Optimization concepts and techniques in structural design

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Optimization Concepts and Techniques in Structural Design

Concepts et techniques d'optimisation

Grundlagen und Methoden

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1. INTRODUCTION

Since the early 1960's structural optimization has established itself as an important area of research in structural engineering and it is now gradually becoming an integral part of the technological expertise of practising designers. The phrase "structural optimization" conjures a very specific impression in the minds of most engineers relating to the application of numerical search techniques to mathematical models of certain types of structures. Whilst this impression is correct in detail it is by no means the whole story. The history of architecture and structural design has been characterized from the very earliest times by efforts to design structures which were in some respect "better" than their predecessors; to use materials and resources more efficiently and to design structures which were as well-suited as possible to their functions. This continual search for improvement is essentially an optimization process, to use the term in its widest sense, and it pervades all present-day engineering activities.

On a national and regional scale governments seek to invest in new industries, resources and communications so as to obtain maximum benefit from a limited investment. At the project level an engineer strives to produce efficient, cheap and reliable designs for projects to satisfy functional requirements. The contractor wishes to use his resources of manpower, machinery and capital in the most efficient manner and the client is concerned whether his structure will be economical to run, to use and to maintain. The whole process of project planning, design, construction and operation is governed by the need to produce the best possible solution. Traditionally, emphasis has been placed on the intuitive skills and ability of experienced planners and engineers to produce near-optimal solutions to problems. However, the demands of an increasingly technologically-based population now require that projects are larger, more expensive and more complex. Financial pressures towards cost economy are increasing. Restrictions imposed by aesthetic, social, environmental and technological factors are ever more stringent. A result of this increasing complexity of the design process is that traditional reliance upon the skills of individual designers must change to meet present-day circumstances.

Consequently, more assistance is now sought from the digital computer, with its speed and power to solve complex problems rapidly, to meet these demands.

Since the digital computer was first used on structural engineering problems over twenty years ago it has completely transformed methods of structural analysis, so that now the use of matrix stiffness and flexibility methods as well as finite element techniques is commonplace in structural design offices throughout the world. However, as it became possible to analyse increasingly complex structures it also became more difficult to interpret the results of the analysis in a logical way for design purposes. It was, therefore, a natural trend in research to try to use the computer to produce design-oriented information rather than analysis-oriented information for the designer to interpret. Thus over the last ten or fifteen years research into aspects of computer-aided structural design has considerably increased. Once it was recognized that the engineering planning and design process is an optimum-seeking one it was also natural to see whether the computer could not also be used to produce not merely design information but optimum design information so that the structural engineer could play his part effectively in the planning and design of efficient, cheap and reliable structures. The computer, therefore, afforded the possibility of helping with the two basic problems mentioned above of the increasing technological complexity of structural design and the growing pressures towards cost control and economical designs.

However, just as the computer revolutionized structural analysis by demanding new techniques suited to its abilities a similar revolution was needed in design techniques if the computer was to be of real use in this area. In the early 1960's Schmit[1] and others gave a first insight into a new computer-oriented approach to structural design by examining the application of newly-developed techniques of mathematical programming and optimization to problems of structural design. Thus structural optimization in its modern form was born. With the passing of time it is perhaps difficult to realize that in the early 1960's the philosophy of structural optimization was really very novel. The early research referred to above clearly demonstrated what had not previously been self-evident: That the largely ad hoc processes of structural design which appeared to have little formal logic may in fact be expressed formally and in mathematical terms, and that there was as much rigour and logic in solving an optimum structural design problem as there was in solving a structural analysis problem. Perhaps the largest contribution which the study of optimization has given to structural engineering is that it has put structural design on a formal, mathematical basis and by so doing has unified a previously fragmentary and ill-disciplined subject.

Coming some fifteen years after Schmit's original work this paper examines some of the more recent developments in structural optimization techniques. It shows that there are still many difficulties to overcome for the full potential of optimization to be realized and it is hoped that it will give an insight into the way in which structural optimization creates a greater understanding of the nature of the design process and of structural behaviour.

2. A HIERARCHY OF STRUCTURAL OPTIMIZATION PROBLEMS

The rationalizing and formalizing influence which optimization has had upon structural design now means that structural optimization problems may be conventionally expressed as:

$$\left.\begin{array}{llll} \text{Minimize, or Maximize} & g_o\,(x_i) & & i = 1, \ldots, N \\ \text{Subject to constraints} & g_j\,(x_i) \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} 0 & & j = 1, \ldots, M \\ & x_i \geq 0 & & i = 1, \ldots, N \end{array}\right\} \quad (1)$$

The variables in this problem, x_i, $i = 1, \ldots, N$, usually represent physical parameters of the structure to be designed such as dimensions, spacings, bar sizes, plate thicknesses, etc. These variables are under the designer's control and he wishes to find "best", or optimal values for all of them. The objective function $g_o\,(x_i)$ which is to be extremized represents some evaluable criterion of efficiency of the structure. The efficiency criteria most frequently used are such things as minimum structural weight, minimum cost, maximum factor of safety, etc. The choice of objective function will be commented upon later. The M constraints $g_j\,(x_i)$ may be equalities or inequalities and they originate from many sources. They specify, for example, the mechanical behaviour of the structure under load, the known properties of the materials used, requirements of relevant codes of practice, fabricational requirements, geometric and layout requirements, etc. All relevant restrictions and requirements upon the structure must appear among the constraints. The non-negativity condition upon each variable is necessary to ensure that values are obtained for all the problem variables which are real and feasible in an engineering sense. Sometimes integer values or values from a discrete set may be required.

Generally structural optimization problems are large and non-linear except for all but the simplest structures. This is seen if a single, very much simplified structural element is examined. Fig. 1 shows the cross-sectional shape of a typical reinforced concrete T-beam.

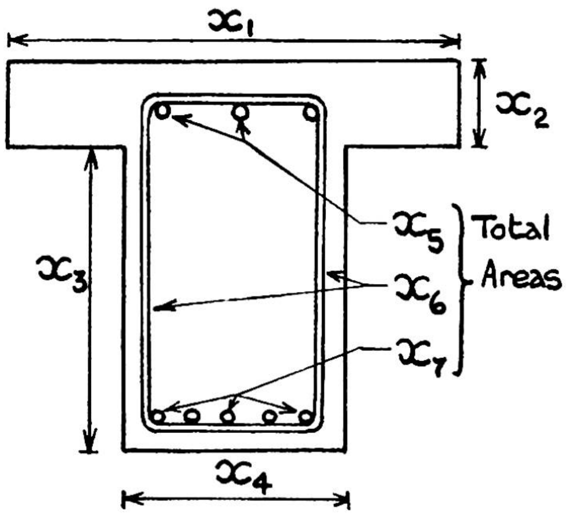

FIG. 1

To design this beam of known length and loading so as to minimize a simple cost function involves finding optimal values for the seven variables shown. The cost function g_o is fairly simple to write down. It involves the cost of the concrete and of the steel, both roughly proportional to the volumes of concrete and steel, and it also involves the cost of shuttering the beam during pouring. This cost depends upon the perimeter of the cross-section. The objective function is then a simple one but is non-linear in the variables. Many constraints are necessary to ensure that the concrete and steel can adequately carry the bending stresses in the beam and also resist shearing stresses. Codes of practice prescribe maximum permissible values for these stresses and also prescribe a large number of other permissible values for such things as deflections, crack widths, bar spacings, etc. There may be fabricational constraints which limit, for example, the bar areas to be not less than some available size. The designer may also add constraints himself if the beam has to fit some restricted location within a structure. Typically to fully describe the optimum design problem, between ten and twenty constraints may be needed in the seven variables and because of the functions involved in these constraints they are almost all highly non-linear ones.

Problems of this size and complexity can now be solved fairly rapidly by a variety of methods which will be outlined later in this paper. However, this example is merely a single beam element. Most real-world structures have many elements such as beams, columns, slabs, panels, etc., and to fully describe the optimum design problem for such real-world structures may require many hundreds of variables and constraints. The general characteristics of structural optimization problems are therefore that they are large, multivariate, non-linear, constrained problems.

Because problem size and complexity become enormous when all variables and constraints are lumped together in a single problem a hierarchy of problems has developed. The hierarchy is as follows:

1. Topology of the structure
2. Geometry of the structure
3. Overall sizes of structure members
4. Detailed design of elements

The logic and implications of this hierarchy can be demonstrated by reference to the beam example which falls into category 4 of the hierarchy as it is a detailed design of an element. Suppose this beam was one of a known number of beams supporting a deck slab longitudinally. By optimizing the cost of each beam in turn cost savings would accrue over a non-optimized design and may be considerable. However, the inquiring designer will ask himself whether the specified number of beams is itself optimal. If more longitudinal beams were used the load to be carried by each would be reduced and so the necessary size would also be reduced. The cost savings to be gained by using a larger number of smaller beams or a smaller number of larger beams might well be far greater than anything achieved by merely paring down upon detailed design sizes. This type of problem falls into category 3 of the hierarchy.

If the deck and beams form part of a bridge over several piers then the interested designer will soon find himself considering the cost savings to be made by varying the distances between piers to arrive at an optimum geometric arrangement in category 2 of the hierarchy. This promises the possibility of even greater cost economy. Finally category 1 which holds out the greatest savings of all, is concerned with topology. Was the decision to build a deck on concrete beams over supporting piers itself optimal? Would not an alternative structural form be more efficient? Why not use a steel box-section deck?

This example demonstrates the logic of the hierarchy fairly well and also demonstrates two general features of it. Starting from the lowest category 4, the higher up the hierarchy that optimization can be used the greater the potential for economy becomes but also the more difficult optimization becomes to implement. This second feature is reflected by the fact that very little work has been published on the optimum topologies of real-world structures, the work of Michell[2] being of theoretical interest rather than practical use. Indeed, the present state of the art of structural optimization is that a vast amount of research work has been published on methods for category 4 problems, and it can be stated that most problems in this category can be solved fairly rapidly. Much work has also been done in category 3 but success there has been less general and new techniques are required. The available literature in category 2 is fairly small but has significantly increased in the last two or three years. Much more work remains to be done in this category and in category 1 where significant practical literature is almost non-existent.

3. APPROXIMATION CONCEPTS

Very often approximations are used in the formulation of optimum design problems for two reasons. Firstly they can be used to hold problem sizes down to a level at which computer solution does not become inordinately expensive. Secondly they can be used to decompose a large design problem into a series of smaller problems. Another simple example demonstrates this. Consider the problem of the optimum elastic design of the beam/column framework of a multi-storey building such as that in Fig. 2. Each member of this framework may have up to ten detailed dimension variables and perhaps ten to twenty constraints. A complete optimum design problem for such a structure including both overall member sizes and detailed dimensions (i.e., categories 3 and 4) would therefore involve hundreds of variables and constraints and would be impossibly expensive in computation time.

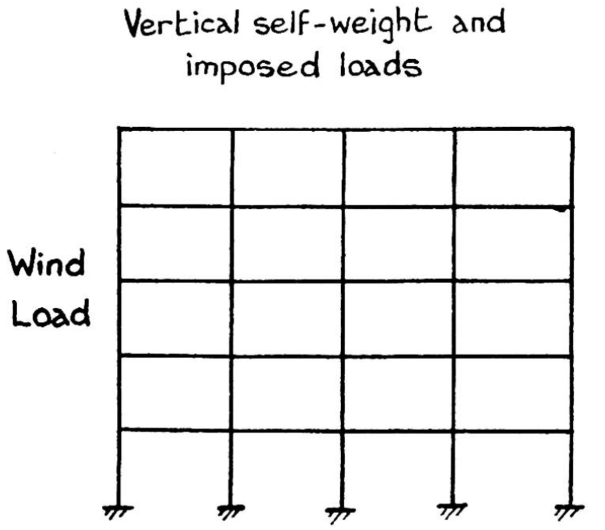

FIG. 2

Approximations can be used to effectively decompose or separate out the large problem into a single category 3 program and many simple category 4 problems. This is done by selecting for each beam or column element of the structure a single parameter and relating the cost, weight and behaviour of the element to this single parameter. In this particular example a suitable parameter might be stiffness, E I. Each element has thus been approximated and obviously in problems generally care must be taken to ensure that the approximate element really does behave as the real element does. The framework is then reassembled using approximate elements and a single category 3 problem may be formulated. This would consist of finding a complete set of optimal stiffness values for the framework which minimizes the cost of the framework while satisfying as constraints the equations of equilibrium and compatibility for all the applied load cases.

Having found the set of optimal stiffness values an analysis of the frame using these values determines all the beam and column moments and forces. Each element may then be designed separately as a category 4 problem so that its cost is minimized over all five to ten detailed variables. The loadings are those obtained in the analysis and the constraints would be the normal detailed constraints plus an additional one that the stiffness of the element must be equal to the value obtained for the approximate stiffness parameter in the overall size optimization.

Decomposition by means of approximations such as this is very widely used with considerable success. Frequently, in the aerospace industry very complex structures such as wing boxes, fin and tail structures, etc., are approximated as assemblies of membrane plates and shear panels for which rapid minimum weight design programs are available. One note of warning however; the last stage of any optimum design problem should always be a detailed analysis of the optimum design to ensure that approximations have not caused errors to be introduced. Indeed, on a more general basis it should perhaps be made clear that optimization is not intended to replace the designer. The objective is to provide him with information on what the most efficient solution to the problem posed might be. Only the designer can know whether the problem he posed is a complete one and whether it was representative of the real-world structure. Optimization can only suggest a possible design which the designer is then at liberty to modify or reject or accept after further analysis. There are always factors in the mind of a designer which he cannot quantify but which nevertheless influence his designs. In using structural optimization to assist him the designer should attempt to formulate a representative problem only. According to how much he feels has been put into the problem or omitted he alone will know how much reliance to place upon the suggested optimum design and also how much he must alter it to satisfy himself regarding the unquantifiable factors such as aesthetics. Structural optimization produces preliminary design information and should never be expected to produce a final design.

In this section on approximation concepts it is useful to comment further upon the choice of objective function to be extremized. In purely technical problems involving reliability, dynamic response, etc., the correct objective function is usually fairly obvious. In more general problems cost or weight usually provide a direct measure of the efficiency of a design. In the aerospace and shipbuilding fields weight is generally of paramount importance partly because it directly reflects an element of cost but more because it directly affects the operational efficiency of the vehicle. In civil engineering weight is perhaps not so important and cost seems to be the vital factor in design. Some structural design applications of optimization in the civil field are frequently criticized because weight has been used as an objective function or, if cost has been used then not enough of the elements of cost arising from fabrication, erection, labour, etc., have been used or the cost coefficients are invalid in an inflating economy. Sometimes these criticisms are valid but frequently the solution is relatively insensitive to variations in the objective function. What is important in constructing an objective function is to ensure that all the variable major elements are included with coefficients of the right order. If, in the future the price of steel was to reach the present price of gold then designs for steel structures would change only minimally since labour costs would likewise have escalated along with costs of all other materials. Cost objective functions are almost always approximations in themselves and as was stated earlier the important thing is to ensure that they are truly representative approximations.

To conclude this section on approximation concepts the occasional requirement that variables must have integer values or values picked from a discrete set is examined. These requirements considerably complicate the solution of problems. A useful way of handling them is first of all to ignore them and solve the problem using continuous variables. The solution of this problem will then be in the approximate area of the solution for integer or discrete variables in most cases. Pathological examples can be constructed mathematically in which the integer/discrete optimum is completely different from the continuous optimum but this does not occur in real-world structural optimization. Having found a continuous optimum integer programming may be used to find the integer/discrete optimum in that region, or, as is perhaps more appropriate, the designer in his analysis, checking and modification can round the continuous solution to suitable discrete values. This continuous approximation to discrete functions is most useful by virtue of the fact that it has reduced the area of search for a discrete optimum to a small region around the continuous optimum. Had this discrete search been made over the whole feasible design space it would be a very lengthy procedure.

In the field of steel civil engineering structures geometric similarity may be used to construct continuous functions representing the section properties of rolled steel sections which are available only in a range of discrete sizes. Consider, for example, the range of available wide flange beams. Select a characteristic cross-section parameter such as section area, A. The assumption is now made that all beams in the range of available sizes are geometrically similar in shape but differ only in scale. Then with this assumption it can be shown that the section modulus Z of a beam in this range is related to its area A by the relationship:

$$Z = C_1 A^{\frac{3}{2}} \tag{2}$$

Similarly the moment of inertia, I and the depth d are given by:

$$\left.\begin{aligned} I &= C_2 A^2 \\ d &= C_3 A^{\frac{1}{2}} \end{aligned}\right\} \tag{3}$$

and similar relationships hold for all section properties which may be related to a single characteristic parameter. The coefficients C_1, C_2, C_3, etc., are all constants for the particular range of sections and may be found by examining any discrete beam within the set.

This continuous approximation has essentially replaced the set of discrete beams by a single variable A to which all section properties are related by known functions (2),(3), etc. If variable A is used in an optimization problem the optimal value A^* will correspond very closely to perhaps two or three beams within the set from which a discrete member may be selected. The assumption of geometric similarity is not absolutely valid for all sets of available rolled sections but is sufficiently accurate for the purpose here.

Thus far this paper has examined some of the major concepts of structural optimization and some of the general techniques involved in problem formulation for real-world structures. Very often it is difficult to formulate a representative mathematical model for the design of some structures particularly in the architectural area where the component of subjectivity in design is far greater than objectivity. However, having formulated a problem in the general form of problem (I) it is necessary to be able to solve it, and so solution methods for structural optimization problems will now be considered.

4. SOLUTION METHODS IN STRUCTURAL OPTIMIZATION

It is important in any survey of methods of structural optimization to differentiate between mathematical and structural optimization techniques. Although problems of optimum structural design can be expressed in mathematical form (problem (I)), the ideal solution technique from the viewpoint of a structural engineer is quite different from what a mathematician would consider ideal. Basically the structural engineer is interested in the structure which the problem represents rather than the problem itself. He is interested in the results of the optimization rather than the means whereby they were obtained. Any optimization method for use in engineering problems must therefore be flexible enough to solve as wide a range of problems as possible - frequently problems will alter several times as new constraints and variables are introduced by the engineer to more accurately represent his real-world design problem. The method used should be robust in operation and reliable - the engineer wants useable results and is frustrated by a solution method which in operation is very sensitive to the mathematics of a problem. Any method should be comparatively easy to use and should require a minimum of pre-solution computer programming and preparation. The engineer is comparatively little bothered by the need to differentiate between global and local optimality of solutions since a great many structural optimization problems display very flat, plateau-like optima. Similarly, extreme accuracy of the solution is not necessary since the mathematical problem itself is only an approximation to a real-world structural design and the designer is aware that the optimum solution is only a guide for him and will probably require modification in accordance with factors not included in the mathematical formulation.

The mathematician, however, has a different viewpoint and is interested in the means whereby results are obtained. He is happy to develop a method which solves a very limited class of problems if it is efficient on those problems. Flexibility of a method is of comparatively little interest. Methods are often reported in the mathematical literature which have low reliability - they are efficient on some problems yet fail to produce results for purely mathematical reasons on problems which to all intents and purposes look very similar. Such methods are of little interest to an engineer who wants results all the time. By training, the mathematician concentrates upon such factors as solution accuracy, speed and accuracy of convergence to the solution, and differentiation between local and global optima.

It is not the intention to disparage the approach of the mathematician in developing optimization techniques. Without this work structural optimization would not be at its present stage of development. It should be stressed however that it is very necessary to examine all mathematical techniques carefully to determine whether they are suitable for the needs of the structural engineer. Very often the structural engineer uses relatively simple and crude techniques to solve problems, not out of ignorance of more sophisticated methods but because he can place reliance upon the results obtained. This point is often mis-understood but is an essential difference between mathematical and structural optimization.

The methods now described are those which, after careful examination and thorough testing by structural engineers, have been established as suitable for optimum structural design problems. The criticisms levelled at them are likewise based upon the performance of the methods on problems arising in the design of real-world structures. At this stage it is useful to restate the basic characteristics of most structural optimization problems which are that they are large problems with many variables and a large number of non-linear constraints.

4.1 Unconstrained Methods

It may seem odd to commence a survey of methods of structural optimization with methods for solving unconstrained problems since structural design problems almost always have a large number of constraints. However, many of the concepts of unconstrained methods are useful in constrained problems and also there are methods which transform problems having constraints to unconstrained problems. Unconstrained optimization can be formally expressed as:

$$\text{Minimize} \quad g_o\,(x_i) \qquad i = 1, \ldots, N \qquad (4)$$

It is not necessary to consider maximization separately since this can be effected by minimizing the negative of $g_o\,(x_i)$. The N variables x_i represent an N-dimensional infinite design space in which all values are feasible. In order to find the minimum value of the function g_o the classical theory of optimization examines all stationary points of g_o, i.e., solves the set of equations:

$$\frac{\partial}{\partial x_i}\, g_o\,(x_i) = 0 \qquad i = 1, \ldots, N \qquad (5)$$

The minimum must be one of the set of possible solutions of (5) and it can be found by substitution of all solutions of (5) into (4), the lowest result being chosen. However, there are very many circumstances under which this classical approach just does not work. Typically, it does not work if $g_o\,(x_i)$ is a non-analytic function or if some of the derivatives in (5) are discontinuous. The science of optimization theory stems from the very frequent failure of the classical approach to solve problems.

In the absence of a successful classical approach to problem (4) a logical method of finding the minimum value of g_o would seem to be to evaluate g_o at a series of trial sets of values of the variables and to numerically select the lowest result. This can be done by imposing a 'grid' over the design space and evaluating g_o numerically at each grid intersection point. However, if the likely range of each variable x_i is divided into equal divisions so as to give 10 trial values for each variable the total number of values of g_o which must be evaluated is 10^N. If there are many variables, i.e., if N is large, this number is very large and the method uses too much computer time. A random search in which trial points are selected according to a statistically random sequence is likewise inefficient because of the large number of trial evaluations of g_o which must be made even to locate a point near to the optimum.

A much better search strategy is to try to ensure that each trial evaluation of g_o is made according to a set of rules which give a good likelihood that g_o will be reduced. A vital concept in this context is that of the gradient of g_o. The gradient of g_o with respect to some variable x_i is simply the first partial derivative of g_o for x_i, i.e.,

$$\frac{\partial}{\partial x_i} g_o\ (x_i)$$

Obviously if the gradient of g_o for variable x_i is negative then if the value of x_i is increased g_o can be expected to decrease. If, at each new evaluation of $g_o\ (x_i)$ all the first partial derivatives can also be evaluated then a new trial point at which the value of g_o could be expected to decrease can easily be found by either increasing or decreasing each value of x_i according to the sign of its partial derivative. Very many numerical search strategies are based upon using gradient information to produce a new search direction in which new trial evaluations may be made with maximum likelihood that g_o will decrease.

The steepest gradient method is one such strategy. The N gradients of g_o at a particular trial point represent an N-dimensional plane which is exactly tangential to the surface of g_o at the trial point. The steepest gradient method finds that direction upon the tangent plane in which the slope of the plane is maximum. Then by placing a new trial point somewhere along this direction in a decreasing sense g_o can be expected to decrease in value by more than if the trial were placed in any other direction. The steepest gradient method is a frequently used one in engineering design because of its reliability and its ease of implementation. However, though it always finds a minimum it can often converge very slowly. This is because the tangent plane is essentially a linear approximation to the surface of g_o which is exact only at the trial point. Once a new trial point is selected, even in the direction of the steepest gradient, at some distance from the original trial point the non-linearity of g_o may render the tangent plane approximation inaccurate, leading to very slow convergence.

For this reason directions other than the steepest gradient direction are often used to form the basis of a search procedure. Methods based upon conjugate directions are typical of these. Within the scope of this paper it is not possible to examine these methods in detail but their objectives should be stressed. The purpose of such methods is to improve the rate of convergence of methods like the steepest gradient which, although logical, is sometimes very slow.

All methods of search which used gradient information as well as trial evaluations are collectively termed first-order methods because they require first partial derivatives of g_o. In order to improve upon the efficiency of these methods many second-order methods have been proposed and used. These methods use the second partial derivatives of g_o, (i.e., information about the local curvature of g_o) in order to speed convergence. Once again space precludes further examination of these methods other than to comment that because they use more information about the local behaviour of g_o they are consequently more efficient. For a very readable account of many different methods for unconstrained minimization reference should be made to the work of Sargent[4].

Several comments should be made on the general applicability of zeroth-, first- and second-order methods to structural optimization. From the point of view of speed and efficiency second-order methods are obviously strong candidates. However, these methods require the prior and recurring evaluation of not only the function g_o but also all its first and second partial derivatives. This can be very time consuming. If g_o is a function of 50 variables x_i, $i = 1, \ldots 50$, then there is at each trial point a single function evaluation, 50 first partial derivative evaluations and the matrix of second partial derivatives requires the evaluation of 2500 elements. The computer time and space required for preparatory work and general housekeeping operations can be considerable in the implementation of second-order optimization methods. First-order methods also suffer from this criticism but less so. Carpenter and Smith[5] have compared the performance of zeroth-, first- and second-order methods on a selection of simple structural optimization problems by the SUMT method[8]. This comparative study is valuable as it brings out very clearly the advantages and the disadvantages of each method. They conclude that on the problems they examined the first-order method of Fletcher-Powell[6] was preferable for larger problems, Newton's second-order method for small analytic problems and the behaviour of Powell's method[7], a frequently used one, was generally poor. However, it is also fair to comment that although this sort of information is very useful it is inevitably problem-dependent. For some problems the first and second partial derivatives may be easy to obtain while for others they may be obtainable only by numerical difference techniques which can be very laborious. The choice of methods is therefore a complex one but, bearing in mind that Carpenter and Smith compared only three methods which have been available for at least ten years, their conclusions form a very useful guide.

4.2 Penalty Function Methods

The above section on unconstrained methods is necessary for an understanding of penalty function methods which solve a constrained problem by means of a sequence of unconstrained problems. Penalty function methods have been widely used in structural optimization and are among the more popular methods. Consider the equality-constrained problem:

$$\left.\begin{array}{lll} \text{Minimize} & g_o\,(x_i) & i = 1, \ldots, N \\ \text{Subject to} & g_j\,(x_i) = 0 & j = 1, \ldots, M \end{array}\right\} \qquad (6)$$

The penalty function approach replaces problem (6) by the unconstrained problem:

$$\text{Minimize } F = g_o\,(x_i) + \sum_{j=1}^{M} P_j \left[g_j\,(x_i) \right]^2 \qquad i = 1, \ldots, N \qquad (7)$$

in which values of P_j, $j = 1, \ldots, M$, are positive constants. The function F is therefore composed of the original objective function g_o plus the value of each constraint multiplied by a penalty factor P_j. Starting with some known set of factors P_j, F is minimized using unconstrained techniques. All values of P_j are then considerably increased and another unconstrained minimization of F is performed. This process continues for increasing values of the penalty factors and has the effect of forcing each of the constraint functions g_j towards the value zero. Thus as values of P_j are increased the results of the sequence of unconstrained minimizations of F tend towards the solution of problem (6).

Penalty function methods are also applicable to inequality constrained problems. If the constraints g_j in problem (6) are written as:

$$g_j\ (x_i) \leqslant 0 \qquad j = 1, \ldots, M \qquad (8)$$

a suitable interior penalty function problem is:

$$\text{Minimize } F = g_o\ (x_i) - P \sum_{j=1}^{M} [g_j\ (x_i)]^{-1} \qquad i = 1, \ldots, N \qquad (9)$$

where P is a positive penalty factor. Starting with a large value of the penalty factor P an unconstrained search is carried out from a feasible starting point, (i.e., values of x_i which do not violate any of the constraints - hence an interior or feasible point). The solution of this search cannot be at a point which causes any constraint value g_j to be zero otherwise F would be infinite. This time the effect of P is to keep the solution away from the constraint boundaries. Further unconstrained searches are carried out using a sequence of decreasing values of P thus the process can progressively approach any constraint boundary, where $g_j = 0$, if it wishes or can remain feasible, $g_j < 0$, if this is advantageous. The results of the sequence of problems thus converges to the minimum of g_o with constraints given by (8).

Much work has been published on penalty function methods and both interior and exterior methods (the search is always in the infeasible region) have been widely studied. A popular method is the SUMT method of Fiacco and McCormick[8] to which reference should be made. Lootsma[9] has given a comprehensive review of the topic and its use in structural optimization has been championed by Fox[10] and Moe[11] in particular.

The chief disadvantage of the approach is that although it converts constrained optimization problems to the much simpler unconstrained form, it requires a considerable amount of time to solve the unconstrained problem many times. Furthermore the composite unconstrained objective function F contains all the constraint functions g_j and since we have seen that structural optimization problems usually have many constraints F can be very large. In solution of the unconstrained minimization if first-order or second-order methods as described in section 4.1 are used, partial derivatives of all the constraint functions g_j must be evaluated since they appear in F. Thus considerable time is necessary for the evaluation of derivatives and for this reason the penalty function approach cannot really be deemed suitable for large structural optimization problems. It has been used very effectively on the detailed design of structural elements or components such as beams, plates, panels, etc., where the number of variables is perhaps a maximum of 15 and the number of constraints is of the same order. For more complex problems its efficiency can sometimes be rather poor.

4.3 Constrained Numerical Search

In section 4.1 methods of unconstrained numerical search were considered. Here the feasible region is infinite. When constraints are present, however, they limit the feasible region which is hedged around by constraints which must not be violated. The only way of knowing whether a particular trial point is or is not feasible is to evaluate all constraints at that point and check them for violation. If methods of gradient search are to be used it is likewise vital to know if a particular search direction points into an infeasible region. This too involves checking all constraints and the derivatives of constraints. It is obvious, therefore, that the presence of many non-linear constraints which is characteristic of structural optimization problems causes considerable difficulties for any numerical optimization method. The success and efficiency of any search technique depends upon the nature of the constraints.

The simplest case is where all constraints are linear equalities. Here as many variables may be eliminated (expressed in terms of other variables) as there are equality constraints and the problem is then reduced to an unconstrained one in a reduced number of variables. The linear equality constraints positively help the solution process. When the problem is constrained by linear inequalities slack variables may be added to convert the constraints to equalities and the solution can then be carried out as above by unconstrained methods. Linear constraints can therefore be handled quite easily and efficiently.

In structural optimization the constraints are usually and unfortunately non-linear inequalities. It is the author's opinion that no numerical search method for non-linear inequality constraints has yet been developed which can be advocated on grounds of reliability and efficiency as suitable for anything but the smallest structural optimization problems, i.e., about 5 - 10 variables and a similar number of constraints. It is almost always possible to solve problems more efficiently by the penalty function approach of section 4.2 or by the methods to be outlined in the following sections. Many methods have been proposed and they founder generally on the need for many trials and derivative evaluations. These are necessary because of the difficulty experienced in locating and following a non-linear boundary in N-dimensional space. The only direct search method for non-linear constraints which could possibly be an exception is that based upon the Simplex method of Nelder and Mead[12] which is a zeroth-order method making trial evaluations at the vertices of a N-dimensional regular figure which 'spins' through the feasible design space.

4.4 Linear Programming

Throughout this paper it has been emphasized that structural optimization problems are generally highly non-linear. There is, however, a major exception to this generalization which arises in the optimum plastic design of structures. Consider the beam/column framework of a multistorey building such as that shown in Fig. 2. The design of such a framework on a fully-plastic basis consists of finding a set of fully-plastic moments M_p for all members of the framework so that a prescribed factor of safety against collapse is achieved. The optimum design is one in which the set of M_p values also minimizes the weight or cost of the frame. For a frame of given layout member lengths are known and it is possible to approximate the cost function for the frame as a linear function of the M_p values of all the members. It should be noted that this is only an approximation but is a very reasonable one to make in order to solve the design problem which is in category 3 of the hierarchy described in section 2.

The constraints upon the problem are those of structural mechanics: It is necessary to ensure that in any possible collapse mode the work done by the factored applied loads does not exceed the energy capacity of the rotations at plastic hinges in the frame. This requirement leads to constraints in which linear functions of the M_p values are bounded above by a known set of constants. For completeness there should be one linear constraint for each possible collapse mode. In this problem, then, both the objective function and all the constraint functions are linear and linear programming may be used to find the optimal set of M_p values.

Optimum fully-plastic design has received much attention in the past since linear programming may often be used for which efficient and reliable computer package programs are available. However, the objective function is not truly linear and from a structural point of view the constraints are incomplete since several non-linear effects have to be omitted. For instance, elastic instability of the frame, the reduction of plastic moment capacity due to axial load, and change of geometry effects cannot be included. There are also considerable problems involved in ensuring that the actual collapse mode of the frame is present among the constraints. Much research effort has been devoted to overcoming some of these difficulties which is ample testimony to the ease of use and popularity of linear programming.

Linear programming deals with problems similar to problem (I) in which all functions g_j, $j = 0, \ldots, M$ are linear. Almost any textbook on optimization or operational research gives several solution techniques so no specific reference is made here. Several important features of linear programming can be mentioned, however. The first is that very large problems can be solved very efficiently. Many thousands of variables and constraints can be handled effectively. It is perhaps for this reason alone that linear programming is popular as a sequential method, (see section 4.5), for structural optimization. It is very worthwhile trying to force a problem into a linear format if possible because very large problems can then be solved. Secondly the duality theory of linear programming has the advantage that it gives insight into the nature of the problem which normally remains obscure. Linear duality may be summarized as follows:

Primal Problem

$$\text{Minimize } W = \sum_{i=1}^{N} C_i \, x_i$$

Subject to

$$\sum_{i=1}^{N} a_{ji} \, x_i \geq b_j \qquad j = 1, \ldots, M$$

$$x_i \geq 0 \qquad i = 1, \ldots, N$$

Dual Problem

$$\text{Maximize } Y = \sum_{j=1}^{M} b_j \, \lambda_j$$

Subject to

$$\sum_{j=1}^{M} a_{ji} \, \lambda_j \leq C_i \qquad i = 1, \ldots, N$$

$$\lambda_j \leq 0 \qquad j = 1, \ldots, M$$

Thus each primal linear programming problem in variables x_i has a dual problem in variables λ_j such that the solution of either one is exactly equivalent to solution of the other. A problem with few variables and many constraints has an equivalent dual problem with many variables but few constraints, and so the easier of the two problems may be solved.

Space precludes any further consideration of duality other than to mention that the physical interpretation of dual variables for a real-world structural design problem remains a very fertile area of research. Many authors[13,14] have examined linear duality for problems arising in fully-plastic design of structures and as a result have contributed greatly to an understanding of the basic behaviour of structures. As a final advantage of linear programming it should be noted that restrictions that variables take integer values or values from a discrete set can be accommodated by linear programming in a rigorous manner. This does not generally apply to other methods, apart from Dynamic programming.

4.5 Sequential Linear Programming

Because linear programming is an efficient and reliable technique which can solve very large problems it has been used widely in a sequential manner for the solution of non-linear problems. The way this is done is as follows. Consider problem (I) with all functions g_j, $j = 0, \ldots, M$ being non-linear. Take a feasible trial point $\bar{x}_i$, $i = 1, \ldots, N$, and evaluate at this point the values of all functions g_j and the values of all first partial derivatives of all functions g_j. Each of the functions g_j may then be replaced by a linear approximation which has the same value and gradients at the trial point. The approximating functions can be easily derived from a first-order Taylor series expansion about the trial point. The resulting approximate problem is now a linear programming one and may be solved as such. The result of this linear programming problem then gives a new trial point at which a new approximating linear problem can be constructed. The solution of problem (I) is then approached by means of a sequence of linear programs.

The sequential linear programming method can be criticised in several ways. Firstly, there is no proof that the sequence of linear approximations will converge to the optimum of problem (I). Conceptually, convergence is often assumed but if the degree of non-linearity in the original problem is high convergence of the sequence may be very slow. Also, a linear approximation to a non-linear problem may be very inaccurate and the results of each linear program may be highly infeasible for the original non-linear problem. In order to ameliorate this move limits are often used which add extra constraints to prevent the linear search from going too far into infeasible regions. This increases the number of cycles of iteration and has the effect of imposing convergence to a point which is not necessarily the optimum of problem (I). The use of linear programming implies that the optimum will always be found at a vertex of the linear constraints but this is not necessarily true of general non-linear problems.

However, despite these criticisms sequential linear programming remains a very popular method of structural optimization and although its performance sometimes leaves much to be desired it has been successfully used on a very wide range of large and complex structural design problems. Pope[15] has given a good description of sequential linear programming in AGARDograph 149 which is entitled 'Structural Design Applications of Mathematical Programming Techniques' and gives excellent background material in the area of structural optimization. A big advantage of the SLP method is that it can tackle large and complex problems unsuited to any other technique. It has frequently been linked to finite element analysis programs to give an iterative optimum design capability. In such cases considerable amounts of computer time are necessary and the final results are sometimes only approximate optima but this method is the only way in which complex problems can be solved. Examples in the field of structural design are many but two may be referred to as they typify the performance of the method on very complex problems[16,17].

4.6 Dynamic Programming

Dynamic programming solves a very special class of problems in which the objective is to extremize the performance of a serial system.

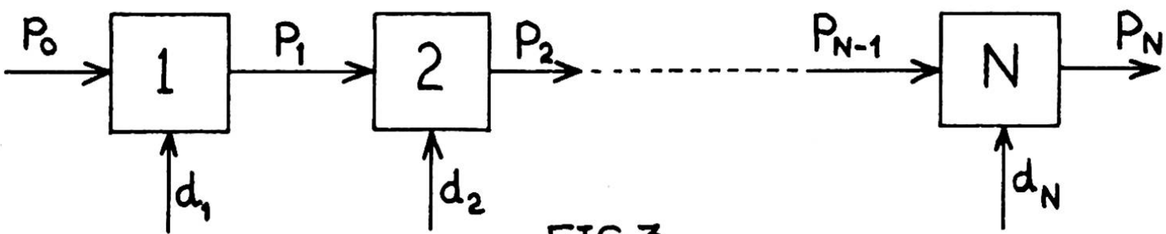

FIG.3

In Fig. 3 each box represents a stage in the serial system. Consider box I. Input to this stage is the value of the performance p_0. By giving different values to the decision variable d_1 the output performance p_1 of stage I is varied. This output p_1 of stage I is the input to stage 2 and the system performance at the end of stage 2, p_2, is modified from the input value p_1 by decision variable d_2. By the final stage N, the final performance criterion p_N is therefore a function of the initial performance p_0 and the N decision variables

d_1 ..., d_N. The objective is to find values for p_O and all the N decision variables which extremize the final performance p_N. Dynamic programming is a very efficient technique for solving such problems and was developed by Bellman[18].

The dynamic programming method itself is not studied here but some characteristics of the problem should be noted. First of all it is necessary to have a serial system such as shown in Fig. 3 in which decisions taken at a particular stage affect only performance after that stage and not before it. No 'looping-back' is possible. Although Fig. 3 shows only one decision variable at each stage it is possible, though more expensive in time, to have multiple decisions at each stage. The reason that dynamic programming is mentioned in connection with structural optimization is that it is very rapid and efficient in solving problems which fall into the suitable class. Unfortunately very few structural optimization problems have the necessary sequential decision characteristics. Palmer[19,20] has applied the method to optimizing the geometry of transmission towers and frameworks with some success. As a general comment upon the dynamic programming method for structural optimization it can be said that it works very well indeed on suitable problems but few structural problems are suitable.

4.7 Geometric Programming

Geometric programming is perhaps the most recently developed mathematical programming method to appear on the structural optimization scene. Like linear programming and dynamic programming, geometric programming solves a particular class of problems but unlike these other methods the class of suitable problems is quite large and many optimum structural design problems are suitable. Geometric programming in its simplest form solves the problem:

$$\left.\begin{array}{lll} \text{Minimize} & g_o\,(x_i) & i = 1, \ldots, N \\ \text{Subject to} & g_j\,(x_i) \leq 1 & j = 1, \ldots, M \\ & x_i \geq 0 & i = 1, \ldots, N \end{array}\right\} \quad (10)$$

with the restrictions on the mathematical form of the functions g_j that:

$$g_j\,(x_i) \equiv \sum_{t=1}^{T_o} C_{jt} \prod_{i=1}^{N} x^{a_{jti}} \qquad j = 0, \ldots, M \qquad (11)$$

$$\text{and } C_{jt} > 0 \qquad \text{for all } j, t \qquad (12)$$

The class of suitable problems is therefore governed by the requirement that all functions be <u>sums</u> of terms, each term involving <u>products</u> of variables raised to known powers. Generally all constraints are non-linear inequalities and such problems can be very difficult to solve directly. Geometric programming does not attempt direct solution but uses theorems of geometric duality, described by Duffin et al[21] to construct an equivalent dual problem. Using dual variables δ the geometric dual problem exactly equivalent to (10), (11) and 12) is:

$$\text{Maximize } V\,(\delta) = \prod_{j=0}^{M} \prod_{t=1}^{T_j} \left(\frac{C_{jt}\,\lambda_j}{\delta_{jt}}\right)^{\delta_{jt}}$$

$$\text{Subject to the constraints} \quad \sum_{t=1}^{T_o} \delta_{ot} = 1 \qquad (13)$$

$$\sum_{j=0}^{M} \sum_{t=1}^{T_j} a_{jti}\, \delta_{jt} = 0 \qquad i = 1, \ldots, N$$

in which

$$\lambda_o = 1$$

$$\lambda_j = \sum_{t=1}^{T_j} \delta_{jt} \qquad j = 1, \ldots, M$$

$$\delta_{jt} \geq 0 \qquad \text{all } j, t$$

The important feature of the geometric dual problem (13) is that the constraints are linear equalities. As was mentioned in section 4.3 linear equality constraints actually assist in maximizing V (δ) and methods of unconstrained search can be adapted to solve the dual problem numerically. The geometric programming method therefore uses duality theorems to convert problems of a difficult type to solve directly into problems with linear equality constraints which are much easier to solve. Since the appearance of Duffin, Peterson and Zener's book[21] in 1967 the method has been considerably extended and requirement (12) that all coefficients must be positive has now been removed in more recent versions of the method. Also constraints of reversed sign can be handled. Templeman and Winterbottom[22] have summarized these recent developments in a paper which describes a computer program for geometric programming and its application to structural design.

Advantages of the method are that it is a non-linear one which works effectively on highly non-linear problems. The class of suitable problems is clearly defined and is as easy to recognize as linear programming. A standard format of problem input is available which makes solution by means of computer package programs relatively easy. It has been shown that very many structural optimization problems are naturally suitable for solution by geometric programming. The detailed design of almost all types of structural elements (category 4 of the hierarchy described in section 2) can be expressed as geometric programming problems. Equations (2) and (3) show that the physical properties of a beam or column can be related to a single variable, A, raised to different powers. Terms such as these typify category 4 problems and they are in the form of (11) which is ideal for geometric programming. Templeman[23] gives several examples of the optimum design of structural components such as beams, corrugated plates and integrally-stiffened compression panels and also shows how geometric approximations may be made to more general problems in structural optimization. Non-standard problems can then be solved by a converging sequence of geometric programming problems in a conceptually similar way to sequential linear programming as described in section 4.5. This has the potential advantage that more representative non-linear approximations rather than linear ones are made to non-linear problems. A final advantage of the method is that the dual problem sheds new light on the optimum design and much insight can be gained by attempting to interpret physically the dual of a structural design problem. This point was made previously in connection with linear duality and it seems likely that research now being carried out in several centres into interpretation of primal/dual systems could in the future be of great benefit, leading to new structural optimization techniques and enhanced understanding of the design process and of structural behaviour.

Finally, the disadvantages of the geometric programming method should be mentioned. Firstly, it is the most difficult to understand of the methods described here. Secondly, if a computer package is not readily available to a prospective user of geometric programming then the effort involved in programming the method is very large. At present there are only a few suitable computer programs available. Thirdly the method is still relatively in its infancy, its use is not widespread and much of its great potential is still to be realized. Finally, the performance of the method on very large non-linear problems is dominated rather by computer storage and hence also by run-time. Factual evidence of this is as yet small but it may be that there is a fairly high limit upon the size of geometric programming problems which can be solved within normal economical limits.

5. OPTIMALITY CRITERION METHODS

All the methods described in section 4 attempt to solve structural optimization problems of the general form of problem (I) by mathematical and numerical search methods. The philosophy adopted is that nothing is assumed about the location or nature of the optimum. The optimum is reached by some purely numerical search which is based upon the mathematical form of problem (I) rather than the real-world structural design which problem (I) represents. The structural engineer, however, sometimes feels that he knows much more about the optimum structure than is present in its mathematical equivalent and that this knowledge might be useful in deriving a search method based upon structural principles rather than upon a mathematical abstraction. The term optimality criterion methods covers such approaches. Optimality criterion methods solve problems such as problem (I) by search methods so strictly this section could be numbered 4.8 and considered with all the other methods. The concept is so different, however, as to warrant a separation from the other methods.

Optimality criterion methods are absolutely problem-dependent and a particular criterion applies only to the optimum design of a particular type of structure under very specific conditions. The philosophy of the optimality criterion approach is first of all to investigate the nature of optimum structures of some specific type to try to establish a condition or set of conditions satisfied only by the optimum structure and which are not satisfied by any other, non-optimal design. For example such conditions might be that for a particular type of structure under restrictions on stresses and displacements the optimum structure always has a recognizable distribution of some form of energy among its components and that this distribution is peculiar only to the optimum design. Having found some structural criterion of optimality it is then necessary to devise some iterative algorithm which, starting with a non-optimal structure, will successively redesign the structure so that a structure which satisfies the optimality criterion and hence is optimal will be found.

For particular classes of structures the optimality criterion approach solves the mathematical problem (I) very indirectly by completely replacing the problem by an analogous one of iteratively redesigning a structure so that it satisfies some pre-established criterion of optimality. In a sense optimality criterion methods are dual methods in that problem (I) is solved by solving a completely different but equivalent 'dual' problem. The nature of this duality between mathematical programming and optimality criterion methods is almost totally unexplored but holds out considerable promise for further research work.

In developing an optimality criterion method for a particular class of structures there are two distinct phases. First of all a relevant and unique criterion of optimality for the class of structures must be found. This is generally very difficult to do since such criteria are rarely obvious. Had optimality criteria been obvious then mathematical programming would never have been necessary for optimum structural design. It is the requirement that the criterion be unique to the optimum structure which is perhaps most difficult to satisfy and indeed at the present time the technical literature holds examples of many so-called optimality criteria which are not optimal at all in that they can also be satisfied by non-optimal structures. The second phase is that of developing a recursion relationship which will produce an iterative redesign algorithm so that the optimality criterion can be satisfied. This too can frequently be difficult to develop and it runs the risk like any numerical search method of being unwieldy to operate or slow to converge. The optimality criterion approach is a logical one which has great appeal to structural engineers since it is based upon structural rather than mathematical principles. It holds great promise for the future although it can never entirely replace mathematical programming methods but, like some of the methods described earlier, it is still in its infancy and it still has to realize much of its potential.

At present very few rigorous, well-tested optimality criterion methods exist although many have been proposed and the range of optimum structural design problems which they cover is small. Generally they apply to overall sizing of structural members in a multimember system such as a truss or frame, i.e., category 3 problems in the hierarchy of section 2. Some authors, notably Prager[24], have developed optimality criteria for structural components such as beams, sandwich plates, etc. For such structures the optimality criterion generally is concerned with energy distributions in the parts of the structure. Prager has concluded that volume integral of energy density in each part is proportional to the volume of the part in many cases with single constraints. When multiple constraints are present the optimality criterion becomes more complicated but this energy distribution pattern is still an optimality criterion although in a modified form. Variations upon this energy-density optimality criterion have been proven to be applicable to truss structures, notably by Venkayya[25] and many others. Indeed many types of structures may be designed by optimality criteria methods with constraints upon stresses or displacements or dynamic stiffness, and the big potential advantage which the optimality criterion approach has is that very large problems can be designed this way, given a suitable criterion and algorithm, whereas mathematical programming methods are often suitable only for relatively smaller structures. A major difficulty is encountered when structures are to be designed subject to multiple constraints of different types. For example, a truss may be required to satisfy both stress constraints for which a criterion is available and multiple displacement constraints for which another criterion is known. Both criteria are not usually satisfied simultaneously and although a composite optimality criterion can be devised it is considerably more difficult to devise a rapid redesign algorithm and the solution process can be very slow.

There are therefore many difficulties associated with the optimality criterion approach and these are not always immediately obvious. The idea of developing structurally based rather than mathematically based optimization methods is, however, very appealing and holds out great hope for the future but much more work remains to be done in order to realize this potential.

6. CONCLUSIONS

Structural optimization is at present a thriving area of research and development. The philosophy is so very obviously right since the structural engineer has historically been guided by the need and desire to produce structures which are in some respect 'better' or more efficient than those which have gone before. The electronic computer has enabled design and optimum design to be put on a more formal and rigorous basis and is the means by which the goals of structural optimization may be achieved. However, it is fair comment to say that the actual methods of optimization at present available are not entirely adequate for very many of the structural optimization problems which surround us today. Mathematical programming methods can sometimes be laborious in operation and are often restricted to only small to medium sized problems. Some methods offer more promise for the future than others and mathematical programming is still very much alive. In the author's opinion better methods will be developed in the future perhaps based upon duality. Optimality criterion methods have great appeal to the structural engineer yet at present they too are in their infancy and require much more development.

In section 2 of this paper a hierarchy of structural optimization problems was discussed. Research up to the present time has tended to be concentrated mainly in categories 3 and 4 of this hierarchy - the easier problems. Categories 1 and 2 are as yet little-explored and the potential rewards offered for methods of solving such problems are very great. By about 1970 a watershed had been reached in structural optimization. Prior to this date research had concentrated upon marrying existing solution methods to structural design problems and it became evident that the marriage was only partially successful. Very many pressing problems remained to be solved. Since 1970 progress in structural optimization has been along new lines of approach. The simpler problems are now things of the past and only the harder ones remain. It is significant that interest in these harder problems of structural optimization is unabated. Over the last five years many new lines of approach have been opened up and although progress has often been slow the full potential of the new methods can now be clearly seen as a future goal. To achieve this potential is the object of structural optimization today.

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SUMMARY

The paper considers the economic objectives of structural optimization and shows that it has put design on a formal and rigorous basis. The size and type of structural optimization problems is then examined and a hierarchy of problem categories is discussed. Approximation methods are considered which enable complex structural problems to be posed concisely. Seven mathematical programming methods are described and critically discussed in the context of a specification for a suitable optimization technique for engineering use. Optimality criterion methods are then examined and the paper concludes that presently available structural optimization techniques have yet to achieve their full potential.

RESUME

L'article traite des buts économiques de l'optimisation structurale et montre que le dimensionnement est posé sur une base formelle et rigoureuse. Le type et l'importance des problèmes d'optimisation structurale sont examinés; une hiérarchie des catégories de problèmes est discutée. Des méthodes d'approximation permettent de poser des problèmes structuraux complexes de façon concise. Sept méthodes de programmation mathématique sont présentées et comparées dans le cadre de directives pour une technique d'optimisation appropriée à l'usage de l'ingénieur. Des critères de méthodes d'optimisation sont discutés. L'article conclut que les techniques actuelles d'optimisation structurale peuvent encore être améliorées.

ZUSAMMENFASSUNG

Der Beitrag behandelt die wirtschaftlichen Aspekte bei der Optimierung von Tragwerken und zeigt, dass die Bemessung auf einer formalen und strengen Grundlage beruht. Grösse und Typen baulicher Optimierungsprobleme werden untersucht und eine Rangordnung der Probleme diskutiert. Es werden Näherungsmethoden untersucht, welche gestatten, komplexe bauliche Probleme rasch zu lösen. Sieben mathematische Programmierungsmethoden werden beschrieben und hinsichtlich des Anwendungsbereiches kritisch verglichen. Sodann werden verschiedene Optimierungskriterien untersucht; der Beitrag schliesst mit der Feststellung, dass die gegenwärtig erzielte Optimierungstechnik ihre vollen Möglichkeiten noch nicht erreicht hat.