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Critical Appraisal of Safety Criteria and Their Basic Concepts

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Since the first Meeting in Vienna (1928) the problem of structural safety, in one form or another, has been on the agenda of all Congresses of the International Association for Bridge and Structural Engineering. The only difference at this Congress is that the purpose of the Introductory Report on this subtheme is the presentation of a survey of the present status rather than of a summary of submitted individual contributions.

1. General Considerations

There are three aspects of structural analysis:

(i) the determination of the operating (external) forces ("load-analysis");

(ii) the determination of the internal forces and stresses ("stress-analysis");

(iii) the determination of the necessary dimensions on the basis of relevant failure mechanisms of the structure and material parameters ("strength and safety analysis").

The principal emphasis in structural research and development has always been on stress-analysis. In fact, for the large majority of structural engineers the other two aspects do not seem really significant, and are usually not considered as research subjects but rather as matters of specification writing and materials testing. However, mainly under the influence of developments in the field of aircraft- and space-structures, there is a growing realization that load analysis and safety analysis are both integral parts of structural analysis, of an importance at least equal to that of stress analysis, because, on the one hand, no matter how elaborate this analysis, its results are only as good as the load analysis underlying it while, on the other, it seems absurd to strive for more and more refinement of methods of stress-analysis if, in order to determine the dimensions of the structural elements, its results are subsequently compared with so-called "working stresses", derived in a rather crude manner by dividing the values of somewhat dubious material parameters obtained in conventional materials tests by still more dubious empirical numbers called safety factors.

For a number of years structural engineers in different countries have attempted to come to grips with the basic problem of structural safety [1–12]; progress has, however, been relatively slow because the large majority of practicing civil and structural engineers is convinced that "engineering intuition" and conventional specifications are adequate for the design of safe and economical structures. Where organized attempts have been made to develop modern concepts of safety analysis and introduce them into engineering practice as, for instance, by a Committee of the Institution of Structural Engineers in London headed by SIR ALFRED PUGSLEY [13], whose recent book [4] provides an excellent introduction to and survey of the field of structural safety, by a Committee of the International (European) Council for Building Research headed by the late PROF. TORROJA [14], and by a Committee of the American Society of Civil Engineers headed by OLIVER G. JULIAN [15] and the author [16], these attempts have ended in a compromise between the conviction of a minority of the committee members that only a radically new approach to structural safety based on probabilistic concepts could provide a rational foundation of safety analysis, and the refusal of the majority to accept a probabilistic interpretation of the safety concept and its implications. The proposed modifications of existing safety standards contain therefore only half-hearted references to the necessity of developing a probabilistic approach and avoid any reference to the concept of an "acceptable risk of failure", which is the key to a rational approach to structural safety.

The principal reason for the rather wide-spread lack of research interest in problems of load-analysis and safety analysis among civil and structural engineers is the fact that, with few exceptions, civil engineering structures do not operate at the limit of the current "state of the art" but stay comfortably within it. Thus failures resulting from inadequate over-all design are extremely rare (arising mainly under unexpected condition of dynamic instability), such failures being usually traceable to mistakes in the design of details, particularly connections, while the economic requirements are not very stringent, since overdesign of such structures does not adversely affect their operation and is therefore not of major consequence. It is only when a structure must be designed to operate at the limit of its capacity and every unwarranted increase of its resistance, by increasing its structural weight, adversely affects not only its cost but interferes seriously with its effective operation, that the conventional approach to load- and safety analysis fails because of the existing precarious balance between safety, operational capacity and/or economy.

It is the result of the rapid development of manned flight that the advanced types of aircraft structures, at the time of their design, are usually close to the limiting condition of their operational capacity, so that only extreme care in the selection of the expected operating loads, coupled with a specified finite risk of failure, considered as "acceptable" and demonstrated by full-scale tests, produces structures that satisfy the operational requirements. Similar, though still more severe conditions are characteristic of the design of space structures, with the result that load and reliability analysis of aircraft and space structures have, within recent years, become areas of research of vital interest to the aircraft and space industries; theoretically and experimentally this research is dominated by probability concept and statistical methodology. The literature in this field is growing so rapidly [17] that a special Abstracting Service has been found necessary to keep the profession informed of advances in research and development [18].

One consequence of this mushrooming concern with reliability in the aircraft and space industries has been the recent wide-spread introduction of reliability courses in the engineering curriculum at Universities throughout the United States. It has also brought about a modest increase of interest in load- and safety analysis among civil and structural engineers. The principal trend in the current development is the gradual, although rather reluctant acceptance, by an increasing group in the profession, of the probabilistic interpretation of the safety factor, as well as of the resulting relation to a numerical value of the risk of functional or structural failure by which this factor acquires a rational meaning. This is demonstrated by the recent increase in the number of published papers in which the essential ideas of the probabilistic interpretation of structural safety are restated and applied to specific problems [19–27].

According to this interpretation the concept of structural safety can be put on a rational basis commensurate with the development of modern methods of stress analysis only through the consideration of the statistical dispersion of the operating loads as well as of the structural resistance. It does neither imply nor specifically advocate a reduction of conventional safety factors, but only attempts to remove the concept of structural safety from the realm of metaphysics to that of physical reality, in which the closest approach to a constant physical parameter is a unimodal frequency distribution.

It must be admitted, however, that the reluctance to accept this approach does not appear to be quite unjustified since the replacement, in actual design, of the well-tried conventional concept of "permissible" or "working" stresses, with their implication of absolute safety, by "safety factors" derived on the basis of probabilistic concepts and associated with a definite risk of failure, raises a number of theoretical and practical problems.

The main theoretical problems are:

(a) the existence of non-random phenomena affecting structural safety which cannot be included in a probabilistic approach, and

(b) the impossibility of observing the relevant random phenomena within the ranges that are significant for safety analysis, and the resulting necessity of extrapolation far beyond the range of actual observation.

The main practical problems are:

(a) the assessment and justification of a numerical value for the "acceptable risk" of failure and

(b) the codification of the results of the rather complex probabilistic safety analysis in a simple enough form to be usable in actual design.

While these are, in fact, serious problems, it is necessary to realize that conventional design procedures do not assure "absolute safety", nor can they produce structures of uniform safety in all its parts. It is quite easily demonstrated by considering the statistical dispersion of the operational loads [28] and of the relevant material resistance [29] that structures designed to current codes have, in fact, a non-zero probability of failure which, more-over, is different in different parts of the structure; for steel structures, such as highway bridges or transmission towers, it is of the order of 10^{-4} to 10^{-6}, for concrete structures of the order of 10^{-3} to 10^{-5} for a single application of the design load [30]. The implication of "absolute safety" is thus not more than a convenient fiction. Acceptance of this fiction makes it impossible, however, to arrive at a design of uniform safety, since it is the acceptable risk of failure rather than the value of the safety factor which provides a rational measure of the safety, and on which a design procedure for uniform safety must be based. This risk can be expressed by different criteria [31] and the choice of the criterion will depend on whether the risk can be assumed to remain practically constant during the operational life of the structure or whether it is a function of the age of the structure. It is only in the first case that the "probability of encountering failure" or the "mean waiting time" between failures can be used as alternative criteria. In the second case, in which the carrying capacity of the structure must be assumed to decrease with time or with the number of load applications (creep, corrosion, fatigue) the risk criterion must reflect the accumulation of damage implied in the failure mechanism.

2. Probabilistic Concept of Structural Safety

The probabilistic interpretation of structural safety is based on the representation of the loads and other forces acting on the structure by a statistical population of forces of known distribution, while its carrying capacity is represented by that of a statistical population of (nominally identical) structures. The probability of failure p_F under a single applied load or load pattern refers to this population of structures of statistically variable carrying capacity R, every one of which is subject to a single load or load pattern out of the population of statistically varying loads S. Hence p_F expresses the proportion of structures expected to fail in this random "matching" of load and structural resistance and, therefore, the probability that any one of the structures will fail under a single load application. This probability is not a direct measure of the safety of a structure subject to a random sequence of loads taken from the load population. Such a measure is provided by the "reliability function" $L_N(n)$ which is defined as the probability that the life of the structure measured in terms of the number N of load applications to failure exceeds n, the number of applied loads, or

$$L_N(n) = Pr\{N > n\} \tag{2.1}$$

so that the probability of failure before or at the n-th load application is

$$F_N(n) = 1 - L_N(n) = Pr\{N \leq n\} \tag{2.2}$$

The probability that a structure will fail at the n-th load application is obviously

$$f_N(n) = Pr\{N = n\} = F_N(n) - F_N(n-1) \tag{2.3}$$

so that the probability that a structure that has survived $(n - 1)$ load applications will fail at the n-th application is

$$h_N(n) = f_N(n)/L_N(n-1)\,. \tag{2.4}$$

The function $h_N(n)$ represents the "risk of failure" or failure rate.

If, in first approximation, n is treated as a continuous variable, Eq. (2.3) can be expressed in the form

$$f_N(n) \cong \frac{d}{dn} F_N(n) \tag{2.5}$$

and therefore

$$h_N(n) = -\frac{d}{dn} \ln L_N(n) \tag{2.6}$$

or

$$L_N(n) = exp\left[-\int_0^n h_N(\xi)d\xi\right] \tag{2.7}$$

which establishes the relation between the reliability and risk functions.

Under the simplifying assumptions that the probability of failure p_F is independent of n, the probability of surviving n load application can be expressed by

$$L_N(n) = (1 - p_F)^n \tag{2.8}$$

and therefore from Eq. (2.6)

$$h_N(n) \cong p_F = T_F^{-1} \tag{2.9}$$

where T_F denotes the "return period" of failures or expected number of load applications ("waiting time") between failures.

From Eq. (2.7) the reliability function

$$L_N(n) = exp\,(-\,np_F) \sim (1 - np_F) \quad \text{and} \quad F_N(n) \sim np_F \tag{2.10}$$

when $np_F << 1$; Eq. (2.10) defines the reliability function for chance failures and the relation between $F_N(n)$ and p_F.

The relation between the safety factor and the probability p_F follows simply from the definition of the safety factor ν as a statistical variable of probability density $p_\nu(\nu)$ and associated function $P_\nu(\nu) = \int_0^\nu p_\nu(t)dt$ formed by the ratio

$$\nu = R/S \tag{2.11}$$

where $R > 0$ denotes the structural resistance or carrying capacity and $S > 0$ the applied load, both considered as statistical variables with probability densities $p_R(R)$ and $p_S(S)$ and associated probability functions $P_R(R) = \int_0^R p_R(t)dt$ and $P_S(S) = \int_0^S p_S(t)dt$. The probability of failure p_F is therefore

$$p_F = Pr\{\nu < 1\} = P_\nu(1) \tag{2.12}$$

where the probability function of the quotient ν is obtained in terms of the functions $P_R(R)$ and $p_S(S)$ in the form [32]

$$P_\nu(\nu) = \int_0^\infty P_R(\nu t)p_S(t)dt\,. \tag{2.13}$$

Hence

$$p_F = P_\nu(1) = \int_0^\infty P_R(t)p_S(t)dt = \int_0^\infty Pr\{R < t\} \cdot Pr\{S = t\}dt \tag{2.14}$$

or alternatively,

$$p_F = \int_0^\infty Pr\{S > t\} \cdot Pr\{R = t\}dt = \int_0^\infty [1 - P_S(t)] \cdot p_R(t)dt\,. \tag{2.15}$$

The tacit assumption underlying Eqs. (2.11) to (2.15) is statistical independence of the variables S and R; this assumption is nearly enough valid for most engineering structures that can be designed without dynamic (aerolastic, seismic) analysis.

That Eqs. (2.14) or (2.15) provide a relation between p_F and a "safety factor" can be easily demonstrated in the simple case of exponential distributions of R and S for $R > 0$ and $S > 0$. Assuming $P_R(R) = 1 - exp(-\alpha R)$ and $1 - P_S(S) = exp(-\beta S)$ and therefore $p_R(R) = \alpha\, exp(-\alpha R)$ and $p_S(S) = \beta\, exp(-\beta S)$ it follows from Eq. (2.14) that

$$p_F = \int_0^\infty (1 - e^{-\alpha t})\beta e^{-\beta t} dt = \frac{\alpha}{\alpha + \beta} = \frac{1}{1 + \bar{R}/\bar{S}} = \frac{1}{1 + \nu_0} \qquad (2.16)$$

since the expectations (mean values) of R and S are $E[R] = \bar{R} = \alpha^{-1}$ and $E[S] = \bar{S} = \beta^{-1}$ and $\nu_0 = \beta/\alpha = \bar{R}/\bar{S}$ defines the ratio between the mean values of R and S, which represents a measure of the central tendency of the distribution of the (statistically variable) safety factor ν defined by Eq. (2.13): $P(\nu) = (1 + \nu_0/\nu)^{-1}$; this measure will be referred to as the "central" safety factor.

The oversimplification in the assumption of an exponential distribution of R and S precludes the actual use of Eq. (2.16). Nevertheless, this equation illustrates the most significant aspect of all relations between p_F and ν_0: when a "central" safety factor is used in design, its value must be extremely high in order to ensure a small enough probability of failure. Conventional design is, however, not based on a "central" safety factor, since the specification values of load and carrying capacity are usually determined under the tacit assumption that the applied (design) load is a "maximum", while the (design) carrying capacity is based on a "minimum" of the relevant material parameter. However, unless the forces acting on the structure have a functionally defined relevant upper limit (maximum storage capacity, maximum crowd density, maximum locomotive weight), a "maximum" load S_{max} and a "minimum" carrying capacity R_{min} can be rationally defined only in probabilistic terms as the load intensity $S_{max} = S_q$ that is exceeded with arbitrarily small probability $q = Pr\{S > S_q\}$ and the resistance $R_{min} = R_p$ that is not attained with arbitrarily small probability $p = Pr\{R < R_p\}$. Introducing the parameters ξ_p and η_q such that $R = \xi_p \bar{R}$ and $S_q = \eta_q \bar{S}$ and defining a "conventional" safety factor

$$\bar{\nu} = R_{min}/S_{max} = R_p/S_q = (\xi_p/\eta_q)\nu_0 \qquad (2.17)$$

the relation between $\bar{\nu}$ and ν_0 can be easily determined. Since $\xi_p < 1$ while $\eta_q > 1$, the factor $\nu_0 >> \bar{\nu}$, which explains the general discrepancy between the values of safety factors used in conventional analysis and the "central" safety factors derived in probabilistic safety analysis based on mean, median or modal values of loads and material parameters.

Existing observations of the dispersion of relevant material parameters [33] suggest that under conditions in which a reasonably high level of quality control exists a logarithmic-normal distribution describes this dispersion fairly well. On the other hand the selection of most (design) load-spectra is such that

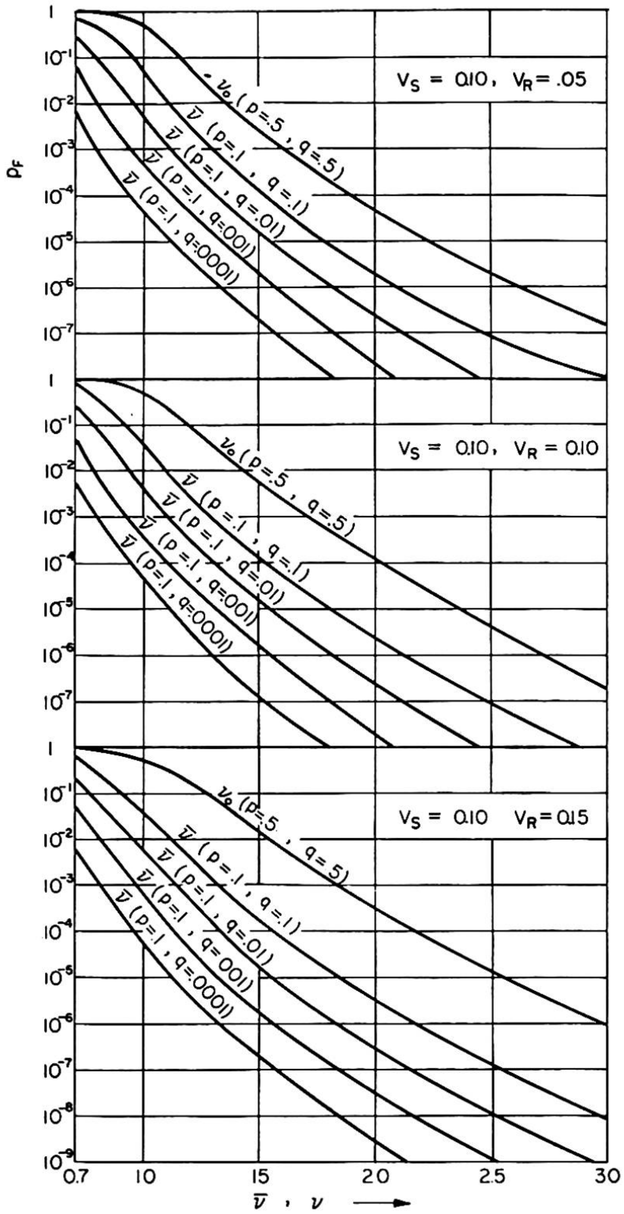

Fig. 1a

only the higher range of the load intensities is considered, which should therefore approximate a distribution of extreme (largest) values. The exclusive consideration of this range of load intensities obviously reduces the number of load applications to be considered in the reliability analysis so that if $F_N(n)$ is specified a higher value of p_F is admissible (see Eq. 2.10) than if application of the complete spectrum of intensities were considered. The relations $p_F(\nu_0)$ and $p_F(\bar{\nu})$ have been evaluated for the above assumptions and the results summarized in Fig. 1. In Fig. 2 the relation $p_F(\bar{\nu})$ for $p = 0.1$ and $q = 0.01$ and a characteristic set of coefficients of variation of S and R is compared with those obtained for logarithmic-normal and extremal distributions of both S and R [34]. The comparison illustrates the effect of the assumed form of the dispersion of S and R on the relation $p_F(\bar{\nu})$.

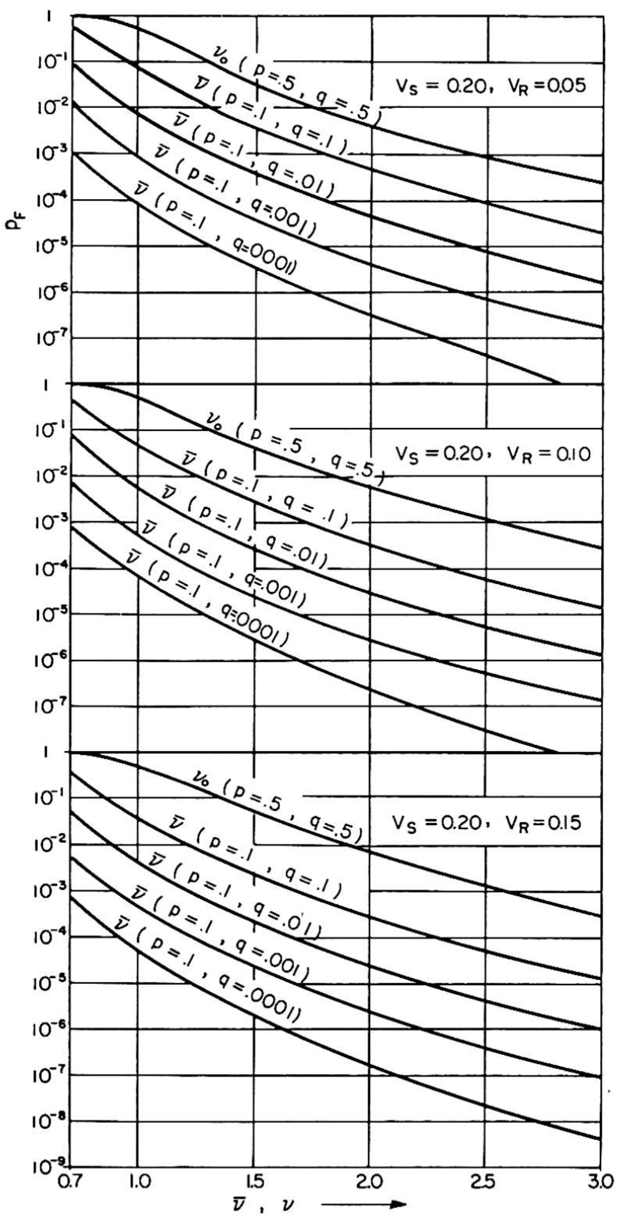

Fig. 1b

The outlined safety analysis implies the existence of a single "failure mechanism" which, once developed, produces failure of the structure. This "failure mechanism" may be related to a maximum (admissible) value of reversible (or irreversible) deformation, exceedance of which might be considered as "functional failure" and thus as a condition of "unserviceability". Or the failure mechanism may be related to fracture of a critical section or, more frequently, to a condition of instability of the structure (kinematic collapse, buckling).

In recent reliability analysis a differentiation is frequently made between structures failing in one of the above modes and considered as "single member" or "weakest link" structures, and multiple-member or "redundant" structures, which are assumed to fail by consecutive failure of the redundants with inter-

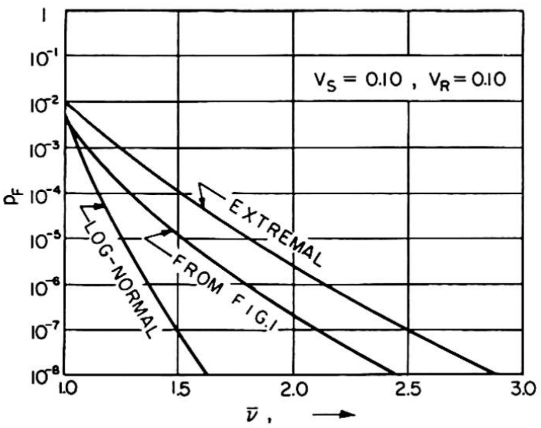

Fig. 2a

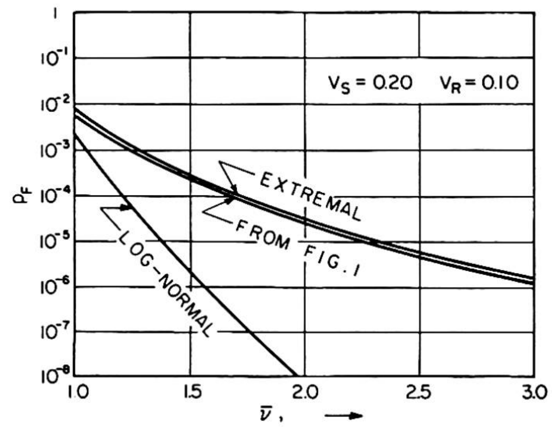

Fig. 2b

mediate load redistribution [35]. However, the consideration of this "chain-reaction" type of structural failure is usually of no practical significance because the failure, whether by fracture or by yielding, of one redundant member immediately increases the probability of failure of the remaining members so that the probability of survival of the structure is not significantly affected, unless the number of redundant members is exceptionally large.

The "redundancy" referred to above implies that the applied load is simultaneously carried by all members of the structure, as in the case of a bundle of parallel threads, as distinct from a non-redundant structure that can be represented by a chain in which every member carries the full load. Neither of the two models reflect the character of bridge- and other structures, members of which are not all stressed to the same intensity by the same load or load pattern. The maximum load intensity in different members is produced by different load patterns which are independent of each other. In a statically determinate structure the probability of failure of each member must be determined separately in order to locate the "critical" member or element which is that with the highest value $F_N(n) \sim np_F$. The "failure mechanism" of a statically determinate structure is that involving failure of the critical member or, in the case of several members with the same value $F_N(n)$, failure of any one of the critical members.

The evaluation of the probability of failure of a m-times statically indeterminate structure requires the consideration of the m consecutive states of decreasing indeterminacy through which the structure passes in the transition to its final failure mechanism in which the m redundant members or elements have been eliminated and the critical member of the resulting determinate structure fails. The difficulty arises from the fact that the process of transition from the indeterminate structure to the final failure mechanism is not unique, nor is this failure mechanism itself unique, since there is usually more than one particular group of $(m + 1)$ members the failure of which can be identified with failure of the structure.

If, under a specified load pattern, the final failure mechanism is attained by simultaneous failure of $(m + 1)$ members or elements, the probability of failure of each of which can be evaluated and is equal to p_F, the probability of failure of the structure in this mechanism is $kp_F{}^{(m+1)}$ if k alternative independent groups of $(m + 1)$ members leading to the same failure mechanism can be identified. Thus the probability of survival under n independent applications of the load pattern according to Eq. (2.8).

$$L_N(n) = [1 - kp_F{}^{(m+1)}]^n \tag{2.18}$$

or, for $kp_F{}^{(m+1)} << 1$,

$$L_N(n) \sim 1 - nkp_F{}^{(m+1)} \tag{2.19}$$

and

$$F_N(n) \sim nkp_F{}^{(m+1)} < np_F \tag{2.20}$$

unless k is unusually large and m very small.

The probability of failure p_F of any of the k groups of $(m + 1)$ members or elements of a statically indeterminate structure can therefore be larger for a specified value $F_N(n)$, than that of a critical member of the determinate structure.

The above equations represent, however, only a first rough approximation to the problem of structural reliability of statically indeterminate structures, and their actual evaluation presupposes a detailed study of the possible failure mechanisms of the structure considering not only failure under a population of independent loads but also failure due to consecutive loads each of which produces "partial" failure in fewer than $(m + 1)$ members or elements. Very little research work has been done on this problem [36].

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