Creep failure of nonlinear rotational shells

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DISCUSSION PRÉPARÉE / VORBEREITETE DISKUSSION / PREPARED DISCUSSION

Creep Failure of Nonlinear Rotational Shells

Rupture par fluage de voiles minces axisymétriques non-linéaires

Kriechbruch nichtlinearer Rotationsschalen

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1. Introduction

The authors have established [1] a criterion of the attainment of critical states in linear viscoelastic bodies. The idea of the criterion can also be extended to the range of nonlinear viscoelastic behavior, if the phenomenon of failure is considered as a critical state.

The criterion is founded on an energetical basis and for a certain group of nonlinear viscoelastic materials it states that such a critical state as, for example, creep rupture depends in general on a function of the accumulated energy and the dissipated power accompanying the deformation process. Thus, if W_E stands for the accumulated energy and $\dot{W}_D$ is the dissipated power, the condition of creep rupture is stated as follows:

$$f(W_E, \dot{W}_D) = \text{const.} \tag{1.1}$$

In some cases, however, the accumulated part of energy may be neglectfully small. Moreover, there are materials which are not able to accumulate energy at all as, for example, the pure creeping ones. In these cases it seems reasonable to represent the criterion (1.1) in the following different form:

$$f(W_D) = \text{const.} \tag{1.2}$$

where W_D is the deviatoric dissipated energy per unit volume of the body. It follows from the condition (1.2) that the dissipated energy is considered as a certain measure of the attainment of the critical state. The correct form of this condition should be founded on experimental results.

The problem of attainment of a critical state as, for example, creep rupture may turn out to be essential when analysing the conditions occurring for thin-walled metallic structures under high pressure, especially, high temperature containers, pneumatic structures, etc.

For such problems, we apply the criterion (1.2) to geometrically nonlinear rotational shells in the membrane state under internal pressure in order to evaluate the critical time of failure as a consequence of the creep process. Accordingly, we assume that the material of shells exhibits pure creep only.

2. Physical and geometrical equations

In general, we assume that an isotropic incompressible material of shells creeps according to the integral law [2]

$$e_{ij} = N s_{ij} , \tag{2.1}$$

where e_{ij} stands for the creep strain tensor, s_{ij} is the stress deviator and N denotes a nonlinear integral operator of the form

$$N s_{ij} = - \int_{t_0}^{t} s_{ij}(\tau) \partial_\tau H\left[t, \tau, s(\tau)\right] d\tau \quad . \tag{2.2}$$

Here, H is the generalized creep function depending on the effective stress

$$s(t) = \frac{3}{2}\left[s_{ij}(t)\, s_{ij}(t)\right] \quad , \tag{2.3}$$

t standing for time, t_0 being the initial instant and $\partial_\tau = \partial/\partial\tau$. As shown in [2] the generalized creep function H covers both the linear and nolinear range of creep. However, in the present paper we use only its nonlinear representation.

In particular, the creep function may be assumed in such a form as to satisfy the following condition:

$$\partial_\tau H\left[t, \tau, s(t)\right] = F\left[s(\tau)\right] \partial_\tau C(t-\tau) \ , \tag{2.4}$$

where F is a nonlinear magnifying factor and C the creep factor. The last representation of the creep function is very useful when considering the non-steady states of creep in which the deformation stabilizes after an infinite period of time.

For the state of creep of metallic materials, the derivative of the creep factor becomes a constant, i.e., C is a linear function of time

$$C(t-\tau) = c(t-\tau) \ , \tag{2.5}$$

where c is a constant. In the last case, Eq.(2.2) takes the form

$$N_0 s_{ij} = \int_{t_0}^{t} s_{ij}(\tau) F_0\left[s(\tau)\right] d\tau \tag{2.6}$$

As it is seen from Eq.(2.6), we assume that the state of stress is variable with time. We shall show later that in the case of creep of nonlinear shells in membrane state, the stress state is always a non-steady one in the presence of a constant internal pressure. The stresses are found to drop to zero in an infinite period of time.

According to Eqs.(2.2) and (2.6), the initial conditions at t equal t_0 are assumed to be of zero value, i.e., there are no deformations at the initial instant. However, these conditions may not be zero, if we consider a creep process at $\bar{t} > t_0$. In this case there is an initial deformation state expressed instantaneously by the values of integrals within the limits $t_0, \bar{t}$.

The equations (2.1) and (2.6) can be written in terms of strain rates as follows:

$$\dot{e}_{ij} = \dot{N} s_{ij} \quad , \qquad \dot{e} = \dot{N}_0 s_{ij} , \tag{2.7}$$

where the dots over the operators are symbolic. For example, in the case of the second relation (2.7) we find

$$\dot{N}_0 s_{ij} = s_{ij}(t) F_0 \left[s(t) \right] . \tag{2.8}$$

We apply the physical equations (2.1) or (2.6) in order to investigate the critical states of rotational membranes of small and constant thickness which deform under constant internal pressure. In deriving the geometrical relations for such membranes we assume that in the time-interval considered, the strain tensor and strain rate tensor are small quantities, the rotation angles being also small; the normal component of displacement is supposed to have a finite value. Further, we assume that the undeformed surface is generated by the revolution of a plane curve which does not imply any singularities. In order to simplify the equations we restrict ourselves to shallow rotational membranes. For such membranes we obtain a set of two equations of equilibrium [2]

$$d_\rho(\rho\sigma_1) = \sigma_2 , \tag{2.9}$$

$$\sigma_1\left(\bar{k}_1 - d_\rho^2 \bar{w}\right) + \sigma_2\left(\bar{k}_2 - \frac{1}{\rho} d_\rho \bar{w}\right) = \frac{p}{h} , \tag{2.10}$$

where σ_{ij} and σ_2 are the stresses in the directions of the main curvatures k_1 and k_2 , respectively, $\bar{w}$ is the displacement normal to the surface, p the constant internal pressure, h the thickness of the membrane and ρ denotes the surface coordinate. The symbol d_ρ represents the derivative $d/d\rho$.

To the set of Eqs.(2.9) and (2.10) we now join the equation of compatibility of deformations [2]

$$\rho d_\rho e_2 + e_2 - e_1 = -\frac{1}{2}\left(d_\rho \bar{w}\right)^2 + d_\rho\left(\rho \bar{k}_2 \bar{w}\right) - \bar{k}_1 \bar{w} , \tag{2.11}$$

where $e_1 = e_{11}$, $e_2 = e_{22}$ are the main strains.

Further, we introduce the following substitutions:

$$r = \left(\frac{\rho}{R}\right)^2 , \quad z = \frac{r}{D}\sigma_1 , \quad D = \frac{p}{g} , \quad g = \frac{h}{R} ,$$
$$w = \frac{\bar{w}}{h} , \quad k_1 = R\bar{k}_1 , \quad k_2 = R\bar{k}_2 , \tag{2.12}$$

where R is the value of the larger radius of main curvatures.

On the basis of Eq.(2.12), the stress deviator components expressed by stress components become

$$s_1 = s_{11} = \frac{1}{3}(3\sigma_1 - 2\sigma_2) = \frac{1}{3}D\left(3\frac{z}{r} - 2d_r z\right) , \tag{2.13}$$

$$s_2 = s_{22} = \frac{1}{3}(3\sigma_2 - 2\sigma_1) = \frac{1}{3}D\left(4d_r z - 3\frac{z}{r}\right) . \tag{2.14}$$

By introducing the quantities of Eq.(2.12), we satisfy Eq.(2.9) and Eq.(2.10) takes the form

$$\frac{z}{r}\left[k_1 - 4g\sqrt{r}\, d_r(\sqrt{r}\, d_r w)\right] + \left(2d_r z - \frac{z}{r}\right)(k_2 - 2g d_r w) = 1. \tag{2.15}$$

If ~~the~~ use of the physical relations (2.1) is made, then the main strains become

$$e_1 = Ns_1 = N\left[\frac{1}{3}D\left(3\frac{z}{r} - 2d_r z\right)\right], \quad e_2 = Ns_2 = N\left[\frac{1}{3}D\left(4d_r z - 3\frac{z}{r}\right)\right], \tag{2.16}$$

and the condition of compatibility may be written as follows [see Eq.(2.11)]:

$$2\sqrt{r}\, d_r(\sqrt{r} N s_2) - N s_1 = -2g^2 r (d_r w)^2 + 2g\sqrt{r}\, d_r(\sqrt{r} k_2 w) - g k_1 w . \tag{2.17}$$

The set of equations (2.15) and (2.17) is a system determining two unknown functions: the non-dimensional stress function z and the non-dimensional deflection w . Thus, the solution of the above system of equations gives the solution of the problem of creeping nonlinear membranes.

In the particular case of physical relations (2.6), we put in Eq.(2.17) N_0 instead of N. In this very case the condition of compatibility may be presented in terms of strain rates and the relations (2.8) applied.

3.Concept of analogy

It has been found [2] that in the case of purely creeping nonlinear rotational membranes it is possible to obtain the creep solution by separating the variables r and t in the fundamental set of Eqs.(2.15) and (2.17). Then the time-independent set of equations is analogous to the corresponding system of the instantaneous problem, if only the nonlinear functions of deviatoric stress intensity are of similar forms in both cases. The time-dependent set of equations can be solved in a closed form. It follows from the last solution that the creep process of nonlinear membranes is always unsteady. If the form of the generalized creep function is assumed according to Eq.(2.4), where the creep factor is expressed by exponential functions, then the solution describes a stabilizing process of creep. On the other hand, if the particular case of Eq.(2.6) is considered and creep is unlimited, a complete relaxation of stresses occurs after an infinite period of time and strains become infinite. From the physical point of view such a state of the membrane cannot occur and at a certain finite time-instant the creep rupture takes place. We assume as a measure of reaching this point the value of dissipated energy during the creep process. Thus, if a certain critical value of energy is dissipated through creep resistance, the membrane is considered as collapsed. From an appropriate condition of the form (1.2) it is then possible to find the critical time of creep failure.

4. Shallow spherical membrane

We shall consider in detail a shallow spherical membrane with the radius of curvature R . For such a membrane we obtain the following system of equations

$$\sigma_1(\bar{k} - d^2_\varrho \bar{w}) + \sigma_2\left(\bar{k} - \frac{1}{\varrho} d_\varrho \bar{w}\right) = \frac{p}{h} \quad , \qquad \bar{k} = 1/\bar{R} \ , \tag{4.1}$$

$$\varrho d_\varrho e_2 + e_2 - e_1 = -\frac{1}{2}(d_\varrho \bar{w})^2 + d_\varrho(\varrho \bar{k} \bar{w}) - \bar{k}\bar{w} \quad , \tag{4.2}$$

and Eq. (2.9) .

The equation (4.1) may be at once integrated by using Eq. (2.9), the constant of integration being equal zero. Thus, instead of Eq. (4.1), we obtain

$$d_\varrho \bar{w} = \varrho\left(\bar{k} - \frac{p}{2h\sigma_1}\right) . \tag{4.3}$$

Further, we introduce the following notations:

$$r = \left(\frac{\varrho}{l}\right)^2 , \quad z = \frac{r}{D}\sigma_1 \ , \quad D = p\left(\frac{l}{2h}\right)^2 , \quad w = \frac{\bar{w}}{h} \ , \tag{4.4}$$
$$k = 1/R \ , \qquad R = \bar{R}/h \ , \qquad g = h/l$$

where l is the maximum value of the variable ϱ (for $\varrho = l$, $r = 1$).

With the above notations, the stress deviator components have the form of Eqs. (2.13) and (2.14), and the strains are given by the formula (2.16).

Considering the radius of curvature R as time dependent, we differentiate Eq. (4.2) with respect to time and thus obtain

$$\varrho d_\varrho \dot{e}_2 + \dot{e}_2 - \dot{e}_1 = -d_\varrho \dot{\bar{w}} d_\varrho \bar{w} + d_\varrho(\varrho \dot{\bar{k}} \bar{w}) + d_\varrho(\varrho \bar{k} \dot{\bar{w}}) - \dot{\bar{k}} w - \bar{k}\dot{\bar{w}} \tag{4.5}$$

Finally, by using the second of Eqs. (2.7) and introducing the notations (4.4), we represent Eqs. (4.3) and (4.5) in the following form:

$$d_r w = \frac{1}{2}\frac{k}{g^2} - \frac{r}{z} \ , \tag{4.6}$$

$$2r d_r \dot{N}_0\left[\frac{1}{3}D\left(4 d_r z - 3\frac{z}{r}\right)\right] + \dot{N}_0\left[\frac{1}{3}D\left(4 d_r z - 3\frac{z}{r}\right)\right] - N_0\left[\frac{1}{3}D\left(3\frac{z}{r} - 2 d_r z\right)\right.$$
$$\left. + 2r\left[d_r \dot{w}(2g^2 - d_r w - k) + \frac{k}{\dot{k}} d_r w\right] = 0 \ . \tag{4.7}$$

Here the operator $\dot{N}_0$ is given by Eq. (2.8).

It is seen from the set of Eqs. (4.6) and (4.7) that the displacement w can be easily eliminated from the second equation by means of the first one.

Now, let us assume that the nonlinear function of effective stress appearing in Eq. (2.8) is a power function of the form

$$F_o[s(t)] = \frac{3}{2} B s^{n-1}(t) \quad , \tag{4.8}$$

where B and n are physical constants, the last being an odd natural number.

According to Eq. (2.3) and Eqs. (2.13), (2.14), the effective stress is expressed as follows

$$s^2(r,t) = D^2 \Omega(r,t) \quad , \tag{4.9}$$

where

$$\Omega(r,t) = \Omega(z) = 4(d_r z)^2 - 6 d_r z \frac{z}{r} + 3\left(\frac{z}{r}\right)^2 . \tag{4.10}$$

Introducing Eq. (4.8) together with Eqs. (4.9) and (4.10) and eliminating the displacement in Eq. (4.7), we finally obtain

$$\Omega^{\frac{1}{2}(n-3)}\left[8\Omega d_r^2 z + (n-1)(4d_r z - 3\frac{z}{r}) d_r \Omega\right] = 2\gamma\left[\frac{\dot{z}}{z}\left(\frac{r}{z}\right)^2 - \left(\frac{2Dk}{p}\right)^2\right]\frac{k}{\dot{k}} \tag{4.11}$$

where

$$\gamma = \frac{p}{BD^{n+1}} . \tag{4.12}$$

The method of solution of the problem for a creeping membrane is founded on the basis of an analogy as stated above. We assume the solution of Eq. (4.11) in the form

$$z(r,t) = z^o(r)\, \varphi(t) \quad , \tag{4.13}$$

and put

$$k(t) = k^o / \varphi(t) \quad . \tag{4.14}$$

If we introduce the solution (4.13) into Eq. (4.11), then after separating the variables we find

$$\frac{1}{\gamma}\left[\left(\frac{2DK}{p}\right)^2 - \left(\frac{r}{z^o}\right)^2\right]^{(-1)} \Omega_o^{\frac{1}{2}(n-1)}\left[8\Omega_o d_r^2 z^o + (m-1)(4d_r z^o - 3\frac{z^o}{r}) d_r \Omega_o\right] = \tag{4.15}$$

$$= -2\,\frac{\dot{\varphi}(t)}{[\varphi(t)]^{n+3}} = \lambda \; ,$$

where

$$\Omega_o(r) = \Omega_o(z^o) = 4(d_r z^o)^2 - 6 d_r z^o \frac{z^o}{r} + 3\left(\frac{z^o}{r}\right)^2 , \tag{4.16}$$

λ being a constant.

The time-independent part of Eq. (4.15) is analogous to the equation for an instantaneous problem, if only the physical equation is of a form analogous to Eq. (4.8). Thus, if the solution of the instantaneous problem is known, we are able to obtain the creep solution in a formal way. The time-independent solution is obtained by representing the stress function in the form of power series.

On the other hand, the time-dependent part of Eq.(4.15) may be written as follows:

$$\dot{\varphi}(t) + \frac{1}{2}\lambda\left[\varphi(t)\right]^{n+3} = 0 . \tag{4.17}$$

The variables in Eq.(4.17) are separable and the solution is given by the formula

$$\varphi(t) = \varphi_0\left[1+\frac{1}{2}\lambda\varphi_0^{\,n+1}(n+2)(t-\bar{t})\right]^{\left(-\frac{1}{n+2}\right)} , \tag{4.18}$$

where the constant of integration $\varphi_0 = \varphi(\bar{t})$. According to the solution (4.18) we consider as initial instant of the observed creep process a certain intermediate time-point at which the past creep effects are taken into account instantaneously.

In order to obtain the appropriate solution for the displacement w, we assume the last in the form

$$w(r,t) = w^{o}(r)\psi(t) , \tag{4.19}$$

and by putting it into Eq.(4.6) we obtain

$$\left[\frac{1}{2}\frac{k}{g^2} - \frac{r}{z^o}\right]^{(-1)} d_1 w^o = \frac{1}{\varphi(t)\psi(t)} = 1 . \tag{4.20}$$

From the last result we obtain the relation between the two time functions ψ and φ

$$\psi(t) = \left[\varphi(t)\right]^{(-1)} . \tag{4.21}$$

As may be seen from Eq.(4.18), the function φ tends to zero with time tending to infinity. This means that the stresses [see Eq.(4.13)] drop to zero and their relaxation is complete after an infinite period of time. On the other hand, the function ψ increases infinitely with time and thus the displacement w [see Eq. (4.19)] becomes infinite.

5. Critical time of failure

In order to find the critical time of failure we use the criterion for the critical creep state in the form (1.2) where the function f is assumed as a linear one. Thus, we obtain the condition

$$W_D = \text{const.} = K^2 , \tag{5.1}$$

where K^2 is the critical value of dissipated energy through the creep resistance.

The power of dissipation is given by the relation containing the stress components and strain rate components

$$\dot{W}_D = \sigma_{ij}(r,t)\,\dot{\varepsilon}_{ij}(r,t) , \tag{5.2}$$

and the condition takes the form

$$W_D = \int_{\bar{t}}^{t^*} \sigma_{ij}(\tau)\,\dot{\varepsilon}_{ij}(\tau)\,d\tau = K^2 \tag{5.3}$$

Here, by t^* we denote the time-instant at which the creep rupture takes place. Evidently, the initial instant $\bar{t}$ should also be considered as a certain critical time-point as, for example, the instant of reaching the stage at which the elastic effects can be neglected. In this case, the value of dissipated energy $W_D(\bar{t}) = \bar{W}$ characterizes the process up to this stage.

In the particular case of a spherical membrane, the condition (5.3) takes the form

$$W_D = \int_{\bar{t}}^{t^*} \left[\sigma_1(r,\tau)\,\dot{\varepsilon}_1(r,\tau) + \sigma_2(r,\tau)\,\dot{\varepsilon}_2(r,\tau)\right] d\tau = K^2, \tag{5.4}$$

where $\sigma_1(r,t) = \frac{D}{r} z^{\circ}(r)\varphi(t), \quad \sigma_2(r,t) = D\left[2d_r z^{\circ}(r) - \frac{1}{r} z^{\circ}(r)\right]\varphi$

$$\dot{\varepsilon}_1(r,t) = \varphi^n(t) L^{\circ}\left[\tfrac{1}{3} D\left(3\frac{z^{\circ}}{r} - 2d_r z^{\circ}\right)\right], \quad \dot{\varepsilon}_2(r,t) = \varphi(t) L^{\circ}\left[\tfrac{1}{3} D\left(4d_r z^{\circ} - 3\frac{z^{\circ}}{r}\right)\right]. \tag{5.5}$$

In the Eq.(5.5) L° is related to the operator $\dot{L}$ as follows

$$\dot{L} = \varphi^{n-1}(t) L^{\circ} \tag{5.6}$$

On the basis of Eqs.(5.4) and (5.5), the condition (5.4) may now be written

$$W_D = W^{\circ}(r) \int_{\bar{t}}^{t^*} [\varphi(\tau)]^{n+1}\, d\tau = K^2, \tag{5.7}$$

where W° stands for a time-independent energetical coefficient the value of which can easily be evaluated on the basis of Eqs. (5.4) and (5.5).

In order to obtain the critical time of creep rupture, we calculate the value of the integral

$$\int_{\bar{t}}^{t^*} [\varphi(\tau)]^{n+1}\, d\tau = \frac{K^2}{W^{\circ}}, \tag{5.8}$$

by substituting the function φ according to the solution (4.18).

Denoting by

$$x = 1 + \frac{1}{2}\lambda\varphi_0^{\,n+2}(n+2)(\tau - \bar{t}), \quad A = \frac{1}{2}\lambda\varphi^{n+2}(n+2), \tag{5.9}$$

we obtain, instead of Eq.(5.8),

$$\frac{\phi_o^{n+1}}{A}\int_1^{x^*} x^{-\frac{n+1}{n+2}}\,dx = \frac{K^2}{W^o}\,, \tag{5.10}$$

where $x^* = x(t^*)$.

Carrying out the integration in Eq.(5.10), we finally obtain

$$t^* = \bar{t} + \frac{1}{A}\left\{\left[1+\frac{1}{2}\lambda\varphi_o\frac{K^2}{W}\right]^{n+2} - 1\right\} \tag{5.11}$$

It is seen from the result obtained that, if the criterion (5.3) is applied, we are able to predict the critical time of creep rupture and thus bound the unlimited creep process predicted by the creep solution. Since only dissipation is involved during the process, it seems reasonable to found the prediction of creep rupture on the basis of the amount of dissipated energy which thus constitutes a certain measure of reaching this critical state.

[1] Z.Bychawski, W.Olszak, Energetic interpretation of critical states in viscoelastic bodies (in Polish), IBTP Reports, No.2, Warsaw, 1967.

[2] Z.Bychawski, W.Olszak, Rheological states of geometrically nonlinear rotational membranes, The Second IUTAM Symposium on the Theory of Thin Shells, Copenhagen, 1967.

SUMMARY

On the basis of the authors criterion of attainment of critical states in viscoelastic bodies, the problem of creep failure of nonlinear rotational shells is investigated. For a spherical membrane the critical time of failure is found by introducing the dissipated energy through creep resistance as a measure of attainment of this state.

RÉSUMÉ

En se basant sur la condition des états critiques proposée par les auteurs, on considère le problème de la rupture par fluage pour les voiles minces nonlinéaires. Pour une membrane, le temps critique de rupture est calculé en introduisant l'énergie dissipée par la résistance de fluage comme une mesure pour atteindre cet état.

ZUSAMMENFASSUNG

Die Verfasser haben ein Kriterium für das Erreichen des kritischen Zustandes infolge Kriecherscheinungen formuliert und dasselbe zur Analyse des Problems des Kriechbruches von nichtlinearen Schalen im Membranzustand angewandt. Als Resultat findet man die kritische Zeit, in welcher Kriechbrucherscheinungen in einer sphärischen Membran eintreten. Als entsprechendes Mass wird dabei voraussetzungsgemäss die durch den Kriechwiderstand zerstreute (dissipierte) Energie eingeführt.