Optimum design of steel frame subjected to dynamic earthquake forces

Optimum Design of Steel Frame Subjected to Dynamic Earthquake Forces

Calcul optimal de cadres métalliques soumis aux forces dynamiques des tremblements de terre

Optimierung von Stahlrahmen unter dynamischer Erdbebenlast

HIDETAKE ANRAKU
Researcher
Technical Research Institute, Ohbayashi-Gumi Ltd.
Tokyo, Japan

1. INTRODUCTION

The mathematical programing technique has already been adopted for the optimization of the structures subjected to the dynamic excitations.[1,2] Most of these optimizations were dealt with beams, trusses or frames, subjected to simple excitations such as harmonic waves or shock waves, and designed under rather simple elastic constraints.

However, in case of earthquake loadings it becomes important to estimate the dynamic forces correctly using the available model for the elastic design, and to take into account the inelastic behaviour of structures during the very strong ground motion.

Considering these problems, this paper presents a method for the automated minimum weight design of wide-flange steel frames which gives the optimum distribution of the moment of inertia of used members.

2. DYNAMIC ANALYSIS

An idealized dynamic model consist of bedrock, ground layres and a structure is considered (see Fig. 1). Ground excitations are given by the model presented by Kanai and Tajimi, and the dynamic response of the structure to this ground motion is estimated by means of the random vibration theory and Davenport's equation which gives the expected maximum value of a random process.

2.1 Vibration of Ground Surface

Kanai and Tajimi has presented the idea that spectrum observed at bedrock is characterized by a constant pattern (white noise), while the spectrum at the ground surface is amplified by the vibration property of the ground layre and showed a power spectrum of this ground surface as follows:[3]

$$S(p) = \sum_{k=1}^{r} \frac{1 + 4h_{gk}^2 (\frac{p}{w_{gk}})^2}{1 + (4h_{gk}^2 - 2)(\frac{p}{w_{gk}})^2 + (\frac{p}{w_{gk}})^4} s_k S_o \qquad (1)$$

where h_{gk} and w_{gk} are ground damping factor and predominant frequency, respectively, S_o is a constant power spectrum density function and where s_k is a factor which measures predominence of each component. This excitation of ground surface becomes Gaussian process of zero mean.

2.2 Dynamic Response of Structure

The variance of elastic response of the structure subjected to the ground motion mentioned above can be obtained by means of random vibration theory. Let σ_Q and $\sigma_{\dot{Q}}$ be the variance of story shear force and its time derivative, respectively.

Following Davenport[4], the mean value of possible maximum elastic response of story shear force can be given as

$$Q = (2\ln vT)^{\frac{1}{2}} + \frac{0.5772}{(2\ln vT)^{\frac{1}{2}}} \qquad (2)$$

where $v = \frac{1}{2\pi}\frac{\sigma_{\dot{\theta}}}{\sigma_{\theta}}$

and T represents the duration of the strong earthquake excitation which is fixed 10 seconds in this paper.

For very strong ground motion, the response of the structure is considered to be inelastic, and the relative displacements of each floor are estimated following the idea of Newmark and et al.[5] Equating the inelastic potential energy of deformation to the elastic one which can be obtained supposing that the structure responses elastically, the maximum ductility factor of floor drift, μ, can be obtained as follows (see Fig. 2);

$$\dot{\mu} = \frac{1}{2} + \frac{1}{2}\left(\frac{Q}{Q_y}\right)^2 \qquad (3)$$

where Q_y may be thought of as the yield level of the story shear force, and can be obtained by means of a simple plastic analysis assuming the mechanism of beam collapse type or column collapse one for each story.

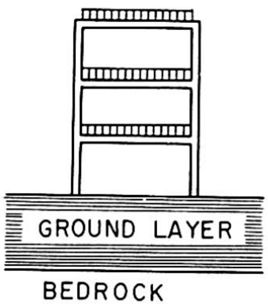

Fig. 1 Dynamic Model

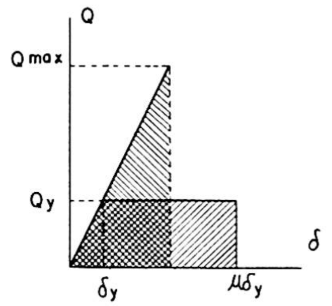

Fig. 2 Definition of Ductility Factor

3. DESIGN CONSTRAINTS

For the moderate earthquakes which give such a dynamic force as usually presented in the design code, the members of the frame are designed elastically in accordance with the design code of steel structure of Architectural Institute of Japan (A.I.J). On the other hand, for the very severe earthquake, which is rarely expected during their service lives, the frame is designed plastically relying on the energy absorption which due to their inelastic deformation. In this design procedure, the maximum ductility factor given by Eq.(3) is constrained less than the allowable value which is fixed 4 in this paper.

To satisfy these ductility requirements, it is necessary for the frame to prevent the weakening of the load-deflection curves caused by the lateral or local buckling of members and P - Δ effects.

These problems are taken into account according to the plastic design code of steel structure of A.I.J.. Namely, lateral buckling is prevented by the correctly designed stiffners, and local buckling is prevented by selecting the members which are on market to satisfy the width-thickness ratio of plate elements imposed by the code mentioned above, or designing each member in-accordance with these requirements after the optimum stiffness distribution of frame member is decided. Moreover to avoid the excessive P - Δ effects, the slenderness ratios and the axial compressive forces of columns are ristricted by the code requirements.

4. OPTIMIZATION

Wide-flange steel members on market are supposed to be mainly used in the design. The moment of inertia of them, I, are the design variables and objective function is the total weight of structural members. The emprical relationships between member properties which are required in the design code and moment of inertia of economical series of the steel wide-flange section was obtained by plotting them. The calculation was proceeded using these equations and treating the moment of inertia as continious design variable.

Sequential linear programming (S.L.P.) technique was successfully adopted for the optimization of the frames. Objective function and constraint equations were approximately linearlized, and using linear programming technique, the optimumly modefied design variables were obtained at each design step. Repeating this procedure, the optimum solusion, namely the distribution of moment of inertia of members which gives the minimum weight of structural members, was obtained.

5. SENSITIVITY ANALYSIS

To optimize the structure by means of S.L.P. technique, the change of member stress and deformation caused by the modefication of each members must be quantitively estimated as the first order derivative of these values with respect to the design variables.

Let P be the vector of external nodal forces of global coordinate, and X and K be the corresponding nodal displacement vector and stiffness matrix. Using these notations

$$X = K^{-1} P \tag{4}$$

Therefore, the derivative of nodal displacements with respect to design variable, I, is obtained as follows;

$$\frac{\partial X}{\partial I} = \frac{\partial}{\partial I} K^{-1} \cdot P + K \frac{\partial}{\partial I} P \tag{5}$$

The second term of the right hand side of the above equation contain the derivative of the dynamic loads which vanish in the static problems. If these values are obtained, the sensitivity coefficients of the stresses and deflections can be evaluated applying the same procedure adopted for the static problems.

As the dynamic loads which is evaluated by means of random vibration theory become the explicit function of natural frequencies and mode vectors of structure,[6] if the sensitivity coefficients of these values are evaluated, then that of these dynamic loads can be obtained without difficulty.

6. NUMERICAL EXAMPLE

The method previously mentioned is applied to the design of three-story frames of equal span length, 6m, and equal story height, 3m, with uniformly distributed load, w, on beams, subjected to the four types of ground motions whose characteristics are decided by the parameters presented in Table 1. Frames are designed both elastically for the power S_0 of Eq.(1), and plastically for the power of α times of S_0 so that the story drifts should be less than allowable ductility factor 4, and beam collapse type mechanism is

TYPE	T_{g1}	h_{g1}	β_1	T_{g2}	h_{g2}	β_2
I	0.3	0.6	1.0			
II	1.0	0.6	1.0			
III	0.1	0.3	0.2	1.0	0.3	0.8
IV	0.1	0.3	0.2	1.5	0.3	0.8

Table 1 Ground Parameter

considered for the calculation of yield levels of story shear forces. Steel used is SS41 whose yield stress is 2.4 ton/cm^2.

6.1 Three story one bay frames subjected to the ground motion of type I is optimized for W = 5 ton/m and S_o = 5 cm/rad/sec^3. In Fig. 3, the maximum stresses and the maximum ductility factors of each story corresponding to the final design are presented for α equal 5 and 7 respectively. Where the maximum stress is defined as the value in the most severely violated constraint equation for elastic design whose allowable limit is normalized as unity. For the case of α equals 5, the member size is desided by the elastic constraints and the response ductility factors of each story are scattering. On the other hand, for the case of α equals 7, the beams are not fully stressed for elastic design constraints and for the plastic design constraints they are equally fully constrained. Therefore it can be pointed out that for the optimum design of earthquake resistance structures, it become important to consider the constraints for the inelastic deflection expected during the very strong earthquakes.

6.2 Three story one bay frame subjected to the ground motion of type III and IV is optimized for w = 2 ton/m, S_o = 2 cm/rad/sec^3 and α = 7. The maximum stresses of each member defined previously and the maximum ductility factors for the final design are presented in Fig. 4. This shows that the optimum member size restricted by both elastic and plastic constraints.

The acceleration response spectrum to these ground motions is presented in Fig. 4 with the values of the spectrum correspond to the fundamental frequencies of the structure of initial and final design. This shows that even if the initial design is at the valley of the response spectrum, or final design is at the vicinity of the maximum, this optimization technique can be successfully adopted.

Neglecting the derivative of dynamic forces which is used in Eq. (5), the optimization is also carried out for the same model. The final result obtained starting from the same initial design mentioned above is presented in Fig. 6. Compared with the above analysis, much more iterative calculations are carried out and the real optimum solusion can not be obtained. This too happen for the optimization of the structure subjected to the ground motion which have more moderate response spectrum showing the importance of sensitivity analysis of dynamic forces for these analysis.

6.3 Three story one bay and three bays frames are optimized for α = 7 by changing the parameters concerend with the distributed load and ground motion. The ductility factors of story drift correspond to the final design are shown in Table 2 with these parameters. Each story yield almost equally fully restricted by the constraints of plastic deformation. Therefore it can be pointed out that for this kind of structures, the optimum design correspond to such a structure whose response ductility factors against very strong ground motion are almost equal for all story.

7. CONCLUSION

As a result of this study, following conclusions can be pointed out.

(1) The analysises of some examples shows the validity of the optimization technique mentioned above together with the importance of the sensitivity analysis of dynamic forces.

(2) The constraints concerened with the plastic deformation against the very strong ground motions must be considered together with the constraints for the elastic strength.

(3) For the type of structure dealt with in this paper, the minimum weight design correspond to such a structure whose response ductility factors against very strong ground motion are almost equal for all story.

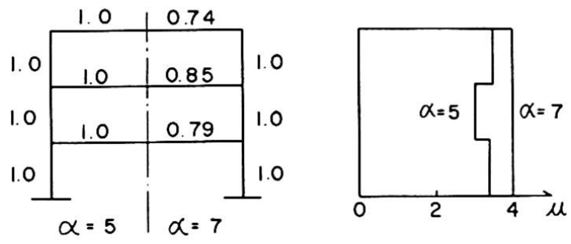

Fig.3 The Maximum Stress and Ductility Factor

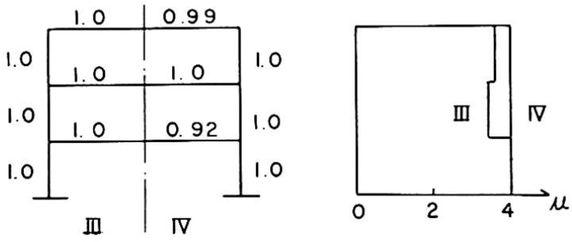

Fig.4 The Maximum Stress and Ductility Factor

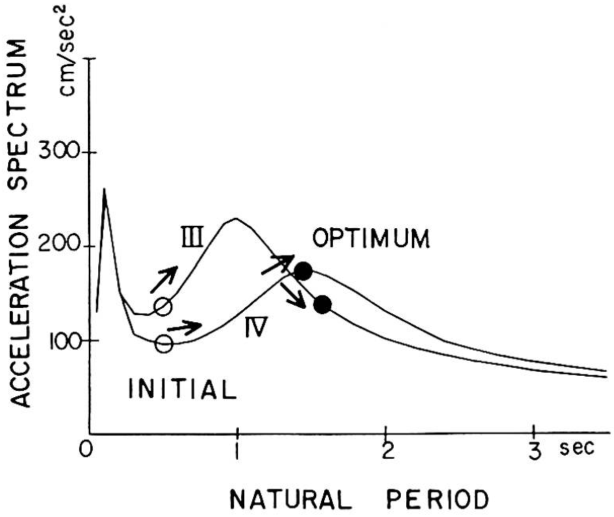

Fig.5 Optimization Process for Different Type Ground Motion

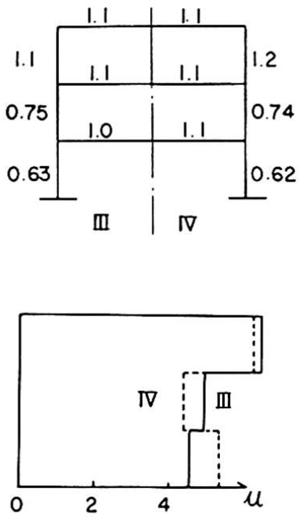

Fig.6 The Maximum Stress and Ductility Factor

SPAN	ω	TYPE	S_o	μ		
				1	2	3
1	10	I	5	4.0	4.0	4.0
1	50	I	5	4.08	4.08	4.05
1	50	II	5	4.04	4.04	4.04
1	5	III	2	4.0	4.0	4.0
1	5	IV	2	4.0	3.68	3.92
1	2	IV	2	4.0	4.0	4.0
1	50	IV	5	3.99	3.82	3.96
3	10	II	5	4.04	4.08	4.08
3	30	I	5	3.92	3.96	3.92
3	30	II	5	4.12	4.04	4.04

Table 2 The Maximum Ductility Factor

3g. 7 VB

REFERENCES

1. Raymond M. Brach : "Optimum Design for Sudden Loading," Journal of the Engineering Mechanics Division, ASCE, Vol 94, No. EM6, Proc. Paper 6272, Dec., 1968.

2. R.L. Fox., and M.P. Kapoor : "Structural Optimization in the Dynamic Response Regime : A computational Approach." AIAA Structural Dynamics and Aeroelasticity Specialist Conference, Neworleans, 1969.

3. H. Tajimi : "A Statistical Method of Determining the Maximum Respons of a Building Structure during Earthquake." Proceedings of the Second World Conference on Earthquake Engineering, Tokyo, Japan.

4. A.G. Davenport : "Note on the Distribution of the Largest Values of a Random Function with Application to Gust Loading." Proceedings, Institution of Civ. Engrs. Vol 28, June, 1964.

5. A.S. Veletsos and N.M. Newmark : "Effect of Inelastic Behaviour on the Response of Simple Systems to Earthquake Motions." Proceedings of the Second World Conference on Earthquake Engineering. Tokyo, Japan.

6. B. Kato, Y. Nakamura and H. Anraku : "Optimum Earthquake Design of Shear Buildings." Journal of the Engineering Mechanics Division, ASCE, Vol 98, No. EM4, 1972.

SUMMARY

The minimum weight design of unbraced steel frames subjected to dynamic earthquake loads is presented. Random vibration theory is adopted to elastic member strength and plastic story deflection, the sequential linear programming technique is successfully adopted to obtain the optimum design. Several examples are presented with the analysis and comparisons are drawn.

RESUME

On présente le dimensionnement, pour un poids minimum, de cadres métalliques soumis aux forces dynamiques des tremblements de terre. La théorie des vibrations aléatoires permet de déterminer le comportement "dynamique" de la structure. La programmation linéaire séquentielle donne le dimensionnement optimal dans des conditions de comportement élastique des éléments et de comportement plastique du cadre soumis à la déflection.

ZUSAMMENFASSUNG

Für unausgesteifte Stahlrahmen, die durch Erdbebenwirkung beansprucht sind, wird die Berechnungsmethode des "minimalen Gewichts" abgeleitet. Die "Random"-Vibrationstheorie erlaubt es, das dynamische Verhalten des Tragwerks festzustellen. Unter Annahme "elastischer" Kräfte und plastischer Verformungen liefert die fortschreitende lineare Programmierung das gesuchte Optimum. Beispiele werden gezeigt und Vergleiche angestellt.