IIa

Minimum Weight Plastic Design of Multi-story Plane Frames for Five Sets of Design Loads

Calcul plastique, pour un poids minimum, de cadres plans à plusieurs étages, avec cinq groupes de cas de charge

Plastische Bemessung auf Minimalgewicht von mehrstöckigen ebenen Rahmen für fünf Belastungszustände

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1. Introduction Several classes of general solutions to the problem of minimum weight plastic design of multi-story multi-span plane frames subjected to a class of *one* set of practical design loads have been derived by the senior author [1] by applying Foulkes' theory [2] and by extending it to a more general theory [3] which incorporates the axial force-bending moment interaction yield conditions. The present authors have further extended the result of [1] so as to incorporate the reaction constraints in [4]. These analytical general solutions are of theoretical and practical interests. Firstly, they serve to clarify even partially the general features of the minimum weight designs. Secondly, once an analytical method is developed for simpler problems based upon the moment yield condition [1], their general solutions would provide a good lead to the general solutions to more complex problems based upon interaction yield conditions [3]. Thirdly, they will provide good initial feasible solutions for neighborhood problems.

In this paper, *a kinematical restricted maximization procedure* is developed by combining the *primal-dual* method of LP [5] with a semi-inverse approach similar to the idea of [1] and then applied to the problem of minimum weight plastic design of multi-story multi-span plane frames subjected to five sets of design loads.

2. Formulation of the Design Problem Fig.1 shows a multi-story multi-span plane frame to be designed by Foulkes' theory [2] and the five sets of design

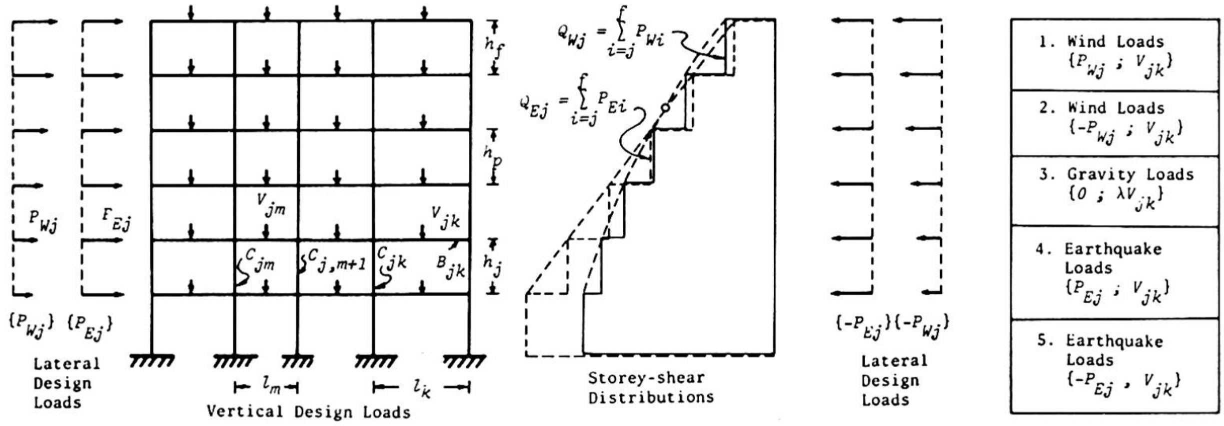


Fig.1 Design Load Distributions, Notation and 5 sets of Design Loads

loads. The fully-plastic moments of *(j,k)*-beam and *(j,k)*-column are denoted by $B_{j,k}$ and $C_{j,k}$, respectively. Without loss of practical generality, it may be assumed that the story-shear force distributions defined in Fig.1 be such that

$$Q_{Ej} \geqq Q_{Wj} \quad for \quad j=1,2,\ldots,p, \text{ and} \qquad Q_{Ej} \leqq Q_{Wj} \quad for \quad j=p+1,\ldots,f. \tag{1}$$

The factor λ for the design gravity loads is assumed to be $\lambda \leqq 2.0$.

The design problem for five sets of design loads is treated in the following three or four steps:

(i) Solve the basic problem for the two sets of co-directional lateral design loads 1 and 4, i.e., for $\{P_{Wj};V_{j,k}\}$ and $\{P_{Ej};V_{j,k}\}$,
(ii) Construct a statically admissible bending moment field for the two sets of design loads 2 and 5, i.e., for $\{-P_{Wj};V_{j,k}\}$ and $\{-P_{Ej};V_{j,k}\}$,
(iii) Construct a statically admissible bending moment field for the design gravity loads 3, i.e., for $\{0;\lambda V_{j,k}\}$,
(iv) If the step (ii) or (iii) is not possible, modify the collapse mechanism locally and find the corresponding modified design.

The basic problem (i) may be stated in terms of the static variables defined in Fig.2(a) as follows:

$$\textit{Minimize} \qquad G = g\{\sum_{k=1}^{s} l_k \sum_{j=1}^{f} B_{jk} + \sum_{j=1}^{f} h_j \sum_{k=1}^{s+1} C_{jk}\}, \quad (g\text{: constant}) \tag{2}$$

$$\left.\begin{aligned} \textit{subject to:}\ & \sum_{k=1}^{s+1}(c^B_{Ijk} + c^T_{Ijk}) = h_j Q_{Ij}\ , \quad b^L_{Ijk} + b^R_{Ij,k-1} = c^B_{Ij+1,k} + c^T_{Ijk}\ , \\ & \tfrac{1}{2}(b^L_{Ijk} - b^R_{Ijk}) + \tfrac{1}{4} l_k V_{jk} \leqq B_{jk} \\ & -B_{jk} \leqq b^L_{Ijk} \leqq B_{jk}\ , \ -B_{jk} \leqq b^R_{Ijk} \leqq B_{jk}\ , \ B_{jk} \geqq 0, \\ & -C_{jk} \leqq c^B_{Ijk} \leqq C_{jk}\ , \ -C_{jk} \leqq c^T_{Ijk} \leqq C_{jk}\ , \ C_{jk} \geqq 0, \end{aligned}\right\} \tag{3a-i}$$

where b^L_{Ijk}, b^R_{Ijk}, c^B_{Ijk} and c^T_{Ijk} are free variables. In the expression (2), f and s denote the numbers of stories and spans, respectively. In the constraints (3), the first subscript I denotes the kind of design loads and is to be either E or W. The second and third subscripts refer to the story number from below and member number from left, respectively. For the sake of brevity, the equations of moment equilibrium about interior and exterior joints have been written in one and the same form with the convention that all the undefined quantities with respect to non-existent members shall be disregarded and dropped as null. This convention will also be used hereafter, unless otherwise stated.

3. Kinematical Restricted Maximization Procedure-Semi-Inverse Primal-dual Method.

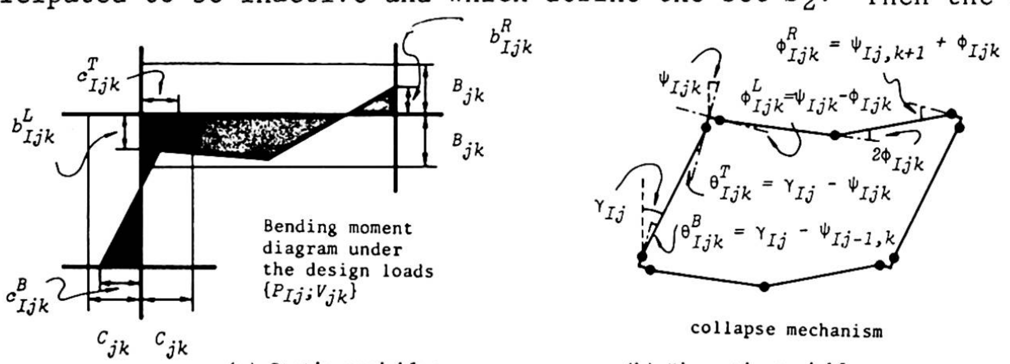
The idea of the proposed approach may be summarized by referring to Fig.3 as follows. A design problem formulated as a linear programming problem [6] of a *mixed* type [5], may often be such that a certain set of constraints may be anticipated to be *inactive* due to the nature of the problem. From the original primal problem

$[PO]$: *Minimize* $\{G(\mathrm{x}) \mid \mathrm{x} \in S_1 \cap S_2\}$

of a large size, a subproblem

$[PS]$: *Minimize* $\{G(\mathrm{x}) \mid \mathrm{x} \in S_1\}$

may be derived by tentatively disregarding a certain set of constraints which are anticipated to be inactive and which define the set S_2. Then the dual problem to



(a) Static variables *(b) Kinematic variables*

Fig.2 Definitions of the variables

[PS], i.e.

[DS]: *Maximize* $\{D(u)\,|\,u\in V\}$

must involve a smaller number of dual variables and a greater number of equality constraints. Therefore, if the solution u^o to [DS] can be found more easily compared to the solution to the dual problem of [PO], then the corresponding solution x^o to [PS] may also be readily found simply by solving the set of simultaneous linear equations derived from the duality theorem of *LP*. It remains then to check if $x^o\in S_2$. The procedure may also be called "a semi-inverse primal-dual method."

4. A Class of General Solutions to the Problem (i) It is now shown that the kinematical maximization procedure is fruitful for rectangular frames due to their regularity in the optimality criteria-based collapse mechanism. Let

$$S_2:\ (b^L_{Ijk},\ b^R_{Ijk})\geq -B_{jk},\ (c^B_{Ijk},\ c^T_{Ijk})\geq -C_{jk},\ B_{jk}\geq 0,\ C_{jk}\geq 0, \qquad (4a\text{-}d)$$

Then the dual problem [DS] may be written in terms of the kinematic variables defined in Fig.2(b) as follows:

$$\textit{Maximize}\quad D=\Theta\{\sum_{j=1}^{f} h_j(Q_{Wj}\gamma_{Wj}+Q_{Ej}\gamma_{Ej})+\sum_{k=1}^{s}\sum_{j=1}^{f}\frac{1}{2}l_kV_{jk}(\phi_{Wjk}+\phi_{Ejk})\} \qquad (5)$$

$$\textit{subject to}\quad \gamma_{Ij}\geq \underset{k}{Max.}\{\psi_{Ijk},\ \psi_{Ij-1,k}\}$$

$$\psi_{Ijk}\geq\phi_{Ijk}\geq 0\quad (k=1,2,\ldots,s),\qquad \psi_{Ij,s+1}\geq-\phi_{Ijs}$$

$$\{(\psi_{Wjk}+\psi_{Wj,k+1}+2\phi_{Wjk})+(\psi_{Ejk}+\psi_{Ej,k+1}+2\phi_{Ejk})\}=\theta l_k \qquad (6a\text{-}d)$$

$$\{(2\gamma_{Wj}-\psi_{Wj-1,k}-\psi_{Wjk})+(2\gamma_{Ej}-\psi_{Ej-1,k}-\psi_{Ejk})\}=\theta h_j$$

The inequalities (6a, b) restrict the directions of plastic hinge rotations and the equalities (6c, d) are the generalized Foulkes conditions defined by Chan [6] and Prager [7]. The latter will be referred to as *FCP* conditions.

The equations (6d) indicate that $\psi_{Wjk}+\psi_{Ejk}\equiv\psi_j$ (independent of k). The problem defined by (5) and (6) may then be simplified to a problem in terms of ψ_{Wjk}, γ_{Wj}, ϕ_{Wjk} and ψ_j only. After some manipulation on the inequalities, γ_{Wj} may be expressed in terms of ψ_{Wjk} and ψ_j only, and then ψ_{Wjk}, in terms of ψ_{Wpk} and ψ_j only. Finally, for those problems in which the load conditions:

$$h_jQ_{Ij}+h_{j+1}Q_{I,j+1}\geq\sum_{k=1}^{s}l_kV_{jk}\ ,\quad \begin{pmatrix}1\leq j\leq p-1 & \textit{for}\ \ I=E,\\ p+1\leq j\leq f & \textit{for}\ \ I=W,\end{pmatrix}$$

$$h_pQ_{Ep}+2h_{p+1}Q_{Ep+1}-h_{p+1}Q_{Wp+1}\geq\sum_{k=1}^{s}l_kV_{pk}\ , \qquad (7a,b)$$

and the geometrical conditions:

$$l_m\leq l_k\leq 2l_m\ ,\ l_m\leq 2h_1\ (l_m=\underset{k}{Min.}\{l_k\})\ , \qquad (8a,b)$$

are satisfied, the problem [DS] may be reduced to the following form:

$$\textit{Maximize}\quad D^*=\Theta(-\Delta M_p\xi+\Delta M_{p+1}\eta), \qquad (9)$$

$$\textit{subject to}\quad \xi\equiv\underset{k}{Max.}\{\psi_{Wpk}\},\quad \eta\equiv\underset{k}{Min.}\{\psi_{Wpk}\},\quad (\xi\geq\eta)$$

$$\xi\leq\frac{1}{2}h_{p+1}\theta+Min.\{0,\eta\},\ \ Max.\{\frac{1}{2}l_m\theta,\xi\}\leq\frac{1}{2}h_{p+1}\theta+\eta, \qquad (10a\text{-}d)$$

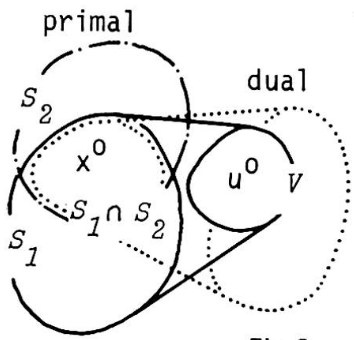


Fig. 3

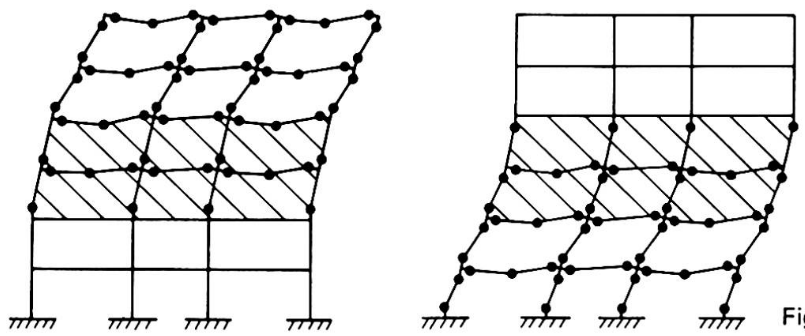


Fig. 4

$$0 \leqq \psi_{Wpk} \leqq \frac{1}{2} l_m \Theta, \quad (k=1,2,\ldots,s), \quad a \leqq \psi_{Wp,s+1} \leqq b,$$
$$a \equiv Max.\{-(l_s - l_m)\Theta/2, \ -\Psi_{Wps}\} \leqq 0$$
$$b \equiv Min.\{l_s\Theta/2, \ l_m\Theta - \psi_{Wps}\} \geqq l_m\Theta/2 \qquad (10e\text{-}h)$$

where $\Delta M_p \equiv h_p(Q_{Ep} - Q_{Wp}) \geqq 0$ and $\Delta M_{p+1} \equiv h_{p+1}(Q_{Wp+1} - Q_{Ep+1}) \geqq 0$. The solution to this reduced problem may readily be derived as summarized in Table 1. In those problems where (7) and (8) are satisfied, the generalized Foulkes mechanism defined by the *FCP* conditions can thus be constructed as shown in Fig.4 for Case (B) as an example.

The solution to the problem [*PS*] corresponding to this problem[*DS*] may also be derived straightforwardly. By assuming that some statical restrictions defined and checked later will be satisfied, the resulting bending moment diagram may be understood best by conceiving it as the result of superposition of the constituent elementary moment diagrams (with equal corner values for $k \neq m$) shown in Fig.5. Such a decomposition was first introduced in [1]. Each diagram is referred to as "frame moment diagram." The minimum weight plastic design corresponding to Table 1 may be compactly summarized as Table 2 in terms of "Maximum Story-Shear Force Design" defined by

$$B^*_{jk} \equiv Max.\{B^W_{jk}, \ B^E_{jk}\}, \qquad C^*_{jk} \equiv Max.\{C^W_{jk}, \ C^E_{jk}\} \qquad (11a,b)$$

where $\{B^W_{jk}, C^W_{jk}\}$ and $\{B^E_{jk}, C^E_{jk}\}$ denote the designs only for $\{P_{Wj}; V_{jk}\}$ and $\{P_{Ej}; V_{jk}\}$, respectively, derived by means of [1]. B^*_{jk} and C^*_{jk} are given by

$$B^*_{jk} = \frac{1}{4} l_k V_{jk} \ (k \neq m); \quad C^*_{jk} = \frac{1}{4}(l_{k-1} V'_{j,k-1} + l_k V'_{jk}); \ (k \neq m, \ m+1)$$
$$B^*_{jk} = \frac{1}{4}(h_j Q_{Ij} + h_{j+1} Q_{I,j+1} - \sum_{k \neq m} l_k V_{jk}), \ \left(\begin{array}{l} 1 \leqq j \leqq p-1 \ \ for \ \ I=E \\ p+1 \leqq j \leqq f \ \ for \ \ I=W \ \ and \end{array}\right)$$
$$j=p, \ I=E \ \ for \ \ \Delta M_p \geqq \Delta M_{p+1}, \ and \ j=p, \ I=W \ \ for \ \ \Delta M_p \leqq \Delta M_{p+1};$$
$$C^*_{jn} = \frac{1}{4}(h_j Q_{Ij} - \sum_{k \neq n-1, n} l_k V'_{jk}), \ \left(\begin{array}{l} 1 \leqq j \leqq p \ \ for \ \ I=E, \\ p+1 \leqq j \leqq f \ \ for \ \ I=W, \\ n=m, \ m+1 \end{array}\right)$$
$$V'_{jk} \equiv \sum_{i=j}^{f} (-1)^{i-j} V_{ik}\ . \qquad (12a\text{-}e)$$

The yield inequalities in (3) provide restrictions on the design loads in accordance with the classification of the solutions listed in Table 2. These

Table 1 Generalized Foulkes Mechanism

<table>
<tr><th colspan="2"></th><th>γ_{Wj}</th><th>γ_{Ej}</th><th>ψ_{Wjk}</th><th>ψ_{Ejk}</th><th>$\phi_{Wjk}(k \neq m)$</th><th>$\phi_{Ejk}(k \neq m)$</th><th>$\phi_{Wjm}, \ \phi_{Ejm}$</th></tr>
<tr><td colspan="2">$j=1$</td><td rowspan="2">0</td><td>$\frac{1}{2}(h_1+\frac{1}{2}l_m)\Theta$</td><td rowspan="2">0</td><td rowspan="2">$\frac{1}{2}l_m\Theta$</td><td rowspan="2">0</td><td rowspan="2">$\frac{1}{2}(l_k-l_m)\Theta$</td><td rowspan="11">0</td></tr>
<tr><td colspan="2">$j=2, \ldots, p-1$</td><td>$\frac{1}{2}(h_j+l_m)\Theta$</td></tr>
<tr><td rowspan="4">$j=p$</td><td>(A)</td><td>0</td><td>$\frac{1}{2}(h_p+l_m)\Theta$</td><td>0</td><td>$\frac{1}{2}l_m\Theta$</td><td>0</td><td>$\frac{1}{2}(l_k-l_m)\Theta$</td></tr>
<tr><td>(B)</td><td>$\frac{1}{2}(l_m-h_{p+1})\Theta$</td><td>$\frac{1}{2}(h_p+h_{p+1})\Theta$</td><td>$\frac{1}{2}(l_m-h_{p+1})\Theta$</td><td>$\frac{1}{2}h_{p+1}\Theta$</td><td>*</td><td>*</td></tr>
<tr><td>(C)</td><td>$\frac{1}{2}l_m\Theta$</td><td>$\frac{1}{2}h_p\Theta$</td><td>$\frac{1}{2}l_m\Theta$</td><td>0</td><td>$\frac{1}{2}(l_k-l_m)\Theta$</td><td>0</td></tr>
<tr><td>(D)</td><td>$\frac{1}{2}h_p\Theta$</td><td>$\frac{1}{2}l_m\Theta$</td><td>$\frac{1}{2}h_p\Theta$</td><td>$\frac{1}{2}(l_m-h_p)\Theta$</td><td>*</td><td>*</td></tr>
<tr><td rowspan="4">$j=p+1$</td><td>(A)</td><td>$\frac{1}{2}h_{p+1}\Theta$</td><td>$\frac{1}{2}l_m\Theta$</td><td rowspan="5">$\frac{1}{2}l_m\Theta$</td><td rowspan="5">0</td><td rowspan="5">$\frac{1}{2}(l_k-l_m)\Theta$</td><td rowspan="5">0</td></tr>
<tr><td>(B)</td><td>$\frac{1}{2}l_m\Theta$</td><td>$\frac{1}{2}h_{p+1}\Theta$</td></tr>
<tr><td>(C)</td><td>$\frac{1}{2}(h_{p+1}+l_m)\Theta$</td><td>0</td></tr>
<tr><td>(D)</td><td>$\frac{1}{2}(h_p+h_{p+1})\Theta$</td><td>$\frac{1}{2}(l_m-h_p)\Theta$</td></tr>
<tr><td colspan="2">$j=p+2, \ldots, f$</td><td>$\frac{1}{2}(h_j+l_m)\Theta$</td><td>0</td></tr>
</table>

restrictions may be summarized as shown in Table 3, where

$$M_{Ij} \equiv \frac{1}{4}(h_j Q_{Ij} + h_{j+1} Q_{I,j+1} - \sum_{k=1}^{s} l_k V_{jk}) \tag{13}$$

It may now be concluded that the present solutions (A~D) are the rigorous solutions to the problems in which all the geometrical and loading conditions are satisfied.

5. Design for Five Sets of Design Loads.

It may readily be confirmed that a statically admissible bending moment field for $\{-P_{Ej};\ V_{jk}\}$ and $\{-P_{Wj};\ V_{jk}\}$ can be constructed just by inverting the frame moment diagrams as shown in Fig.6.

For design gravity loads, it is convenient to consider again the decomposed moment diagram with the respectively equal corner values $\lambda l_k V'_{jk}/8$, as shown in Fig.7. The conditions that the bending moment diagram given by superposing the elementary diagrams in Fig.7 be statically admissible in a frame designed by the procedure in Section 4, lead again to further restrictions on the design gravity loads. An examination of these restrictions indicates that there are a number of practically useful design solutions within the range defined by them.

6. Concluding Remarks

It may now be concluded that, for the class of design problems in which all the previous and supplementary conditions are satisfied, the solutions (A~D) are the rigorous minimum weight plastic designs. The present designs have apparently clarified the nature of minimum weight plastic designs. While these designs must be modified for practical use so as to satisfy a number of structural requirements, the present solutions will at least provide a basis

Table 2

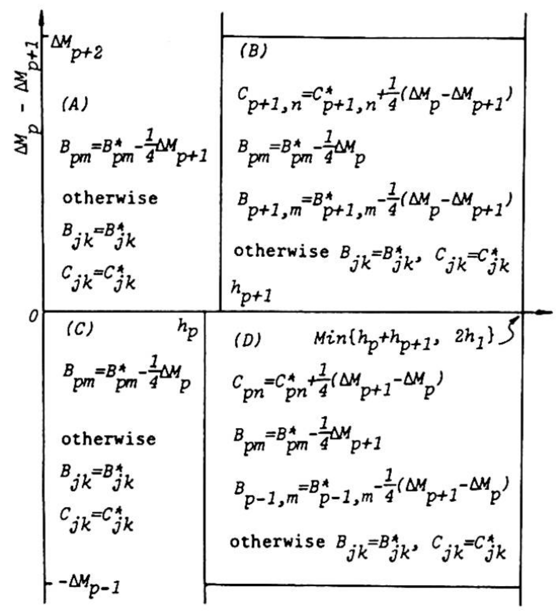


Table 3

		(A)	(B)	(C)	(D)
	$j=1,2,\ldots,p-2$	$M_{Ej} \geq 0$			
	$j=p-1$	$M_{Ej} \geq 0$		$M_{Ep-1} \geq \frac{1}{4}(\Delta M_{p+1}-\Delta M_p)$	
	$j=p$	$M_{Ep} \geq \frac{1}{4}\Delta M_{p+1}$	$M_{Ep} \geq \frac{1}{4}\Delta M_p$	$M_{Wp} \geq \frac{1}{4}\Delta M_p$	$M_{Wp} \geq \frac{1}{4}\Delta M_{p+1}$
	$j=p+1$		$M_{Wp+1} \geq \frac{1}{4}(\Delta M_p-\Delta M_{p+1})$		
	$j=p+2,\ldots,f$	$M_{Wj} \geq 0$			
	$k \neq m, m+1$; $j=1,2,\ldots,f$	$l_{k-1}V'_{j,k-1}+l_k V'_{jk} \geq 0$			
$n=m,m+1$	$j=1,\ldots,p-1$	$h_j Q_{Ej} \geq \sum_{k \neq n,n-1} l_k V'_{jk}$			
$n=m,m+1$	$j=p$		$h_p Q_{Ep}+\frac{1}{4}(\Delta M_p-\Delta M_{p+1}) \geq \sum_{k \neq n,n-1} l_k V'_{pk}$		
$n=m,m+1$	$j=p+1$	$h_j Q_{Wj} \geq \sum_{k \neq n,n-1} l_k V'_{jk}$			$h_{p+1}Q_{Wp+1}+\frac{1}{4}(\Delta M_{p+1}-\Delta M_p) \geq \sum_{k \neq n,n-1} l_k V'_{p+1,k}$
$n=m,m+1$	$j=p+2,\ldots,f$	$h_j Q_{Wj} \geq \sum_{k \neq n,n-1} l_k V'_{jk}$			

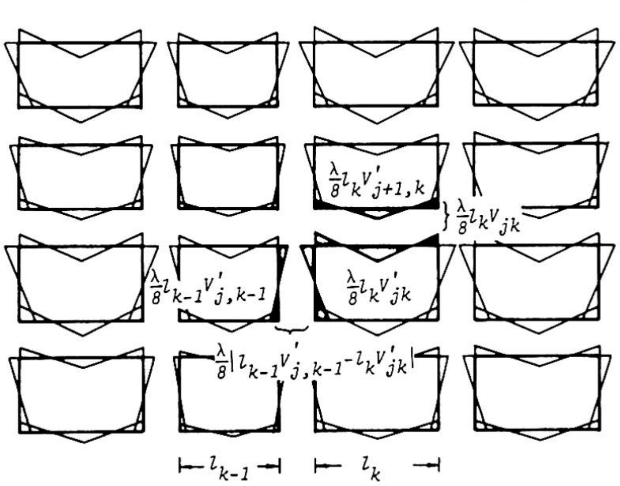


Fig. 7 Decomposed Moment Diagram for Gravity Loads

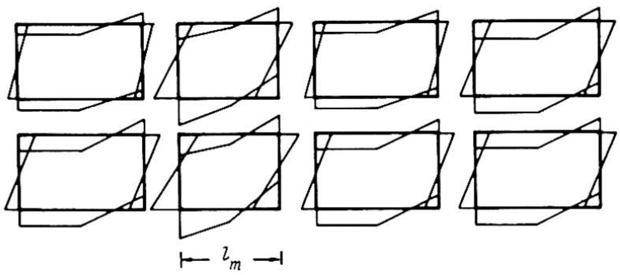


Fig. 5 Frame moments under the wind load (P_{Wj})

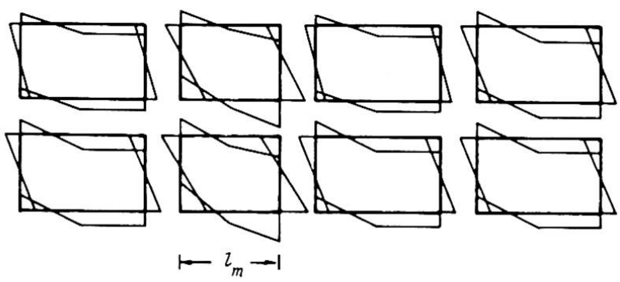


Fig. 6 Frame moments under the wind load ($-P_{Wj}$)

for initial designs useful in such countries where fairly large lateral design loads must be assigned for PLASTIC DESIGN so that frames can withstand against strong winds and strong motion earthquakes. The present solutions may be said to be a class of the most fundamental designs in the sense that a number of useful designs to practical neighborhood problems can be derived by appropriate but mostly local modifications. Three cases:

(a) $h_f P_{Wf} \leqq \sum_{k=1}^{s} l_k V_{fk}$, (b) $l_m \geqq 2h_1$ and (c) $\exists l_k \geqq 2l_m$ have been treated in [8].

The present solution and the solutions in [1, 3, 4] indicate that a frame designed by these solutions would collapse in an extremely deteriorated overcomplete mechanism under a designated set of design loads according to the rigid-plastic analysis. It is therefore necessary to confirm the safety of such a frame against possible collapse due to inelastic instability according to a more refined theory of large-deflection elastic-plastic analysis. For this purpose, static and dynamic large-displacement analyses have been carried out on minimum weight frames in [9~11] under alternating lateral loads well beyond their static stability limits and under strong motion earthquake disturbances, respectively.

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SUMMARY

A kinematical restricted maximization procedure has been developed by combining the primal-dual method of linear programming with a semi-inverse approach. Some general solutions to practical problems of minimum weight plastic design have been derived analytically by applying the proposed method.

RESUME

Une procédure cinématique de maximisation limitée a été développée par combinaison de la méthode primale-duale de la programmation linéaire avec une approche semi-inverse. Quelques solutions générales pour des problèmes pratiques de dimensionnement plastique, conduisant à un poids minimum ont été obtenues analytiquement par application de la méthode proposée.

ZUSAMMENFASSUNG

Ein begrenztes kinematisches Maximierungsverfahren wird bei einer Kombination der "primal-dual"-Methode der linearen Programmierung mit einem "semi-inversen" Verfahren entwickelt. Allgemeine analytische Lösungen praktischer Probleme der plastischen Bemessung auf Minimalgewicht werden durch Anwendung der vorgelegten Methode gefunden.