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The Influence of Member Quality on the Safety of Space Trusses

L'influence de la qualité des éléments sur la sécurité de treillis spatiaux

Einfluss der Qualität der Bauelemente auf die Sicherheit von Raumfachwerken

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1. INTRODUCTION

In this paper an answer is sought to the question "How does the admissable load of a double-layer grid change, when it contains a certain percentage of 'inferior' members"? In this context, 'inferior' means that for the load-deflection relation of the member concerned the valid situation is that of figure 1b, and not that of figure 1a.
It is assumed that such inferior members retain their stiffness, do not break and are distributed over the entire construction in a random manner.

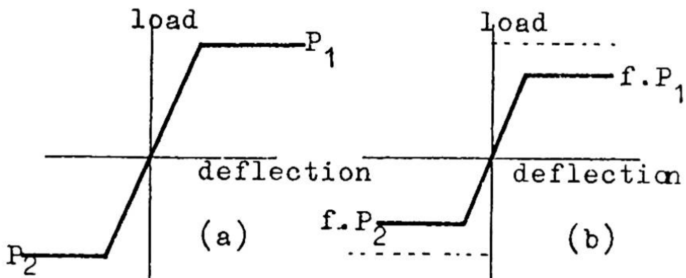

figure 1

In the research project, two apporaches were used: (a) a Monte Carlo simulation programme based on the displacement method; (b) a theoretical approach based on probability calculus.

2. MONTE CARLO SIMULATION

Whereas for the double-layer grid not only the probability of collapse had to be determined but the probability distribution of collapse loads had also to be found, in order to compare it subsequently with the probability distribution obtained theoretically, it was decided to perform an analogous simulation. The variable was the location of inferior members (random), and our assumptions related to their number and degree of inferiority (= factor f in figure 1b). It was furthermore assumed that in respect of all the members f be equal.
To enable this analogous simulation to be performed, we designed a computer programme, based on the displacement method, which programme calculates the collapse load of a three-dimensional pin-jointed truss by an incremental method.

To make this programme useful for practice, i.e. to keep the calculation time for each collapse load low, we derived a modification algorithm which, with regard to a yielding or a buckling member, permitted the pertinent absence of stiffness to be accounted for in the load vector as an imaginary extra load (figure 2), and not in the stiffness matrix. This extra load, $\Delta \underline{k}$, may be calculated from:

$$\left[\Delta S^{-1} - S^{-1}\right]_{m*m} \cdot \Delta \underline{k} = \underline{v} \qquad \ldots\ldots (1)$$

where:

ΔS = change in stiffness matrix because of yielding or buckling of a member (stiffness of the member)
S = terms of original stiffness matrix belonging with collapsed member
$\underline{v}$ = previous displacement vector
m = number of collapsed members.

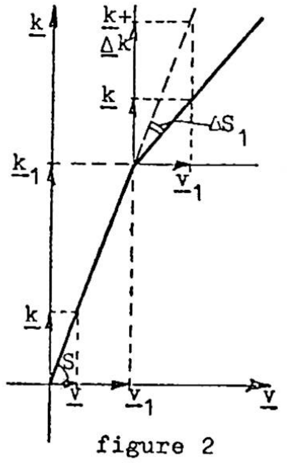

figure 2

From eq (1), we could easily find $\Delta \underline{k}$, because $[\Delta S^{-1} - S^{-1}]$ represents a symmetric matrix (Maxwell). We only had to add one extra column to the matrix for each subsequent member collapsed.
For inversion of the new matrix, the inverse of the preceding one was used (bordering method [1]). Based on this algorithm [2], a very rapid computer programme was evolved. A double-layer grid with 682 elements and 211 Knots (about 600 equations with a half-bandwidth of 39) called for about one minute of calculation time on a CDC 6600 computer for each collapse load calculated (about 50 collapsed members), this is hardly more than a normal elastic calculation.
With the aid of this programme, we made calculations for some types of double-layer grids of various sizes, with various percentages of inferior members and different factors of weakening. The results have inter alia been used to verify whether a statistical approach of the problem was valid.

3. THEORETICAL DERIVATION

It was found possible to approach the problem theoretically, when the structure has one or more clearly discernible mechanisms of collapse in the very state of collapse. In the double-layer grid of figure 3, we distinguish two shear mechanisms and one yield mechanism. We assume that the members do not break and that their connections are stronger than the members themselves.
We now schematize the structure in its state of collapse as is shown in figure 4. The elements in a collapse mechanism we call "critical" elements. For each collapse mechanism, the probability g, that k critical, inferior members (or that k inferior, critical elements) are concerned is found from:

$$g(k) = \frac{\binom{K}{k}\binom{N-K}{S-k}}{\binom{N}{S}} \qquad \ldots\ldots (2)$$

where N is the number of elements, S the number of inferior, K that of critical elements, and $\binom{K}{k} = \frac{K!}{(K-k)! * k!}$

With each value of k belongs a collapse load, so that the pertinent probability distribution, i.e. the hypergeometric distribution, is at the same time the probability distribution of the collapse load of the mechanism considered.

When the number of critical members is small, over against the number of inferior ones, eq. may be approximated through a binominal distribution; with $P = \frac{K}{N}$ we then find:

$$g(k) = \binom{S}{k} (P)^k (1 - P)^{S - k} \qquad \ldots\ldots (3)$$

When (N - K) > 50 and P < 0.2, the binomial distribution changes to a Poisson distribution:

$$g(k) = \frac{\left(\frac{S.K}{N}\right)^k}{k!} e^{-\left(\frac{S.K}{N}\right)} \qquad \ldots\ldots (4)$$

And when P > 0.1 and P(1 - P) K > 8, the binomial distribution changes to the normal distribution:

$$\mu = \frac{S.K}{N} \quad \text{and} \quad \sigma = \sqrt{P(1 - P)K} \qquad \ldots\ldots (5)$$

When, over against the number of critical members, that of the inferior ones is small, we find the same equations except that S and K change places.

Of these latter two distributions (4) and (5), the summated probabilities g(1) + g(2) + ... + g(k) may be read from a table.

The procedure for calculating the sought probability of collapse may be itemised, with reference to figure 3, as follows:

1. For each individual mechanism of collapse, determine the collapse load; this load will then be in equilibrium with the total load bearing capacity of the critical members.

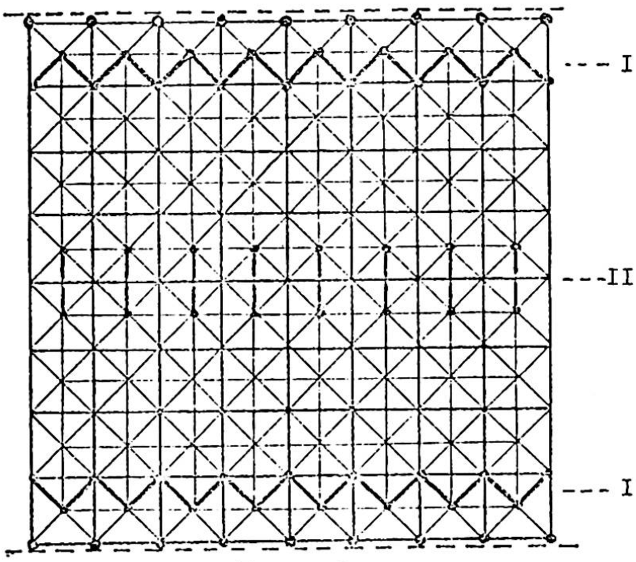

figure 3

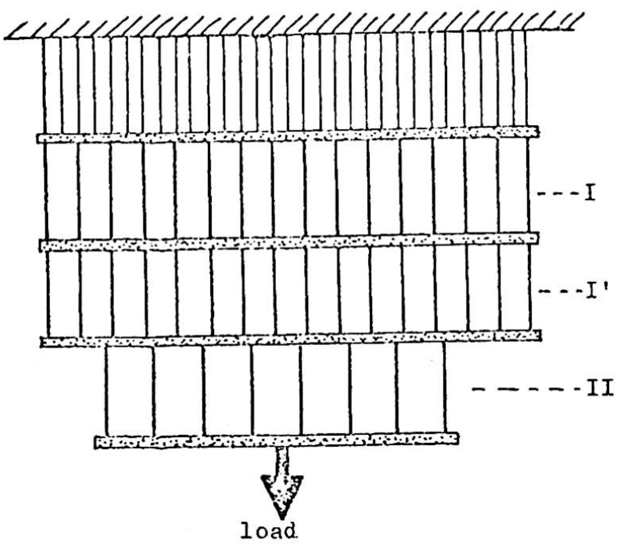

figure 4

2. For each individual collapse mechanism, the lower bound of the probability distribution of the collapse load; all the inferior members are in the mechanism concerned (LB(1) and LB(2) in fig. 5). Primarily we had assumed the factor of weakening, f, to be equal for all the members. If for each collapse mechanism, in other words: for each type of member, this factor is the same then only the lower bound of the distribution is subject to change.
3. From the results found in 2, calculate the number of members that should be inferior, at least, in order to enable the finding of a contribution towards a probability of collapse, i.e. the number of intervals between $B_{collapse}$ and B_{use} (B = load) when the distance between $B_{collapse}$ and the lower bound is apportioned into as many intervals as there are members in the mechanism.
4. Calculate the probability of collapse for each individual mechanism. Figure 5 presents an elucidation. Beside the load-deflection diagram for the structure, it shows the probability distribution of the collapse load of the two mechanisms sketched. For each mechanism, the probability distribution equals the shaded area of the curve indicating the part of probability distribution that lies below the permissible load. The probabilities that either mechanism I or mechanism I' (cf figure 4) occurs are summated to $P_{(1)}$.

We now know the probabilities of occurrence for the individual mechanisms. We will next find the collapse probability, of the entire structure, if we may assume that the mechanisms do not affect one another, i.e. are stochastically independent, with the aid of:

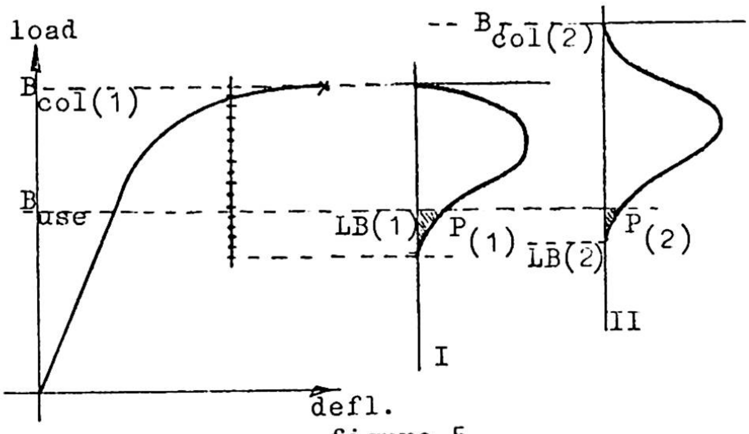

figure 5

$$P_{(collapse)} = P_{(1)} + P_{(2)} - P_{(1)} \cdot P_{(2)} \qquad \ldots\ldots (6)$$

where:

$P_{(1)}$ = the collapse probability of mechanism 1;
$P_{(2)}$ = the collapse probability of mechanism 2;
$P_{(1)} \cdot P_{(2)}$ = the probability that mechanisms 1 and 2 occur simultaneously.

When the number of inferior members is about equal to the number of members in each mechanism, the probability of simultaneous occurrence of several mechanisms is nil. Accordingly, $P_{(1)} \cdot P_{(2)}$ will be larger or smaller in dependence upon N, K and S.

It should be noted that eq. (6) applies to uncorrelated mechanisms only. However, different forms of correlation are possible. In fact, members may belong to several mechanisms of collapse at the same time; deformations arising from the collapse of members in one mechanism may affect another mechanism; members from adjacent mechanisms may influence each other, and so on. Of these forms of correlation we shall here discuss the last-mentioned one only.

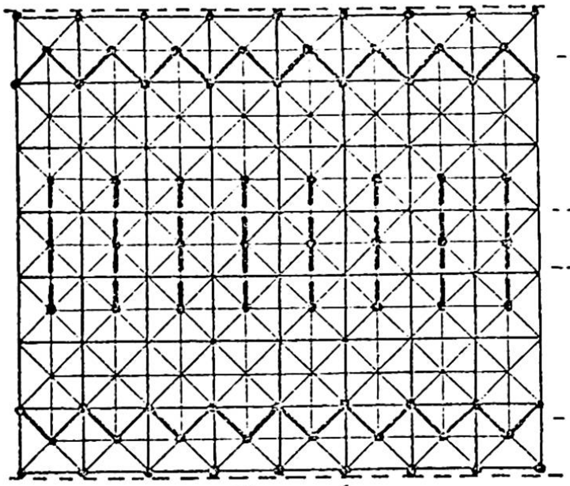

figure 6

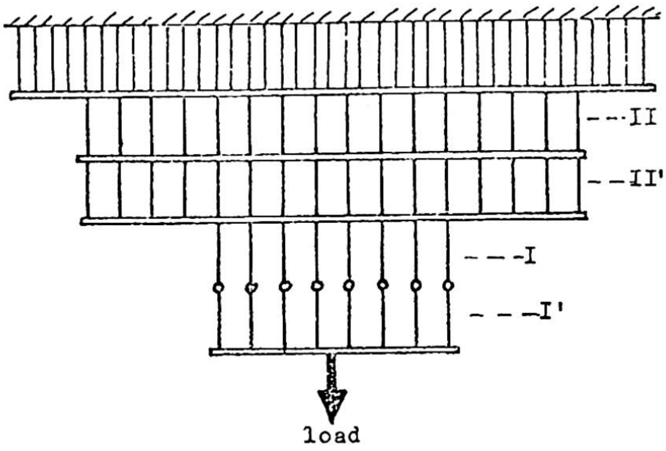

figure 7

When (cf figure 6) a member from mechanism I has collapsed, the adjacent member from mechanism I' is relieved and this remains whole.
For this structure, figure 7 then applies. The probability of collapse may be determined for the "complex" mechanism, cf I and I' in figure 7.
For the example in figure 6, the collapse load of the non-weakened mechanism is 323 kgf/m^2. The permissible load is assumed to be 175 kgf/m^2. The number of members N = 448, K = 16 and; when we take f = 0.5 and S = 200, we find:

1. for the number of strings of members that must be inferior:
$$\frac{f \cdot x + 8 - x}{8} \cdot 323 = 178; \text{ so that } x = 8;$$
2. for the probability that there are 8,9 ...i...16 inferior members in the mechanism:
$$g(i) = \frac{\binom{K}{i}\binom{N-K}{S-i}}{\binom{N}{S}}$$
3. for the probability that 8 inferior strings of members occur for i inferior members:
$$h(i) = \frac{\binom{8}{i-8} \cdot 2^{(16-i)}}{\binom{16}{i}}$$

As a result, we find for the collapse probability of mechanism: I - I':
$$P_{(collapse)} = \sum_{i=8}^{16} g(i) \cdot h(i) = 0.052$$
For a shear mechanism we thus find that at least eleven elements must be inferior: $g(11) + g(12) + \ldots + g(16) = 0.042$
Assuming that the two shear mechanisms are independent, we find for the probability of collapse:
$$P_{(collapse)} = 0.042 + 0.042 - 0.042 \cdot 0.042 = 0.082$$
Analogously, the combination of mechanisms I - I' and II - II' yields:
$$P_{(collapse)} = 0.130$$
Using the Monte Carlo simulation programme, we found a collapse probability of 0.139, which - considering that only the major mechanisms had been concerned in the calculation - indicates that we found an acceptable answer.

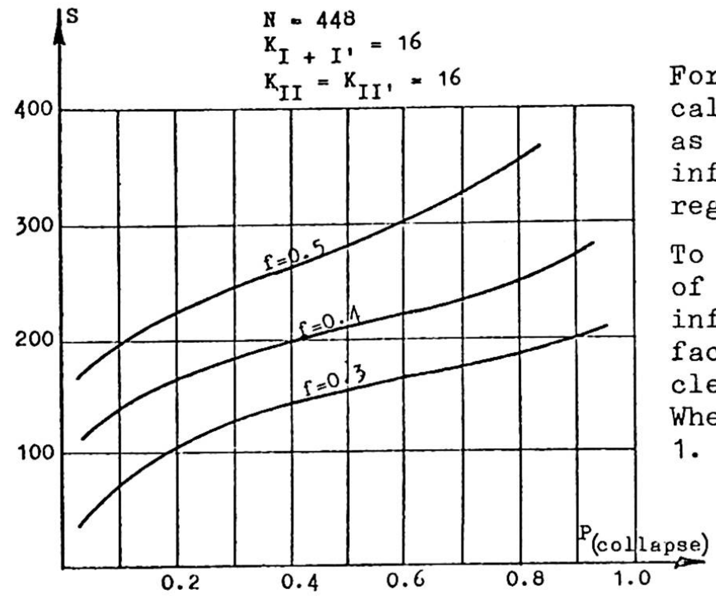

figure 8

For this trial structure, we finally calculated the collapse probabilities, as a function of the number of inferior members (cf figure 8), with regard to several factors of weakening.

To yield a reasonably high probability of collapse, either the number of inferior members must be large or the factor of weakness very low; this is clear from the curve.
Whereas:
1. At a constant percentage of inferior and critical members, the probability of collapse considerably reduces when the structure grows in size;
2. In practice, this type of grids is usually supported on four sides, so that the number of critical members goes up and, consequently, the probability of collapse diminishes, we suggest that the following conclusion is appropriate.

4. CONCLUSION

Members with, for example, welding defects affect the safety of the investigated multiple staticaly indeterminate double-layer grids very slightly.

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SUMMARY

Using an analogous Monte Carlo simulation programme that is based on the displacement method it is shown that a statistic prediction can be made about the probability that double-layer grids in which "inferior" members occur, for example due to welding defects, may collapse. For a trial structure is next calculated the probability of collapse with regard to several factors of weakening as a function of the number of inferior members used. It is found that, to yield a reasonably high probability of collapse, either the number of inferior members should be very large or the members very bad indeed.