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## Basic Optimum Design Diagrams of Highway Plate Girders

Diagrammes fondamentaux pour le calcul optimum de ponts-poutres à âme pleine

Grunddiagramme für den optimalen Entwurf von Vollwand-Brückenträgern

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### 1. INTRODUCTION

In the structural system optimization process multistage systematic comparisons and decisions in such as behavior analysis, fabrication, erection, appearance etc. have to be made with regard to a number of optimized solutions of a wide variety of different structural schemes. For the progress of such a design process, the developments of the optimum design programs for specific structures are the essential efforts. The design approaches centered on this problem have been developed principally based upon the use of a combination of mathematical programming or rigorous numerical search techniques and structural analysis method. However the man-machine communication using these sophisticated optimization programs causes numerical complexity and large computer costs considerably in the large scale structural systems, accordingly, to avoid the difficulties and to make a realistic large scale system optimization process reliable, more practical and efficient design approaches have been expected [7,8].

On this problem a simplified optimum design process for minimum cost design of constant-depth highway plate girders is presented in this paper on the basis of the data manipulation of system optimum design diagrams developed by the graphical optimum design method [1,2] and the fundamental characteristics of the diagrams are discussed.

### 2. DESIGN CONDITIONS

The bridge types to be objective in this paper are 1st class simple span, 2 and 3 span continuous highway welded plate girders with constant web height throughout the bridge length. The range of bridge length, width, span ratio and the number of segments are indicated in Table 1. The arrangements of the main girders and thicknesses of reinforced concrete slabs are assumed as shown in Fig. 1 and Table 2 with due regard to the load distributions and appearances of the bridge superstructures.

The design variables to be determined are the steel type,  $M$ , all sectional dimensions,  $X$ , and the length,  $l$ , to be used for each girder segment.

Design criteria imposed on the steel girder section are constraints on allowable stresses, plate thicknesses for stabilities of the girder and minimum rigidities of vertical and horizontal stiffeners, and which, as well as the displacement constraints of the girder, are taken from the "Specifications for Steel Highway Bridges" [5]. Discrete constraints on commercial availability of plate thicknesses are also considered. The steel types available for the design are assumed as SS41, SM50 and SM58 steels which physical and economical characteristics are described in the Specifications and the "Table of Prime Costs for Steel Highway Bridges" [6] respectively.

The total cost of the girder,  $TCOST$ , is assumed to consist of material cost,  $CM$ , fabrication cost,  $CFF$ , and welding cost,  $CWF$ , which are evaluated with reference to Ref. [6].

$$TCOST = \sum_{i=1}^{NS} COST_i \times l_i$$

$$= \sum_{i=1}^{NS} [CM_i(X, BP, SP, EX) + CFF_i(X, KS, CP) + CWF_i(X, WL, CP)] \times l_i \quad (1)$$

in which NS = number of segments, COST = minimum cost per unit length of girder segment, BP = base price of steel plate per unit weight, SP = extra charge for steel type, EX = extra charge for plate thickness and width, KS = number of workmen to fabricate unit weight of steel plate except welding, CP = daily wage of a workman, WL = welding length in specific size. SP and the charge for pre-heating in welding vary with steel type used.

3. EFFECTS OF PRICE RATIOS

Since the optimum solution and the total cost of a girder with specific bridge length and width are complex functions of the design factors such as X, BP, EX, CP, KS, etc. as described in the previous section, it seems to be considerably difficult to presume the effects to the optimum solutions due to the changes of the design factors. However the ratio of CP to BP may be supposed to be the most effective factors. Consequently at first stage of this study the effects due to the changes of unit price ratios CP/BP to the optimum solutions are investigated.

Table 3 shows the examples of optimum solutions for the girders at various price ratios. As seen clearly in this table the optimum steel type, M, moment of inertia, I, and length, L, to be used for each girder segment are extremely insensitive to a wide variety of the unit price ratios except the case CP/BP=0.0 which is an unconsiderable case in the practical design problems. These phenomena can be seen at every span lengths and bridge types in the minimum cost design of highway plate girders and it has been cleared that the optimum design diagrams presented in the following sections may be applicable for a wide range of the ratios of CP to BP with sufficient accuracy.

4. SL(BL)-OPTIMUM WH RELATIONSHIPS

The minimum total cost of the plate girder with specific SL, BW and NS is changed concavely with web height on the whole, but locally several minimum solutions exist as noted in Ref. [1,2]. The optimum web height, WH<sub>opt</sub>, therefore should be decided by comparing the results in each discrete web

Table 1. Bridge Types, Lengths, Widths, Span Ratios and Segment Numbers

BRIDGE TYPE	BRIDGE LENGTH	BRIDGE WIDTH	SPAN RATIO	SEGMENT NUMBER
SIMPLE SPAN G.	20-40 <sup>m</sup>	6,7,8,9 <sup>m</sup>	—	5,7,9
2 SPAN CONT. G.	40-80	6,7,8,9	1.00:1.00	8,10,12,14
3 SPAN CONT. G.	60-120	6,7,8,9	1.00:1.00:1.00 1.00:1.30:1.00	13,15,23

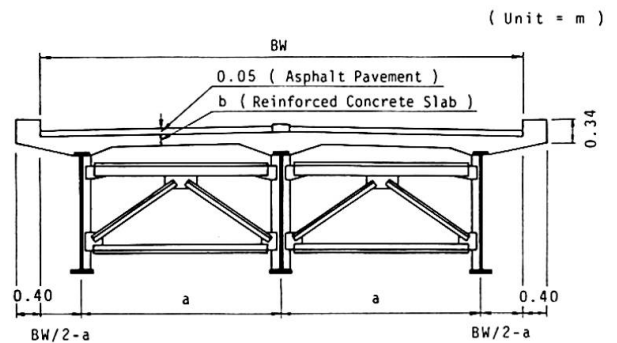


Fig. 1. Cross Section of Girder Bridge

Table 2. Girder Spaces (a) and Thicknesses of Reinforced Concrete Slabs (b)

Bridge Width = BW (m)	6.00	7.00	8.00	9.00
a (m)	2.4	2.8	3.3	3.8
b (cm)	19.0	20.0	22.0	23.0

Table 3. Examples of Optimum Solutions for the Girders at various Price Ratios (BW = 8.00m, WH = 170cm)

CP/BP	1 SPAN (SL=30m)						2 SPAN (SL=30m)					
	Seg. 1			Seg. 2			Seg. 2			Seg. 4		
	M <sub>1</sub>	I <sub>1</sub> /I <sub>0</sub>	L <sub>1</sub> /BL	M <sub>2</sub>	I <sub>2</sub> /I <sub>0</sub>	L <sub>2</sub> /BL	M <sub>2</sub>	I <sub>2</sub> /I <sub>0</sub>	L <sub>2</sub> /BL	M <sub>4</sub>	I <sub>4</sub> /I <sub>0</sub>	L <sub>4</sub> /BL
0.0	SS41	0.656	0.098	SM50	0.963	0.228	SS41	0.984	0.097	SS41	0.882	0.447
0.023	SM50	0.501	0.103	50	0.966	0.230	SM50	0.749	0.102	SM50	0.678	0.447
0.045	50	0.504	0.104	50	0.969	0.231	50	0.746	0.101	50	0.708	0.444
0.068	50	0.531	0.110	50	0.964	0.229	50	0.748	0.101	50	0.709	0.444
0.082	50	0.532	0.111	50	0.963	0.229	50	0.752	0.103	50	0.664	0.451
0.098	50	0.533	0.111	50	0.967	0.230	50	0.757	0.104	50	0.665	0.450
0.136	50	0.502	0.103	50	0.946	0.223	50	0.757	0.104	50	0.665	0.451

CP/BP	3 SPAN (BL=80m, η=1.25)						
	Seg. 1		Seg. 3		Seg. 7		
	M <sub>1</sub>	I <sub>1</sub> /I <sub>0</sub>	M <sub>3</sub>	I <sub>3</sub> /I <sub>0</sub>	M <sub>7</sub>	I <sub>7</sub> /I <sub>0</sub>	
SS41	0.564	0.043	SS41	0.725	0.273	SS41	0.969
SM50	0.566	0.043	SM50	0.755	0.270	SM50	0.976
50	0.566	0.043	50	0.755	0.270	50	0.976
50	0.566	0.043	50	0.756	0.270	50	0.977
50	0.560	0.042	50	0.753	0.269	50	0.974
50	0.566	0.043	50	0.756	0.270	50	0.977
50	0.566	0.043	50	0.755	0.270	50	0.978

BP = Base Price of Steel plate (yen/ton)  
 CP = Daily Wage of a Workman (yen/(person-day))  
 M = Opt. Steel Type  
 I = Opt. Moment of Inertia (cm<sup>4</sup>)  
 L = Opt. Distance of variation of Sectional Dimensions from Left End Support

height. Fig. 2 shows  $WH_{opt}$  for every bridge types and bridge widths. They are changed stepwise and keep constant values for fairly wide ranges of span lengths. Discrete web plate thicknesses play an important role in the decision of optimum web heights and the ratios of web heights to thicknesses fairly coincide with the upper limits prescribed by the stability constraints of the girders. It is to be notable furthermore that only 170,200 and 230cm are selected almost as optimum web heights for the plate girders with nonuniform cross sections with a few exceptions of 190cm at the small ranges of 2, 3 span continuous girders.

5. OPTIMUM M, I, L, SDIM

Optimum M, I and L to be used for the segments of the girder with the optimum web height decided in the previous section may be determined efficiently by using graphical optimization algorithms [1,2]. In the algorithms  $M_{opt}$  and  $I_{opt}$  for each segment are determined by comparing the minimum costs of unit length of girder segment for every steel type namely SS41, SM50 and SM58 steels which are evaluated straightly with use of the maximum working bending moment, BM, and the relationships between moment of inertias and maximum resisting bending moments, RBM, and minimum costs, COST, of the girder sections which are provided from the suboptimization of girder elements. According as bending moment increases, higher strength

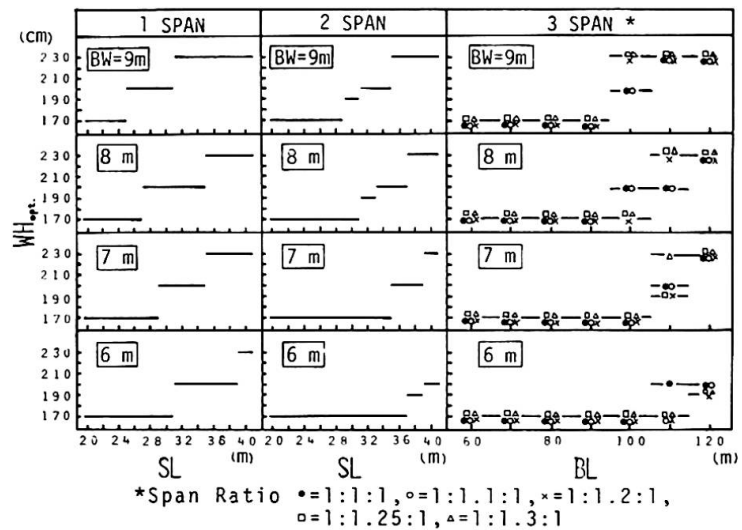


Fig. 2. SL(BL)- $WH_{opt}$ . Relationships for 1,2,3 Span Girders

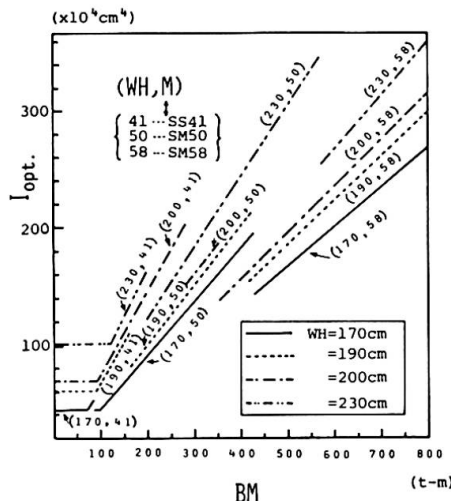


Fig. 3. BM-Opt. M and I Relationships (WH=170,190,200,230cm)

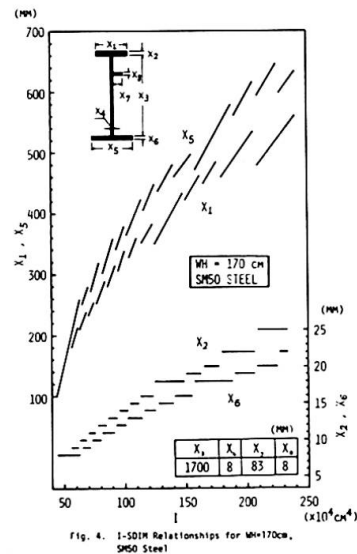


Fig. 4. I-SDIM Relationships for WH=170cm, SM50 Steel



Table 4. Opt.  $L_i/SL(BL)$  for the Bridge Types and Number of Segments

Type of Girder	Number of Seg.	$\xi (=L_i/BL)$ (Symmetric)								
SIMPLE SPAN G.	5	0.104	0.238	0.500						
	7	0.061	0.134	0.234	0.500					
	9	0.053	0.118	0.194	0.298	0.500				
2 SPAN CONT. G.	8	0.070	0.323	0.447	0.500					
	10	0.069	0.330	0.434	0.468	0.500				
	12	0.046	0.101	0.302	0.378	0.453	0.500			
	14	0.046	0.101	0.333	0.431	0.457	0.478	0.500		
3 SPAN CONT. G.	1:1.00:1	13	0.049	0.237	0.309	0.334	0.376	0.455	0.500	
		15	0.031	0.067	0.233	0.305	0.334	0.384	0.463	0.500
	1:1.10:1	13	0.047	0.220	0.295	0.323	0.363	0.439	0.500	
		15	0.030	0.066	0.216	0.291	0.323	0.367	0.466	0.500
	1:1.20:1	13	0.046	0.203	0.280	0.313	0.351	0.429	0.500	
		15	0.029	0.064	0.198	0.279	0.313	0.349	0.434	0.500
	1:1.25:1	13	0.046	0.194	0.273	0.308	0.345	0.424	0.500	
		15	0.028	0.062	0.192	0.272	0.308	0.344	0.428	0.500
	1:1.30:1	13	0.029	0.063	0.180	0.209	0.262	0.287	0.308	0.328
		15	0.354	0.410	0.441	0.500				
	1:1.30:1	13	0.042	0.188	0.263	0.303	0.340	0.420	0.500	
		15	0.027	0.060	0.183	0.264	0.303	0.340	0.424	0.500

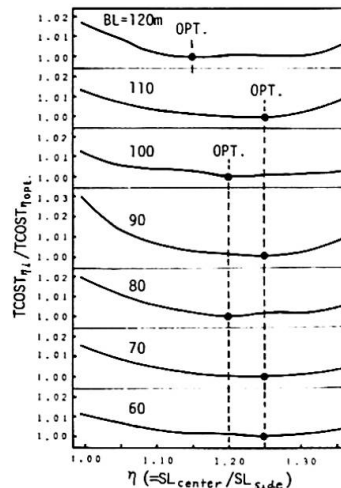


Fig. 5. Span Ratio-TCOST Ratio Relationships for 3 Span Continuous Girders

steels namely from SS41 to SM50 and SM58 are selected as optimum steel types and the corresponding optimum I are therefore varied discretely. The relationships between BM and optimum M, I for  $WH_{opt} = 170, 190, 200, 230\text{cm}$  are shown in Fig. 3. Horizontal lines of the relationships are caused by the constraints on minimum widths and thicknesses for flange plates. The arrangement of optimum steel types and moment of inertias of the girders therefore differ with values of the maximum bending moment diagrams. On the other hand, the ratios of optimum segment lengths do not change so much with span lengths and web heights, as tabulated in Table 4, they may be represented by a particular set of the ratios for a specific segment number and girder type with quite few exceptions. It has been cleared that the increments of total costs of the girders designed with these segment length ratios are less than 0.3% at most compared with the correct solutions. Optimum sectional dimensions, SDIM, of the girder segments can be decided directly from the moment of inertia and optimum sectional dimension diagrams for the web height and steel type as shown in Fig. 4 for  $WH=170\text{cm}$  and  $M=SM50$  steel. Flange widths  $X_2$  and  $X_6$  are increased like as sawteeth with discrete changes of flange plate thicknesses as seen in the figure.

6. SPAN RATIO-TCOST RELATIONSHIPS OF 3 SPAN CONTINUOUS GIRDERS

For the 3 span continuous girder design, the variation in the minimum total cost with span ratio may be one of the interesting features and the relations for the girders with nonuniform cross sections are presented in Fig. 5. According to this result the most economical span ratio for plate girder superstructures with nonuniform cross sections is scarcely varied with bridge lengths and it may be decided as almost  $1.20 \sim 1.25$ . It should be emphasized also, however, the fact that the relative differences of total costs in the range of span ratio from 1.10 to 1.35 are considerably small as less than 0.7%. On the contrary, the optimum span ratio for the girder with uniform cross section reduces with bridge lengths from 1.10 to 0.95 for bridge length  $60\text{m} \sim 120\text{m}$  and the relative differences of total costs are changed so much as the order of several percents.

7. ABM-OPTIMUM WH, M, I, SW RELATIONSHIPS FOR THE GIRDER WITH UNIFORM CROSS SECTION

Optimum solutions of the girders with uniform cross section may be decided directly such as example A or B in Fig. 6 by using the relationships between absolute maximum bending moment, ABM, and optimum design variables, WH, M, I and SW without reference to the bridge types. Different from the cases of girders with nonuniform cross sections, in which only one optimum solution is obtained for each span length, two optimum solutions, one for SM50 steel another for SM58 steel, which give the same minimum total cost are existed in a wide range of absolute maximum bending moments. Optimum web heights for the girder with uniform cross section differ with optimum steel types and they are decided almost as 170, 240, 270cm for SM50 steel and 200, 230, 260cm for SM58 steel.

8. DESIGN PROCEDURE USING OPTIMUM DESIGN DIAGRAMS

By using the design diagrams stated above as data banks, the minimum cost design of highway plate girders may be carried out simply, and the design procedures are shown in Fig. 7. The design process begins by choice of the segment number, NS, of the girder. The optimum web height and segment lengths are decided from SL(BL)-WH<sub>opt</sub> and SL-L/BL relationships respectively. In the next place assumptions are made of moment of inertia and girder weights at each segment, the girder is then analyzed. The optimum steel type, moment of inertia and weight to resist the maximum bending moment at each segment are decided straightly from BM-Opt. M, I, SW relationships for the optimum web height. The girder analysis should be repeated until maximum bending moment, namely optimum M, I, SW, are converged. Optimum sectional dimensions for each segment are decided directly from I-SDIM relationships for the web height and steel type. The total cost of the optimized girder is computed by using I-COST relationships or eq.(1). The entire looping is then carried out again for other NS if necessary. The girders with arbitrary web heights may be designed similarly without the selections of WH<sub>opt</sub>.

9. CONCLUSIONS

This paper has dealt with fundamental optimum design diagrams effective to the minimum cost design of constant depth highway plate girders and an efficient systematic optimum design process based on the data manipulation of the design diagrams has been proposed.

The design diagrams described herein in detail are only that for the optimum web heights at each span length, but the diagrams for other considerable web heights have been developed as well. It has been confirmed also that the diagrams are applicable with accuracy to a wide range of unit price ratios of materials to workmen.

By using the optimum design diagrams as data banks, the minimum cost design of highway plate girders may be carried out by the simple

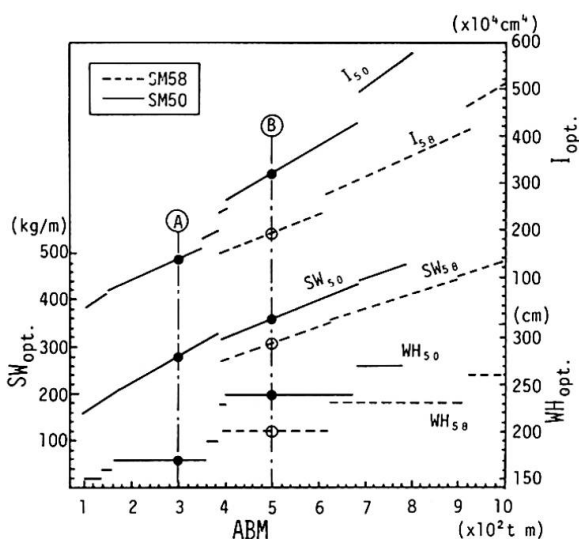


Fig. 6. ABM-Opt. WH, M, I, SW Relationships for the Girder with Uniform Cross Section

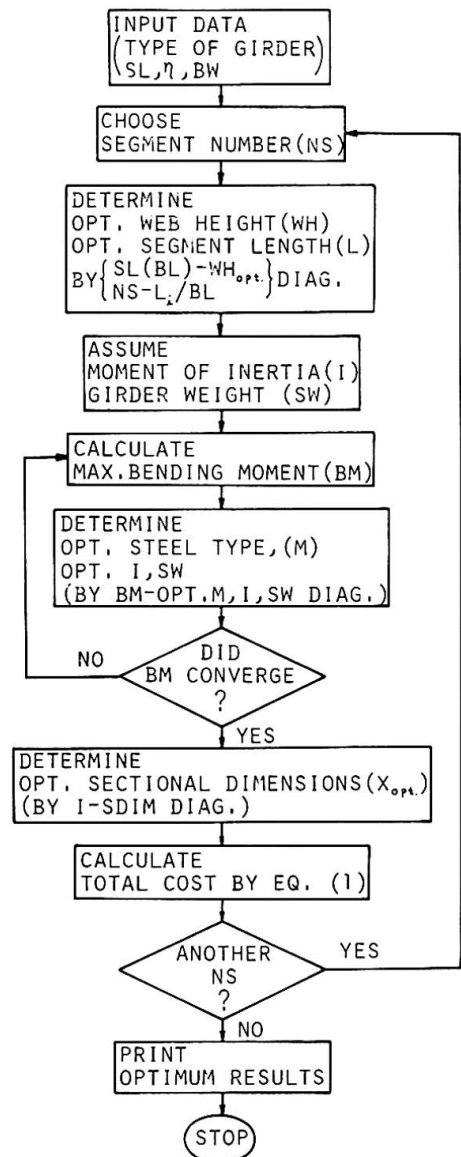


Fig. 7. Flow Chart of the Design Procedures

design procedures. The main computation required for the design is girder analysis only and most part of the design decisions can be made straightly from the diagrams.

The design procedures proposed in this paper may be utilized as one of the element design programs for specific structures in a general purpose system optimization program for highway bridges.

#### 10. REFERENCES

- [1] Ohkubo, S. and Okumura, T., "Graphical Optimization of Steel Girders based on Suboptimization of Girder Elements", Proc. JSCE, No.252, August, 1976, pp.23-34. (in Japanese)
- [2] Ohkubo, S. and Okumura, T., "Structural System Optimization based on Suboptimizing Method of Member Elements" Preliminary Report, 10th Congress, IABSE, Tokyo, Sept., 1976, pp.163-168.
- [3] Ohkubo, S., Okumura, T. and Yamagata, N., "Optimum Design Diagrams of Highway Plate Girders with Nonuniform Cross Sections", Memoirs of the Ehime University, Sect. III, Engineering, Vol. VIII, No.3, Feb., 1977. (in Japanese)
- [4] Okumura, T. and Ohkubo, S., "Optimum Design of Steel Continuous Girders using Suboptimization of Girder Elements", Proc. JSCE, No.215, July, 1973, pp.1-14. (in Japanese) and Proc. 20th National Symposium on Bridge and Structural Engineering, Tokyo, March, 1975, pp.209-229.
- [5] Japan Road Association, "Specifications for Steel Highway Bridges", Tokyo, Feb., 1973. (in Japanese)
- [6] Japan Association of Steel Bridge Construction, Steel Rib and Bridge Association, "Table of Prime Costs for Steel Highway Bridges", 1971. (in Japanese)
- [7] Moses, F., "System and Geometrical Optimization for Linear and Non-linear Structural Behaviour", Introductory Report, IABSE 10th Congress, Tokyo, 1976, pp.61-75.
- [8] Gellatly, R.A. and Dupree, D.M., "Examples of Computer-aided Optimal Design of Structures", Introductory Report, IABSE 10th Congress, Tokyo, 1976, pp.77-105.

#### SUMMARY

The basic optimum design diagrams are presented for the purpose of minimum cost design of 1 to 3 span highway plate girders; the fundamental characteristics of the diagrams are clarified. By using the design diagrams as data banks, minimum cost design of highway plate girders may be carried out in detail by a simple design procedure. The design procedure can be utilized as one of the element design programs in a general purpose system optimization program for highway bridges.

#### RESUME

On présente les diagrammes fondamentaux et leurs caractéristiques essentielles, pour un calcul, avec frais minimum, d'un pont-route à poutres à âme pleine de 1 à 3 travées. L'usage de ces diagrammes comme banque de données permet de faire le calcul détaillé selon un procédé simple, qui peut être utilisé comme un des programmes élémentaires d'un système général d'optimisation pour les ponts-routes.

#### ZUSAMMENFASSUNG

Grunddiagramme des optimalen Entwurfes werden mit ihren Charakteristiken dargestellt. Sie erlauben eine Berechnung mit Minimalkosten von 1 bis 3 feldrigen Strassenbrücken aus Stahl-Vollwandträgern. Diese als Datenbank verwendeten Diagramme gestatten ein Minimalkosten-Entwurf bis ins Detail. Das Verfahren lässt sich als eines der elementaren Entwurfsprogramme im allgemeinen System-Optimierungs-Programm für Strassenbrücken benutzen.

## A Contribution to the Optimum Design of Prestressed Plane Cable Structures

Une contribution au calcul optimal de structures planes de câbles prétendues

Ein Beitrag zur Optimierung von ebenen vorgespannten Seiltragwerken

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A research for the optimum design of prestressed plane cable structures is here briefly referred.

The starting point are P. Pedersen's (1) and G.N. Vanderplaats and F. Moses's (2) papers, where trusses are optimized when cross-sectional areas and joint coordinates are the design variables. The techniques here used must be substantially changed to be applied to prestressed cable structures. This is due to initial prestress (the necessary prestress to grant the wanted design requirements) which is to be kept constant under different loading conditions while cross-sectional areas and the geometry of the structure change. This need really complicates the computational process since, keeping the initial prestress constant, big changes in bar forces correspond to small displacements in joints.

Two kinds of prestressed plane cable structures have been optimized: a standard scheme<sup>1</sup> [Fig. 1,a] and a new type of prestressed cable-stayed structure<sup>2</sup> [Fig. 1,b] whose good static and dynamic behaviour had been shown in a previous work (3).

Both types of structures satisfy the necessary and sufficient conditions for fully stressed design, when at least two load systems are considered. Two uniformly distributed loads, a downward load ( $p_{\max}$ ) and an upward load ( $p_{\min}$ ), are here considered besides prestressing.

The problems relating to prestressed cable structures are of non-linear type. Yet, linear theory has been used because the very many checks carried on by the non-linear theory (3), (4) showed that results from linear analysis are valid, at least as far as localization of good structural parameters is concerned.

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1. Henceforth indicated by TSHS (two surface hanging structure).

2. Abbreviated by PCSS .

IIb - PRESTRESSED PLANE CABLE STRUCTURES

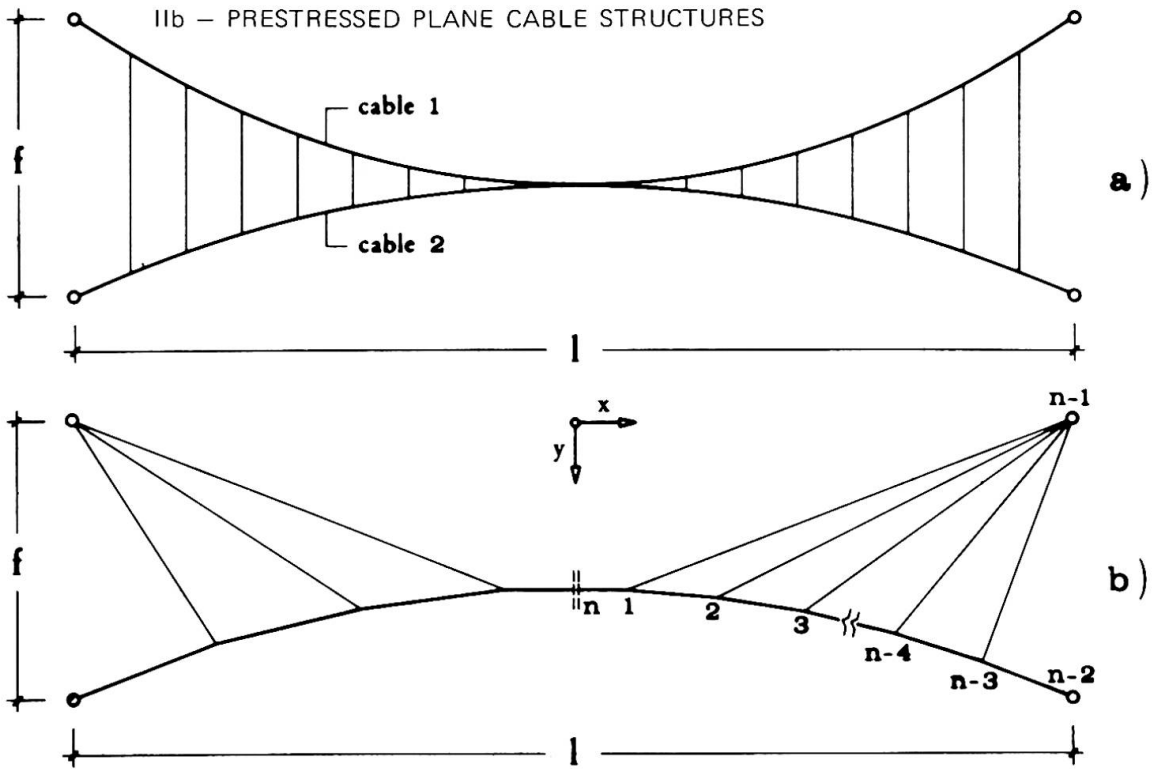


Fig. 1

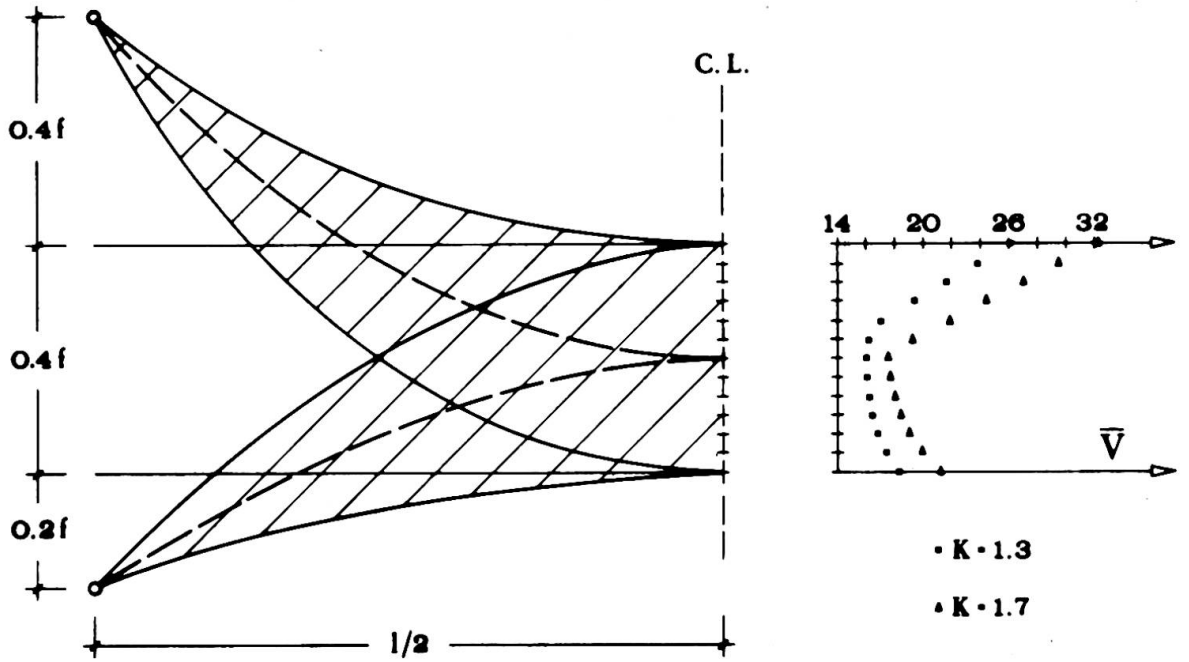


Fig. 2

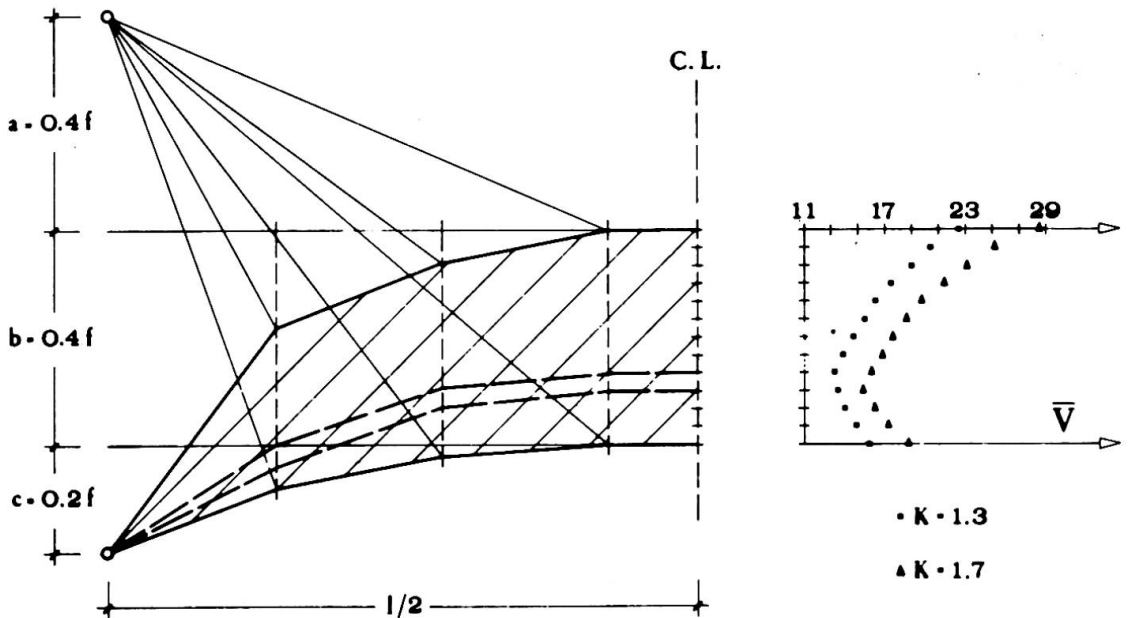


Fig. 3

The TSHS are studied with continuous approach: the relevant computer program (5) allows direct calculation of the areas of main cables and connecting roads once the geometry is fixed.

The PCSS are calculated by a discrete approach, specifically by displacement method: bar cross-sectional areas, when the geometry is fixed, are calculated by successive iterations (6) with a somewhat rapid convergence.

As an exemplification some results are graphically reported in Fig. 2 and in Fig. 3.

In either cases  $f/l = 0.10$ ,  $\bar{p} = |p_{\max}| / |p_{\min}| = 5.00$ ,  $K = 1.3 \div 1.7$  ( $K$  is a number which quantifies the initial prestress) and  $\bar{V} = (V\sigma_a) / (|p_{\min}|l^2)$ , where  $V$  is the volume of structure and  $\sigma_a$  is the allowable normal stress.

As it known, the fully stressed design of redundant structures coincides with the optimum design in peculiar situations only, which do not occur here.

The global optimum is determined by a process of successive iterations similar to Pedersen's (1). For each iteration, the problem of moving the design variables (cross-sectional areas and joint displacements) is worked out as a Linear Programming problem. Moreover, thanks to preliminary researches developed by fully stressed design, which gives solutions quite near to the global optimum, local minima can be avoided.

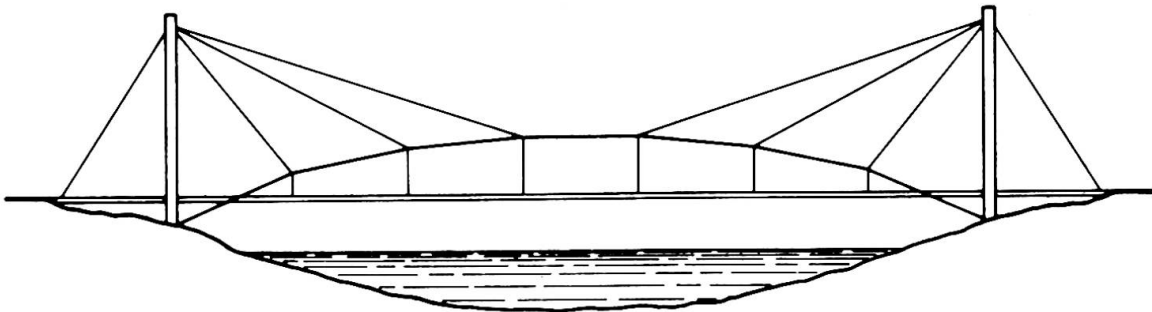


Fig. 4

The optimization process has emphasized the advantages of PCSS over TSHS when volume is the merit function. On the other hand, PCSS have a much better static and dynamic behaviour than TSHS, as it has been largely demonstrated (3), (4).

The extension of the proposed scheme [Fig. 1,b] to long span bridges [see Fig. 4] seems therefore reasonable. To this aim, it is necessary to conform the computational technique already formulated. This will be the purpose of future researches, where the effects of the moving loads and of the wind will be specifically considered.



## REFERENCES.

1. P. PEDERSEN, On the Optimal Layout of Multi-purpose Trusses. Computers & Structures, Vol. 2, 1972.
2. G.N. VANDERPLAATS and F. MOSES, Automated Optimal Design of Structures. Journ. of the Struct. Div. ASCE, Vol. 98, ST 3, 1972.
3. F. SELLERI e O. SPADACCINI. Su una struttura a stralli nel quadro delle tensostrutture piane. Atti dell'Ist. di Scienza delle Costr., Univ. di Pisa, n. 208, Vol. XIV, 1974.
4. O. SPADACCINI, Aspetti costruttivi di una tensostruttura a stralli: funi di bordo e problemi di montaggio. Atti dell'Ist. di Scienza delle Costr., Univ. di Pisa, n. 213, Vol. XIV, 1975.
5. F. SELLERI and O. SPADACCINI, Optimal Design of Prestressed Plane Cable Structures. J. Struct. Mech. (To appear).
6. F. SELLERI, Progetto ottimo di una tensostruttura piana strallata. Atti dell'Ist. di Scienza delle Costr., Univ. di Pisa, n. 215, Vol. XIV, 1975.

## SUMMARY

The optimization of prestressed plane cable structures is presented and in particular the optimum design of a new kind of structure, whose good static and dynamic behaviour has already been reported on. Some conclusions and suggestions for future developments are given.

## RESUME

On présente l'optimisation de structures planes de câbles prétendues et particulièrement le calcul optimal d'une nouvelle structure, dont une étude a déjà souligné le bon comportement statique et dynamique. Quelques conclusions et idées sont présentées pour de futurs développements.

## ZUSAMMENFASSUNG

Es wird über die Optimierung von ebenen vorgespannten Seiltragwerken berichtet und besonders über die optimale Geometrie eines neuartigen Tragwerks; das gute statische und dynamische Verhalten dieses Tragwerks wurde schon früher hervorgehoben. Einige Bemerkungen über die zukünftigen Entwicklungsmöglichkeiten beschliessen diesen Beitrag.

Optimierung der Abmessungen vorgespannter Stahlvollwandträger

Optimization of Dimensions of Prestressed Steel Girders

Optimisation des dimensions de poutres métalliques précontraintes

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Stahlvollwandträger, die durch ein gerades hochfestes Zugband in der Ebene des Untergurtes vorgespannt sind, bilden das meistverwendete Grundelement vorgespannter Stahlkonstruktionen.

Bei der Ableitung rechnerischer Beziehungen für den optimalen Entwurf des durch gerades Zugband vorgespannten Vollwandträgers ist es notwendig, die Beziehungen unter einzelnen Querschnittscharakteristiken des Trägers zu kennen. Mit genügender Genauigkeit ist es möglich, alle Querschnittsgrößen in Abhängigkeit von der Trägerquerschnittsfläche  $F$  unter Anwendung folgender drei Parameter auszudrücken: Trägerasymmetrie  $A = W_{x1}/W_{x2} = h_2/h_1$ , Stegbeiwert  $\varphi_{st} = F_{st}/F$ , Stegchlankheit  $\lambda = h/d$ .

Bei der Ableitung der Beziehungen für den optimalen Entwurf des vorgespannten Vollwandträgers wird vom Festigkeitsgesichtspunkt ausgegangen. Die grösste Tragfähigkeit hat der Träger mit der optimalen Asymmetrie, in dem /bei verschiedenen Belastungsstadien/ möglichst viele Stellen ausgenützt sind, bei voll ausgenütztem Zugband - Bild 1.

In der Tabelle I. sind übersichtlich die Grundbeziehungen für die Berechnung der geometrischen Grössen und der Kraftgrössen des asymmetrischen Querschnittes I, beim optimalen Entwurf des vorgespannten Trägers angeführt. Die optimale Zugbandlänge  $l_v$  folgt aus den Festigkeitsbedingungen des Trägers an den Stellen des Zugbandanfangs und Zugbandendes.

Als erste sind die Lösungsergebnisse des vorgespannten Trägers mit freier /nicht vorgeschriebener/ Untergurtfläche  $F_2$  angeführt,

d. h. die Fläche  $F_2$  ist ausser Festigkeitsbedingung durch nichts

beschränkt. Der Entwurf des vorgespannten Vollwandträgers wird beschleunigt, wenn die Bemessungsbeiwerte  $\varphi$ ,  $\varphi_1$ ,  $\varphi_{st}$ ,  $\varphi_2$ ,  $\varphi_v$  und

$\psi_v$  /Tabelle I./ und das Verhältnis  $l_v/l$  berechnet und verein-

facht werden in der Form eines Berechnungshilfsmittels für oft vorkommende Grundbelastungen. Einige Typen aus den Hilfsmitteln für den Entwurf sind auf den Bildern 2a und 2b angeführt. Teil des Hilfsmittels zur Bestimmung der Zugbandlänge  $l_v$  ist am Bild 3.

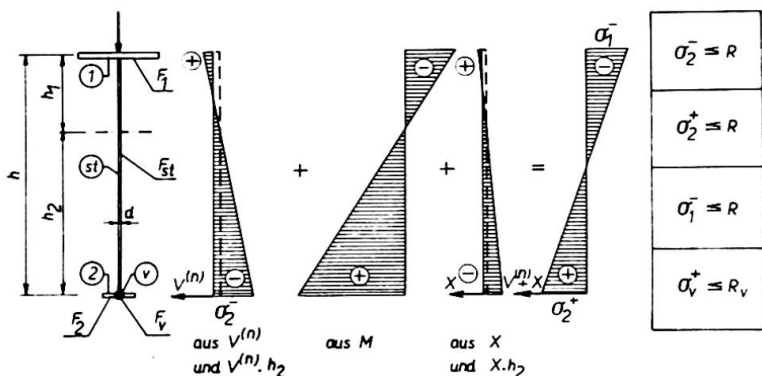


Tabelle I.

$$F = \varphi \sqrt[3]{\frac{M^2}{R^2 \lambda}}$$

$$\varphi = \sqrt[3]{\frac{1}{C^2}}$$

$$C = \sqrt[3]{\varphi_{st} \frac{3A - \varphi_{st}(A+1)}{3(A+1)}}$$

$$F_1 = \varphi_1 F$$

$$\varphi_1 = \frac{A}{A+1} - \frac{\varphi_{st}}{2}$$

$$F_{st} = F \varphi_{st}$$

$$\varphi_{st} = \frac{2[1 - \varphi_2(A+1)]}{A+1}$$

$$F_2 = \varphi_2 F$$

$$\varphi_2 = \frac{1}{A+1} - \frac{\varphi_{st}}{2}$$

$$F_v = F \frac{R}{R_v} \varphi_v$$

$$\varphi_v = D \left( 1 - \frac{n_{vd}}{n_{vh}} \right) \cdot \frac{A-1}{A+1}$$

$$D = \frac{6A - \varphi_{st}(A+1)^2}{(A+1)[6A - \varphi_{st}(A+1)]}$$

$$v^{(n)} = F R \psi_v$$

$$\psi_v = \frac{D}{n_{vh}}$$

Bild 1. Spannungsbild bei einstufiger Vorspannung.  $v^{(n)}$  - Normwert der Vorspannkraft; M - maximales Berechnungsbiegemoment aus der Belastung am Träger ohne Zugband; X - statisch unbestimmte Komponente der Zugbandkraft aus der Berechnungsbelastung; R - Berechnungsbeanspruchung/Index v ist für das Zugbandmaterial/

Auf den Bildern 2a und 2b ist der Bereich mit  $F_2 < 0,04 \cdot F$  bei den betreffenden Bemessungsbeiwerten gestrichelt angezeigt; es hat keinen Zweck  $F_2 < 0,04 \cdot F$  zu entwerfen. In diesem Falle wird die Grösse der Trägeruntergurthfläche  $F_2$  durch die Beziehung

$$F_2 = \varphi_2 F = \left[ \frac{1}{A+1} - \frac{\varphi_{st}}{2} \right] F$$

gebunden /vorgeschrieben/.

Die Berechnung der notwendigen Bemessungsbeiwerte für diesen zweiten Fall, d. i. vorgespannter Träger mit vorgeschriebener /gebundener/ Untergurthfläche  $F_2$ , ist ähnlich wie im ersten Fall, aber es muss auch die oben angeführte Bedingung erfüllt sein.

Wie man aus der Beziehung für  $\varphi_{st}$  /Tabelle I./ sieht, befindet sich in ihr nur eine Unbekannte und zwar Querschnittsasymmetrie A, da  $\varphi_2$  gewählt wird; es bleibt nur der Beiwert A zu berechnen.

Durch Vergleich der Beziehungen für die Berechnung der Kraft X im Zugband, die aus den Festigkeits- und Verformungsbedingungen bestimmt sind, erhalten wir schliesslich die Beziehung

$$\frac{\varrho}{1 + (A-1) \frac{6A - 2[1 - \varphi_2(A+1)]}{6A - (A+1)2[1 - \varphi_2(A+1)]} - \frac{n_{vd}}{n_{vh}}} - \alpha \frac{A\{6A - 4[1 - \varphi_2(A+1)]\}}{\{6A - 2[1 - \varphi_2(A+1)](A+1)\} \cdot \left\{ (A-1) \frac{6A - 2(1 - \varphi_2(A+1))}{6A - 2[1 - \varphi_2(A+1)](A+1)} - \frac{n_{vd}}{n_{vh}} \right\}} = 0$$

aus der die optimale Asymmetrie A in Abhängigkeit vom den gewählten Parameter E,  $E_v$ , R,  $R_v$ ,  $\varphi_2$ ,  $n_{vd}/n_{vh}$  und von der Belastungsart bestimmt wird;  $n_v$  ist der Koeffizient der Vorspanngenauigkeit  $n_{vd} \leq 1,0$  - im allgemeinen 0,9,  $n_{vh} \geq 1,0$  - im allgemeinen 1,1/,  $\varrho = ER_v/E_v R$ . Nach Bestimmung des Wertes A werden mit seiner Hilfe



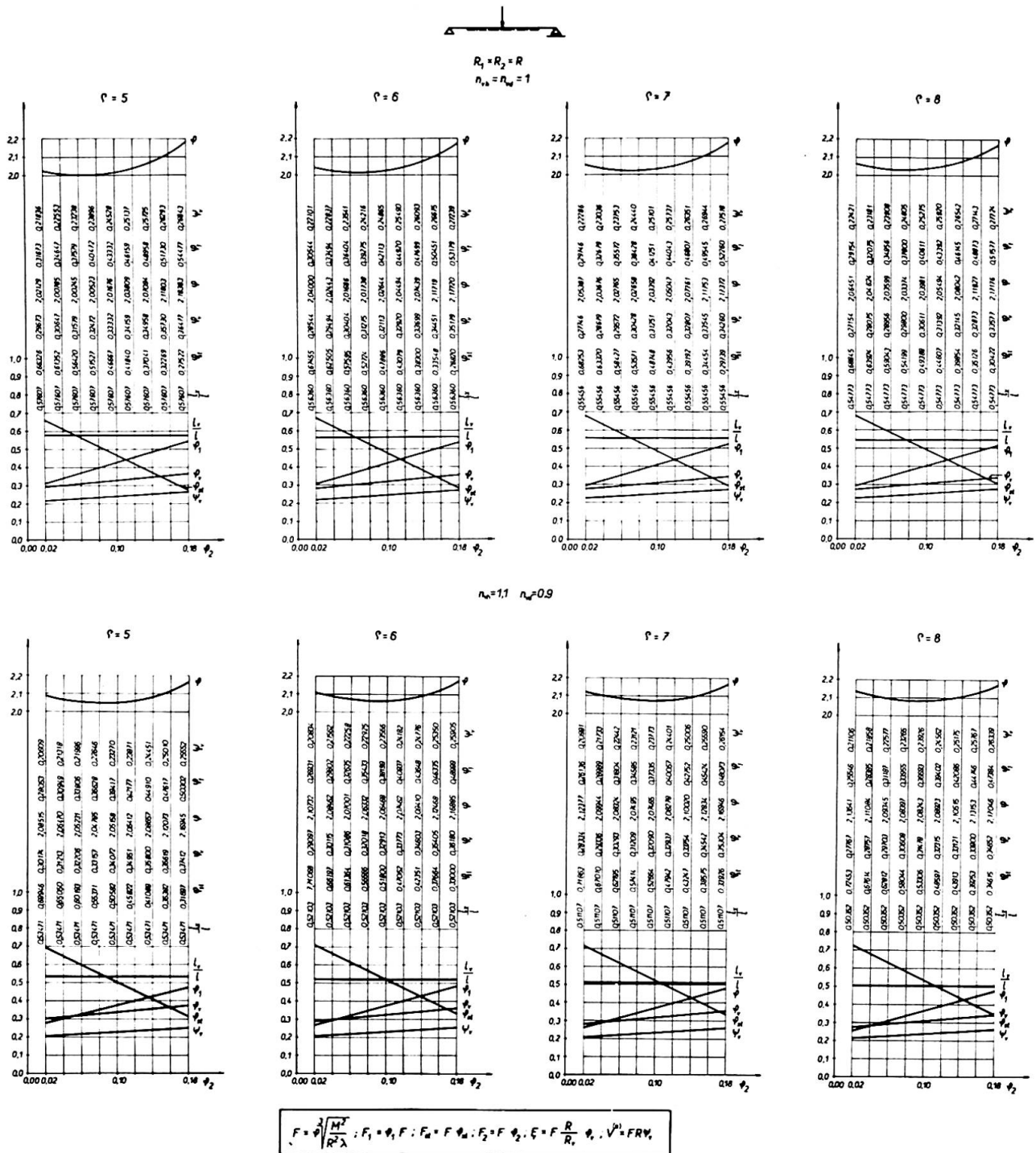


Bild 4. Bemessungsbeiwerte und Beziehungen für den Entwurf des vorgespannten Stahlträgers; Wert  $\phi_2$  ist gewählt

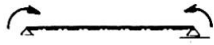
III. Meždunarodnaja konferencija po predvaritelno naprjažennym metalličeskim konstrukcijam. Doklady - tom I., Leningrad 1971.

/2/ Ferjenčik, P.: Navrhovanie a realizácie predpätých kovových plnostenných nosníkov. Bratislava, KKDK ŠvF ŠVST, 1972.

/3/ Ferjenčik, P.: Optymalne parametry sprężonych, stalowych dźwigarow pełnościennych. In: Konstrukcje metalowe, bjuletyn branżowy, Mostostal, Warszawa 1973, č. 3.



Tabelle II. Hilfs- und Ergebnisbeiwerte für den Entwurf des vorgespannten Stahlträgers; Wert  $\varphi_2$  ist gewählt.  $R_1 = R_2 = R$  und  $n_{vh} = n_{vd} = 1,0$



$\varphi$	$\varphi_2$	A	$\alpha$	$\varphi_{st}$	$\varphi$	C	D	$\varphi$	$\varphi$	$\varphi$	$\varphi$
3	0,02	2,97355	1,00000	0,46332	0,49667	0,40425	0,16555	1,82907	0,51667	1,00000	0,16555
	0,04	3,07985	1,00000	0,41021	0,50978	0,39591	0,16992	1,85466	0,54978	1,00000	0,16992
	0,06	3,18765	1,00000	0,35759	0,52240	0,38391	0,17413	1,89311	0,58240	1,00000	0,17413
	0,08	3,29714	1,00000	0,30542	0,53457	0,36777	0,17819	1,94808	0,61457	1,00000	0,17819
	0,10	3,40850	1,00000	0,25366	0,54633	0,34682	0,18211	2,02579	0,64633	1,00000	0,18211
	0,12	3,52193	1,00000	0,20228	0,55771	0,31997	0,18590	2,13758	0,67771	1,00000	0,18590
	0,14	3,63762	1,00000	0,15125	0,56874	0,28544	0,18958	2,30666	0,70874	1,00000	0,18958
	0,16	3,75573	1,00000	0,10054	0,57945	0,23978	0,19315	2,59090	0,73945	1,00000	0,19315
	0,18	3,87646	1,00000	0,05013	0,58986	0,17424	0,19662	3,20545	0,76986	1,00000	0,19662
4	0,02	2,27839	1,00000	0,57005	0,38994	0,38125	0,19497	1,90192	0,40994	1,00000	0,19497
	0,04	2,34116	1,00000	0,51859	0,40140	0,38011	0,20070	1,90571	0,44140	1,00000	0,20070
	0,06	2,40367	1,00000	0,46760	0,41239	0,37632	0,20619	1,91848	0,47239	1,00000	0,20619
	0,08	2,46602	1,00000	0,41702	0,42297	0,36969	0,21148	1,94136	0,50297	1,00000	0,21148
	0,10	2,52830	1,00000	0,36684	0,43315	0,35995	0,21657	1,97622	0,53315	1,00000	0,21657
	0,12	2,59060	1,00000	0,31700	0,44299	0,34673	0,22149	2,02614	0,56299	1,00000	0,22149
	0,14	2,65297	1,00000	0,26749	0,45250	0,32950	0,22625	2,09617	0,59250	1,00000	0,22625
	0,16	2,71548	1,00000	0,21828	0,46171	0,30746	0,23085	2,19515	0,62171	1,00000	0,23085
	0,18	2,77820	1,00000	0,16935	0,47064	0,27937	0,23532	2,33997	0,65064	1,00000	0,23532
5	0,02	2,05107	1,00000	0,61550	0,34449	0,36644	0,20869	1,95282	0,36449	1,00000	0,20869
	0,04	2,10151	1,00000	0,56484	0,35515	0,36773	0,21309	1,94824	0,39515	1,00000	0,21309
	0,06	2,15142	1,00000	0,51463	0,36536	0,36668	0,21921	1,95197	0,42536	1,00000	0,21921
	0,08	2,20089	1,00000	0,46482	0,37517	0,36314	0,22510	1,96461	0,45517	1,00000	0,22510
	0,10	2,25000	1,00000	0,41538	0,38461	0,36695	0,23076	1,98727	0,48461	1,00000	0,23076
	0,12	2,29880	1,00000	0,36627	0,39372	0,34785	0,23623	2,02178	0,51372	1,00000	0,23623
	0,14	2,34737	1,00000	0,31748	0,40251	0,33549	0,24150	2,07112	0,54251	1,00000	0,24150
	0,16	2,39574	1,00000	0,26897	0,41102	0,31939	0,24661	2,14015	0,57102	1,00000	0,24661
	0,18	2,44396	1,00000	0,22072	0,41927	0,29883	0,25156	2,23725	0,59927	1,00000	0,25156
6	0,02	1,93850	1,00000	0,64061	0,31938	0,35709	0,21292	1,98675	0,33938	1,00000	0,21292
	0,04	1,98316	1,00000	0,59042	0,32957	0,35958	0,21971	1,97755	0,36957	1,00000	0,21971
	0,06	2,02719	1,00000	0,54067	0,33932	0,35988	0,22621	1,97647	0,39932	1,00000	0,22621
	0,08	2,07069	1,00000	0,49131	0,34868	0,35787	0,23245	1,98385	0,42868	1,00000	0,23245
	0,10	2,11374	1,00000	0,44231	0,35768	0,35341	0,23845	2,00050	0,45768	1,00000	0,23845
	0,12	2,15638	1,00000	0,39363	0,36636	0,34630	0,24424	2,02779	0,48636	1,00000	0,24424
	0,14	2,19868	1,00000	0,34525	0,37474	0,33626	0,24982	2,06796	0,51474	1,00000	0,24982
	0,16	2,24067	1,00000	0,29715	0,38284	0,32291	0,25523	2,12459	0,54284	1,00000	0,25523
	0,18	2,28241	1,00000	0,24930	0,39069	0,30569	0,26046	2,20363	0,57069	1,00000	0,26046
7	0,02	1,87138	1,00000	0,65652	0,30347	0,35075	0,21876	2,01061	0,32347	1,00000	0,21876
	0,04	1,91266	1,00000	0,60665	0,31334	0,35396	0,22381	1,99844	0,35334	1,00000	0,22381
	0,06	1,95330	1,00000	0,55720	0,32279	0,35506	0,23056	1,99432	0,38279	1,00000	0,23056
	0,08	1,99337	1,00000	0,50814	0,33185	0,35395	0,23704	1,99846	0,41185	1,00000	0,23704
	0,10	2,03293	1,00000	0,45942	0,34057	0,35052	0,24326	2,01149	0,44057	1,00000	0,24326
	0,12	2,07206	1,00000	0,41102	0,34897	0,34458	0,24926	2,03455	0,46897	1,00000	0,24926
	0,14	2,11079	1,00000	0,36292	0,35707	0,33589	0,25505	2,06949	0,49707	1,00000	0,25505
	0,16	2,14917	1,00000	0,31508	0,36491	0,32412	0,26065	2,11929	0,52491	1,00000	0,26065
	0,18	2,18724	1,00000	0,26750	0,37249	0,30881	0,26606	2,18877	0,55249	1,00000	0,26606
8	0,02	1,82682	1,00000	0,66750	0,29249	0,34620	0,21936	2,02820	0,31249	1,00000	0,21936
	0,04	1,86591	1,00000	0,61785	0,30214	0,34988	0,22660	2,01397	0,34214	1,00000	0,22660
	0,06	1,90433	1,00000	0,56862	0,31137	0,35150	0,23352	2,00075	0,37137	1,00000	0,23352
	0,08	1,94216	1,00000	0,51977	0,32022	0,35100	0,24017	2,00968	0,40022	1,00000	0,24017
	0,10	1,97948	1,00000	0,47125	0,32874	0,34824	0,24655	2,02028	0,42874	1,00000	0,24655
	0,12	2,01632	1,00000	0,42305	0,33694	0,34306	0,25270	2,04053	0,45694	1,00000	0,25270
	0,14	2,05274	1,00000	0,37514	0,34485	0,33526	0,25863	2,07208	0,48485	1,00000	0,25863
	0,16	2,08880	1,00000	0,32750	0,35249	0,32452	0,26437	2,11754	0,51249	1,00000	0,26437
	0,18	2,12451	1,00000	0,28009	0,35990	0,31044	0,26992	2,18110	0,53990	1,00000	0,26992
9	0,02	1,79509	1,00000	0,67553	0,28446	0,34277	0,22124	2,04169	0,30446	1,00000	0,22124
	0,04	1,83262	1,00000	0,62605	0,29394	0,34678	0,22862	2,02592	0,33394	1,00000	0,22862
	0,06	1,86950	1,00000	0,57698	0,30301	0,34879	0,23567	2,01816	0,36301	1,00000	0,23567
	0,08	1,90576	1,00000	0,52828	0,31171	0,34870	0,24244	2,01849	0,39171	1,00000	0,24244
	0,10	1,94149	1,00000	0,47992	0,32007	0,34642	0,24894	2,02733	0,42007	1,00000	0,24894
	0,12	1,97674	1,00000	0,43187	0,32812	0,34179	0,25520	2,04559	0,44812	1,00000	0,25520
	0,14	2,01156	1,00000	0,38410	0,33589	0,33461	0,26124	2,07476	0,47589	1,00000	0,26124
	0,16	2,04599	1,00000	0,33660	0,34339	0,32460	0,26708	2,11720	0,50339	1,00000	0,26708
	0,18	2,08006	1,00000	0,28933	0,35066	0,31138	0,27273	2,17672	0,53066	1,00000	0,27273
10	0,02	1,77135	1,00000	0,68166	0,27833	0,34011	0,22266	2,05234	0,29833	1,00000	0,22266
	0,04	1,80773	1,00000	0,63231	0,28768	0,34436	0,23014	2,03540	0,32768	1,00000	0,23014
	0,06	1,84345	1,00000	0,58337	0,29662	0,34665	0,23730	2,02646	0,35662	1,00000	0,23730
	0,08	1,87855	1,00000	0,53479	0,30520	0,34688	0,24416	2,02556	0,38520	1,00000	0,24416
	0,10	1,91311	1,00000	0,48655	0,31344	0,34495	0,25075	2,03308	0,41344	1,00000	0,25075
	0,12	1,94718	1,00000	0,43861	0,32138	0,34073	0,25710	2,04984	0,44138	1,00000	0,25710
	0,14	1,98082	1,00000	0,39095	0,32904	0,33401	0,26323	2,07723	0,46904	1,00000	0,26323
	0,16	2,01405	1,00000	0,34355	0,33644	0,32454	0,26915	2,11746	0,49644	1,00000	0,26915
	0,18	2,04692	1,00000	0,29640	0,34359	0,31195	0,27487	2,17405	0,52359	1,00000	0,27487



- /4/ Ferjenčík, P. - Toháček, M.: Podklady pre revíziu a doplnenie ON 73 1405 "Smernice pro navrhování předpjatých ocelových konstrukcí". Tretia dielčia etapa ulohy P 12-124-003-02/2.e Predpäté kovové a lanové konštrukcie. Bratislava, KKDK SvF SVST, 1975.
- /5/ Ferjenčík, P. - Toháček, M.: Die Vorspannung im Stahlbau. Theorie und Konstruktionspraxis. Bauingenieur-Praxis, Heft 38. Verlag von W. Ernst und Sohn, 1975.

#### ZUSAMMENFASSUNG

Der Aufsatz befasst sich mit der Optimierung der Abmessungen vorgespannter Stahlvollwandträger. Angeführt werden Berechnungshilfsmittel für den Entwurf, wobei die Trägeruntergurtfläche  $F_2$  im ersten Fall frei und im zweiten Fall gebunden ist.

#### SUMMARY

This article deals with the optimization of dimensions of prestressed steel girders. Calculation means are given for the design, whereas in the first case the area of the lower flange  $F_2$  is assumed to be free and in the second case it is assumed to be given.

#### RESUME

L'article traite de l'optimisation des dimensions des poutres métalliques précontraintes. Des tables auxiliaires de calcul sont présentées, avec la section de la semelle inférieure  $F_2$  libre en premier cas et liée en deuxième cas.