \tag{34}$$

The same results can be obtained using the spectral density analysis from the section 2.3. In this case the load (24) must be taken for a function of the time only, see [4].

3.2. Random Moving Distributed Load. As a next example we shall solve a simple beam loaded by an infinitely long random strip which is moving with constant velocity c along the beam. The analogous deterministic case was solved in [4] where not only the movement of the continous load p (measured per unit length) but also the effects of its inertia mass $\mu_p = p/g$ were taken into account.

The load is assumed to have the following form

$$p(x,t) = p(x-ct)\, r(t) \tag{35}$$

The first of the components p(x-ct) is a random variable in the moving coordinate system $\xi = x-ct$ while the second r(t) is a random function of time. The mean values of these two functions are assumed to be constant

$$E[p(\xi)] = p \quad , \qquad E[r(t)] = 1$$

so that the load (35) may be written as

$$p(x,t) = p + \overset{o}{p}(x,t) = [p + \overset{o}{p}(\xi)] \cdot [1 + \overset{o}{r}(t)] \tag{36}$$

where $\overset{o}{p}(x,t) = \overset{o}{p}(\xi) + p\overset{o}{r}(t) + \overset{o}{p}(\xi)\overset{o}{r}(t)$. Then with respect to (12) the covariance of the load is

$$K_{pp}(x_1,x_2,t_1,t_2) = K_{pp}(\xi_1,\xi_2) + p\,K_{pr}(\xi_2,t_1) + K_{ppr}(\xi_1,\xi_2,t_1) +$$

$$+ pK_{pr}(\xi_1,t_2) + p^2K_{rr}(t_1,t_2) + p\,K_{prr}(\xi_1,t_1,t_2) + K_{ppr}(\xi_1,\xi_2,t_2) +$$

$$+ p\,K_{prr}(\xi_2,t_1,t_2) + K_{pprr}(\xi_1,\xi_2,t_1,t_2) \tag{37}$$

where $\xi_i = x_i - ct_i$, $i = 1, 2$. Let us assume approximately that the functions $p(\xi)$ and $r(t)$ have no cross-correlation of the second up to the fourth order; then (37) reduces to

$$K_{pp}(x_1,x_2,t_1,t_2) = K_{pp}(\xi_1,\xi_2) + p^2\,K_{rr}(t_1,t_2) \tag{38}$$

where $K_{pp}(\xi_1,\xi_2)$ is the covariance of the load function in the moving coordinate system ξ and $K_{rr}(t_1,t_2)$ is the covariance in the time coordinate.

As an example let us assume the covariances of these functions in the following form

$$K_{pp}(\xi_1,\xi_2) = 2\pi S_p\,\delta(\xi_2-\xi_1)\ , \qquad K_{rr}(t_1,t_2) = 2\pi S_r\,\delta(t_2-t_1) \tag{39}$$

where S_p and S_r are the constant spectral densities (wide-band spectra). Putting (38) and (39) into (13) the covariance of the generalized deflection may be evaluated; hence

$$K_{q_{(j)}q_{(k)}}(t_1,t_2) = \frac{1}{V_{(j)}V_{(k)}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\ell}\int_0^{\ell} h_{(j)}(\tau_1)h_{(k)}(\tau_2)v_{(j)}(x_1)v_{(k)}(x_2)\,\cdot$$

$$\cdot\left\{2\pi S_p\,\delta[x_2-x_1-c(t_2-\tau_2-t_1+\tau_1)] + 2\pi S_r p^2\,\delta(t_2-t_1-\tau_2+\tau_1)\right\}dx_1dx_2d\tau_1d\tau_2 \tag{40}$$

The limits of integration with respect to time τ are considered from 0 to ∞ in accordance with (10) and because the movement of the load has an infinitely long duration.

Neglecting the cross-correlation $K_{q_{(j)}q_{(k)}}(t_1,t_2) = 0$ for $j \neq k$ the variance may be calculated from (40) and (14) :

$$\sigma_v^2(x,t) = K_{vv}(x,x,t,t) = \sum_{j=1}^{\infty} v_{(j)}^2(x)\,K_{q_{(j)}q_{(j)}}(t,t) =$$

$$= \sum_{j=1}^{\infty}\frac{4\pi S_p c}{\mu^2}\,\frac{1}{\ell^{2(1+\varkappa)}\,\bar\omega_{(j)}^2\bar\omega'_{(j)}\bar\omega_b D}\,\sin^2\frac{j\pi x}{\ell}\left\{\frac{\bar\omega_{(j)}^2\bar\omega'_{(j)}\bar\omega_b\ell}{c} + \frac{j^2\pi^2c^2/\ell^2}{D}\left\{\bar\omega'_{(j)}\left[D\,+\right.\right.\right.$$

$$\left.+\,4\,\bar\omega_b^2(\bar\omega_b^2 - 3\,\bar\omega_{(j)}^2 + j^2\pi^2c^2/\ell^2)\right]\left(1 - e^{-\bar\omega_b\ell/c}\cos\bar\omega'_{(j)}\ell/c.\cos j\pi\right) -$$

$$- \bar{\omega}_b \left[D + 4\, \bar{\omega}'^2_{(j)} (\bar{\omega}'^2_{(j)} - 3\bar{\omega}^2_b - j^2\pi^2 c^2/\ell^2) \right] e^{-\bar{\omega}_b \ell/c} \sin \bar{\omega}'_{(j)} \ell/c . \cos j\pi \Big\}\Big\} +$$

$$+ \sum_{j=1}^{\infty} \frac{4 S_r p^2 (1 - \cos j\pi)}{\mu^2 j^2 \pi^2 \bar{\omega}^2_{(j)} \bar{\omega}_b (1+\varkappa)^2} \sin^2 \frac{j\pi x}{\ell} \qquad (41)$$

where $\bar{\omega}^2_{(j)} = \omega^2_{(j)} (1 - \alpha^2 \varkappa / j^2)/(1 + \varkappa)$, $\bar{\omega}'^2_{(j)} = \bar{\omega}^2_{(j)} - \bar{\omega}^2_b$

$\bar{\omega}_b = \omega_b /(1 + \varkappa)$, $D = (\bar{\omega}^2_{(j)} - j^2\pi^2 c^2/\ell^2)^2 + 4 j^4 \bar{\omega}^2_b \pi^2 c^2/\ell^2$

$$\varkappa = \mu_p / \mu \qquad (42)$$

The result (41) does not depend on time so that the vibration of the beam is a random process stationary in time. The coefficient of variance for the centre of the beam can be approximately brought to the following form, see [4]

$$C_v(\ell/2,t) = \sigma_v(\ell/2,t)/E[v(\ell/2,t)] = C_p\, C_{vp} + C_r \cdot C_{vr} \qquad (43)$$

where C_p and C_r are the analogous coefficients of variance of functions $p(\xi)$ and $r(t)$ respectively and the expressions C_{vp} and C_{vr} are represented in Figs. 2 and 3 as functions of α, β and mass parameter $\varkappa$, see (33), (34) and (42).

4. Application of the Theory and Experimental Results

The theory presented above can be applied to bridge structures assuming that their moving load is a random function. The solution is shown for two typical cases which concern (a) short span bridges and (b) long span bridges.

(a) The load of short span bridges or short longitudinal beams is idealized by a concentrated force of random time variation moving along a beam. Structures of this type are usually loaded by one axle of the vehicle only.

(b) The load of long span bridges is idealized by an infinitely long random strip (35). The first component of this load $p(\xi)$ expresses the random distribution of the static load in the bridge span direction while the second component $r(t)$ interprets the true dynamic effect of the load. The large span bridges are usually loaded by a series of axles resulting either from continous highway traffic or from a railway train whose length is supposed to be much longer than the span of the bridge.

In reality the traffic loading is - generally speaking - an unknown random process. Therefore a solution was given also for the problem inverse to that given in the present paper (see [5]. The probability analysis [5] starts with the known statistic characteristics of the response v(x,t) giving the input characteristics for p(x,t) as a result. The statistic characteristics of any particular bridge (i.e. the beam deflections or stresses in some points) can be measured without difficulties under service conditions and on this basis the load characteristics can be evaluated.

As an example the Fig. 4 shows a covariance function measured on a railway bridge.

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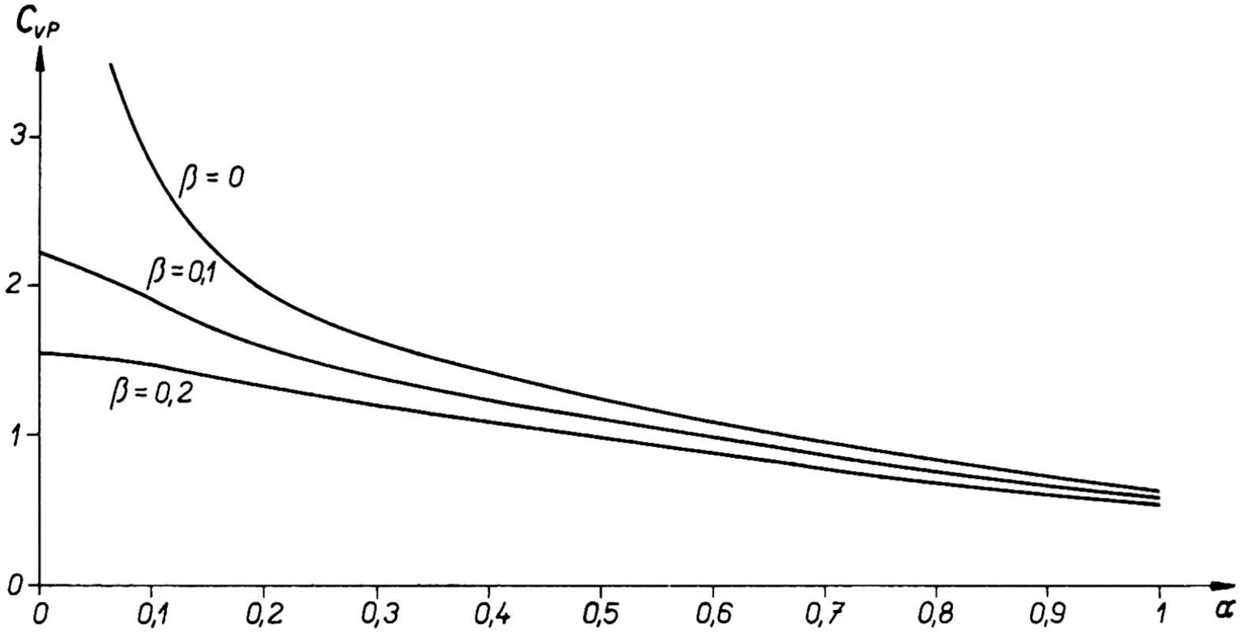

Fig.1. The coefficient C_{vP} as a function of the velocity parameter α and damping parameter β

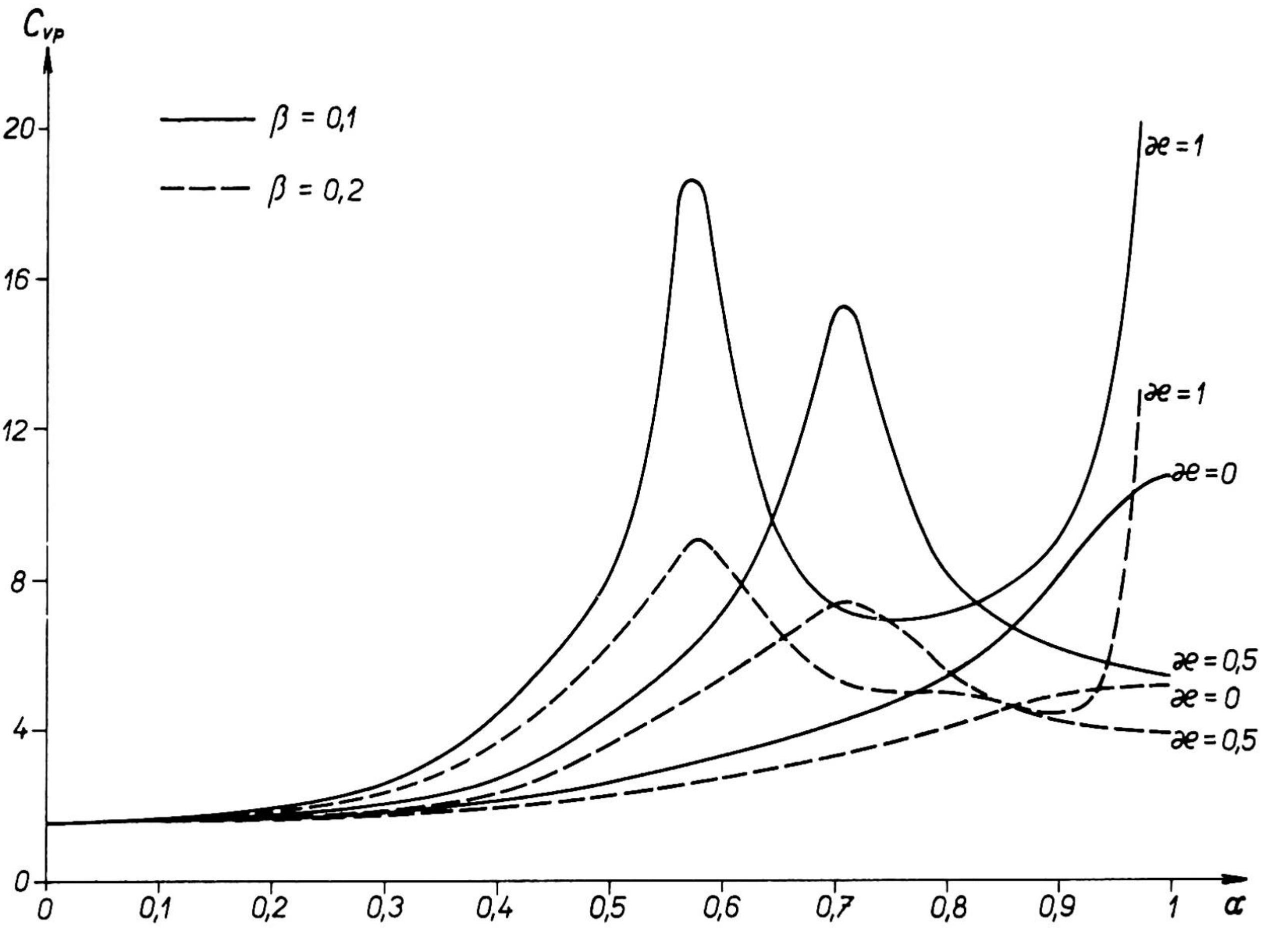

Fig.2. The coefficient C_{vp} as a function of the velocity parameter α, damping parameter β and mass parameter $\varkappa$

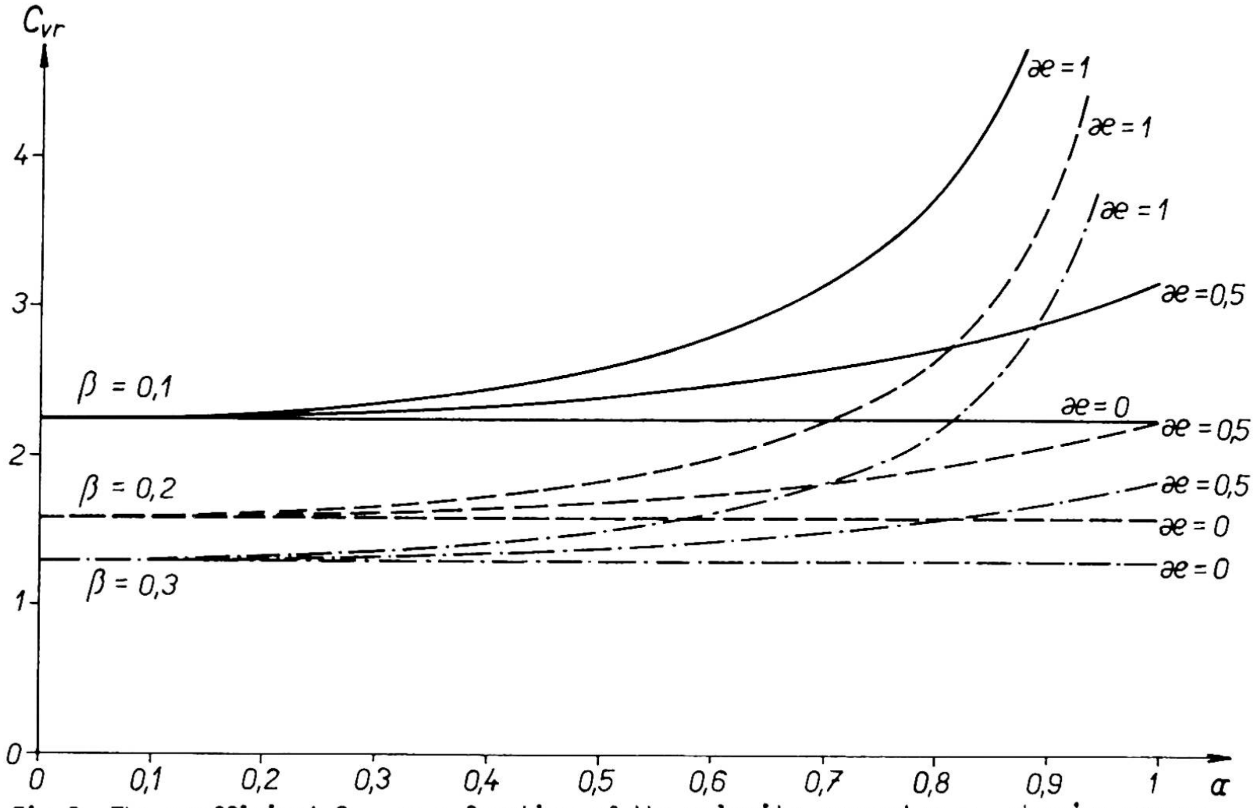

Fig.3. The coefficient C_{vr} as a function of the velocity parameter α , damping parameter β and mass parameter æ

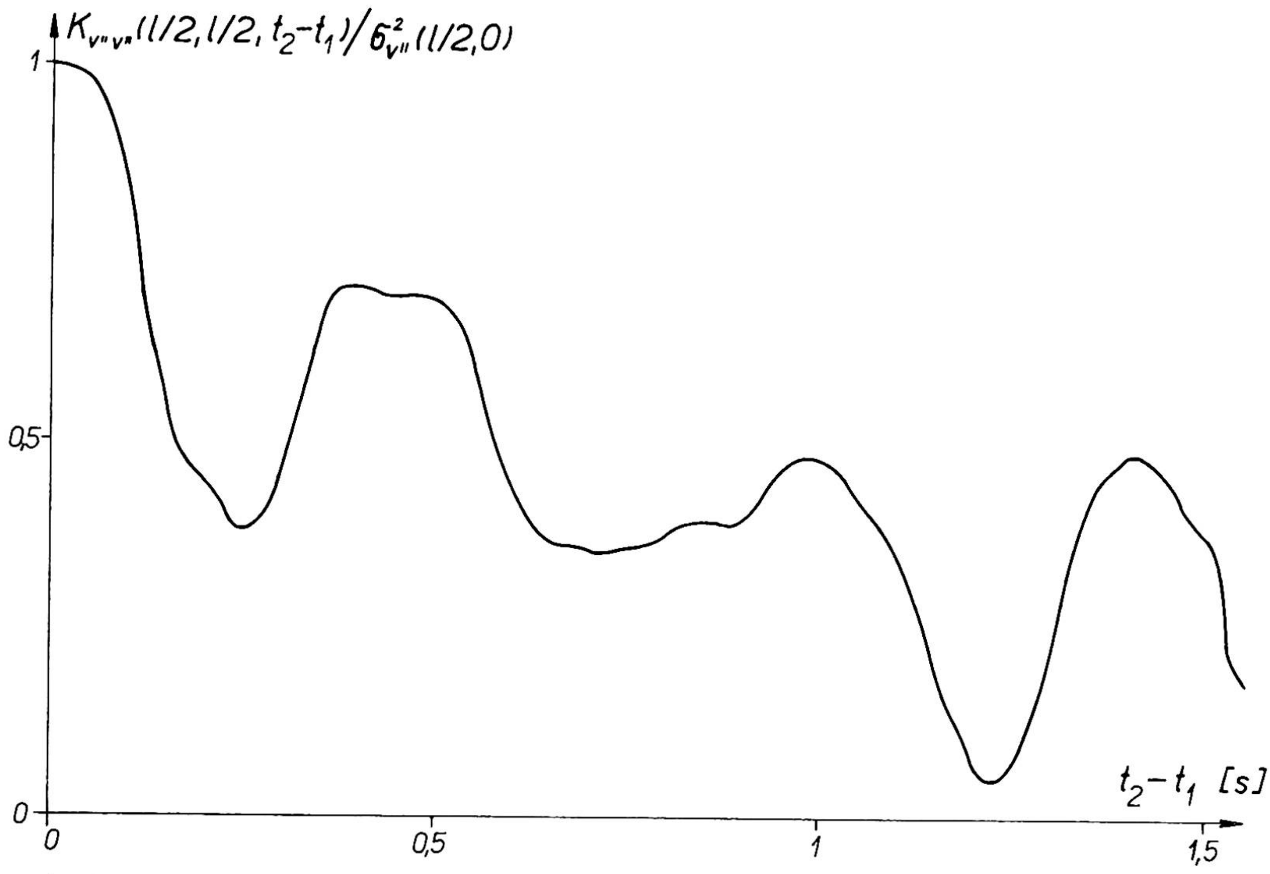

Fig.4. The experimental covariance measured from the stresses in the longitudinal beam on a steel-railway bridge

SUMMARY

The traffic loading of bridges is considered as a nonstationary random process. Starting from the statistic characteristics of the load the theory supplies information as to the statistic characteristics of the deflections or stresses in a bridge , i.e. the mean value, the covariance, the spectral density, the variance or the coefficient of variance.

The solution is shown for two typical cases which concern small and large span bridges. In the former case the load is idealized by a concentrated force of random time variation moving with constant velocity along a simply supported beam. The random effects of this load are decreasing with increasing velocity and damping (Fig. 1).

In the latter case the load is idealized by an infinitely long random strip which is moving again with constant velocity along a simple beam. This type of load induces in the beam a stationary random vibration the amplitudes of which are increasing with decreasing damping and for velocities approaching the critical speed which depends also on the mass of the traffic load (Figs. 2 and 3).

RÉSUMÉ

Le traffic sur un pont est considéré comme une charge stochastique non-stationnaire. En partant des caractéristiques statistiques de cette charge, la théorie donne des informations concernant les caractéristiques statistiques des déformations ou des tensions dans un pont, c-à-d. la valeur moyenne, la covariance, la densité spectrale, la variance ou le coëfficient de variance.

Deux cas typiques ont été traités pour un pont court, resp. long. Dans le premier cas, la charge est idéalisée par une force concentrée, variable arbitrairement avec le temps et voyageant avec une vitesse constante le long d'une poutre simple. L'effet arbitraire de cette charge décroît avec vitesse et amortissement croissant (fig. 1).

Dans le deuxième cas, la charge est idéalisée par une charge répartie stochastique infiniment longue voyageant sur la poutre simple avec une vitesse constante. Cette charge provoque une vibration stationnaire arbitraire, dont les amplitudes croissent inversément avec l'amortissement et augmentent avec des vitesses approchant la vitesse critique, qui dépend également de la masse de la charge de traffic (voir fig. 2 et 3).

ZUSAMMENFASSUNG

Die Verkehrslast von Brücken wird als nichtstationärer, zufälliger Vorgang aufgefasst. Ausgehend von den statistischen Charakteristiken der Last liefert die Theorie Auskunft über die statistischen Charakteristiken der Verformungen oder Spannungen einer Brücke, die da sind der Hauptwert, die Kovarianz, die Verteilungsdichte, die Varianz oder der Koeffizient der Varianz.

Die Lösung wird an zwei ausgeprägten Beispielen mit einer kurzen und einer langen Brücke gezeigt. Im ersterwähnten Fall ist die Belastung durch eine Einzellast idealisiert, die sich bei zufälliger Zeitvariation mit konstanter Geschwindigkeit entlang des einfachen Balkens bewegt. Die zufällige Wirkung dieser Last ist verschwindend bei wachsender Geschwindigkeit und Dämpfung (Fig. 1).

Im letzteren Fall ist die Belastung durch einen unendlich langen Streifen idealisiert worden, der sich wiederum mit konstanter Geschwindigkeit bewegt. Dies bewirkt im Balken eine stationäre, zufällige Schwingung, deren Amplitude mit abnehmender Dämpfung und mit Geschwindigkeiten, die sich der kritischen nähern, welche von der Lastmasse abhängt, wächst.