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II

Progrès dans l'optimisation structurale

Fortschritte in der Optimierung von Tragwerken

Progress in Structural Optimization

II a

Concepts et techniques d'optimisation

Grundlagen und Methoden

Optimization Concepts and Techniques in Structural Design

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IIa

Optimum Design of Steel Frame Subjected to Dynamic Earthquake Forces

Calcul optimal de cadres métalliques soumis aux forces dynamiques des tremblements de terre

Optimierung von Stahlrahmen unter dynamischer Erdbebenlast

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1. INTRODUCTION

The mathematical programing technique has already been adopted for the optimization of the structures subjected to the dynamic excitations.[1,2] Most of these optimizations were dealt with beams, trusses or frames, subjected to simple excitations such as harmonic waves or shock waves, and designed under rather simple elastic constraints.

However, in case of earthquake loadings it becomes important to estimate the dynamic forces correctly using the available model for the elastic design, and to take into account the inelastic behaviour of structures during the very strong ground motion.

Considering these problems, this paper presents a method for the automated minimum weight design of wide-flange steel frames which gives the optimum distribution of the moment of inertia of used members.

2. DYNAMIC ANALYSIS

An idealized dynamic model consist of bedrock, ground layres and a structure is considered (see Fig. 1). Ground excitations are given by the model presented by Kanai and Tajimi, and the dynamic response of the structure to this ground motion is estimated by means of the random vibration theory and Davenport's equation which gives the expected maximum value of a random process.

2.1 Vibration of Ground Surface

Kanai and Tajimi has presented the idea that spectrum observed at bedrock is characterized by a constant pattern (white noise), while the spectrum at the ground surface is amplified by the vibration property of the ground layre and showed a power spectrum of this ground surface as follows:[3]

$$S(p) = \sum_{k=1}^{r} \frac{1 + 4h_{gk}{}^2(\frac{p}{w_{gk}})^2}{1 + (4h_{gk}{}^2-2)(\frac{p}{w_{gk}})^2 + (\frac{p}{w_{gk}})^4} s_k S_o \qquad (1)$$

where h_{gk} and w_{gk} are ground damping factor and predominant frequency, respectively, S_o is a constant power spectrum density function and where s_k is a factor which measures predominence of each component. This excitation of ground surface becomes Gaussian process of zero mean.

2.2 Dynamic Response of Structure

The variance of elastic response of the structure subjected to the ground motion mentioned above can be obtained by means of random vibration theory. Let σ_Q and $\sigma_{\dot{Q}}$ be the variance of story shear force and its time derivative, respectively.

Following Davenport[4], the mean value of possible maximum elastic response of story shear force can be given as

$$Q = (2\ln vT)^{\frac{1}{2}} + \frac{0.5772}{(2\ln vT)^{\frac{1}{2}}} \qquad (2)$$

where $v = \frac{1}{2\pi} \frac{\sigma_{\dot{Q}}}{\sigma_{Q}}$

and T represents the duration of the strong earthquake excitation which is fixed 10 seconds in this paper.

For very strong ground motion, the response of the structure is considered to be inelastic, and the relative displacements of each floor are estimated following the idea of Newmark and et al.[5] Equating the inelastic potential energy of deformation to the elastic one which can be obtained supposing that the structure responses elastically, the maximum ductility factor of floor drift, μ, can be obtained as follows (see Fig. 2);

$$\mu = \frac{1}{2} + \frac{1}{2}\left(\frac{Q}{Q_y}\right)^2 \qquad (3)$$

where Q_y may be thought of as the yield level of the story shear force, and can be obtained by means of a simple plastic analysis assuming the mechanism of beam collapse type or column collapse one for each story.

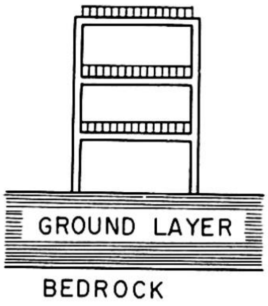

Fig. 1 Dynamic Model

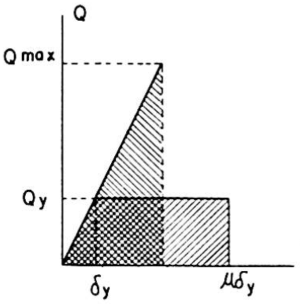

Fig. 2 Definition of Ductility Factor

3. DESIGN CONSTRAINTS

For the moderate earthquakes which give such a dynamic force as usually presented in the design code, the members of the frame are designed elastically in accordance with the design code of steel structure of Architectural Institute of Japan (A.I.J). On the other hand, for the very severe earthquake, which is rarely expected during their service lives, the frame is designed plastically relying on the energy absorption which due to their inelastic deformation. In this design procedure, the maximum ductility factor given by Eq.(3) is constrained less than the allowable value which is fixed 4 in this paper.

To satisfy these ductility requirements, it is necessary for the frame to prevent the weakening of the load-deflection curves caused by the lateral or local buckling of members and P – Δ effects.

These problems are taken into account according to the plastic design code of steel structure of A.I.J.. Namely, lateral buckling is prevented by the correctly designed stiffners, and local buckling is prevented by selecting the members which are on market to satisfy the width-thickness ratio of plate elements imposed by the code mentioned above, or designing each member in-accordance with these requirements after the optimum stiffness distribution of frame member is decided. Moreover to avoid the excessive P – Δ effects, the slenderness ratios and the axial compressive forces of columns are ristricted by the code requirements.

4. OPTIMIZATION

Wide-flange steel members on market are supposed to be mainly used in the design. The moment of inertia of them, I, are the design variables and objective function is the total weight of structural members. The emprical relationships between member properties which are required in the design code and moment of inertia of economical series of the steel wide-flange section was obtained by plotting them. The calculation was proceeded using these equations and treating the moment of inertia as continious design variable.

Sequential linear programming (S.L.P.) technique was successfully adopted for the optimization of the frames. Objective function and constraint equations were approximately linearlized, and using linear programming technique, the optimumly modefied design variables were obtained at each design step. Repeating this procedure, the optimum solusion, namely the distribution of moment of inertia of members which gives the minimum weight of structural members, was obtained.

5. SENSITIVITY ANALYSIS

To optimize the structure by means of S.L.P. technique, the change of member stress and deformation caused by the modefication of each members must be quantitively estimated as the first order derivative of these values with respect to the design variables.

Let P be the vector of external nodal forces of global coordinate, and X and K be the corresponding nodal displacement vector and stiffness matrix. Using these notations

$$X = K^{-1} P \qquad (4)$$

Therefore, the derivative of nodal displacements with respect to design variable, I, is obtained as follows;

$$\frac{\partial X}{\partial I} = \frac{\partial}{\partial I} K^{-1} \cdot P + K \frac{\partial}{\partial I} P \qquad (5)$$

The second term of the right hand side of the above equation contain the derivative of the dynamic loads which vanish in the static problems. If these values are obtained, the sensitivity coefficients of the stresses and deflections can be evaluated applying the same procedure adopted for the static problems.

As the dynamic loads which is evaluated by means of random vibration theory become the explicit function of natural frequencies and mode[6] vectors of structure, if the sensitivity coefficients of these values are evaluated, then that of these dynamic loads can be obtained without difficulty.

6. NUMERICAL EXAMPLE

The method previously mentioned is applied to the design of three-story frames of equal span length, 6m, and equal story height, 3m, with uniformly distributed load, w, on beams, subjected to the four types of ground motions whose characteristics are decided by the parameters presented in Table 1. Frames are designed both elastically for the power S_0 of E_q.(1), and plastically for the power of α times of S_0 so that the story drifts should be less than allowable ductility factor 4, and beam collapse type mechanism is

TYPE	T_{g1}	h_{g1}	β_1	T_{g2}	h_{g2}	β_2
I	0.3	0.6	1.0			
II	1.0	0.6	1.0			
III	0.1	0.3	0.2	1.0	0.3	0.8
IV	0.1	0.3	0.2	1.5	0.3	0.8

Table 1 Ground Parameter

considered for the calculation of yield levels of story shear forces. Steel used is SS41 whose yield stress is 2.4 ton/cm^2.

6.1 Three story one bay frames subjected to the ground motion of type I is optimized for $W = 5$ ton/m and $S_0 = 5$ cm/rad/sec^3. In Fig. 3, the maximum stresses and the maximum ductility factors of each story corresponding to the final design are presented for α equal 5 and 7 respectively. Where the maximum stress is defined as the value in the most severely violated constraint equation for elastic design whose allowable limit is normalized as unity. For the case of α equals 5, the member size is desided by the elastic constraints and the response ductility factors of each story are scattering. On the other hand, for the case of α equals 7, the beams are not fully stressed for elastic design constraints and for the plastic design constraints they are equally fully constrained. Therefore it can be pointed out that for the optimum design of earthquake resistance structures, it become important to consider the constraints for the inelastic deflection expected during the very strong earthquakes.

6.2 Three story one bay frame subjected to the ground motion of type III and IV is optimized for $w = 2$ ton/m, $S_0 = 2$ cm/rad/sec^3 and $\alpha = 7$. The maximum stresses of each member defined previously and the maximum ductility factors for the final design are presented in Fig. 4. This shows that the optimum member size restricted by both elastic and plastic constraints.

The acceleration response spectrum to these ground motions is presented in Fig. 4 with the values of the spectrum correspond to the fundamental frequencies of the structure of initial and final design. This shows that even if the initial design is at the valley of the response spectrum, or final design is at the vicinity of the maximum, this optimization technique can be successfully adopted.

Neglecting the derivative of dynamic forces which is used in Eq. (5), the optimization is also carried out for the same model. The final result obtained starting from the same initial design mentioned above is presented in Fig. 6. Compared with the above analysis, much more iterative calculations are carried out and the real optimum solusion can not be obtained. This too happen for the optimization of the structure subjected to the ground motion which have more moderate response spectrum showing the importance of sensitivity analysis of dynamic forces for these analysis.

6.3 Three story one bay and three bays frames are optimized for $\alpha = 7$ by changing the parameters concerend with the distributed load and ground motion. The ductility factors of story drift correspond to the final design are shown in Table 2 with these parameters. Each story yield almost equally fully restricted by the constraints of plastic deformation. Therefore it can be pointed out that for this kind of structures, the optimum design correspond to such a structure whose response ductility factors against very strong ground motion are almost equal for all story.

7. CONCLUSION

As a result of this study, following conclusions can be pointed out.

(1) The analysises of some examples shows the validity of the optimization technique mentioned above together with the importance of the sensitivity analysis of dynamic forces.

(2) The constraints concerened with the plastic deformation against the very strong ground motions must be considered together with the constraints for the elastic strength.

(3) For the type of structure dealt with in this paper, the minimum weight design correspond to such a structure whose response ductility factors against very strong ground motion are almost equal for all story.

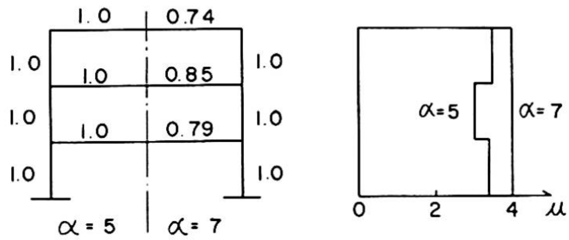

Fig.3 The Maximum Stress and Ductility Factor

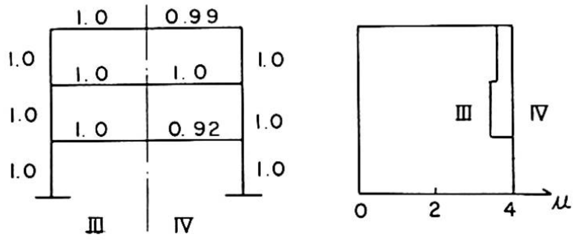

Fig.4 The Maximum Stress and Ductility Factor

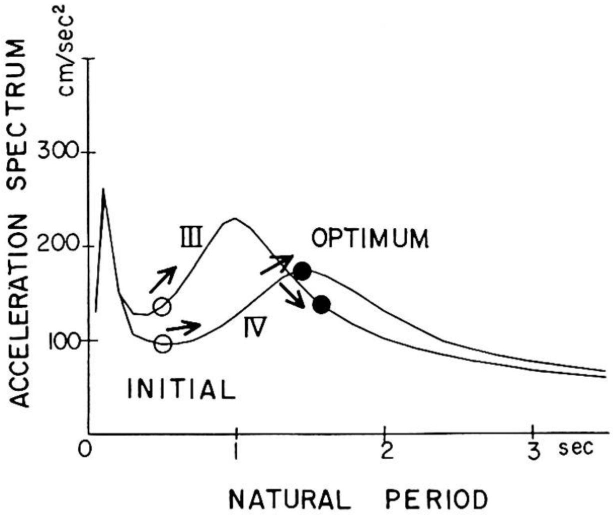

Fig.5 Optimization Process for Different Type Ground Motion

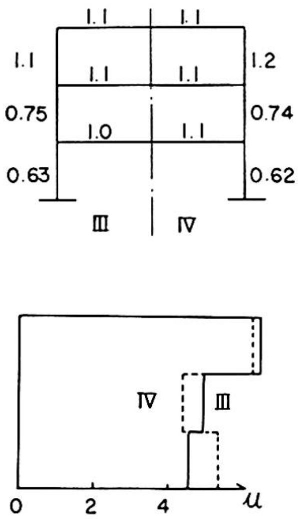

Fig.6 The Maximum Stress and Ductility Factor

SPAN	ω	TYPE	S₀	μ		
				1	2	3
1	10	I	5	4.0	4.0	4.0
1	50	I	5	4.08	4.08	4.05
1	50	II	5	4.04	4.04	4.04
1	5	III	2	4.0	4.0	4.0
1	5	IV	2	4.0	3.68	3.92
1	2	IV	2	4.0	4.0	4.0
1	50	IV	5	3.99	3.82	3.96
3	10	II	5	4.04	4.08	4.08
3	30	I	5	3.92	3.96	3.92
3	30	II	5	4.12	4.04	4.04

Table 2 The Maximum Ductility Factor

3g. 7 VB

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SUMMARY

The minimum weight design of unbraced steel frames subjected to dynamic earthquake loads is presented. Random vibration theory is adopted to elastic member strength and plastic story deflection, the sequential linear programming technique is successfully adopted to obtain the optimum design. Several examples are presented with the analysis and comparisons are drawn.

RESUME

On présente le dimensionnement, pour un poids minimum, de cadres métalliques soumis aux forces dynamiques des tremblements de terre. La théorie des vibrations aléatoires permet de déterminer le comportement "dynamique" de la structure. La programmation linéaire séquentielle donne le dimensionnement optimal dans des conditions de comportement élastique des éléments et de comportement plastique du cadre soumis à la déflection.

ZUSAMMENFASSUNG

Für unausgesteifte Stahlrahmen, die durch Erdbebenwirkung beansprucht sind, wird die Berechnungsmethode des "minimalen Gewichts" abgeleitet. Die "Random"-Vibrationstheorie erlaubt es, das dynamische Verhalten des Tragwerks festzustellen. Unter Annahme "elastischer" Kräfte und plastischer Verformungen liefert die fortschreitende lineare Programmierung das gesuchte Optimum. Beispiele werden gezeigt und Vergleiche angestellt.

Optimization Techniques under Random Loading Effects

Techniques d'optimisation et effets des charges aléatoires

Optimierungstechnik bei Wirkung von Zufallsbelastungen

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1. INTRODUCTION

The developments that have taken place in the last few years in the field of optimization techniques applied to structural problems were restricted mainly to structures subjected to deterministic loadings. The reasons for the lack of research activities towards the analysis of structures under the effects of random loadings could be attributed to the mathematical complication involved in the procedure and the non-availability of sufficient and reliable data regarding the past histories of the random exciting force.

In this paper a simplified approach is reported to deal with the structural optimization problems under non-stationary loadings by making use of the upper bound probability of failure of the structure. The analysis is carried out in two phases:

(A) to obtain an expression for the probability that the response of the structure at a critical zone reaches for the first time an upper limit value with time-dependent control-barriers, interms of their rate of uperossings; and

(B) to seek an approximate solution to the optimization problem, using the result obtained in phase (A), with the probability of failure, the natural frequency of vibration and the frequency response function of the system as restraints.

2. PHASE (A).

The estimation of the upper and lower bound probabilities of failure of a structure in a closed internal of time, has been a field of great interest among engineers dealing with random vibration problems. J.J Coleman[1] for the first time, suggested an approximate solution to estimate the upper bound value interms of the expected rates of the threshold crossings of the response process at positive and negative slopes. However, the process of independent arrivals of failure, as assumed by Coleman, is unacceptable especially for narrow band random process, such as the response of lightly damped dynamic systems. Besides, for low damped structural systems, crossings of response process tend to occur in 'clumps' of dependent crossings and hence the expected rate of threshold crossings should be replaced by the average clumping rate. M Shinozuka[2] has developed a method applicable to stationary and non-stationary cases as well, to estimate the upper and lower bounds for the probability of the first excursion failure within an arbitrary semi-closed time internal (o, t) and constant barriers without the assumption of independent threshold crossings. When the computed values of the upper and lower bounds are sufficiently close to each other, they are just as valuable as the mathematically exact values of the probability as a basis for making engineering decisions. In a paper[3] published later, Shinozuka has further extended his solution to take into account the effects of time-dependent barriers also.

The solution to the above problem with time-dependent barriers, presented in this paper is a modification to Shinozuka's approach with a different interpretation, interms of the expected rate of crossings of the response-barriers.

Following Shinozuka's expression for the upper - bound probability of failure of the structure,

$$P_r\,[t\,;-Y_2(t)\,,Y_1(t)] < P_r\,[t\,;-Y_2(t),\propto] + P_r\,[t\,;-\propto,Y_1(t] \\ -P_r\,[\{x(t_1) < -Y_2(t_2)\}\{x(t_2) > Y_1(t)\}] \text{ - - - -}(1)$$

where x(t) represents the response of the system at a critical zone and the failure of the system, for the first time, is defined as when $x(t) \geqslant Y_1(t)$, or $x(t) \leqslant -Y_2(t)$, in which $Y_1(t)$ and $Y_2(t)$ are positive barriers of response process.

Let $N[Y_1(t),t]$, hereafter referred as N_1, represents a random variable denoting the number of crossings of $Y_1(t)$ from below during the internal (0,t). The probability that $N[Y_1(t),t]$ takes a value 'r' during (0,t), $P_r[N_1=r]$, can be expressed as:

$$P_r\,[N_1=r] = P_r\,[N_1=r\,;\,x(0) \geqslant Y_1(0)] + P_r\,[N_1=r\,;\,x(0) < Y_1(0)] \text{ - - - -}(2)$$

Also,

$$P_r\,[t\,;-\propto,Y_1(t)] = P_r\,[x(0) > Y_1(0)\,,N_1 \geqslant 0] + P_r\,[x(0) < Y_1(0),N_1 \geqslant 1] \\ + P_r\,[x(0) = Y_1(0),N_1 \geqslant 0] \text{ - - - - - - -}(3)$$

Equation (3) can further be simplified as :

$$P_r\,[t\,;-\propto,Y_1(t)] = P_r\,[x(0) > Y_1(0)] + P_r\,[x(0) < Y_1(0)]\,P_r[N_1 \geqslant 1 \,|\, x(0) < Y_1(0)] \\ \leqslant P_r\,[x(0) > Y_1(0)] + P_r\,[x(0) < Y_1(0)] \sum_{S=1}^{\propto} S\,P_r[N_1 = S \,|\, x(0) < Y_1(0)] \text{ -}(\,$$

Equation (4) with the help of equation (2) finally reduces to,

$$P_r[t\,;-\propto,Y_1(t)] \leqslant P_r\,[x(0) > Y_1(0)] + E[N_1] - P_r\,[x(0) \geqslant Y_1(0)]\,E[N_1 \,|\, x(0) \geqslant Y_1(0)] \\ \text{- - -}(5)$$

in which E denotes the expected value.

If $N[-Y_2(t),t]$, hereafter referred as N_2, represents a random variable denoting the number of crossings of $-Y_2(t)$ from above during an internal $(0,t)$,

$$Pr[t; -Y_2(t), Y_1(t)] < Pr[x(0) < -Y_2(0)] + Pr[x(0) > Y_1(0)] + Pr[x(0) > -Y_2(0)]\, E[N_2 | x(0) > -Y_2(0)] + Pr[x(0) < Y_1(0)]\, E[N_1 | x(0) < Y_1(0)] - Pr[\{x(0) < -Y_2(0)\}\{x(t) > Y_1(t)\}] \; ---- (6)$$

Equation (6) in effect represents the best upper bound probability of failure of the structure interms of the rate of crossings of the time-dependent barriers of response process.

In case the response process starts from zero origin, such that $Pr[x(0) = 0] = 1$, equation (6) further simplifies to :

$$Pr[t; -Y_2(t), Y_1(t)] < E[N_1] + E[N_2] - Pr[\{x(t_1) < Y_2(t_1)\}\{x(t_2) > Y_1(t_2)\}] \;..(7)$$

The approach presented above, to estimate the upper bound value becomes significant in dealing with those problems where a stationary process for a finite time internal is observed, as in certain control system problems.

3. PHASE (B).

An approximate solution to the structural optimization problem is attempted in this phase, making use of the results obtained in phase (A), with the probability of failure of the structure and the system-characteristics as restraints.

Let $Z(d)$ be the objective function to be minimised subject to the condition,

$$Pr\left[\bigcup_{i=1}^{k} \{ S_i(x(d,t)) \geq r_i \} \right]_j \leq [p_f]_j \quad \ldots\ldots\ldots\ldots (8)$$

and $S_j(x(d,t)) \leq r_j \quad \ldots\ldots\ldots\ldots (9)$

and $\omega_{il} \leq \omega_i \leq \omega_{iu} \quad \ldots\ldots\ldots\ldots (10)$

where $S_i(x(d,t))$ is the frequency response function of the system; $x(d,t)$ represents the response (stress, strain or displacement) at a critical zone to random excitation;

ω_{il}, ω_{iu} are the lower and upper limits of the natural frequency of vibration of the structure, respectively;

$[p_f]_j$ denotes the upper limit of the probability of failure under mode j.

Let $Pr[S_i(x(d,t)) \geq r_j] = p_i(d) \quad \ldots\ldots\ldots\ldots (11)$

For example, if the safety of the structure is analysed on the basis of the external load acting on it and its internal resistance, say F and R respectively, both treated as statistically independent normal distributions, then,

$$p(d) = \frac{1}{\sqrt{2\pi}} \int_{\rho}^{\infty} e^{-x^2/2}\, dx, \quad \ldots\ldots\ldots\ldots (12)$$

where $\rho = \dfrac{\bar{R} - \bar{F}}{\sigma_R} \dfrac{1}{\sqrt{1 + (\sigma_F/\sigma_R)^2}} \quad ------------ (13)$

in which $\overline{R}$ and $\overline{F}$ are respectively the mean value of the resistance and the load; σ_R^2 and σ_F^2 are their variance.

Equation (8) now reduces to,

$$P_r\left[\bigcup_{i=1}^{k}\{S_i(x(d,t)) \geqslant r_i\}\right]_j = \left[\sum_{i=1}^{k} p_i(d)\right]_j, \quad \ldots\ldots\ldots\ldots (14)$$

the limit of summation of the time variable being from $-\infty$ to ∞ . It follows,

$$\left[\sum_{i=1}^{k} p_i(d)\right]_j \leqslant [p_f]_j, \quad j = 1, 2, \ldots n. \quad \ldots\ldots\ldots\ldots (15)$$

In the case of non-stationary random excitations, for example, ground acceleration due to earthquakes, the left hand side of equation (15) may be replaced by the upper bound value of the probability of failure of the structure as obtained in phase (A).

4. CONCLUSIONS.

Since a knowledge of the rate of crossings of the time-dependent response-barriers is an essential pre-requisite to the present analysis, a rigorous statistical analysis of the past records of the random exciting force is warranted to achieve a high level of accuracy. A large class of optimization problems in control system engineering could be advantageously studied using this method.

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SUMMARY

A general solution to deal with structural optimization problems under non-stationary random loadings is presented, with the upper bound probability of failure of the structure within time-dependent barriers and the system characteristics as restraints.

RESUME

Une technique générale d'optimisation des structures est présentée pour le cas de charges aléatoires. Les caractéristiques du système et les valeurs supérieures de la probabilité de ruine en fonction du temps sont prises en considération.

ZUSAMMENFASSUNG

Es wird eine allgemeine Lösung der Bauoptimierungsprobleme für nicht stationäre Unfallsbelastungen dargestellt, mit der oberen Grenze der Versagenswahrscheinlichkeit innerhalb zeitabhängiger Grenzen und den Systemcharakteristiken als Einschränkungen.

IIa

Optimisation des structures: Critères prépondérants et méthode de prédimensionnement en structure métallique

Optimierung von Tragwerken: Entscheidende Kriterien und Verfahren zur Vorbemessung im Stahlbau

Structural Optimization: Prevailing Criteria and Proportioning Approach in Steel Structures

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1 - INTRODUCTION

L'idéal que cherchent à atteindre tous ceux qui sont associés à l'art de construire est de réaliser l'ouvrage qui donnera les meilleures garanties de service dans des conditions requises de sécurité et au meilleur prix.

L'optimisation envisagée ainsi n'est aujourd'hui pas accessible par des méthodes déductives. Elle demeure un art. Cependant, pour les démarches qu'il doit faire en vue de cette optimisation, l'ingénieur dispose de moyens de plus en plus élaborés. Les critères qu'il faudra respecter dans ces choix sont dans la pratique imposés par les autorités responsables de la sécurité, par les maîtres d'ouvrage et par les maîtres d'oeuvre. On les trouve exposés soit dans les textes règlementaires [1,2] , soit dans des cahiers des charges.

Par utilisation des techniques de programmation linéaire, le projeteur peut dans la pratique optimiser sa structure en poids, tout en satisfaisant un certain nombre de critères aux états limites ultimes. Un programme de dimensionnement optimal de structures à barres, visant ces objectifs, a été réalisé dans le cadre de travaux entrepris au CTICM et nous montrerons un certain nombre d'exemples qui mettent en lumière l'influence que peut avoir le respect des critères de vérification sur l'optimisation de la structure.

2 - RAPPEL DES DIFFERENTS CRITERES A SATISFAIRE AUX ETATS LIMITES ULTIMES

Un état limite ultime est atteint lorsqu'un des phénomènes suivants se produit :

a) perte d'équilibre de la structure
b) transformation de tout ou d'une partie de la structure en un mécanisme
c) instabilité de forme :
  - d'ensemble de la structure,
  - individuelle d'une barre
d) déformations excessives
e) cumul de déformations sous charges répétées
f) rupture d'un élément (fragilité ou par fatigue).

Un état limite d'utilisation est atteint lorsque la structure devient inapte aux fonctions normales pour lesquelles elle est conçue, en particulier lorsque les déformations excessives entraînent une interruption du service normal de la structure ou des désordres dans les éléments non structuraux.

Dans le cadre actuel français de la philosophie de la sécurité, pour vérifier la sécurité vis-à-vis des états limites, le projeteur multiplie les valeurs (caractéristiques ou nominales) des actions par des facteurs appelés coefficients de pondération. Les valeurs de ces coefficients dépendent de l'état limite considéré (état limite d'utilisation ou état limite ultime) du type d'action envisagé (actions permanentes ou variables) et de la combinaison d'actions étudiée (intervention simultanée d'actions variables).

Ainsi, pour la vérification à l'état limite ultime, on est conduit à considérer les plus défavorables des combinaisons d'actions données dans le tableau ci-dessous :

Tableau 1

Cas de combinaisons d'actions		
prenant en compte une des trois actions Q, S, W.	$\frac{3}{2} Q + \frac{4}{3} G$ $\frac{3}{2} S + \frac{4}{3} G$ $\frac{3}{2} W + \frac{4}{3} G$	$\frac{3}{2} Q + G$ $\frac{3}{2} S + G$ $\frac{3}{2} W + G$
prenant en compte simultanément deux des trois actions Q, S, W.	$\frac{17}{12} (S_r+W) + \frac{4}{3} G$ $\frac{17}{12} (W+Q) + \frac{4}{3} G$ $\frac{17}{12} (Q+S) + \frac{4}{3} G$	$\frac{17}{12} (S_r+W) + G$ $\frac{17}{12} (W+Q) + G$ $\frac{17}{12} (Q+S) + G$
prenant en compte simultanément trois des actions Q, S, W.	$\frac{4}{3} (Q + S_r + W + G)$	$\frac{4}{3} (Q+S_r+W) + G$
prenant en compte les actions climatiques extrêmes	$Q + S_{re} + W_e + G$ $Q + S_e + G$	$Q + S_{re} + W_e + G$ $Q + S_e + G$

SYMBOLES

- Majuscules

Q = action permanente
G = surcharge
S = neige
W = vent

- Indices

e = extrême
r = réduite

A l'état limite d'utilisation, la vérification doit être effectuée en considérant les combinaisons les plus défavorables des actions non pondérées.

Selon l'état limite considéré, la vérification consiste en particulier à contrôler si la structure satisfait aux critères de déformations, d'instabilité ou de résistance.

Il a paru utile, dans le cadre de cet article, de bien souligner les principes sur la manière de prendre en compte la sécurité dans l'optique des règlements actuels français. Car il est de l'opinion des auteurs que ces considérations sont de nature à avoir une influence très importante, non seulement sur la façon dont on entend poser le problème de l'optimisation, mais aussi sur la nature des résultats de cette optimisation.

L'étude et la mise au point d'un projet de construction passent toujours par trois phases essentielles, à savoir :

. le choix des dispositions générales de la construction,
. la détermination des dimensions de tous les éléments composants,
. la vérification que les dimensions adoptées sont acceptables et -en particulier- confèrent à la construction un degré de sécurité suffisant.

En ce qui concerne la première phase, on admet généralement que seul le choix des dispositions générales de l'ouvrage et de sa conception constitue oeuvre d'imagination créatrice, pour laquelle l'intuition et l'expérience de l'architecte et du constructeur jouent un rôle essentiel.

La question qui nous préoccupe dans le cadre de cet exposé est de savoir s'il existe des méthodes pratiques qui permettent de déterminer un choix préalable des sections ou composants d'une structure quelconque et qui, d'une part satisfont à l'ensemble des critères de vérification que nous venons de décrire brièvement et d'autre part, conduisent à une optimisation de poids de la structure.

3 - TECHNIQUES D'OPTIMISATION DES STRUCTURES

Le cadre réduit de cet article ne nous permet pas d'exposer les fondements de la méthode utilisée ni le détail de sa formulation en termes de programmation linéaire. Cette étude a fait l'objet de plusieurs publications [5,6,7] où l'on trouvera la formulation du problème de prédimensionnement optimal en termes de de programmation linéaire, avec le choix de la fonction objective (que l'on peut linéariser) et la prise en compte, d'une part de l'interaction effort normal-moment fléchissant et d'autre part du flambement.

L'approche utilisée par les auteurs se distinguent d'autres méthodes itératives [3,4] de type "heuristique", qui abordent le problème de la recherche d'un optimum au travers d'un processus complexe "d'itération-contrôle-modification" permettant de prendre en considération de nombreux critères de vérification (contrainte, stabilité, déformabilité) de la structure étudiée. Ces méthodes présentent, à défaut d'un manque de généralités et d'une incertitude sur l'optimum atteint, l'avantage d'avoir été pensées comme un programme module (PLADS-I PLASTIC ANALYSIS AND DESIGN SYSTEM, écrit dans un système général de langage orienté : ICES INTEGRATED CIVIL ENGINEERING SYSTEM). A ce titre, il a le mérite d'être immédiatement disponible et utilisable par l'ingénieur de bureau d'études.

4 - EXEMPLES D'APPLICATION

Le programme de prédimensionnement automatique des structures permet de prendre en compte la stabilité individuelle des barres et une combinaison quelconque d'états de charges pondérées. Il est cependant nécessaire, pour être en conformité avec les règlements de calcul [1,2] , de contrôler que la solution obtenue satisfait les critères aux états limites d'utilisation et de vérifier les conditions d'instabilité d'ensemble de la structure.

Nons donnons ci-après deux exemples qui démontrent que d'une part, la solution optimale recherchée dépend des critères d'états limites adoptés, selon que le dimensionnement se réfère à un règlement de calcul en élasticité [1] ou en plasticité [2], d'autre part le prédimensionnement est d'autant plus proche de la solution finale optimale que l'on considère ou non les conditions d'instabilité individuelle.

Exemple 1 : A titre d'exemple, nous donnons les résultats obtenus sur la structure donnée à la figure 1a. Les schémas 1b et 1c donnent la valeur de deux combinaisons de charges les plus défavorables pour la structure considérée, à savoir charges permanentes + neige et charges permanentes + neige + vent.

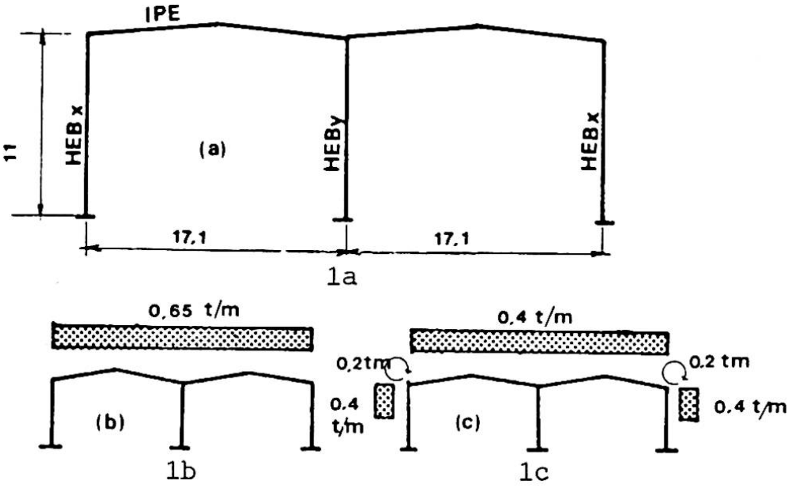

Fig. 1

1a

1b

1c

Les résultats sont résumés dans le tableau de la page suivante.

Tableau 2

		Poids (tonnes)	$\frac{\Delta H}{H}(\frac{1}{150})$	$\frac{\Delta v}{L}(\frac{1}{200})$	Nbre plastification état limite utilisation
Elas. [1]	poteaux HEB 200 traverse IPE 360	3,98	$\frac{1}{199,5}$	$\frac{1}{350}$	0
Plas. [2]	poteaux HEB 200 traverse IPE 300	3,47	$\frac{1}{174}$	$\frac{1}{210}$	2
Pred.	poteaux HEB 180 traverse IPE 360	3,64	$\frac{1}{144}$	$\frac{1}{332}$	

Dans cet exemple particulier pour lequel les conditions d'instabilité au flambement sont vérifiées, l'optimisation est différente selon qu'elle est élastique ou plastique. Dans les deux cas elle satisfait aux conditions de déformabilité aux états limites d'utilisation ; par contre, la présence de 2 rotules plastiques aux états limites d'utilisation n'est pas acceptée en élasticité. Le gain de poids est ici de 12,8%.

Le prédimensionnement initial donnait une solution proche de la solution élastique, mais la condition de déformabilité en tête du poteau n'était pas vérifiée, quoique la condition de flambement du poteau était satisfaisante.

Exemple 2 : Soit la structure donnée en figure 2, avec le cas de charges pondérées considéré. Les résultats du prédimensionnement sont rassemblés dans le tableau 3.

Fig. 2

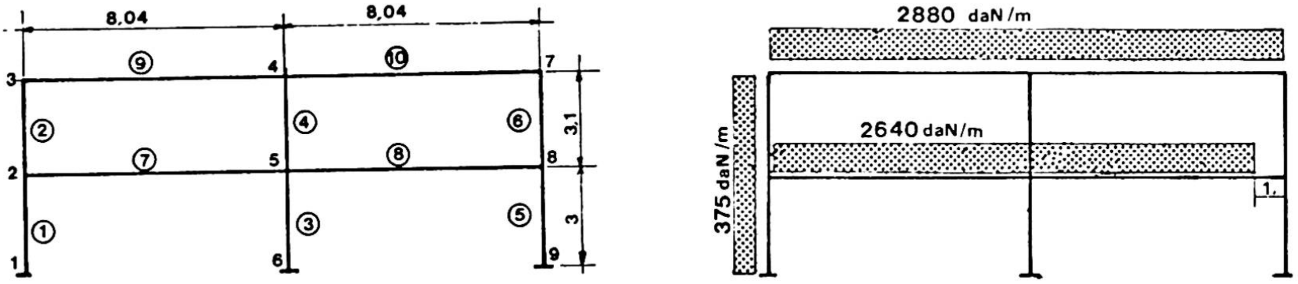

Tableau 3

	Poteaux 1,2,5,6	Poutres 3,4	Poutres 7,8,9,10
sans interaction M, N, ni flambement	IPE 300	HEB 100	IPE 400
avec interaction M, N et flambement	IPE 360	HEB 160	IPE 400
solution opt. selon Elas [1]	IPE 360	HEB 200	IPE 400

L'examen des résultats de ce tableau par le programme de prédimensionnement automatique des structures montre que si le dimensionnement sans interaction M et N est acceptable pour les poutres (c'est-à-dire lorsque la sollicitation de flexion est prépondérante), il n'en est pas de même pour les poteaux où l'effort axial est prépondérant. Il est nécessaire alors d'introduire dans le prédimensionnement les conditions d'interaction entre l'effort normal et le moment fléchissant et les conditions d'instabilité (voir 2ème ligne du tableau 3). L'introduction de ces conditions amène généralement une redistribution des efforts entre les sections et peut conduire aussi à une augmentation des sections simplement fléchies (barres). La 3ème ligne du tableau 3 donne la solution finale compatible avec les exigences d'un règlement élastique [1].

5 - CONCLUSIONS

La méthode mise au point dans le cadre d'études entreprises au CTICM trouve son fondement dans l'application du théorème statique en plasticité et les techniques de programmation linéaire. Elle conduit d'une manière pratique à un prédimensionnement initial correct, à condition toutefois de prendre en considération les conditions d'interaction entre sollicitation de flexion et effort axial et les conditions de stabilité individuelle au flambement des barres.

Il y a lieu cependant de procéder à une vérification de ce prédimensionnement initial, pour contrôler si la structure satisfait aux diverses exigences imposées par les codes de calcul aux états limites d'utilisation.

La fonction à optimiser est le coût total de la structure, c'est-à-dire la somme des coûts des aciers, de la fabrication, du montage et de l'entretien. Une étude factorielle de l'influence de ces divers coûts dans l'établissement d'une fonction économique a été étudiée [8]. Si cette étude a montré qu'il était possible d'améliorer sensiblement la fonction économique, la qualité du dimensionnement n'est cependant pas accrue dans les mêmes proportions. En particulier, du fait de nombreuses hypothèses au niveau de la prise en compte dans le prédimensionnement de l'instabilité individuelle des barres, le gain de précision du à l'amélioration de la fonction économique est illusoire.

Le programme de prédimensionnement automatique des structures est valable quelle que soit la configuration géométrique de la structure et la nature des charges extérieures appliquées. Cependant, le nombre de sections potentiellement critiques choisies et celui des contraintes résultant des conditions de plastification, d'interaction M et N et d'instabilité de flambement des barres comprimées et fléchies, en limitent l'application pratique à des structures relativement simples (portiques simples, portiques accolés, cadres multi-étagés de 2 niveaux, 3 baies).

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RESUME

L'article expose brièvement l'état des conditions à satisfaire dans le cadre d'une philosophie règlementaire aux états limites. Il est actuellement possible de tenir compte des conditions d'interaction effort normal-moment fléchissant et des conditions de flambement dans l'optimisation des structures à barres. Deux exemples montrent qu'il est important de prendre en considération ces critères si l'on veut aboutir à un prédimensionnement valable.

ZUSAMMENFASSUNG

Der Artikel weist kurz auf die Bedingungen hin, die im Rahmen einer vertretbaren Philosophie der Grenzzustände erfüllt sein müssen. Es ist heutzutage möglich, in der Optimierung von Stabtragwerken der gegenseitigen Wirkung zwischen Normalkraft und Biegungsmoment und dem Knicken Rechnung zu tragen. Zwei Beispiele zeigen, dass es wichtig ist, solche Kriterien in Betracht zu ziehen, wenn eine günstige Vorbemessung erreicht werden soll.

SUMMARY

The paper states briefly the conditions to be satisfied within the framework of an ultimate state design philosophy. It is presently possible to improve the optimization of structures by taking into account interaction between normal force and bending moment and buckling conditions. Two examples show that it is important to consider such criteria, if we want to achieve a proper members selection.

IIa

Minimum Weight Plastic Design of Multi-story Plane Frames for Five Sets of Design Loads

Calcul plastique, pour un poids minimum, de cadres plans à plusieurs étages, avec cinq groupes de cas de charge

Plastische Bemessung auf Minimalgewicht von mehrstöckigen ebenen Rahmen für fünf Belastungszustände

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1. Introduction Several classes of general solutions to the problem of minimum weight plastic design of multi-story multi-span plane frames subjected to a class of *one* set of practical design loads have been derived by the senior author [1] by applying Foulkes' theory [2] and by extending it to a more general theory [3] which incorporates the axial force-bending moment interaction yield conditions. The present authors have further extended the result of [1] so as to incorporate the reaction constraints in [4]. These analytical general solutions are of theoretical and practical interests. Firstly, they serve to clarify even partially the general features of the minimum weight designs. Secondly, once an analytical method is developed for simpler problems based upon the moment yield condition [1], their general solutions would provide a good lead to the general solutions to more complex problems based upon interaction yield conditions [3]. Thirdly, they will provide good initial feasible solutions for neighborhood problems.

In this paper, *a kinematical restricted maximization procedure* is developed by combining the *primal-dual* method of LP [5] with a semi-inverse approach similar to the idea of [1] and then applied to the problem of minimum weight plastic design of multi-story multi-span plane frames subjected to five sets of design loads.

2. Formulation of the Design Problem Fig.1 shows a multi-story multi-span plane frame to be designed by Foulkes' theory [2] and the five sets of design

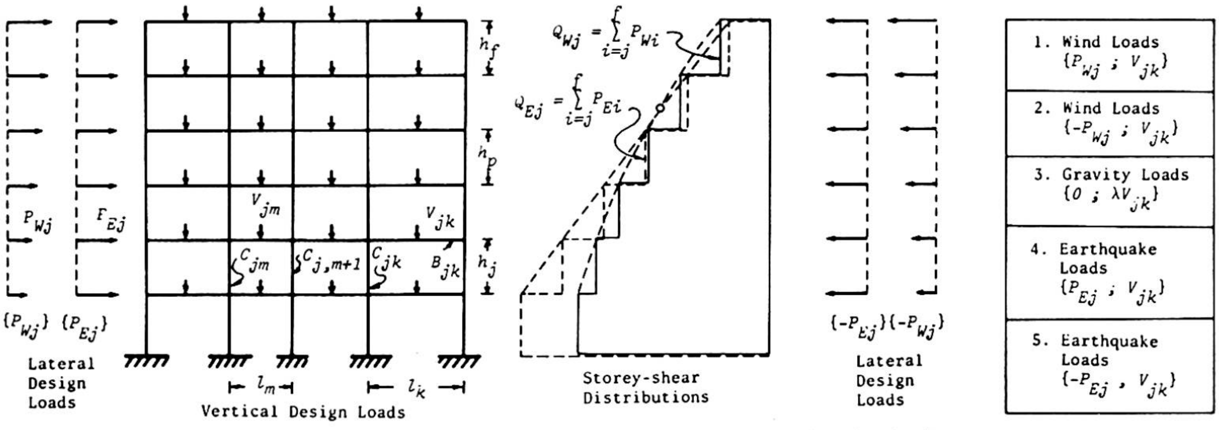


Fig.1 Design Load Distributions, Notation and 5 sets of Design Loads

loads. The fully-plastic moments of *(j,k)*-beam and *(j,k)*-column are denoted by $B_{j,k}$ and $C_{j,k}$, respectively. Without loss of practical generality, it may be assumed that the story-shear force distributions defined in Fig.1 be such that

$$\begin{aligned} &Q_{Ej} \geqq Q_{Wj} \quad \text{for} \quad j=1,2,\ \ldots,\ p,\ \text{and} \\ &Q_{Ej} \leqq Q_{Wj} \quad \text{for} \quad j=p+1,\ \ldots,\ f. \end{aligned} \tag{1}$$

The factor λ for the design gravity loads is assumed to be $\lambda \leqq 2.0$.

The design problem for five sets of design loads is treated in the following three or four steps:

(i) Solve the basic problem for the two sets of co-directional lateral design loads 1 and 4, i.e., for $\{P_{Wj};V_{j,k}\}$ and $\{P_{Ej};V_{j,k}\}$,

(ii) Construct a statically admissible bending moment field for the two sets of design loads 2 and 5, i.e., for $\{-P_{Wj};V_{j,k}\}$ and $\{-P_{Ej};V_{j,k}\}$,

(iii)Construct a statically admissible bending moment field for the design gravity loads 3, i.e., for $\{0;\lambda V_{j,k}\}$,

(iv) If the step (ii) or (iii) is not possible, modify the collapse mechanism locally and find the corresponding modified design.

The basic problem (i) may be stated in terms of the static variables defined in Fig.2(a) as follows:

$$\textit{Minimize} \qquad G = g\{\sum_{k=1}^{s} l_k \sum_{j=1}^{f} B_{jk} + \sum_{j=1}^{f} h_j \sum_{k=1}^{s+1} C_{jk}\}, \quad (g:\ \textit{constant}) \tag{2}$$

$$\left.\begin{aligned} \textit{subject to:}\ &\sum_{k=1}^{s+1}(c^B_{Ijk} + c^T_{Ijk}) = h_j Q_{Ij}\ , \quad b^L_{Ijk} + b^R_{Ij,k-1} = c^B_{Ij+1,k} + c^T_{Ijk}\ , \\ &\tfrac{1}{2}(b^L_{Ijk} - b^R_{Ijk}) + \tfrac{1}{4} l_k V_{jk} \leqq B_{jk} \\ &-B_{jk} \leqq b^L_{Ijk} \leqq B_{jk}\ , \ -B_{jk} \leqq b^R_{Ijk} \leqq B_{jk}\ , \ B_{jk} \geqq 0, \\ &-C_{jk} \leqq c^B_{Ijk} \leqq C_{jk}\ , \ -C_{jk} \leqq c^T_{Ijk} \leqq C_{jk}\ , \ C_{jk} \geqq 0, \end{aligned}\right\} \text{(3a-i)}$$

where b^L_{Ijk}, b^R_{Ijk}, c^B_{Ijk} and c^T_{Ijk} are free variables. In the expression (2), f and s denote the numbers of stories and spans, respectively. In the constraints (3), the first subscript I denotes the kind of design loads and is to be either E or W. The second and third subscripts refer to the story number from below and member number from left, respectively. For the sake of brevity, the equations of moment equilibrium about interior and exterior joints have been written in one and the same form with the convention that all the undefined quantities with respect to non-existent members shall be disregarded and dropped as null. This convention will also be used hereafter, unless otherwise stated.

3. Kinematical Restricted Maximization Procedure-Semi-Inverse Primal-dual Method.

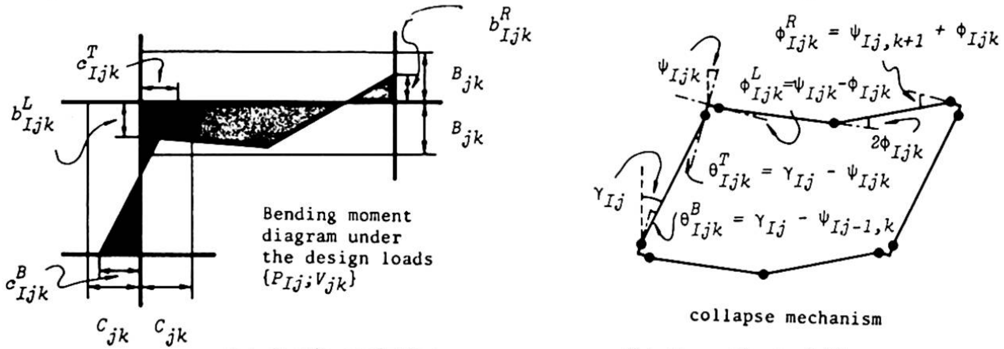
The idea of the proposed approach may be summarized by referring to Fig.3 as follows. A design problem formulated as a linear programming problem [6] of a *mixed* type [5], may often be such that a certain set of constraints may be anticipated to be *inactive* due to the nature of the problem. From the original primal problem

$[PO]$: *Minimize* $\{G(x)\,|\,x \in S_1 \cap S_2\}$

of a large size, a subproblem

$[PS]$: *Minimize* $\{G(x)\,|\,x \in S_1\}$

may be derived by tentatively disregarding a certain set of constraints which are anticipated to be inactive and which define the set S_2. Then the dual problem to



(a) Static variables *(b) Kinematic variables*

Fig.2 Definitions of the variables

[*PS*], i.e.

[*DS*]: *Maximize* $\{D(u)\,|\,u \in V\}$

must involve a smaller number of dual variables and a greater number of equality constraints. Therefore, if the solution u^o to [*DS*] can be found more easily compared to the solution to the dual problem of [*PO*], then the corresponding solution x^o to [*PS*] may also be readily found simply by solving the set of simultaneous linear equations derived from the duality theorem of *LP*. It remains then to check if $x^o \in S_2$. The procedure may also be called "a semi-inverse primal-dual method."

4. A Class of General Solutions to the Problem (i)

It is now shown that the kinematical maximization procedure is fruitful for rectangular frames due to their regularity in the optimality criteria-based collapse mechanism. Let

$$S_2:\ (b^L_{Ijk},\ b^R_{Ijk}) \geq -B_{jk},\ (c^B_{Ijk},\ c^T_{Ijk}) \geq -C_{jk},\ B_{jk} \geqq 0,\ C_{jk} \geqq 0, \qquad (4a\text{-}d)$$

Then the dual problem [*DS*] may be written in terms of the kinematic variables defined in Fig.2(b) as follows:

$$\text{Maximize}\quad D=\Theta\{\sum_{j=1}^{f} h_j(Q_{Wj}\gamma_{Wj}+Q_{Ej}\gamma_{Ej})+\sum_{k=1}^{s}\ \sum_{j=1}^{f}\frac{1}{2}l_k V_{jk}(\phi_{Wjk}+\phi_{Ejk})\} \qquad (5)$$

$$\begin{aligned}\text{subject to}\quad &\gamma_{Ij} \geqq \underset{k}{Max.}\{\psi_{Ijk},\ \psi_{Ij-1,k}\}\\ &\psi_{Ijk} \geqq \phi_{Ijk} \geqq 0 \quad (k=1,2,\ldots,\ s),\qquad \psi_{Ij,s+1} \geqq -\phi_{Ijs}\\ &\{(\psi_{Wjk}+\psi_{Wj,k+1}+2\phi_{Wjk})+(\psi_{Ejk}+\psi_{Ej,k+1}+2\phi_{Ejk})\} = \Theta l_k \qquad (6a\text{-}d)\\ &\{(2\gamma_{Wj}-\psi_{Wj-1,k}-\psi_{Wjk})+(2\gamma_{Ej}-\psi_{Ej-1,k}-\psi_{Ejk})\} = \Theta h_j\end{aligned}$$

The inequalities (6a, b) restrict the directions of plastic hinge rotations and the equalities (6c, d) are the generalized Foulkes conditions defined by Chan [6] and Prager [7]. The latter will be referred to as *FCP* conditions.

The equations (6d) indicate that $\psi_{Wjk}+\psi_{Ejk} \equiv \psi_j$ (independent of k). The problem defined by (5) and (6) may then be simplified to a problem in terms of ψ_{Wjk}, γ_{Wj}, ϕ_{Wjk} and ψ_j only. After some manipulation on the inequalities, γ_{Wj} may be expressed in terms of ψ_{Wjk} and ψ_j only, and then ψ_{Wjk}, in terms of ψ_{Wpk} and ψ_j only. Finally, for those problems in which the load conditions:

$$\begin{aligned}&h_jQ_{Ij}+h_{j+1}Q_{I,j+1} \geqq \sum_{k=1}^{s} l_k V_{jk}\ ,\quad \begin{pmatrix}1 \leqq j \leqq p-1 & for & I=E,\\ p+1 \leqq j \leqq f & for & I=W,\end{pmatrix}\\ &h_pQ_{Ep}+2h_{p+1}Q_{Ep+1}-h_{p+1}Q_{Wp+1} \geqq \sum_{k=1}^{s} l_k V_{pk}\ ,\end{aligned} \qquad (7a,b)$$

and the geometrical conditions:

$$l_m \leqq l_k \leqq 2l_m\ ,\ l_m \leqq 2h_1\ (l_m = \underset{k}{Min.}\{l_k\})\ , \qquad (8a,b)$$

are satisfied, the problem [*DS*] may be reduced to the following form:

$$\text{Maximize}\quad D^{*}=\Theta(-\Delta M_p\xi + \Delta M_{p+1}\eta), \qquad (9)$$

$$\begin{aligned}\text{subject to}\quad &\xi \equiv \underset{k}{Max.}\{\psi_{Wpk}\},\qquad \eta \equiv \underset{k}{Min.}\{\psi_{Wpk}\},\qquad (\xi \geqq \eta)\\ &\xi \leqq \frac{1}{2}h_{p+1}\Theta+Min.\{0,\eta\},\ Max.\{\frac{1}{2}l_m\Theta,\xi\} \leqq \frac{1}{2}h_{p+1}\Theta+\eta,\end{aligned} \qquad (10a\text{-}d)$$

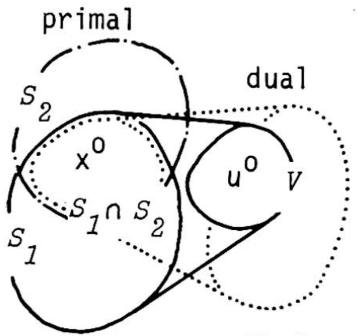


Fig. 3

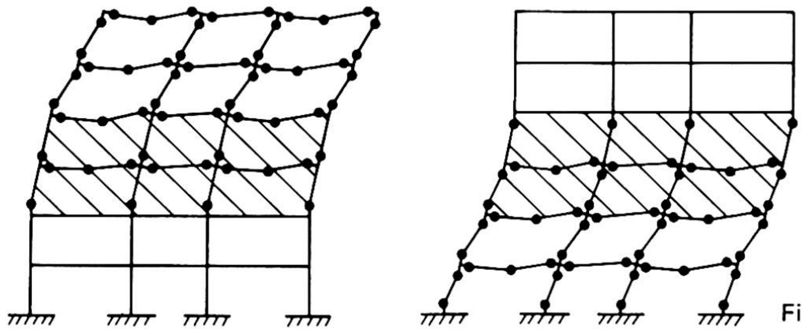


Fig. 4

$$0 \leqq \psi_{Wpk} \leqq \frac{1}{2}l_m\Theta,\ (k=1,2,\ldots,\ s),\ a \leqq \psi_{Wp,s+1} \leqq b,$$
$$a \equiv Max.\{-(l_s-l_m)\Theta/2,\ -\Psi_{Wps}\} \leqq 0$$
$$b \equiv Min.\{l_s\Theta/2,\ l_m\Theta-\psi_{Wps}\} \geqq l_m\Theta/2 \qquad (10e\text{-}h)$$

where $\Delta M_p \equiv h_p(Q_{Ep}-Q_{Wp}) \geqq 0$ and $\Delta M_{p+1} \equiv h_{p+1}(Q_{Wp+1}-Q_{Ep+1}) \geqq 0$. The solution to this reduced problem may readily be derived as summarized in Table 1. In those problems where (7) and (8) are satisfied, the generalized Foulkes mechanism defined by the *FCP* conditions can thus be constructed as shown in Fig.4 for Case (B) as an example.

The solution to the problem [*PS*] corresponding to this problem[*DS*] may also be derived straightforwardly. By assuming that some statical restrictions defined and checked later will be satisfied, the resulting bending moment diagram may be understood best by conceiving it as the result of superposition of the constituent elementary moment diagrams (with equal corner values for $k{\neq}m$) shown in Fig.5. Such a decomposition was first introduced in [1]. Each diagram is referred to as "frame moment diagram." The minimum weight plastic design corresponding to Table 1 may be compactly summarized as Table 2 in terms of "Maximum Story-Shear Force Design" defined by

$$B^*_{jk} \equiv Max.\{B^W_{jk},\ B^E_{jk}\},\qquad C^*_{jk} \equiv Max.\{C^W_{jk},\ C^E_{jk}\} \qquad (11a,b)$$

where $\{B^W_{jk},\ C^W_{jk}\}$ and $\{B^E_{jk},\ C^E_{jk}\}$ denote the designs only for $\{P_{Wj};\ V_{jk}\}$ and $\{P_{Ej};\ V_{jk}\}$, respectively, derived by means of [1]. B^*_{jk} and C^*_{jk} are given by

$$B^*_{jk} = \frac{1}{4}l_kV_{jk}\ (k{\neq}m);\quad C^*_{jk} = \frac{1}{4}(l_{k-1}V'_{j,k-1}+l_kV'_{jk});\ (k{\neq}m,\ m+1)$$
$$B^*_{jk} = \frac{1}{4}(h_jQ_{Ij}+h_{j+1}Q_{I,j+1}-\sum_{k\neq m}l_kV_{jk}),\ \left(\begin{matrix}1 \leqq j \leqq p-1 & for & I=E\\ p+1 \leqq j \leqq f & for & I=W & and\end{matrix}\right)$$
$$j=p,\ I=E\ \ for\ \ \Delta M_p \geqq \Delta M_{p+1},\ and\ j=p,\ I=W\ \ for\ \ \Delta M_p \leqq \Delta M_{p+1};$$
$$C^*_{jn} = \frac{1}{4}(h_jQ_{Ij}-\sum_{k\neq n-1,n}l_kV'_{jk}),\ \left(\begin{matrix}1 \leqq j \leqq p\ \ for\ \ I=E,\\ p+1 \leqq j \leqq f\ \ for\ I=W,\\ n=m,\ m+1\end{matrix}\right)$$
$$V'_{jk} \equiv \sum_{i=j}^{f}(-1)^{i-j}V_{ik}\ . \qquad (12a\text{-}e)$$

The yield inequalities in (3) provide restrictions on the design loads in accordance with the classification of the solutions listed in Table 2. These

Table 1 Generalized Foulkes Mechanism

		γ_{Wj}	γ_{Ej}	ψ_{Wjk}	ψ_{Ejk}	$\phi_{Wjk}(k{\neq}m)$	$\phi_{Ejk}(k{\neq}m)$	$\phi_{Wjm},\ \phi_{Ejm}$
$j=1$		0	$\frac{1}{2}(h_1+\frac{1}{2}l_m)\theta$	0	$\frac{1}{2}l_m\theta$	0	$\frac{1}{2}(l_k-l_m)\theta$	0
$j=2,\ \ldots,\ p-1$			$\frac{1}{2}(h_j+l_m)\theta$					
$j=p$	(A)	0	$\frac{1}{2}(h_p+l_m)\theta$	0	$\frac{1}{2}l_m\theta$	0	$\frac{1}{2}(l_k-l_m)\theta$	
	(B)	$\frac{1}{2}(l_m-h_{p+1})\theta$	$\frac{1}{2}(h_p+h_{p+1})\theta$	$\frac{1}{2}(l_m-h_{p+1})\theta$	$\frac{1}{2}h_{p+1}\theta$	*	*	
	(C)	$\frac{1}{2}l_m\theta$	$\frac{1}{2}h_p\theta$	$\frac{1}{2}l_m\theta$	0	$\frac{1}{2}(l_k-l_m)\theta$	0	
	(D)	$\frac{1}{2}h_p\theta$	$\frac{1}{2}l_m\theta$	$\frac{1}{2}h_p\theta$	$\frac{1}{2}(l_m-h_p)\theta$	*	*	
$j=p+1$	(A)	$\frac{1}{2}h_{p+1}\theta$	$\frac{1}{2}l_m\theta$	$\frac{1}{2}l_m\theta$	0	$\frac{1}{2}(l_k-l_m)\theta$	0	
	(B)	$\frac{1}{2}l_m\theta$	$\frac{1}{2}h_{p+1}\theta$					
	(C)	$\frac{1}{2}(h_{p+1}+l_m)\theta$	0					
	(D)	$\frac{1}{2}(h_p+h_{p+1})\theta$	$\frac{1}{2}(l_m-h_p)\theta$					
$j=p+2,\ \ldots,\ f$		$\frac{1}{2}(h_j+l_m)\theta$	0					

restrictions may be summarized as shown in Table 3, where

$$M_{Ij} \equiv \frac{1}{4}(h_j Q_{Ij} + h_{j+1}Q_{I,j+1} - \sum_{k=1}^{s} l_k V_{jk}) \qquad (13)$$

It may now be concluded that the present solutions (*A~D*) are the rigorous solutions to the problems in which all the geometrical and loading conditions are satisfied.

5. Design for Five Sets of Design Loads.

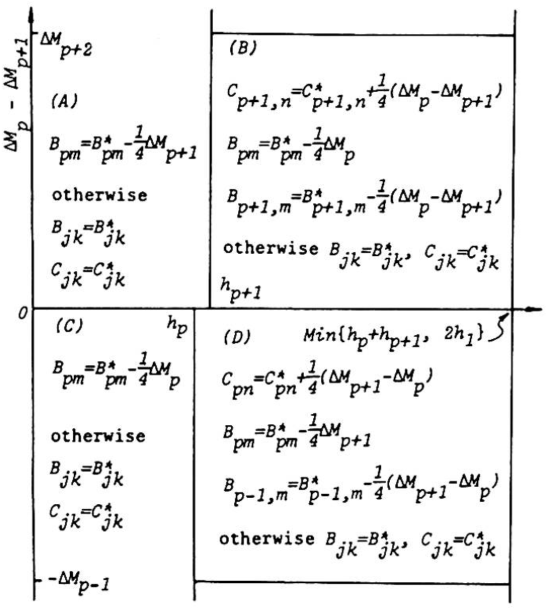
It may readily be confirmed that a statically admissible bending moment field for $\{-P_{Ej};\ V_{jk}\}$ and $\{-P_{Wj};\ V_{jk}\}$ can be constructed just by inverting the frame moment diagrams as shown in Fig.6.

For design gravity loads, it is convenient to consider again the decomposed moment diagram with the respectively equal corner values $\lambda l_k V'_{jk}/8$, as shown in Fig.7. The conditions that the bending moment diagram given by superposing the elementary diagrams in Fig.7 be statically admissible in a frame designed by the procedure in Section 4, lead again to further restrictions on the design gravity loads. An examination of these restrictions indicates that there are a number of practically useful design solutions within the range defined by them.

6. Concluding Remarks

It may now be concluded that, for the class of design problems in which all the previous and supplementary conditions are satisfied, the solutions (*A~D*) are the rigorous minimum weight plastic designs. The present designs have apparently clarified the nature of minimum weight plastic designs. While these designs must be modified for practical use so as to satisfy a number of structural requirements, the present solutions will at least provide a basis

Table 2



(A) $B_{pm}=B^*_{pm}-\frac{1}{4}\Delta M_{p+1}$ otherwise $B_{jk}=B^*_{jk}$ $C_{jk}=C^*_{jk}$	(B) $C_{p+1,n}=C^*_{p+1,n}+\frac{1}{4}(\Delta M_p-\Delta M_{p+1})$ $B_{pm}=B^*_{pm}-\frac{1}{4}\Delta M_p$ $B_{p+1,m}=B^*_{p+1,m}-\frac{1}{4}(\Delta M_p-\Delta M_{p+1})$ otherwise $B_{jk}=B^*_{jk}$, $C_{jk}=C^*_{jk}$
(C) $B_{pm}=B^*_{pm}-\frac{1}{4}\Delta M_p$ otherwise $B_{jk}=B^*_{jk}$ $C_{jk}=C^*_{jk}$	(D) $C_{pn}=C^*_{pn}+\frac{1}{4}(\Delta M_{p+1}-\Delta M_p)$ $B_{pm}=B^*_{pm}-\frac{1}{4}\Delta M_{p+1}$ $B_{p-1,m}=B^*_{p-1,m}-\frac{1}{4}(\Delta M_{p+1}-\Delta M_p)$ otherwise $B_{jk}=B^*_{jk}$, $C_{jk}=C^*_{jk}$

Table 3

		(A)	(B)	(C)	(D)
$j=1,2,\ldots,p-2$		$M_{Ej} \geq 0$			
$j=p-1$					$M_{Ep-1} \geq \frac{1}{4}(\Delta M_{p+1}-\Delta M_p)$
$j=p$		$M_{Ep} \geq \frac{1}{4}\Delta M_{p+1}$	$M_{Ep} \geq \frac{1}{4}\Delta M_p$	$M_{Wp} \geq \frac{1}{4}\Delta M_p$	$M_{Wp} \geq \frac{1}{4}\Delta M_{p+1}$
$j=p+1$			$M_{Wp+1} \geq \frac{1}{4}(\Delta M_p - \Delta M_{p+1})$		
$j=p+2,\ldots,f$		$M_{Wj} \geq 0$			
$k \neq m,\ m+1$ $j=1,2,\ldots,f$		$l_{k-1}V'_{j,k-1} + l_k V'_{jk} \geq 0$			
$n=m,m+1$	$j=1,\ldots,p-1$	$h_j Q_{Ej} \geq \sum_{k\neq n,n-1} l_k V'_{jk}$			
	$j=p$	$h_p Q_{Ep} + \frac{1}{4}(\Delta M_p - \Delta M_{p+1}) \geq \sum_{k\neq n,n-1} l_k V'_{pk}$			
	$j=p+1$	$h_j Q_{Wj} \geq \sum_{k\neq n,n-1} l_k V'_{jk}$			
	$j=p+2,\ldots,f$	$h_{p+1}Q_{Wp+1} + \frac{1}{4}(\Delta M_{p+1} - \Delta M_p) \geq \sum_{k\neq n,n-1} l_k V'_{p+1,k}$			

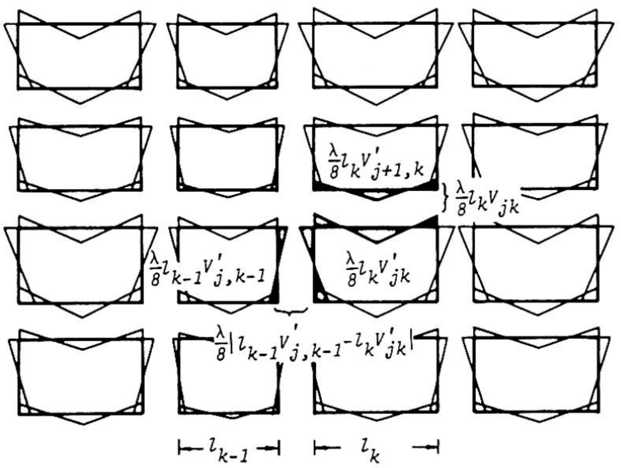


Fig. 7 Decomposed Moment Diagram for Gravity Loads

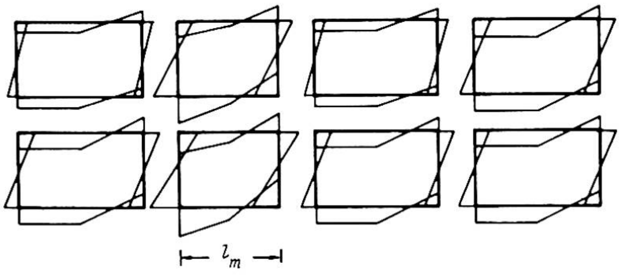


Fig. 5 Frame moments under the wind load (P_{Wj})

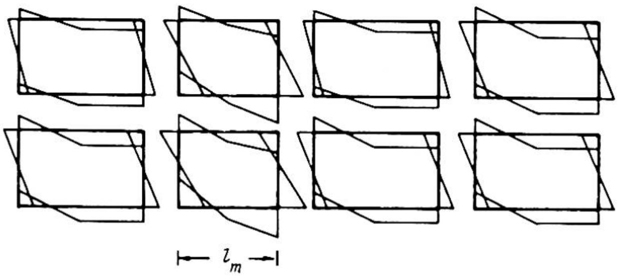


Fig. 6 Frame moments under the wind load ($-P_{Wj}$)

for initial designs useful in such countries where fairly large lateral design loads must be assigned for PLASTIC DESIGN so that frames can withstand against strong winds and strong motion earthquakes. The present solutions may be said to be a class of the most fundamental designs in the sense that a number of useful designs to practical neighborhood problems can be derived by appropriate but mostly local modifications. Three cases:
(a) $h_f P_{Wf} \leq \sum_{k=1}^{s} l_k V_{fk}$, (b) $l_m \geq 2h_1$ and (c) $\exists l_k \geq 2l_m$ have been treated in [8].

The present solution and the solutions in [1, 3, 4] indicate that a frame designed by these solutions would collapse in an extremely deteriorated overcomplete mechanism under a designated set of design loads according to the rigid-plastic analysis. It is therefore necessary to confirm the safety of such a frame against possible collapse due to inelastic instability according to a more refined theory of large-deflection elastic-plastic analysis. For this purpose, static and dynamic large-displacement analyses have been carried out on minimum weight frames in [9~11] under alternating lateral loads well beyond their static stability limits and under strong motion earthquake disturbances, respectively.

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SUMMARY

A kinematical restricted maximization procedure has been developed by combining the primal-dual method of linear programming with a semi-inverse approach. Some general solutions to practical problems of minimum weight plastic design have been derived analytically by applying the proposed method.

RESUME

Une procédure cinématique de maximisation limitée a été développée par combinaison de la méthode primale-duale de la programmation linéaire avec une approche semi-inverse. Quelques solutions générales pour des problèmes pratiques de dimensionnement plastique, conduisant à un poids minimum ont été obtenues analytiquement par application de la méthode proposée.

ZUSAMMENFASSUNG

Ein begrenztes kinematisches Maximierungsverfahren wird bei einer Kombination der "primal-dual"-Methode der linearen Programmierung mit einem "semi-inversen" Verfahren entwickelt. Allgemeine analytische Lösungen praktischer Probleme der plastischen Bemessung auf Minimalgewicht werden durch Anwendung der vorgelegten Methode gefunden.

Optimality Criteria and Dual Methods in Truss Design

Critères d'optimisation et méthodes duales dans le dimensionnement
de treillis

Optimierungskriterien und Dualmethoden in der Berechnung von Fachwerken

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1. INTRODUCTION

In the Introductory Report of the 10th Congress of IABSE Gellatly and Dupree[1] describe the optimality criteria approach to the optimum design of large structural systems. In handling large structural systems the direct solution approach by numerical mathematical programming methods is often excessively slow and cumbersome as a result of the large numbers of variables which must be optimized. The optimality criteria approach is intended to overcome the difficulties posed by having large numbers of variables. Gellatly and Dupree consider the optimality approach to the design of structures in which element mass and stiffness are proportional. Such structures include those composed of axial force bars, membrane plates and shear panels. For this class of structures Gellatly and Dupree derive an optimality criterion, their equation (2), for the minimum weight design of a truss subject to a single displacement constraint. They then use this optimality criterion, (2), to develop a recursion relationship, (8), which allows any arbitrary set of member areas to be modified iteratively so as to eventually produce an optimal set of member sizes. The important time-saving feature of this approach is that at each iteration the existing set of member sizes is altered by applying the simple relationship (8) to each area in turn. There is no complicated numerical search involved.

Gellatly and Dupree then continue to describe a large computer program, OPTIM II, in which this optimality criterion and redesign formula is used to design structures with multiple displacement constraints (stiffness requirements) and also individual member size constraints. They point out that neither the optimality criterion itself nor the redesign formula is valid for anything other than a single displacement constraint but, despite this lack of rigour, OPTIM II still obtains remarkably good numerical results very quickly. This is not disputed here; OPTIM II is an efficient program, but its lack of rigour is perplexing and it makes it difficult to interpret and identify those occasional cases in which OPTIM II performs poorly.

The purpose of this paper is to examine a new dual formulation of optimum design problems for this class of structures. In particular the problem of how best to handle multiple constraints is examined and an interpretation of the dual problem is presented which has considerable relevance in the development of improved optimum design algorithms for large structural systems.

2. THE OPTIMUM DESIGN PROBLEM

For simplicity of notation a truss structure composed only of axial force bars is considered, being typical of the general class of structures with member stiffness proportional to member mass. The minimum weight (minimum volume) design problem can be posed as that of finding the set of member areas A_i, i = 1, ..., N, which

$$\text{Minimize } W = \sum_{i=1}^{N} L_i A_i \tag{1}$$

subject to M independent nodal displacement constraints (Gellatly and Dupree consider only a single generalised stiffness constraint),

$$g_m \equiv \sum_{i=1}^{N} \left(\frac{FUL}{E\delta}\right)_{mi} \frac{1}{A_i} \leqslant 1 \qquad m = 1, \ldots, M \tag{2}$$

and subject also to N member size constraints, one for each member

$$g_{M+i} \equiv \frac{\overline{A_i}}{A_i} \leqslant 1 \qquad i = 1, \ldots, N \tag{3}$$

In constraints (2) F and U are the member actual forces and virtual forces associated with unit displacement in the direction of the nodal constraint. δ_m is the maximum permitted displacement of a node in constraint m, m = 1, ..., M. E is the elastic modulus, and each of constraints (2) is derived from specific applied loads and virtual force systems. In constraints (3) $\bar{A}_i$ is the minimum permissible size of member i, derived either from maximum member stress limits or from fabricational considerations.

In the above formulation it is assumed that F and U are constants, hence $\bar{A}_i$ is also constant. This assumption is valid for statically determinate trusses. It is strictly invalid for indeterminate trusses, however, F, U and hence $\bar{A}_i$ do not usually alter appreciably as members sizes alter and it is common to assume them constant, obtain an altered set of member sizes in some way, update the values of F, U and $\bar{A}_i$, solve again and continue in this iterative fashion until the process converges to an optimum solution. This iterative solution technique is used by both mathematical programming and optimality criterion devotees, the essential difference between them being only the way in which the altered set of member sizes is obtained. It is assumed here that this iterative method for indeterminate structures is used and so in the above formulation F, U, L, E, δ and $\bar{A}$ are all known constants. Our problem is how best to find the optimal set of member sizes.

Recently the present author[2] has shown that there is a dual formulation of the problem expressed in relationships (1), (2) and (3). Derivation of the dual problem is accomplished by exploiting the fact that the Lagrangian function of the above problem has a saddle point as a stationarity condition. A full proof of the dual formulation is given in reference[2] and here it is merely stated as

$$\left.\begin{array}{ll} \text{Maximise } V = \sum\limits_{i=1}^{N} L_i \left\{\sum\limits_{m=1}^{M} \left(\frac{FU}{E\delta}\right)_{mi} \lambda_m + \frac{\overline{A_i}}{L_i} \lambda_{M+i}\right\}^{\frac{1}{2}} & \\ \text{subject to } \sum\limits_{m=1}^{M+N} \lambda_m = 1 & \\ \qquad\qquad \lambda_m \geqslant 0 & m = 1, \ldots, M + N \end{array}\right\} \tag{4}$$

The solution of (4) is equivalent exactly to the solution of the primal problem, (1), (2) and (3). At the solution point (minimum of W, maximum of V) the following transformation relationships hold, with superscript asterisk denoting optimal values,

$$\left.\begin{aligned} &(\text{Minimum}) \quad W^* \equiv V^{*2} \quad (\text{Maximum}) \\ &A_i^* \equiv V^* \left\{\sum_{m=1}^{M} \left(\frac{FU}{E\delta}\right)_{mi} \lambda_m^* + \frac{\overline{A_i}}{L_i} \lambda_{M+i}^*\right\}^{\frac{1}{2}} \qquad i = 1, \ldots, N \end{aligned}\right\} \quad (5)$$

The dual variables in dual problem (4) are the λ_m, m = 1, ..., M + N and it will be noted that there is a dual variable λ_m for each of the primal constraints (2) and (3). The dual variables are therefore similar to the unknown Lagrange multipliers of the primal problem. All λ's must be non-negative; any value of $\lambda = 0$ denotes that the primal constraint to which it corresponds is inactive at the optimum. The single constraint in dual problem (4) requires that all λ's sum to unity.

3. PROBLEMS WITH ONLY DISPLACEMENT CONSTRAINTS

Gellatly and Dupree[1] consider only a single displacement constraint and their equations (2) and (8) represent an optimality criterion and a resizing formula for this problem. Their equation (2) contains a single unknown Lagrange multiplier corresponding to the single constraint. This unknown multiplier may be eliminated by substitution into the constraint which must perforce be active; consequently their resizing formula (8) contains no unknown multipliers. A major difficulty is encountered if this method is extended to multiple displacement constraints. In this case there will be M unknown Lagrange multipliers, one for each constraint, and since it is not known *a priori* which of the multiple displacement constraints are active and which are slack at the optimum it is not possible to eliminate the unknown multipliers by substitution. Consequently when a member resizing formula for multiple constraints is developed corresponding to Gellatly and Dupree's equation (8) it contains all the M unknown Lagrange multipliers. In order to use the resizing formula it is necessary to supply values to all the unknown Lagrange multipliers but there is no way of knowing what these values should be. This constitutes the major difficulty of using optimality criteria methods for multiple constraints. In order to get round this difficulty OPTIM II uses the envelope method which resizes each member according to the single constraint resize formula for each displacement constraint and then selects the largest resulting size. This process seems intuitively logical but has no theoretical rigour.

If the dual approach is examined for multiple displacement constraints only, the dual problem becomes

$$\left.\begin{aligned} &\text{Maximise } V = \sum_{i=1}^{N} L_i \left\{\sum_{m=1}^{M} \left(\frac{FU}{E\delta}\right)_{mi} \lambda_m\right\}^{\frac{1}{2}} \\ &\text{subject to } \sum_{m=1}^{M} \lambda_m = 1 \\ &\qquad\qquad \lambda_m \geqslant 0 \qquad\qquad m = 1, \ldots, M \end{aligned}\right\} \quad (6)$$

At the optimum, we have

$$\left.\begin{aligned} &(\text{Minimum}) \quad W^* = V^{*2} \quad (\text{Maximum}) \\ &A^* = V^* \left\{\sum_{m=1}^{M} \left(\frac{FU}{E\delta}\right)_{mi} \lambda_m^*\right\}^{\frac{1}{2}} \qquad i = 1, \ldots, N \end{aligned}\right\} \quad (7)$$

Problem (6) consists of maximizing V, a non-linear function of the M dual variables λ_m subject only to a single linear equality constraint and non-negativity of the dual variables. This is easily done by classical optimization methods. Once λ_m^*, m = 1, ..., M are known, relationships (7) give the minimum weight and optimal member sizes directly.

Several features of the dual problem can be noted. Firstly the number of dual variables is M, the number of displacement constraints. This means that the dimensionality of the original problem, which had N member size variables, is greatly reduced. Thus a large structure with perhaps 1000 members to be sized and 5 displacement constraints has a dual problem which consists of maximizing a non-linear function V of only 5 variables. In most large structural problems there are usually many more members than displacement constraints so the reduction in dimensionality afforded by the dual problem is of considerable advantage. Secondly, the dual problem itself is of a convenient form for rapid solution. The single linear equality constraint may be eliminated by substitution, converting the problem to one of unconstrained form with non-negativity requirements. First and second derivatives can be easily evaluated which makes solution comparatively simple. Thirdly, the result gives immediate information about which constraints in the primal problem are active and which are slack since a value of $\lambda_m = 0$ corresponds to a slack constraint. Finally the dual approach has the theoretical rigour which is lacking in the envelope method.

A physical interpretation of the primal/dual problems in terms of structural behaviour is illuminating. Consider a structure constrained by M independent displacement constraints, i.e.

$$\left.\begin{array}{ll}\text{Minimize } W & \\ \text{Subject to } g_m \leqslant 1 & m = 1, \ldots, M\end{array}\right\} \quad (8)$$

If each of the M constraints in (8) is multiplied by a multiplier λ_m, m = 1, ..., M, such that the sum of the λ_m's is unity, and all the constraints are then summed into a single surrogate constraint we have

$$\left.\begin{array}{l}\text{Minimize } W \\ \text{Subject to } \sum_{m=1}^{M} \lambda_m g_m \leqslant 1\end{array}\right\} \quad (9)$$

Examination of the dual problems corresponding to (8) and (9) shows them to be identical providing the λ_m's in (9) solve problem (6) optimally. This demonstrates that in responding to multiple constraints the structure apportions its member sizes as if all the independent constraints were surrogated into a single generalised stiffness requirement. The structure therefore responds to a single fictitious surrogated stiffness requirement and, since the λ_m must solve (6), the surrogate stiffness requirement is such that the independent sitffness requirements are combined together in such a way as to maximize their constraining potential.

This physical interpretation may partly help to explain the good results often obtained by the envelope method as used in OPTIM II. The envelope method resizes a a member by applying a single resize formula to each constraint in turn and selects the highest resulting member size. These highest sizes form a resized set. By this means the constraining potential of all the constraints is maximized. This is in the same spirit as the more rigorous dual approach outlined above but is mathematically different and is not rigorous. However, it may be conjectured that the good results obtained by OPTIM II correspond to problems in which the enveloping and surrogation approaches are similar and that the occasional poor performance of

OPTIM II corresponds to problems in which the member sizes obtained by enveloping are very different from those which satisfy the more correct surrogated constraint in (9).

4. PROBLEMS WITH DISPLACEMENT AND MEMBER SIZE CONSTRAINTS

As Gellatly and Dupree demonstrate, a displacement constraint governs the distribution of material throughout the structure. A member stress or size constraint only controls the material in an individual member. Difficulties arise when both types of constraints are present together since the distribution of material required to optimally satisfy a displacement constraint may violate the amount of material required to satisfy one or more of the individual member constraints. There is no optimality criterion of practical use for combined types of constraints. Somewhat *ad hoc* methods are usually used such as active/passive sets of variables as in OPTIM II to handle both types of constraints.

The primal problem concerning us here is that given in (1), (2) and (3) and the corresponding dual problem is given in (4) and (5). On examining the dual problem it at first appears that its dimensionality, (M + N), is greater than that of the primal problem, N. This would negate the advantage which the dual approach has of reducing problem dimensionality. Fortunately, very recent research has shown that the N dual variables corresponding to member size constraints may be effectively eliminated by an iterative process. A brief summary of this now follows.

Consider dual problem (4) for a single displacement constraint (with dual variable λ_0) and a full set of N member size constraints. If we write

$$\overline{W_i} = L_i \overline{A_i} \qquad \overline{\delta_i} = \left(\frac{FUL}{\bar{A}E}\right)_i$$

$$\bar{W} = \sum_{i=1}^{N} \bar{W}_i \qquad \bar{\delta} = \sum_{i=1}^{N} \bar{\delta}_i$$

and if δ is the maximum permissible nodal displacement, dual problem (4) is

$$\left.\begin{aligned} &\text{Maximize } V = \sum_{i=1}^{N} \sqrt{\bar{W}_i} \left\{\frac{\overline{\delta_i}}{\delta} \lambda_0 + \lambda_i\right\}^{\frac{1}{2}} \\ &\text{Subject to } \sum_{i=o}^{N} \lambda_i = 1 \\ &\qquad \lambda_i \geq 0 \qquad\qquad i = 0, \ldots, N \end{aligned}\right\} \quad (10)$$

Necessary conditions for a constrained maximum of V with respect to the N member size dual variables only are that

$$\frac{\partial V}{\partial \lambda_i} = 0 \qquad\qquad i = 1, \ldots, N$$

This leads to

$$\lambda_i{}^* = \frac{\overline{W_i}}{\bar{W}} \left(1 + \frac{\lambda_0}{\delta} \left[\bar{\delta} - \delta - \frac{\bar{W}}{\overline{W_i}} \bar{\delta}_i\right]\right) \qquad\qquad i = 1, \ldots, N \qquad (11)$$

Substituting (11) into V of (10) gives

$$V = \sqrt{\bar{W}}\,\{1 + \lambda_0 \left[\frac{\bar{\delta} - \delta}{\delta}\right]\} \tag{12}$$

If $\bar{\delta} < \delta$ this denotes that member sizes evaluated from the member size constraints alone will satisfy the displacement constraint and hence λ_0 will be zero. We are interested in the case where $\bar{\delta} > \delta$ and the displacement constraint must be active. In this case V as given in (12) is maximized by as large a value of λ_0 as is possible. However, λ_0 may not increase to a value such as to drive any of the λ_i*, i = 1, ..., N in (11) below zero. The highest possible value of λ_0 is therefore that value which first puts any λ_i* equal to zero, i.e.

$$\lambda_0 = \underset{i=1,\ldots,N}{\text{Min}} \left\langle\left(1 - \frac{\bar{\delta}}{\delta} + \frac{\bar{W}}{\overline{W_i}}\ \frac{\overline{\delta_i}}{\delta}\right)\right\rangle \tag{13}$$

This value of λ_0 drives one of the λ_i* to zero. Let the variable driven to zero be λ_N * = 0. This is now eliminated as a slack member size constraint.

A new dual problem may now be formed with λ_N eliminated. This replaces problem (10) and is

$$\left.\begin{aligned} &\text{Maximize } V = \sum_{i=1}^{N-1} \sqrt{\bar{W}_i}\,\{\frac{\overline{\delta_i}}{\delta}\lambda_0 + \lambda_i\}^{\frac{1}{2}} + \sqrt{\bar{W}_N}\,\{\frac{\overline{\delta_N}}{\delta}\lambda_0\}^{\frac{1}{2}} \\ &\text{Subject to } \sum_{i=o}^{N-1} \lambda_i = 1 \\ &\qquad\qquad \lambda_i \geq 0 \qquad\qquad\qquad\qquad i = 0, \ldots, N = 1 \end{aligned}\right\} \tag{14}$$

Problem (14) is treated in a similar way to problem (10). Relationships similar to (11) are established for the λ_i*, this time for i = 1, ,.., N - 1. An expression for V similar to (12) is found and a new value of λ_0 is determined as (13). If the new value of λ_0 is greater than its previous value another of the λ_i* is eliminated, another problem similar to (14) but with (N - 2) values of λ_i is set up and the process is continued in this iterative fashion until the value of λ_0 reduces. The previous iteration's results for all the λ's are then optimal. Relationships (5) then give the minimum weight and optimal member sizes.

The iterative procedure described above forms into a very simple algorithm since the relationships of the types of (11), (12) and (13) are very concise in nature. Using this iterative dual approach the interactions of member size constraints and a displacement constraint may be optimized very rapidly, the dimensionality of the method being essentially unity. An advantage of the method is that it starts essentially with a fully-stressed design (all member size dual variables active and λ_0 = 0). The activity level of the displacement constraint, λ_0, is then progressively increased, knocking out member size constraints as they become slack. In many practical design situations a first requirement is to examine the fully-stressed design and check it against possible displacement limitations. If the displacements are excessive the fully-stressed design needs to be altered in some way so as to optimally satisfy displacement limitations. This is precisely how the dual approach outlined above tackles the problem and it is therefore well suited to implementation in practical optimum design programs.

The treatment above is limited to the combination of a single displacement constraint and member size constraints. If multiple displacement constraints are present the iteration algorithm is more complex and has not yet been fully investigated. However, it has already been shown in this paper that multiple displacement constraints behave as a single surrogated constraint. This suggests a possible solution algorithm in which the multiple constraints are first solved separately and the single surrogate constraint formed and then the above algorithm used to handle the interactions of the surrogate constraint and the member size constraints. This remains to be further investigated.

5. CONCLUSIONS

This paper has examined a dual approach to the optimum design of structures whose elements have stiffness proportional to mass. It has shown that a study of duality gives insight and rationale for some of the successful, non-rigorous approaches to truss design such as the optimality criterion approach used in OPTIM II. It would have been more satisfying to give numerical results confirming the speed and efficiency of the dual algorithms suggested in this paper but space limitations preclude this. Nevertheless it can be stated that the dual approach does provide a means of very rapidly solving optimum design problems for large structural systems. The reduction in dimensionality and the ease with which the dual problems may be manipulated and solved makes the approach a very serious competitor to the much-used, less rigorous optimality criteria methods. From a practical structural engineering point of view it should be stressed that although duality theory and the associated algebra may seem unnecessarily complicated and abstract, the algorithms which may be developed from it are rigorous and are very simple to operate, giving practically useful results very rapidly. Furthermore the dual-based algorithms often tend to be similar to those suggested by engineering intuition. This is very satisfying and a firmer theoretical basis for intuitive design approaches adds considerable strength to them.

As the present author has commented in the Introductory report to the 10th IABSE Congress[3] a major advantage of a study of dual methods is that it sheds new light on well-known problems and enables the nature of the problems to be understood more deeply. Sometimes, as in the case here, this extra insight allows new solution algorithms to be developed. The ultimate usefulness of these algorithms remains to be fully investigated in a continuing program of research.

6. REFERENCES

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SUMMARY

The paper examines a new dual approach to the optimum design of trusses with multiple displacement and member size constraints. Comparison is made with optimality criteria approaches to the same problem. Reductions in problem dimensionality and simple solution algorithms arise from casting the problem into dual space, which also gives insight into some ad hoc, intuitive artifices often employed in the solution of these problems.

RESUME

Une nouvelle méthode duale est présentée pour le dimensionnement optimal de treillis, soumis à des contraintes de déplacements multiples et de types de profils. Une comparaison est faite avec la méthode des critères d'optimisation. Des réductions de la dimension des problèmes ainsi que des algorithmes simples pour leur résolution sont obtenus en situant le problème dans l'espace dual, ce qui permet également d'analyser quelques artifices de calcul souvent utilisés dans la solution de tels problèmes.

ZUSAMMENFASSUNG

Der Bericht behandelt eine neue Dualmethode für die Optimierung von Fachwerken mit mehrfachen Formänderungs- und Formgebungsrestriktionen. Die Ergebnisse werden mit der Methode der Optimalitätskriterien verglichen. Eine Abminderung der Komplexität und einfache Lösungsalgorithmen resultieren aus der Problemprojektion in einem Dualraum, was auch Einblick in gewisse intuitive Verfahren gewährt, die bei der Lösung solcher Probleme oft angewendet werden.

IIa

Die Bedeutung des Kraft- und Weggrössenverfahrens für die Optimierung von Tragwerken nach der Lagrange'schen Multiplikatorenmethode

The role of the Force- and Displacement-Method for the Optimization of Structures with the Lagrangian-Multiplier-Technique

Rôle de la méthode des forces et des déformations dans l'optimisation des structures selon la méthode de Lagrange

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1. Problemstellung

Im Konstruktiven Ingenieurbau stehen heute eine Reihe leistungsfähiger Berechnungsverfahren zur Verfügung. Das Dimensionieren von Tragwerken erfolgt dagegen durch den Ingenieur, wobei Können und Erfahrung eine wesentliche Rolle spielen. Kann man eine Gewichts- oder Kostenfunktion definieren, so läßt sich dieses Problem als Optimierungsaufgabe formulieren, die als Folge der Bemessungskriterien i.a. nichtlinear und nichtkonvex ist. Aus der Vielzahl der Lösungsverfahren zur Bestimmung eines lokalen Minimums [1] wird hier das Verfahren der Optimalitätskriterien betrachtet, das eine problemorientierte Variante der Lagrange'schen Multiplikatorenmethode darstellt.

Dem Optimierungsmodell liegt ein durch n Elemente diskretisiertes Tragwerk zugrunde. Es wird vorausgesetzt, daß für jedes Element i die Elementflexibilität $\underline{f}_i$+) umgekehrt proportional von einer Querschnittsvariablen (Entwurfsvariable) $\alpha_i > 0$ abhängt und daß sich das Gewicht des Tragwerkes als lineare Funktion (Zielfunktion) dieser Entwurfsvariablen darstellen läßt:

$$W = \sum_{i=1}^{n} w_i = \sum_{i=1}^{n} \overline{w}_i \alpha_i \qquad (1)$$

Als Nebenbedingungen werden Spannungs- und Verformungsrestriktionen berücksichtigt, wobei σ_{il}^o und δ_{il}^o die zulässige Spannung des Elementes i bzw. die zulässige Verformung in Richtung des Freiheitsgrades j infolge Lastfall l bedeutet. Zusätzlich kann eine Einschrankung der Variablen durch untere und obere Schranken α_i^u bzw. α_i^o vorgegeben werden. Damit ergibt sich folgende Optimierungsaufgabe:

$$\text{Minimiere} \qquad W = \sum_{i=1}^{n} \overline{w}_i \alpha_i$$

unter Berücksichtigung der Restriktionen

+) Matrizen und Spaltenvektoren werden durch Unterstreichen gekennzeichnet, ein hochgestelltes T bedeutet die Transponierte.

$$\sigma_{il} - \sigma^{o}_{il} \leqq 0 \qquad (i=1,\ldots,n\,;\; l=1,\ldots,p)\ , \tag{2}$$

$$\delta_{jl} - \delta^{o}_{jl} \leqq 0 \qquad (j=1,\ldots,q\,;\; l=1,\ldots,p)\ , \tag{3}$$

$$\alpha^{u}_{i} - \alpha_{i} \leqq 0 \qquad (i=1,\ldots,n)\ , \tag{4}$$

$$\alpha_{i} - \alpha^{o}_{i} \leqq 0 \qquad (i=1,\ldots,n)\ . \tag{5}$$

Es bedeutet q die Anzahl der Freiheitsgrade und p die Anzahl der Lastfälle. Die Spannungen σ und die Verformungen δ sind nichtlineare Funktionen der Entwurfsvarieblen $\underline{\alpha}$, so daß die Restriktionen einen nichtkonvexen Lösungsbereich beschreiben. Da die Problematik bei einem Lastfall bzw. mehreren Lastfällen dieselbe ist, wird im folgenden aus Gründen der Übersichtlichkeit auf den Belastungsindex l verzichtet.

2. Notwendige und hinreichende Optimalitätsbedingungen

Die Herleitung notwendiger Extremalbedingungen der nichtlinearen Optimierungsaufgabe erfolgt mit der verallgemeinerten Lagrange'schen Multiplikatorenmethode [2]. Da sämtliche Variablen $\underline{\alpha}$ nichtnegativ definiert und alle Restriktionen als Ungleichungen gegeben sind, sind diese Bedingungen hinreichend für ein lokales Minimum der Zielfunktion [3]. Bezeichnet man mit $G_j \leq 0$ die allgemeine Form der Restriktionen (2) und (3), so lautet die Lagrange'sche Funktion:

$$J = W + \sum_{j=1}^{m} \lambda_j\, G_j + \sum_{i=1}^{n} \mu_i\, (\alpha^{u}_{i} - \alpha_i) + \sum_{i=1}^{n} \eta_i\, (\alpha_i - \alpha^{o}_{i})\ . \tag{6}$$

Die Lagrange'schen Parameter λ_j, μ_i und η_i sind festgelegt durch:

$$\lambda_j \geqq 0\ , \quad \text{für} \quad G_j \overset{=}{<} 0 \qquad (j=1,\ldots,m) \tag{7}$$

$$\mu_i \geqq 0\ , \quad \text{für} \quad \alpha_i \overset{=}{>} \alpha^{u}_{i} \qquad (i=1,\ldots,n) \tag{8}$$

$$\eta_i \geqq 0\ , \quad \text{für} \quad \alpha_i \overset{=}{<} \alpha^{o}_{i} \qquad (i=1,\ldots,n) \tag{9}$$

Als notwendige und hinreichende Bedingung für einen stationären Wert von W müssen die partiellen Ableitungen von J nach den Variablen $\underline{\alpha}$ verschwinden. Mit $\partial(\ldots)/\partial\alpha_k = (\ldots)_{,k}$ erhält man:

$$W_{,k} + \sum_{j=1}^{m} G_{j,k} - \mu_k + \eta_k = 0 \qquad (k=1,\ldots,n) \tag{10}$$

Mit (8) und (9) folgt:

$$-\sum_{j=1}^{m} \lambda_j\, G_{j,k} \begin{cases} \geqq W_{,k} \\ = W_{,k} \\ \leqq W_{,k} \end{cases} , \quad \text{für} \begin{cases} \alpha_k = \alpha^{o}_{k} \\ \alpha^{u}_{k} < \alpha_k < \alpha^{o}_{k} \\ \alpha_k = \alpha^{u}_{k} \end{cases} \tag{11}$$

Für alle "passiven " Restriktionen $G_j < 0$ ist nach (7) der Lagrange'sche Parameter λ_j gleich Null, so daß in der Optimalitätsbedingung (11) nur die "aktiven" Restriktionen $G_j = 0$ berücksichtigt zu werden brauchen.

3. Rekursionsformeln zur Bestimmung der optimalen Konstruktion

3.1 Aktive Verformungsrestriktionen

Einzelne Verformungsgrößen können mit Hilfe des Prinzips der virtuellen Kräfte berechnet werden. Es gilt:

$$\delta_j = \sum_{i=1}^{n} e_{ij} = \sum_{i=1}^{n} \underline{S}_i^T \underline{f}_i \underline{\tilde{S}}_i \qquad (j=1,\dots,q') \quad , \qquad (12)$$

wobei e_{ij} die virtuelle Verzerrungsenergie, S_i die Schnittgrößen infolge der Belastung, $\tilde{S}_i$ die Schnittgrößen infolge der virtuellen Einheitsbelastung in Richtung der gesuchten Verformungsgröße des Elementes i und q' die Anzahl der aktiven Verformungsrestriktionen darstellt. Als partielle Ableitung nach den Variablen α_k (k=1,...,n) erhält man mit $e_{kj} = \bar{e}_{kj}/\alpha_k$:

$$G_{j,k} = - \bar{e}_{kj} / \alpha_k^2 \quad . \qquad (13)$$

Bezeichnet k ∈ N1 eine "aktive" Variable α_k mit dem Wert $\alpha_k^u < \alpha_k < \alpha_k^o$ und k ∈ N2 eine "passive" Variable mit $\alpha_k = \alpha_k^u$ oder $\alpha_k = \alpha_k^o$, so muß für alle aktiven Variablen k ∈ N1 das Gleichheitszeichen in der Optimalitätsbedingung (11) erfüllt sein. Mit $W_{,k} = \bar{w}_k$ und (13) folgt:

$$\sum_{j=1}^{q'} \lambda_j \bar{e}_{kj} / \alpha_k^2 = \bar{w}_k \qquad (\forall k \in N1) \quad . \qquad (14)$$

Diese Gleichung stellt i.a. ein hochgradig nichtlineares Gleichungssystem mit den Unbekannten λ_j (j=1,...,q') und α_k (k=1,...,n) dar, das nur iterativ gelöst werden kann. Ist nur eine einzige Verformungsrestriktion zu berücksichtigen, d.h.

$$\delta_j^o = \sum_{k \in N1} \bar{e}_{kj} / \alpha_k + \sum_{k \in N2} e_{kj} \quad , \qquad (15)$$

so läßt sich der Lagrange'sche Parameter λ_j eliminieren. Die Gleichungen (14) aufgelöst nach α_k (k ∈ N1) und in (15) eingesetzt, liefert:

$$\lambda_j = \left(\frac{1}{\delta^*} \sum_{k \in N1} \sqrt{\bar{e}_{kj} \bar{w}_k} \right)^2 \quad \text{mit} \quad \delta^* = \delta^o - \sum_{k \in N2} e_{kj} \quad . \qquad (16)$$

Bei mehreren aktiven Verformungsrestriktionen ist eine Bestimmung von λ_j (j=1,...,q') aus (14) nur dann möglich, wenn $\bar{e}_{kj} / \alpha_k^2$ als invariant betrachtet werden. In diesem Fall stellt (14) ein überbestimmtes lineares Gleichungssystem in $\underline{\lambda}$ dar:

$$\underline{G} \, \underline{\lambda} = \underline{E} \qquad (17)$$

mit

$$\underline{G} = \left[\bar{e}_{kj} / \bar{w}_k \, \alpha_k^2 \right] \qquad (18)$$

und $\underline{E} = \left\{ 1,\dots,1 \right\}$ für alle k ∈ N1 und j=1,...,q'. Mit Hilfe der ersten Gauß'schen Transformation kann eine Lösung für $\underline{\lambda}$ gefunden werden. Es gilt:

$$\underline{\lambda} = \left[\underline{G}^T \underline{G} \right]^{-1} \underline{G}^T \underline{E} \quad . \qquad (19)$$

In Bezug auf die ursprüngliche Gleichung (17) stellt $\underline{\lambda}$ die beste Lösung im Sinne der kleinsten Quadrate dar. Mit den bekannten $\underline{\lambda}$-Werten und der Annahme invarianter Größen $\bar{e}_{kj}$ (bei stat. best. Systemen) entkoppelt sich das Gleichungssystem (14), so daß die aktiven Variablen α_k (k ∈ N1) bestimmt werden können:

$$\alpha_k = \left(\sum_{j=1}^{q'} \lambda_j \bar{e}_{kj} / \bar{w}_k \right)^{1/2} \qquad (20)$$

Bei stat. unbest. Systemen sind die Größen $\bar{e}_{kj}$ komplizierte Funktionen von $\underline{\alpha}$. Da sich eine Änderung von α_k in erster Linie auf die Schnittgrößen des

Elementes k auswirkt, kann (20) iterativ angewendet werden, d.h.

$$\alpha_k^{\nu+1} = \left(\sum_{j=1}^{q'} \lambda_j^{\nu} \bar{e}_{kj}^{\nu} / \bar{w}_k \right)^{1/2} , \qquad (21)$$

wobei ν den Iterationsschritt kennzeichnet und λ_j^{ν} $(j=1,\ldots,q'_{\nu})$ für $q'=1$ aus (16) bzw. für $q' \geq 2$ aus (19) mit den Werten $\bar{e}_{kj}^{\nu}$ und α_k^{ν} berechnet wird. Da die passiven Variablen α_k $(k \in N2)$ i.a. nicht im voraus bekannt sind, muß ihre Bestimmung ebenfalls iterativ erfolgen. Dabei können die Schranken $\underline{\alpha}^u$ und $\underline{\alpha}^o$ durch die Bedingungsgleichungen

$$\alpha_k^{\nu+1} = \begin{cases} \alpha_k^o \\ \alpha_k^{\nu+1} \\ \alpha_k^u \end{cases} \quad \text{für} \quad \begin{cases} \alpha^{\nu+1} \geqq \alpha_k^o \\ \alpha_k^u < \alpha_k^{\nu+1} < \alpha_k^o \\ \alpha_k^{\nu+1} \leqq \alpha_k^u \end{cases} \qquad (22)$$

berücksichtigt werden. Alle Variablen, für die α_k^u bzw. α_k^o maßgebend ist, werden in der nächsten Iteration zu den passiven gezählt.

3.2 Aktive Spannungsrestriktionen

Sind ausschließlich Spannungsbeschränkungen vorgeschrieben, so kann die Bestimmung der Variablen $\underline{\alpha}$ nach der bekannten "stress-ratio"- Methode [4] erfolgen, in der jedes Element entsprechend seiner spannungsmäßigen Auslastung dimensioniert wird. Es gilt:

$$\alpha_k^{\nu+1} = \alpha_k^{\nu} \left| \sigma_k^{\nu} / \sigma_k^o \right| \qquad (23)$$

wobei σ_k^{ν} die maßgebende Spannung des Elementes k im ν-ten Iterationsschritt bedeutet. Als Ergebnis erhält man eine sogenannte "voll-beanspruchte" Konstruktion, die in jedem Element die zulässige Spannung ausnutzt, wenn nicht der durch α_k^u festgelegte minimale Querschnitt maßgebend ist.

Bei aktiven Verformungsrestriktionen können Spannungsbeschränkungen berücksichtigt werden, wenn man in jeder Iteration die nach (23) berechneten $\underline{\alpha}$-Werte in der Bestimmungsgleichung (22) als zusätzliche untere Schranken auffaßt.

3.3 Konvergenz des Verfahrens

Die Anwendung der Gleichungen (16), (19), (21) bis (23) verlangt nach jeder Iteration eine vollständige Berechnung der Konstruktion. Um jeweils eine zulässige Lösung zu erhalten, werden sämtliche Variablen $\underline{\alpha}^{\nu}$ mit einem globalen Skalierungsfaktor multipliziert, so daß keine der Restriktionen (2) und (3) verletzt und mindestens eine identisch erfüllt wird. Danach erfolgt die Bestimmung der aktiven Verformungsrestriktionen, wobei alle Verformungen, die im Verlauf des Iterationsprozesses einmal ihren zulässigen Wert erreicht haben, weiterhin zu den aktiven gezählt werden. Ergibt sich jedoch nach (19) ein negativer λ-Wert so muß die entsprechende Restriktion aufgrund der Nichtnegativitätsbedingung (7) wieder eliminiert werden. Erst wenn alle aktiven Verformungen bekannt sind, ist mit einer schnellen Konvergenz zu rechnen. Das Konvergenzverhalten kann durch eine Begrenzung der Schrittweite in aufeinanderfolgenden Iterationen beeinflußt werden. Mit

$$\alpha_k^{\nu+1} = \alpha_k^{\nu} \qquad (k=1,\ldots,n)$$

ist die optimale Konstruktion gefunden, für die das Gewicht ein (lokales) Minimum annimmt.

4. Die Bedeutung des Kraft- und Weggrößenverfahrens

Bisher wurde nur das Iterationsverfahren zur Lösung der Optimierungsaufgabe betrachtet. Über die Lagrange'schen Parameter $\underline{\lambda}$ bei mehreren aktiven Restriktionen wurde im Sinne der kleinsten Quadrate verfügt. Im Vergleich mit anderen Verfahren [4] ergibt sich hierdurch ein stabiles Konvergenzverhalten bei nur wenigen Iterationsschritten. Die wiederholte Berechnung des Tragwerkes nach der Finiten-Elementmethode erfordert bei den vorliegenden Problemen einen erheblichen Rechenaufwand und verdient damit besondere Beachtung. Ohne auf die Möglichkeiten der Ableitung von Elementmatrizen [5] einzugehen, werden hier nur die Lösungsverfahren betrachtet. Diese Verfahren folgen direkt aus den klassischen Minimalprinzipien elastischer Tragwerke.

Das Prinzip vom Minimum der Potentiellen Energie

$$\text{Min} \left\{ \frac{1}{2} \underline{\delta}^T \underline{K}\, \underline{\delta} - \underline{P}^T \underline{\delta} \right\} , \tag{24}$$

mit der positiv definiten Gesamtsteifigkeitsmatrix $\underline{K}$, den Lasten $\underline{P}$ und den Verschiebungen $\underline{\delta}$, liefert als notwendige und hinreichende Bedingung die Grundgleichung der Verschiebungemethode:

$$\underline{K}\, \underline{\delta} = \underline{P} \ . \tag{25}$$

Das Prinzip vom Minimum der Komplementärenergie

$$\text{Min} \left\{ \frac{1}{2} \underline{S}^T \underline{f}\, \underline{S} \;\middle|\; \underline{N}\, \underline{S} = \underline{P} \right\} , \tag{26}$$

mit der Hyperdiagonalmatrix $\underline{f}$ der Elementflexibilitätsmatrizen, den verallgemeinerten Spannungen $\underline{S}$ und der Gleichgewichtsmatrix $\underline{N}$ ergibt die Grundgleichungen der Kraftmethode:

$$\begin{aligned} &\underline{N}\, \underline{S} = \underline{P} && \text{(Gleichgewicht)} , \\ &\underline{B}_x^T \underline{f}\, \underline{S} = 0 && \text{(Verträglichkeit)} . \end{aligned} \tag{27}$$

$\underline{B}_x^T$ ist der Kern der Gleichgewichtsmatrix $(\underline{N}\, \underline{B}_x^T = 0)$.

Den geringsten Aufwand für die einmalige Berechnung eines Tragwerkes erfordert im allgemeinen die Verschiebungsmethode: Der einfache Aufbau, die positive Definitheit und Bandstruktur der q x q Matrix $\underline{K}$ erleichtert die Berechnung. Bei einer mehrmaligen Berechnung des Tragwerkes mit variabler Flexibilität $\underline{f}$ zeigt jedoch die Kraftmethode gewisse Vorteile: Die q Gleichgewichtsgleichungen (27) müssen nur einmalig gelöst werden, die Verträglichkeitsbedingungen lassen sich einfacher darstellen und mit geringerem Aufwand für jede Wiederbemessung lösen. Als Lösung erhält man die n Schnittgrößen $\underline{S}_i$ zur Iteration nach (12). Mit dem in [6] näher beschriebenen Lösungsverfahren kann zudem die Bandstruktur der Gleichgewichtsgleichungen gewahrt werden. Ein genauer Vergleich des numerischen Aufwandes beider Methoden führte zu dem Ergebnis, daß mit steigender Zahl der Wiederbemessungen der Aufwand A_F der Kraftmethode abnimmt. Das Verhältnis des Aufwandes A_D der Verschiebungsmethode zur Kraftmethode nimmt jedoch bei wachsendem n/q ab. In den für die Praxis wichtigen Stabtragwerken ist jedoch i.a. $n/q < 2$. Für ein System mit 1000 Freiheitsgraden der Verschiebung und einem speziellen Elementtyp (s) ergibt sich die in Bild 1 dargestellte Abhängigkeit [6].

Umfangreiche numerische Untersuchungen [7] an den aus der Literatur bekannten optimalen Tragwerken bestätigen in allen Fällen die Überlegenheit der Kraftmethode.

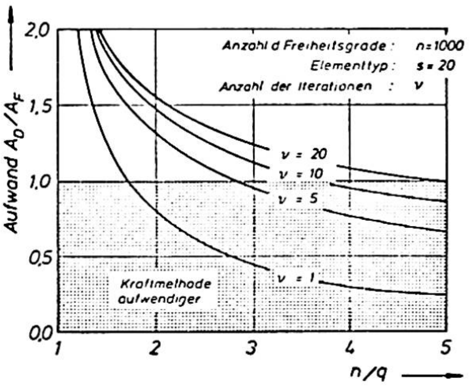

Bild 1: Vergleich der Kraft- und Verschiebungsmethode

Lastfall	Knoten (P_z=-1000 lbs)
1	1
2	1-4,7-13,19-28,37
3	1-37
4	1,4-7,13-19,28-37

Tabelle 1: Belastungsangaben

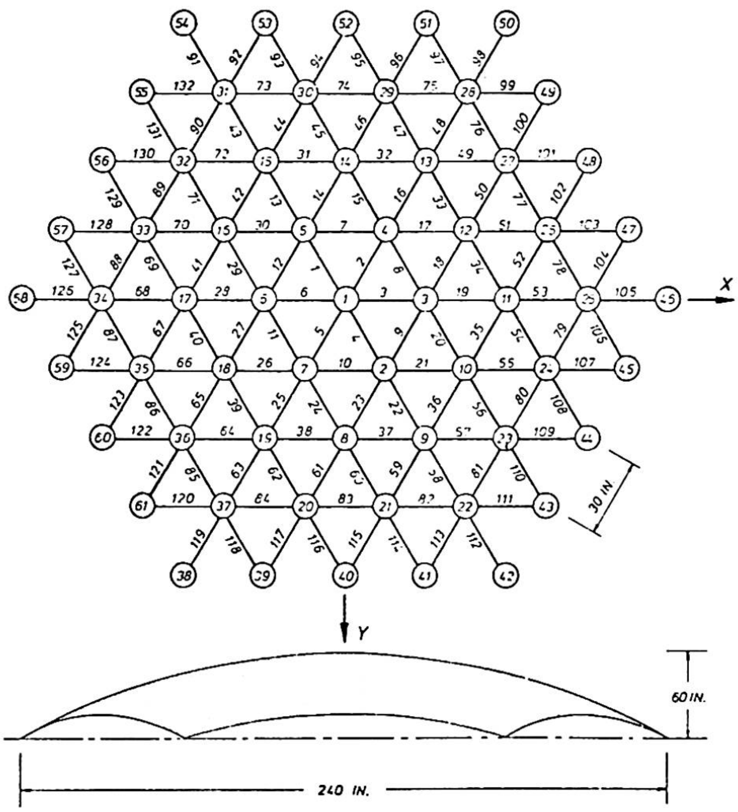

Bild 2: Fachwerkkuppel

5. Numerische Ergebnisse

Die Zuverlässigkeit des Optimierungsverfahrens soll hier an einem ausgewählten Beispiel gezeigt werden. Die in Bild 2 dargestellte Fachwerkkuppel, die in den Knoten 38 - 61 unverschieblich gelagert ist, wird durch vier Lastfälle beansprucht. Die genauen Belastungsangaben sind in Tabelle 1 zusammengestellt. Als Material wird Aluminium mit einem Elastizitätsmodul von E = 10^7 psi und dem spezifischen Gewicht von ρ = 0.1 lbs/in^3 verwendet. Für alle Stäbe beträgt der minimale Querschnitt 0.1 in^2, wobei die zulässige Spannung von ± 25000 psi nicht überschritten werden darf. Die Verschiebungen sämtlicher Freiheitsgrade in z-Richtung werden auf ± 0.1 in. begrenzt. Alle Entwurfsbedingungen sind mit denen aus /8/ identisch.

Ausgehend von einer zulässigen Konstruktion mit querschnittsgleichen Stäben (W_1=358.85 lbs) wird die optimale Kuppel nach 15 Iterationen und einem Gewicht von 161.63 lbs gefunden, das um 10.7% geringer ist als in /8/ . Während zu Beginn der Optimierung nur die Verschiebung von Knoten 1 (LF 1) den maximal erlaubten Wert von -0.1 in. erreicht, sind von der 13. Iteration an 41 Verformungsrestriktionen zu berücksichtigen, die jeweils durch einen der 4 Lastfälle aktiviert wurden. Spannungen waren in keiner Phase des Iterationsprozesses maßgebend. Bild 3 zeigt das stabile Konvergenzverhalten, wobei insgesamt eine Gewichtsreduktion von 55% erreicht wird. Die Querschnittsflächen der optimalen Kuppel, die symmetrisch zu den beiden Achsen 38-50 und 44-56 ausgebildet

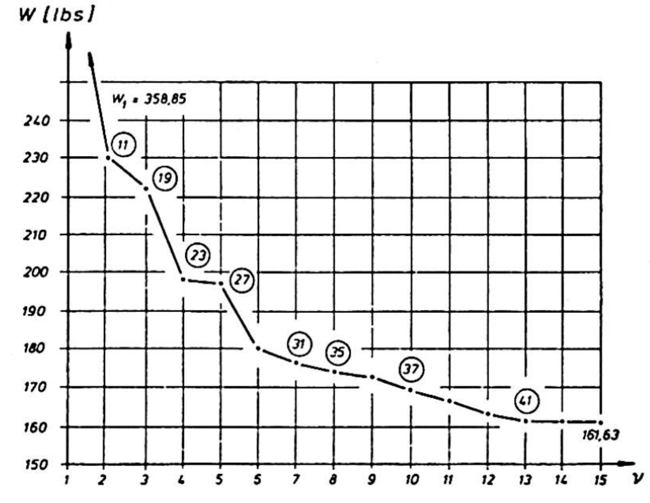

Bild 3: Iterationsverlauf

ist, sind in Tabelle 2 zusammengestellt. Bei n/q=1.19 konnte die Kraftmethode äußerst wirtschaftlich eingesetzt werden. Die Rechenzeit (TR 440) betrug nur 182 sec.

Stab	Fläche	Stab	Fläche	Stab	Fläche	Stab	Fläche
4	1.0176	36	0.4831	62	0.3177	111	0.1003
5	1.1732	37	0.3051	63	0.6572	112	0.2403
9	0.9720	38	0.3514	80	0.3062	113	0.3088
10	0.8322	56	0.3207	81	0.2128	114	0.1429
21	0.2990	57	0.1904	82	0.1003	115	0.5000
22	0.3395	58	0.3378	83	0.1003	116	0.1003
23	0.5773	59	0.3431	84	0.3347	117	0.4381
24	0.4148	60	0.29 6	109	0.1003	118	0.3312
25	0.6776	61	0.5494	110	0.4961	119	0.1003

Tabelle 2: Optimale Querschnittsflächen (in^2) eines Quadranten

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ZUSAMMENFASSUNG

Es wird eine spezielle Anwendung der Lagrange'schen Multiplikatorenmethode, die als Verfahren der Optimalitätskriterien bekannt wurde, dargestellt. Eine lineare Transformation der Lagrange-Parameter führte zu einer schnellen und gleichmässigen Konvergenz.

SUMMARY

A special application of the Lagrangian-Multiplier-Technique, known as the optimality-criterion-method, is presented. A simple linear transformation of the Lagrange parameters leads to fast and uniform convergence.

RESUME

Une application spéciale de la technique des multiplicateurs de Lagrange, dite méthode des critères d'optimisation est présentée. Une transformation linéaire entraîne une convergence rapide et uniforme.

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