Optimum design for structural safety

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Optimum Design for Structural Safety

Dimensionnement optimal pour la sécurité d'une construction

Optimierung für die Bausicherheit

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BACKGROUND

In recent years there have been developments in the area of optimum design of structures which concerned the sizing and proportioning of members for minimum weight or cost. The utilization of digital computers and advances in allied fields of Mathematical Programming and Operations Research led to the formulation of structural optimization as a problem in Mathematical Programming. Given a set of design variables such as depth, thickness, area, moment of inertia all denoted by a vector X_i the design problem becomes:

Minimize $f(X_i)$ (1)

such that $g_j(X_i) \geq 0$ $j=1,2,...,$ Number of constraints (2)

$f(X_i)$ is a function of weight or cost to be minimized while $g(X_i)$ are the design limitations on stress, stability and deflection or any practical fabrication or construction restrictions. It is necessary to be able to compute for any set of design variables X_i the stresses, deflections and stability associated with this design. Recent papers in the structural engineering literature have presented efficient optimum design techniques for a wide class of problems including plate girders, trusses, frames, stiffened plates and cylinders.[1] These works have used Mathematical Programming techniques such as linear programming, dynamic programming, gradient methods and unconstrained minimization.[2] The Mathematical Programming approach to design may be limited in that unlike Professor Courbon's paper entitled, "Optimization of Structures" it does not consider creative changes in design or even large deviations from an initial prescribed design geometry and topology.[3] Nevertheless, it has led to important economies in structural design and has been used in practice particularly for conventional type structures such as plate girder bridges and frames.

This paper considers the problem of optimization within the context of safety. It has been proposed by Professor Freudenthal that a rational approach to safety must be based on reliability or probability of failure.[4] This leads to a new formulation of an optimum design problem which is:

$$\text{Minimize } f(X_i) \tag{3}$$

$$\text{such that } P_f(X_i) \leq P_f \text{ allowable} \tag{4}$$

P_f allowable is the minimum failure probability acceptable for the design and is assumed in this study as given. $P_f(X_i)$ is a function which gives the overall failure probability of the structure as a function of the design variables to be determined. This constraint on $P_f(X_i)$ is the only constraint used on structural behavior although other constraints may be based on construction or other requirements. Several methods for computing $P_f(X_i)$ and utilizing it in an optimum design procedure are presented. Some factors which have motivated this approach to optimization include the following points:

a) Safety as expressed in terms of probability of survival may actually be impaired in current deterministic optimum design programs. This is because existing design codes and safety factors are used to provide protection against stress, deflection and stability-type failure modes. These safety factors were developed over a period of time in practice and were not associated with structures which were optimized. Most mathematical programming optimum designs end up with a larger number of constraints on stress and deflection against their limit than an unoptimized design. It should be expected, therefore, that the probability of failure which is the probability that any failure mode occurs will be higher for an optimized design. An optimization procedure which uses overall structural failure probability as the behavior constraint should produce more balanced designs consistant with the development of rational safety.

b) In order to reach more significant levels of structural optimization it is necessary to compare optimized structures of different configuration, material and geometry. Within this decision context a rational comparison is possible only if the structures have the same level of safety as expressed in terms of probability of failure.

c) The use of new materials such as brittle composites with greater scatter in strength tests and new structural applications in environments with greater uncertainty suggest that there will be more emphasis in the future on designing for probability levels rather than using preassigned safety factors. It, therefore, seems appropriate to formulate the design optimization problem as in equations 3 and 4.

d) Reliability based optimum design may actually facilitate the mathematical optimization problem by replacing the numerous limitations (on member stress

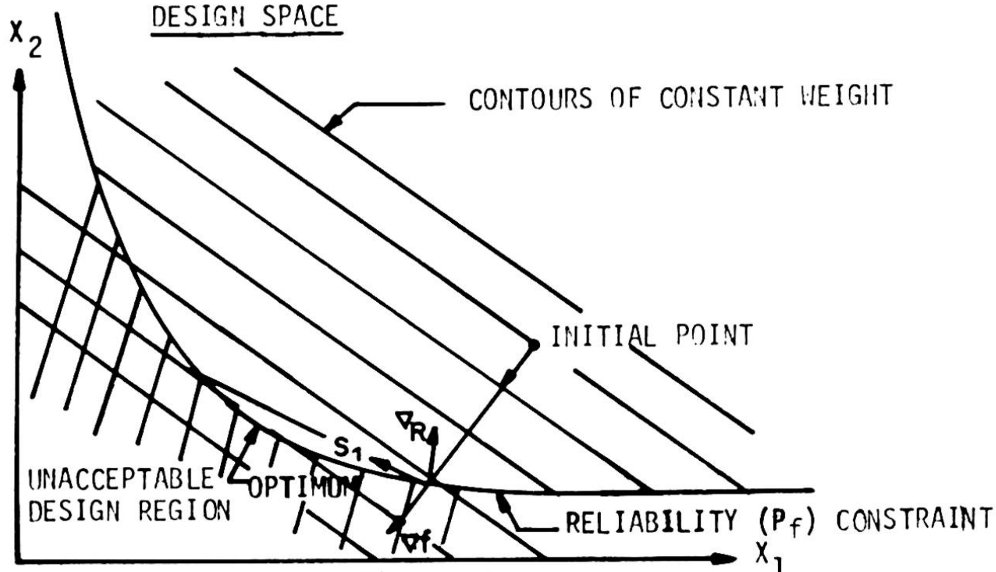

Figure 1

and deflections) in a deterministic design by a single constraint on overall structural failure. The mathematical and computational complexity, however, has been transformed from the minimization aspect to the analysis of failure probability. The problem of minimizing the weight subject to only one constraint as illustrated in Figure 1 with two variables can be handled by several techniques including linear programming approximations and useable feasible gradient moves. Figure 1 illustrates an algorithm for minimization by use of the gradients or normal vectors to the weight function and the reliability constraint. From an arbitrary starting point the design is changed in steps in the direction of the gradient to the weight until the reliability constraint is encountered. Subsequent changes in the design variables are made in a direction s_1 which both reduces weight and avoids violating the reliability constraint. This is a useable feasible direction and methods for determining this direction are well known.[5] The design changes are continued until the constraint gradient and reliability gradient are colinear.

RELIABILITY ANALYSIS AND OPTIMUM DESIGN

Most work in failure probability has concentrated on a problem in which all the strength variability was included in one member and all the load variability was included in one load. Freudenthal presented the probability of failure of this one member one load structure often called the fundamental case of structural reliability including the effect of frequency distributions, standard deviations for load and strength and the safety factor or ratio of mean values.[6] In considering the design or proportioning of members in multi-member multi-load structures, a model is needed to show the effect on failure probability of each of the individual members of the structure and their interaction with all load conditions. Two reliability applications of importance are presented herein. The first is multi-member "weakest link" structures discussed by Professor Freudenthal in which the structure fails if any single element fails under any load condition.[4] The second application is "redundant" structures such as limit designed frames in which failure is the occurrence of any collapse mode each involving more than one element yielding. In both cases loads and strengths are random variables described by known frequency distributions. The reliability analysis computes for a given design the overall failure probability of the structure. The optimum design problem is to proportion member sizes to have minimum weight or cost for a specified allowable failure probability.

"WEAKEST LINK" STRUCTURES

The failure probability of a single member of strength R under a single load condition S can be determined from the following equation:[6]

$$P_f = \int_0^\infty \Pr\{S>\tau\} \Pr\{R=\tau\}\, d\tau = \int_0^\infty [1-F_S(\tau)]\, f_R(\tau)\, d\tau \qquad (5)$$

Pr should be read "probability that". F (τ) is the distribution function and f (τ) the density function. In extensions to multi-member structures under one load condition it has often been proposed that the overall failure probability could be obtained from the following equation:[7,8,9]

$$P_f = 1 - \prod_{i=1}^{N} [(1-P_{fi})] \quad \text{N=Number of members} \qquad (6)$$

P_{fi} is the failure probability of the i^{th} member and P_f the overall failure probability. If the individual P_{fi} are small as is usually the case, then equation 6 becomes:

$$P_f = \sum_{i=1}^{N} P_{fi} \qquad (7)$$

Equation 6 ignores, however, the fact that the events corresponding to member failures are statistically correlated since the stresses in each member are completely correlated since they arise from the same loading. The member failures are not 100% correlated since the strengths are independent random variables. The consideration of correlation loads to a lower value of P_f. If there is only one load condition and N members or element strengths, then it is easy to verify that the equation that gives the failure probability is:

$$P_f = 1 - \int_0^\infty \prod_{i=1}^{N} [1-F_{Ri}(a_i\tau)] \; f_P(\tau)\, d\tau \tag{8}$$

The constant a_i relates the forces or stress levels in member i to the load value of $P=\tau$. Equation 8 is valid for "weakest-link" structures which fail if any member fails. This includes determinate structures and those indeterminate structures with little "fail-safe" probability of survival available after the first member has yielded. This is true for structures with brittle members which can't carry any load after reaching yield load and for all structures for which the load variability greatly exceeds the strength variability. If the structure is subjected to M repeated application of the same loading condition then P_f can be computed by integrating on the density function of the worst load which is:

$$f_{P_{max}}(\tau) = M\,[F_P(\tau)]^{M-1}\, f_P(\tau) \tag{9}$$

The failure probability is then:

$$P_f = 1 - \int_0^\infty \prod_{i=1}^{N} [1-F_{Ri}\,(a_i\,\tau)]\, f_{P_{max}}\,(\tau)\, d\tau \tag{10}$$

If all loads are not of the same load condition but represent distinct load conditions applied at different times then an exact solution for P_f requires an evaluation of a multiple integral based on the joint distribution function of the load conditions. Various bounds have been presented on the failure probability based on evaluating integrals which reflect the importance of statistical dependence between failure modes due to a single load condition on multiple members or a single member acted on by distinct load conditions.[10,11]

An important factor in reliability design of "weakest link" structures is whether the design constraint is based on equation 10 which is exact for the case shown or equation 6 which is an upper bound on P_f and ignores the statistical correlation between failure modes. A previous study showed that this correlation significantly affects the reliability analysis if the variability of the load random variable exceeds that of the strength as in structures designed to resist as their major loading wind and earthquakes.[11] In such instances, a lower bound on P_f which is the largest member failure probability may be used as the design constraint.

In studying optimum design of "weakest-link" structures two factors were under observation. One was the effect of statistical correlation between failure modes on the overall structural weight and the second is the influence of unequal individual member failure probabilities. It should be noted that all previous studies on reliability based optimums have used equation 6 as the basis for computing the failure probability and thus have ignored the correlation.[7,8,9]. To study the effect of correlation, a design is found for a truss with one load condition. All members are assumed to have equal mean loads and, therefore, have the same optimum area. The consideration of the exact value of the P_f constraint in equation 8 including statistical correlation allows each member to be designed for a higher individual failure probability than if equation 6 were used as the constraint and correlation ignored. The higher individual failure probability means a lower weight and the ratio of the two optimum weights based on equation 8 and equation 6 is plotted in Figure 2. With a normal frequency distribution of load and strength for the coefficients of variation shown the maximum weight saving reaches 7.3% for case 1 in a 50 member structure.

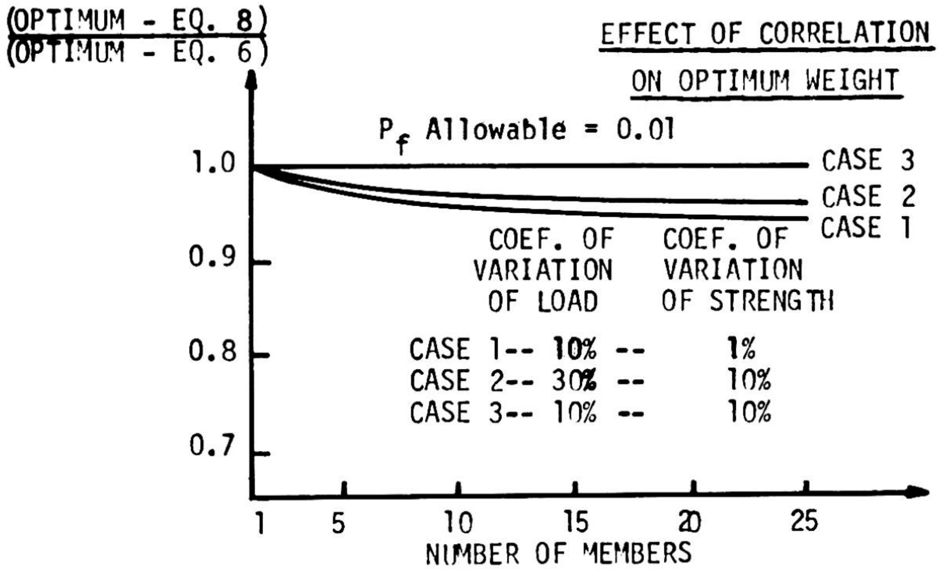

Figure 2

Another indication of the correlation factor becomes evident when the overall failure probability is written as:

$$P_f = a_1 P_{f_1} + a_2 P_{f_2} + ..+ a_i P_{fi} + ...+ a_N P_{fN} \qquad (11)$$

P_{fi} is the failure probability of the i^{th} member and a_i is the percentage of its failure probability that this member contributes to the overall probability. The a_i shown in Figure 3 are computed either by sequentially integrating equation 8 for increasing number of members or by another technique discussed elsewhere.[11] If there were no statistical correlation all a_i would equal 1.0. If there was complete correlation between failure modes and the members were ordered with member 1 having the highest failure probability then a_1 would be 1.0 and all other a's equal zero.

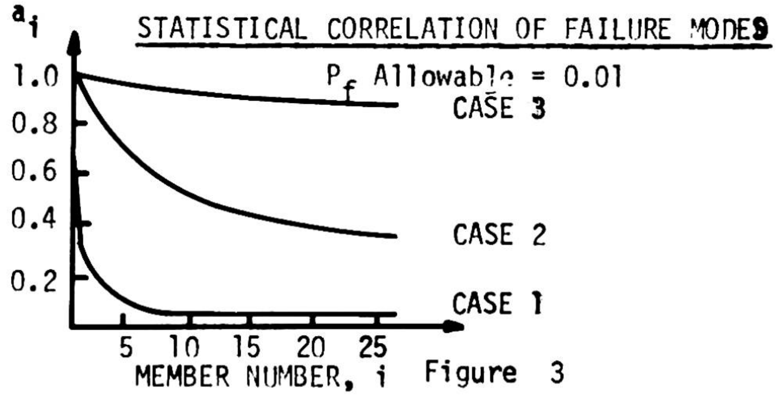

Figure 3

Figure 3 shows the potential weight saving with other frequency distributions and members with unequal lengths and mean applied loads since it indicates the correlation effect and the error introduced into the overall probability expression as the number of members in a structure increases. The magnitude of the weight saving can be obtained from plots of member failure probability vs. member size (or equivalently its safety factor) and is available from the work on the single member single load fundamental case.[6] It is seen from Figure 3 that the correlation factor becomes more significant when the load variability exceeds the strength variability as in structures subject to wind and earthquake loading.

The second factor mentioned above for study is "weakest-link" structures with members of unequal mean loads. An optimum design in this case has unequal safety factor and failure probability for each member. In general minimum weight results if heavier members have higher than the average failure probability and lighter member with lower mean applied loads or stresses have less than one average member failure probability. It has been proposed that the following equation applies to an optimum design.[8]

$$\frac{\text{Weight of member } i}{\text{Total weight}} = \frac{P_{fi}}{P_{f\ \text{allowable}}} \tag{12}$$

This equation was derived and is applicable to cases where equation 7 for P_f is used as the design constraint. Furthermore, it implies that the element P_{fi} depends only on the size of the i^{th} element, or:

$$\frac{\partial P_f}{\partial x_i} = \frac{\partial P_{fi}}{\partial x_i} \tag{13}$$

In general, for a statically indeterminate structure this last equation is not applicable and:

$$\frac{\partial P_f}{\partial x_i} = \sum_{k=1}^{N} \frac{\partial P_{fk}}{\partial x_i} \tag{14}$$

That is the change in any member affects the force distribution and, therefore, the failure probability of every indeterminate member of the structure. It should be noted that the simplicity that equation 7 introduces into the optimization procedure is not lost when the correlation is included in the computation of the failure probability. This simplicity is needed since most optimization methods need the gradient to the constraint. Using equation 11, the components of the gradient of the reliability constraint can be computed as:

$$\frac{\partial P_f}{\partial x_i} = \sum_{k=1}^{N} a_k \frac{\partial P_{fk}}{\partial x_i} \tag{15}$$

It has been found from experience that the a_k which accounts for the correlation does not change much in a small region in which the gradient is determined from the partial derivatives using a finite difference technique. Thus the gradient based on N computations of the form shown in equation 15 need not be obtained from finite difference perturbations of equation 8 of N members and one load but rather from equation 5 which is for one member and one load. The computation time saved in this manner may be significant.

To illustrate an optimum design for a structure with unequal member sizes Table 1 is presented for a determinate truss with 10 members. The formulation of the design problem as a minimum weight design with a failure probability constraint as indicated in equations 3 and 4 was used. Equation 12 can be seen not to be valid at the optimum due to correlation between failure modes not considered in its derivation. Also shown in Table 1 is a design based on equal safety factor for each member such that the overall failure probability of the structure based on equation 7 is equal to the allowable value. The difference between the weight of the optimum design and the equal safety factor design is partly due to the correlation factor but principally due to the Mathematical Programming technique discussed above which proportions the members in an optimum manner using only the single overall failure probability constraint.

TABLE 1 - 10 Member Example[a]

Member	Mean Load Value	Equal Safety Factor: Area in.^{2}	Equal Safety Factor: P_{fi}	Optimum Design: Area in.^{2}	Optimum Design: P_{fi}
1	0.1P	0.274	0.0001	0.297	0.519×10^{-4}
2	0.2P	0.547	0.0001	0.554	0.604×10^{-4}
3	0.3P	0.817	0.0001	0.818	0.958×10^{-4}
4	0.4P	1.09	0.0001	1.09	0.991×10^{-4}
5	0.5P	1.37	0.0001	1.35	1.23×10^{-4}
6	0.6P	1.64	0.0001	1.61	1.61×10^{-4}
7	0.7P	1.92	0.0001	1.86	2.08×10^{-4}
8	0.8P	2.19	0.0001	2.11	2.65×10^{-4}
9	0.9P	2.46	0.0001	2.35	3.25×10^{-4}
10	1.0P	2.74	0.0001	2.59	3.91×10^{-4}

a Mean Load, $\overline{P}$=60,000 lb.; C.V.(P) = 20%
Mean Yield stress, $\overline{\sigma_y}$=40,000 psi; C.V. (σ_y) = 5%
Density = 0.283 lb/in.^{3};
Length L_i=60" for all members.
Both P and σ_y have normal distributions

Weight based on equal safety design factor = 255.6 lb.
Weight based on optimum design = 248.6 lb.
P_f allowable is 0.001

"REDUNDANT" STRUCTURES

In many structures particularly those designed by limit or ultimate design methods several members or elements must simultaneously reach their capacity before the structure is failed. This is the case with some indeterminate trusses and also beams and frames in which mechanisms form at failure. It is assumed that each element strength is an independent random variable.[12]

A failure mechanism occurs if the contribution of load elements exceeds the strength elements for any particular collapse mode. If the contributions are linear this leads to an equation for reserve strength Z_j in a mode j of:

$$Z_j = \sum_{i=1}^{n} a_{ji} M_i - \sum_{k=1}^{L} b_{jk} P_k \qquad \begin{array}{l} i=1, .., n - \text{Critical Elements} \\ k=1, .., L - \text{Loads} \\ j=1, .., m - \text{Collapse Modes} \end{array} \tag{16}$$

Equation 16 would for example govern a frame against the formation of a collapse mechanism. The overall failure probability of the structure is the probability that any Z_j is less than zero and can be written as:

$$P_f = Pr\{Z_1 \le 0\} + Pr\{Z_2 \le 0, Z_1 > 0\} + Pr\{Z_3 \le 0, Z_2 > 0\ Z_1 \ge 0\} + \dots \tag{17}$$

A method presented elsewhere has been developed to compute the probability that each Z_j is less than zero, including the effect of statistical correlation between Z_j present because some load and strength terms appear in more than one collapse equation.[13]

As an example of an optimum redundant design consider a single bent frame with design variables corresponding to the plastic moment capacity of the beam and column. Figure 4 shows the deterministic constraints based on a safety factor approach and the constraint based on failure probability as expressed in equation 17. The weight function used which is shown linearized in Figure 4 is:

$$W = K \sum_{i=1}^{R} (M_i)^{2/3} L_i \tag{18}$$

M_i is the plastic moment capacity, L_i the length of the member, R the number of members and K is a constant.

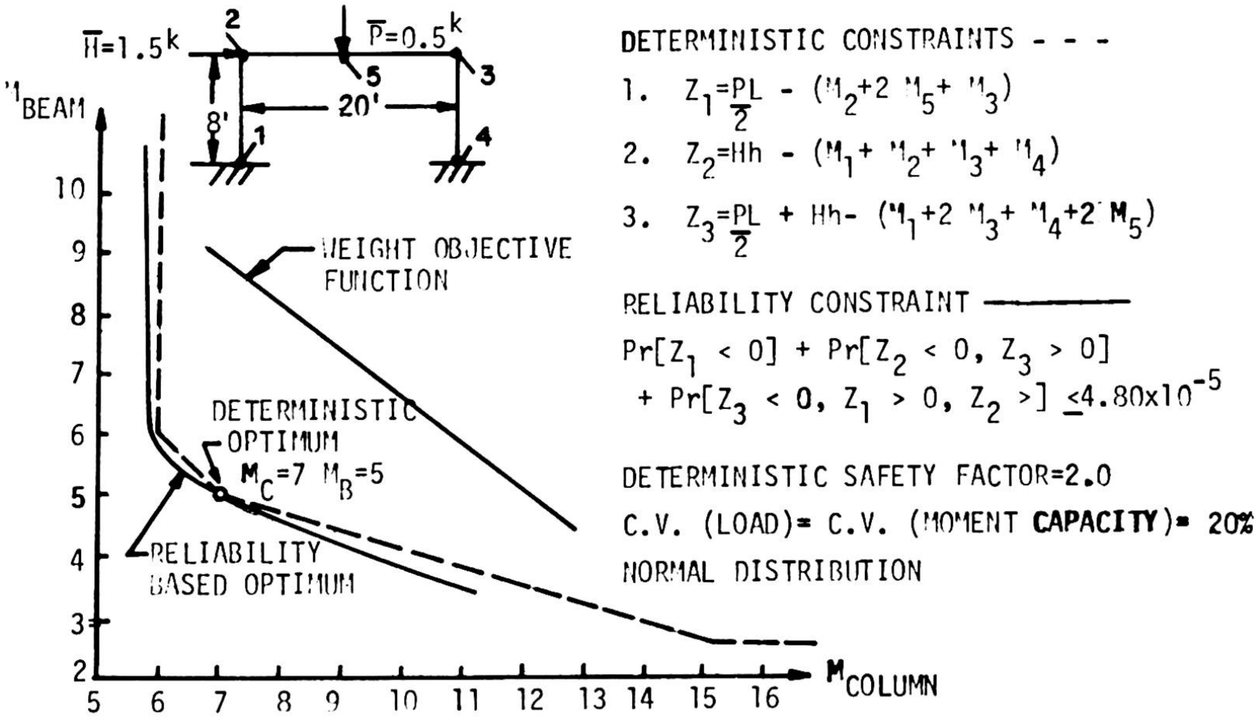

Figure 4 DESIGN SPACE FOR BENT FRAME

The allowable failure probability was set at 4.80×10^{-5} and was chosen based on a failure probability analysis of the optimum design obtained using the deterministic or nonstatistical constraints. Optimum design methods for frames to resist plastic collapse in the deterministic case are well known and since all constraints are linear, a linear programming technique is applicable. A result of a deterministic optimum design is that a theorem for this case shows that the number of collapse modes designed up against their limit in the optimum design equals the number of design variables which is the number of unknown member plastic moment capacities.[14] Although this is acceptable from a deterministic viewpoint and does not violate conventional safety factors, it must be viewed as unsafe from a reliability design viewpoint. The fact that an optimum design with a large number of design variables has an equally large number of failure modes designed to their limit must indicate that the failure probability is increased over a conventional unoptimized safety factor limited design. Consequentially, the replacement of the deterministic constraints by a single reliability constraint should lead to a more balanced optimum and also allows the failure probability to be specified in the constraint. An added benefit that may be seen from Figure 4 is that the design optimized with respect to failure probability is lower in weight than the deterministic optimum with the same failure probability. Further examples of optimum single story frames showed the optimum weight increased as both the allowable failure probability was decreased and the coefficients of variation of load and strength was increased.[15]

To illustrate reliability based optimum design for larger redundant structures the frame shown in Figure 5 with six design variables was studied.

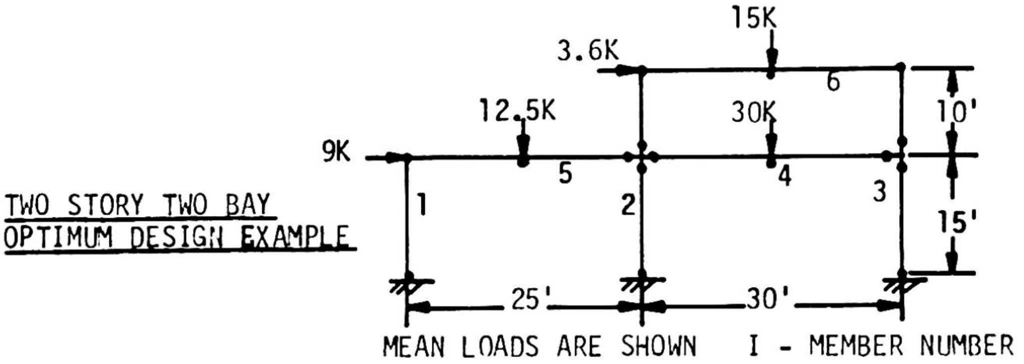

Figure 5

The resulting design moment capacities for the frame in Figure 5 is shown in Table 2 for designs with normal and log normal frequency distribution for load and strength of different coefficients of variation. It is interesting to observe as shown in Table 2 that at least for the examples studied one mode seems to dominate in having a relatively larger failure probability than the other modes.[13]

Table 2

Optimum Design Results of Two Story Two Bay Frame Shown in Figure 4

Example No.	Optimum Design Moment Capacities K-ft Members 1 2 3 / 4 5 6	C.V. Mom. Load %	P_f Allowable	Weight Function	Frequency Distribution	Individaul Collapse Mode Failure Probability in Order of Largest Value First 1 / 4	2 / 5	3
1	29.2 95.8 84.4	0.10	7.78(2)*	312.5	Normal	6.70(2)	6.87(3)	4.50(3)
	175.0 73.2 74.4	0.20				6.26(4)	3.16(4)	
2	27.8 96.3 84.4	0.10	7.80(2)	310.9	Log Normal	4.85(2)	1.55(2)	1.35(2)
	173.8 72.0 77.9	0.20				2.21(3)	1.67(3)	
3	28.0 78.7 71.0	0.20	7.72(2)	297.3	Normal	4.94(2)	1.22(2)	1.16(2)
	170.9 69.4 74.9	0.10				2.81(3)	2.68(3)	
4	27.3 78.3 71.3	0.20	7.16(2)	293.5	Log Normal	4.19(2)	1.64(2)	1.09(2)
	166.4 65.1 74.9	0.20				3.13(3)	2.25(3)	
5	29.1 87.8 72.3	0.15	7.52(2)	300.6	Normal	5.39(2)	1.10(2)	9.89(3)
	170.3 68.0 74.1	0.15				1.71(3)	1.69(3)	

* Exponents of failure probability are shown in parenthesis (m) and should be read as 10-m

DISCUSSION AND CONCLUSIONS

1] The results presented indicate the feasibility of using reliability or probability of failure constraints in solving for optimum multi-member structural designs. By using Mathematical Programming methods to proportion member sizes a design is obtained which has an overall failure probability equal to an allowable value. This approach to design appears more rational than many current optimum design methods which use conventional code safety factors to restrict member dimensions based on stress and deflection limitations. As a result such optimum designs end up with many element constraints active which from a reliability viewpoint reduces its safety below a conventionally unoptimized design. Two examples presented include "weakest-link" structures for which any member failure constitutes failure of the structure and "redundant" structures which fail by forming collapse mechanisms after several members have simultaneously yielded.

2] It is seen from the examples presented that a reliability based optimum design does not have equal safety factor for all elements. In a "weakest-link" structure the heavier members have higher failure probability values than lighter members. This factor is influenced by the degree of statistical correlation between member failures which depends on the ratio of the variability or coefficient of variation of the load to the strength. In an optimum "redundant" structure such as a frame designed to resist formation of a collapse mechanism, the same safety factor is not present for each mechanism at the optimum design. Rather the Mathematical Programming method proportions each member to achieve minimum weight within the constraint of overall failure probability. For the frames studied it was observed that one particular mechanism in an optimum design dominates in its value of failure probability but it is not possible to choose beforehand which mechanism this will be.

3] An important factor influencing the magnitude of the optimum design as well as its member sizes will be the choice of load and strength frequency distributions and their parameters particularly the coefficients of variation. Curves of cost vs. statistical parameters show choice of frequency distribution is not too critical unless the distribution is highly skewed. Another important factor is the choice of an allowable failure probability. This should depend on the function of the structure as well as the failure consequences in social and economic terms and is not considered herein.[16]

4] The computation of failure probability for any frequency distribution as presented herein and its incorporation in an optimum design procedure should stimulate studies of random variables encountered in structural engineering to improve their description. Empirical studies are needed to provide reasonable frequency distributions for static strength, fatigue life, creep rate, floor loading as well as stochastic theories for dynamic phenomena associated with wind, highway and earthquake loading. The use of optimum design techniques illustrated herein should be useful in assessing the importance of changes in the parameters of these frequency distributions in terms of optimum cost or weight rather than in terms of predicted failure probabilities. Failure probabilities are usually expressed in quantities of 10^{-2} to 10^{-9} and small changes in frequency distribution may cause large changes in failure probability. However, the change in optimum weight associated with this change in frequency distribution may be of smaller magnitude.

5] A truly optimum design should consider the behavior of the structure over various types of loading conditions as well as possible strength deteriorations. In a more extensive approach under study an optimum design is to be found which considers all levels of failure including yielding, formation of cracks, large deflections, instability and collapse. Although for some "weakest-link" structures yielding and collapse occur simultaneously, this is not true for most structures.[17] One approach to this problem would be to impose an allowable failure probability which depends on the damage for each failure level and to seek an optimum design which satisfies all probability of failure constraints. Another approach is to combine the constraints into one reliability constraint which would contain the probability of a level of failure occurring multiplied by a factor which includes the associated damage.

6] One question often raised in consideration of failure probability analysis is the meaningfulness of a statement that the failure probability is 0.001 since not enough data could conceivably exist to support this claim. It is easy, however, to see that a statement that P_f of structure A is 0.001 and of structure B 0.00001 is meaningful.[18] In the light of optimum design the P_f statements take on further meaning since they influence the overall structural cost. Furthermore, in a single design it would be possible to use various combinations of P_f allowables, coefficients of variation of load and strength to see how they influence the overall cost. The final structural cost would be based on the best estimate of these parameters in the light of previous experience, empirical studies of existing structures and other economic factors.

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SUMMARY

An optimization procedure is presented in which safety in terms of reliability or probability of failure is used as the controlling design limitation for finding minimum weight structures. Reliability analysis and optimum proportioning in multimember structures is given for "weakest-link" and redundant cases. Examples illustrate the effect on optimum weight of frequency distributions, coefficients of variation and allowable failure probability. Some aspects of the reliability analysis problem are discussed.

RÉSUMÉ

Le rapport présente un procédé d'optimation où la sécurité, en termes d'endurance et de probabilité de ruine, détermine le dimensionnement minimal d'une construction. Dans les structures à éléments multiples, l'analyse de la sécurité et le dimensionnement optimal sont déterminés par le membre le plus faible et par les conditions extrêmes exagérées. Des exemples expliquent l'effet de la répartition des fréquences, des coefficients de variation et de la probabilité de ruine acceptable sur le dimensionnement optimal. Quelques aspects du problème de l'analyse de la sécurité sont discutés.

ZUSAMMENFASSUNG

In diesem Beitrag wird ein Optimierungsverfahren vorgestellt, in welchem der Sicherheitsbegriff in Zuverlässigkeits- oder Wahrscheinlichkeitsraten des Bruches zur Kontrollbegrenzung des minimalen Gewichtes gebraucht wird. Die Zuverlässigkeitsanalyse und die Optimierung (vielstäbiger) hochgradiger Bauwerke ist für das schwächste Glied und extreme Fälle durchgeführt worden. Beispiele zeigen die Wirkung der Häufigkeitsverteilungen, der Streuungsmasse und der zulässigen Bruchwahrscheinlichkeit auf das minimale Gewicht. Einige Merkmale der Zuverlässigkeitsanalyse werden besprochen.