Voilement de coques coniques sous charge hydrostatique

IX

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Beulen von Kegelschalen unter hydrostatischer Belastung

Buckling of Hydrostatically Loaded Conical Shells

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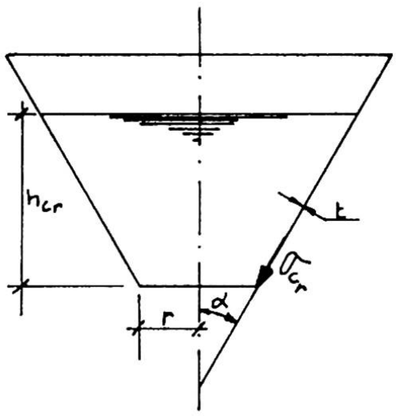
RESUME
Après l'effondrement par instabilité de deux châteaux d'eau en acier, le Laboratoire d'essais sur modèles de la R.U.G. a effectué un programme d'essais concernant la stabilité de coques tronconiques sollicitées par le poids d'un liquide. Les résultats finals sont présentés sous une forme pratique et directement utilisable par les auteurs de projet.

ZUSAMMENFASSUNG
Nach dem Einsturz von zwei stählernen Wassertürmen durch Instabilität ist im Laboratorium für Modellstatik der R.U.G. eine Forschung über die Stabilität abgestumpfter Kegelschalen, belastet durch Flüssigkeitsfüllung, durchgeführt worden. Die endgültigen Ergebnisse sind in einer für den Entwerfer verwendungsfähigen Form dargestellt.

SUMMARY
After the collapse of two steel watertowers due to instability a test program on the stability of truncated conical shells filled with a liquid was carried out at the Laboratory for Model Research of the R.U.G. The final results are presented in a practical form for use by designers.

A number of researchers have studied the stability of conical shells subjected to an axial compressive load which is constant along the axis of the cone and which may or may not act together with a uniform internal pressure.

We tested conical shells which were supported along their narrow base (fig. 1) and which we loaded by filling them with a liquid.



	MYLAR	BRASS	ALUMINIUM	STEEL
Number of models tested	554	69	36	78
t (mm)	0,125-0,250	0,20	0,20	0,31
α	10° 20° 30° 40° 50° 60° and 75°	20° 30° 40° 50° and 60°	20° 30° 40° 50° and 60°	20° 30° 40° 50° and 60°
r/(tcosα)	100 → 3600	200 → 1500	200 → 1500	200 → 1500

FIG. 1

Hence the meridional compressive stresses in the cone wall and the normal hydrostatic pressures on the wall increased with the height of the liquid.

Such a loading occurs in the bottom part of water towers. This part has often a conical form and is the most critical part of the reservoir. The compressive stresses cause the shell to buckle when the liquid reaches a certain critical height h_{cr} .

After the failure of two full size steel water towers, one of which is discussed in [1] and [2], we started a test program in order to determine the critical level of the liquid for a wide range of conical shells. A search of the literature accessible to us had shown that this particular problem had never been investigated systematically and that no reliable data were available about the actual failure load of thin cones loaded in the way described. Our test cones were made of mylar, brass, aluminium and steel. Figure 1 contains the geometrical parameters considered. The models were loaded with water.

Some cones were fully clamped at the bottom ; they gave slightly higher results. Others were elastically restrained against rotation of their lower edge around the base circle and still others were for all practical purposes simply supported.

The geometrical imperfections were measured with a special device and a distinction was made between "good" and "imperfect" cones. When the inward amplitude of the dents, measured from a straight rod of length ℓ_r , did nowhere exceed $0,008\ell_r$, the cone is said to be "good". ℓ_r is the full meridional wave length of the lowest buckle in the vicinity of the small base. ℓ_r was calculated with the BOSOR 4 computer program and turned out to be about $3,6\sqrt{rt}/\sqrt{\cos\alpha}$.

The first three writers reported earlier on these tests [3] and presented the results that were available at the time by plotting the meridional membrane stress σ_{cr}, derived from the experimental h_{cr} by means of the formula

$$\sigma_{cr} = \frac{\gamma h_{cr}(r + \frac{h_{cr}}{3} tg\alpha)\ h_{cr} tg\alpha}{2\ r\ t\cos\alpha} \qquad (1)$$

and made dimensionless by dividing it by a suitable quantity, versus $r/(t\cos\alpha)$.

In the meantime the test program has been completed and the final results are presented now in a more practical form.

The following parameters have to be considered :
E , α , r , t , h_{cr} or σ_{cr} (fig. 1) ,
ν : Poisson's ratio, and
γ : the specific weight of the liquid.

The similitude requirements provide us with the following complete set of independent dimensionless parameters which must normally appear in relations that should be valid for full scale structures as well as models :
$\frac{\sigma_{cr}}{E}$, $\frac{E}{\gamma r}$, $\frac{r}{t}$, α and ν . Any combination of these five parameters is also allowed. We know that E and ν appear in the combination $\frac{E}{1-\nu^2}$ for plate structures. Hence a relation of the kind

$$\frac{(1-\nu^2)\sigma_{cr}}{E} = f\left[\frac{E}{(1-\nu^2)\gamma r}\ ,\ \frac{r}{t}\ ,\ \alpha\right] \qquad (2)$$

presumably exists. We now rearrange equation (1) :

$$\frac{(1-\nu^2)\sigma_{cr}}{E} = \frac{(1-\nu^2)\gamma r}{E tg\alpha}\cdot\frac{h_{cr} tg\alpha}{2r}\left(\frac{r}{t\cos\alpha}+\frac{h_{cr} tg\alpha}{3r}\cdot\frac{r}{t\cos\alpha}\right)\frac{h_{cr} tg\alpha}{r} \qquad (3)$$

and we combine (2) and (3) :

$$f\left[\frac{E}{(1-\nu^2)\gamma r}\ ,\ \frac{r}{t}\ ,\ \alpha\right] = \frac{(1-\nu^2)\gamma r}{2 E tg\alpha}\left(\frac{r}{t\cos\alpha}+\frac{h_{cr} tg\alpha}{3r}\cdot\frac{r}{t\cos\alpha}\right)\left(\frac{h_{cr} tg\alpha}{r}\right)^2 \qquad (4)$$

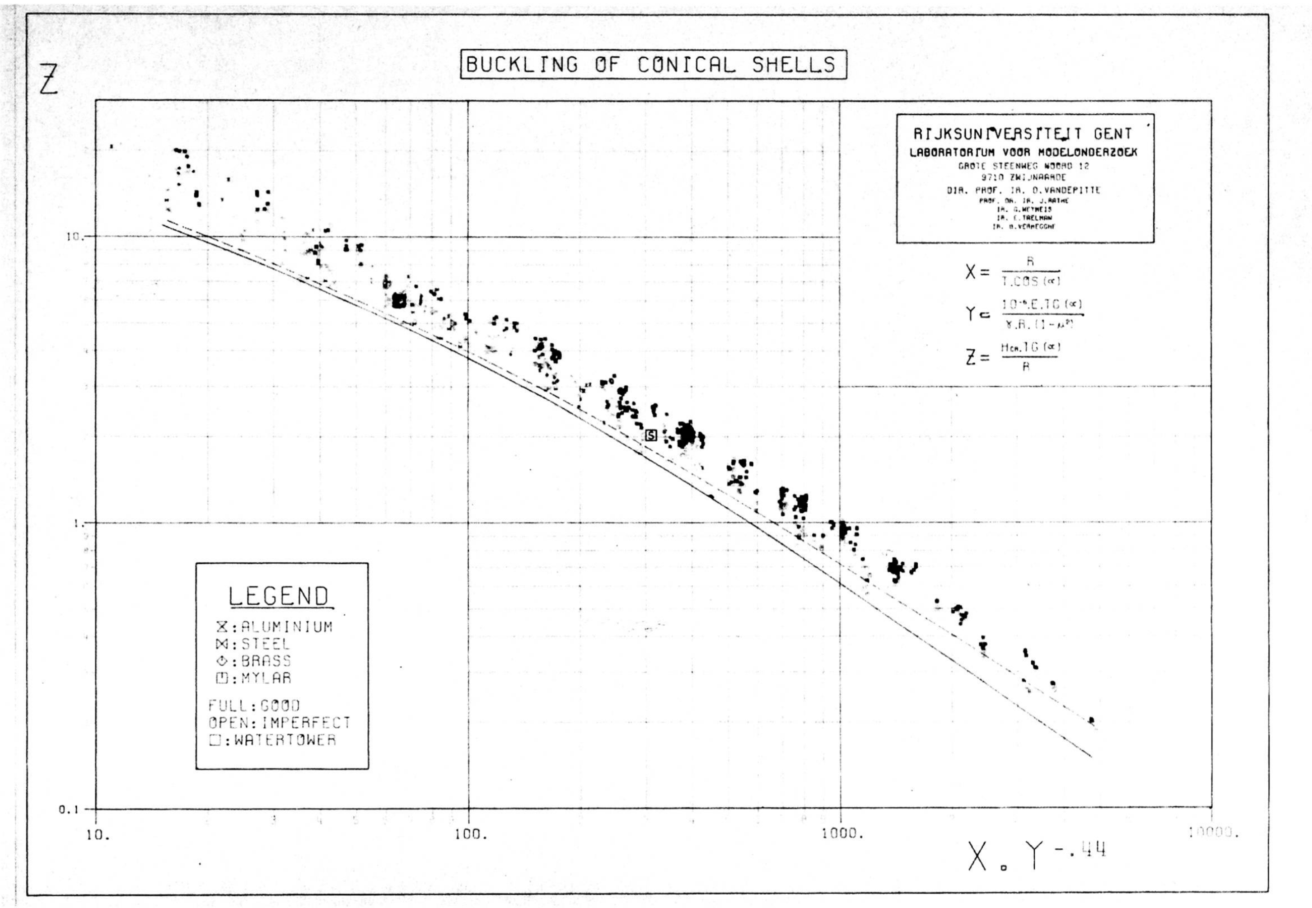
The three variables determining f appear only in the combinations, $\frac{r}{t\cos\alpha}$ and $\frac{E tg\alpha}{(1-\nu^2)\gamma r}$ in the second member of (4), and h_{cr} appears only in the combination $\frac{h_{cr} tg\alpha}{r}$. We conclude that for really perfect cones some relationship should exist between the 3 dimensionless parameters

$$\xi = \frac{r}{t\cos\alpha}\ ,\qquad \eta = 10^{-6}\ \frac{E tg\alpha}{(1-\nu^2)\gamma r}\ ,\qquad \zeta = \frac{h_{cr} tg\alpha}{r} \qquad (5)$$

We computed ξ , η and ζ for the some 700 model cones tested and first plotted the logarithms of the results in a three-dimensional coordinate system. We found that all the representative points nearly lie in the plane defined by the equation

$$\log\zeta = 0{,}75\ \log\xi\ -\ 0{,}33\ \log\eta\ +\ constant \qquad (6)$$

Then we projected the points on the plane containing the $\log\zeta$-axis and perpendicular on the plane defined by (6). This resulted in the two-dimensional representation of ζ versus $\xi.\eta^{-0,44}$


BUCKLING OF CONICAL SHELLS
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X = R / T.COS (α)
Y = 10⁻⁶.E.TG (α) / γ.R.(1 − μ²)
Z = Hcr.TG (α) / R
LEGEND
X : ALUMINIUM
⋈ : STEEL
Φ : BRASS
⊞ : MYLAR
FULL : GOOD
OPEN : IMPERFECT
□ : WATERTOWER
10.
1.
0.1
10.
100.
1000.
10000.
X . Y⁻·⁴⁴

shown in figure 2. This figure illustrates the relatively small scatter of the test data, small, that is, for buckling tests on shells. Note should be taken, however,of the fact that, owing to the representation of the ordinates to a logarithmic scale, the scatter is wider than one might infer from a glance at the diagram.

The lower bounds for the "good" and the "imperfect" shells are also drawn in the diagram. These curves are of practical interest for calculating the critical liquid level for actual conical shells that are loaded hydrostatically. Since all our models collapsed at membrane stresses which, roughly speaking, did not exceed half of the yield stress of the model material, there is no evidence that the results are valid in the elasto-plastic region. No safety margin is incorporated into the lower bound curves.

When only the lower part of a reservoir is conical, then it may be surmised that the critical liquid level is reached when the meridional membrane stress at the bottom becomes equal to the critical stress calculated by means of equation *(1)* from the critical liquid level given in figure 2 for a complete conical shell with the same characteristics.

The letter *S* in the diagram represents a collapsed full scale water tower.

CONCLUSION

The report deals with tests on 740 conical shells loaded with water. It was possible to draw lower bound curves for "good" and "imperfect" cones giving the critical height of the liquid, which causes the shell to buckle, as a function of shell geometry, material properties and specific weight of the liquid. Practical use of the lower bound curves by designers is straightforward. Buckling in the elasto-plastic region is not covered by the test results.

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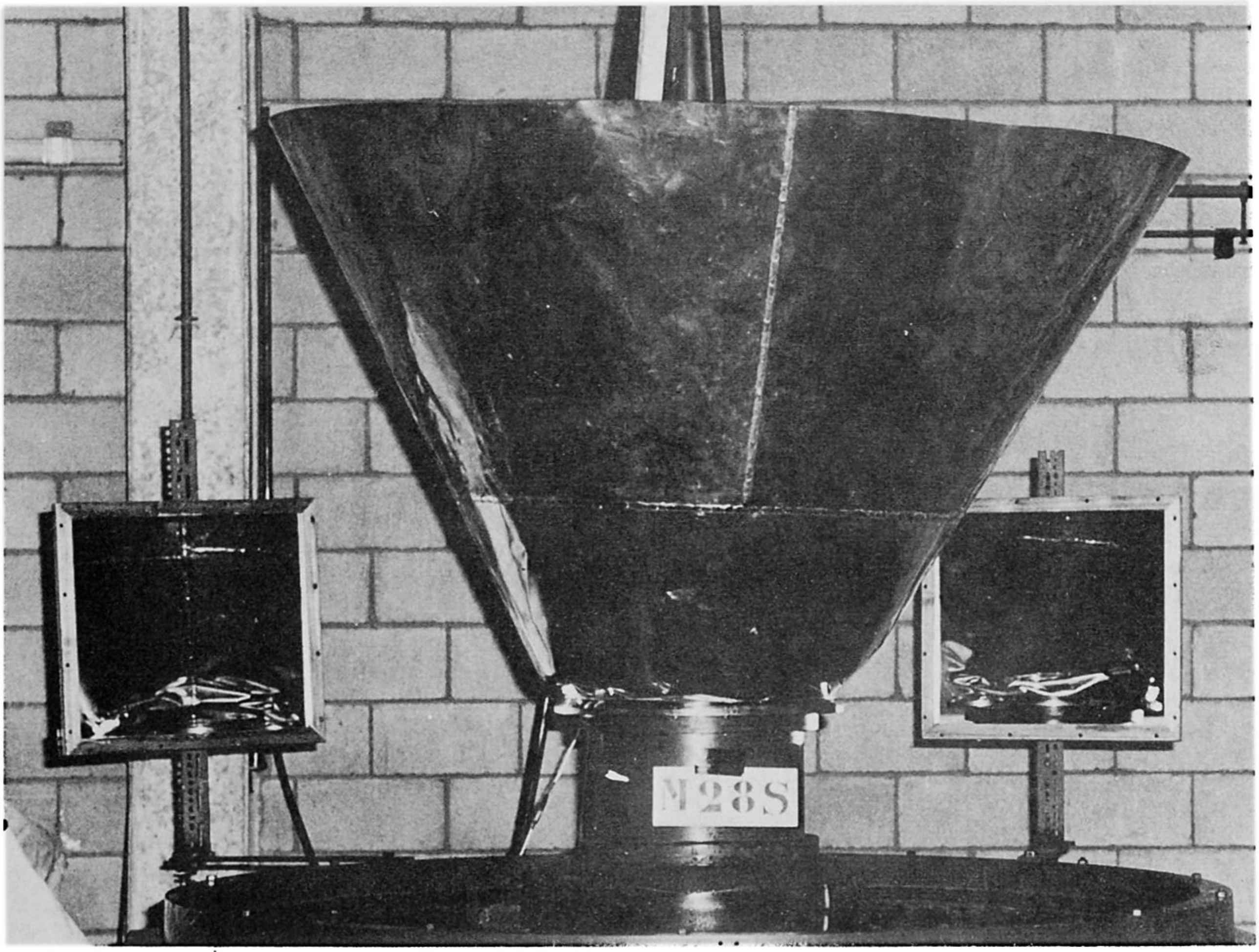

Fig. 3 - Photograph taken during the buckling failure of a brass model - The buckles at the back of the model are visible in the mirrors.