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II

The Stress Distributions of a Partial Uniform Load Applied to Timber

Répartition des tensions dans un élément en bois soumis à une charge répartie locale

Die Spannungsverteilung bei Lasteinleitungen in Holzbauteilen

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SUMMARY
Using Fourier integral, an exact solution for the stresses of isotropic and orthotropic half-infinite plane loaded by a partial uniform load is presented. Numerical results of the stress distributions are given for three layered plywood and laminated delta wood. The results can be applied to the analysis and design of timber connections.

RESUME
On applique la méthode des intégrales de Fourier au calcul de la répartition des tensions dans un demi-plan isotrope et orthotrope soumis à une charge répartie locale. On présente une solution théorique exacte, puis les résultats numériques obtenus dans le cas de bois croisé à 3 couches et de lamellé-collé. Ces résultats peuvent être utilisés pour le calcul et le dimensionnement des attaches d'éléments en bois.

ZUSAMMENFASSUNG
Mittels Fourier-Integrale werden für isotrope und orthotrope Halbebenen die Spannungsverteilungen für eine gleichmässige Teillast abgeleitet. Für dreischichtiges Sperrholz und für geschichtetes Delta-Holz werden numerische Werte angegeben. Die Ergebnisse können für die Analyse und für die Bemessung von Holzverbindungen verwendet werden.

1. INTRODUCTION

Since wood is assumed to be orthotropic, many theoretical paper, in the past, have been written on the subject of orthotropy. An introduction to be elasticity of anisotropic materials was covered in two classical books; one by A.E.H. Love(1), and the other by A.E. Green and Zerna(2). S.G Lekhnitskii's book(3), published in Russia in 1950 and translated into English in 1963, was devoted entirely to the problems of anisotropic bodies. Since then, many technical paper are written including some relating to the problem of concentrated load applied to orthotropic materials(4,5,6,7,8).

This paper presents an exact solution for the stresses of isotropic and orthotropic half-infinite plane loaded by an uniform load with a definite width. Fourier integral is introduced to solve the problem and the solution satisfies the equilibrium and compatibility equations.

The equations for the partial uniform load are derived independently for isotropic and orthotropic cases. The equations of orthotropy are degenerated into the expressions for isotropy when orthotropic constants are replaced by isotropic ones. The numerical results of isotropic case agree quite closely with simple results of other investigators.

The numerical results for orthotropy are evaluated for two kinds of wood and two different orientations of the grain. The types of wood considered are three layered plywood and laminated delta wood. The formal solutions are expressed in terms of closed form. The numerical results are shown in figures.

2. FORMULATION OF GOVERNING EQUATIONS

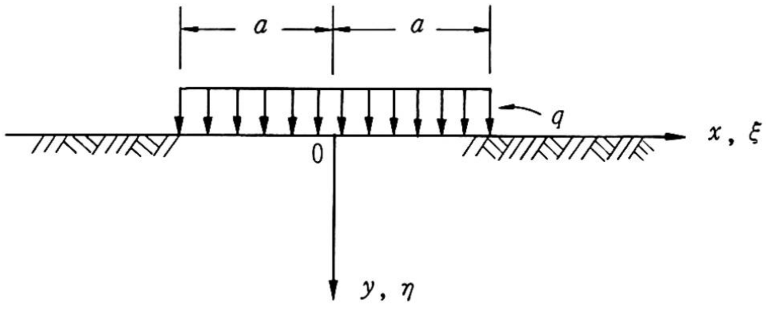

Fig. 1 Partial Uniform Load on an Half- infinite Orthotropic Material

Considering a unit thickness of a half-infinite orthotropic plane as shown in Fig. 1, the partial uniform load on $y=0$ can be expressed in terms of Fourier cosine integral of frequency β' as

$$f(x) = -\frac{2q}{\pi}\int_0^\infty \frac{1}{\beta'}\sin(a\beta')\cos(x\beta')\,d\beta' \tag{1}$$

where "a" represents a half of the loaded length. With the axes of coordinates taken along the principal axes of orthotropy, the governing equation for the plane problem of orthotropy, which is equivalent to the biharmonic equation of the place isotropy can be expressed as

$$\frac{\partial^4 \varphi}{\partial x^4} + (D_1^2 + D_2^2)\frac{\partial^4 \varphi}{\partial x^2 \partial y^2} + D_1^2 D_2^2 \frac{\partial^4 \varphi}{\partial y^4} = 0 \tag{2}$$

where

$$D_1^2 + D_2^2 = \frac{2C_{12} + C_{66}}{C_{22}} \quad , \quad D_1^2 D_2^2 = \frac{C_{11}}{C_{22}} \tag{2a}$$

the elastic constants can be expressed in terms of the moduli as

$$C_{11} = \frac{1}{E_x} \quad , \quad C_{22} = \frac{1}{E_y} \tag{3a}$$

$$C_{12} = -\frac{\nu_x}{E_y} = -\frac{\nu_y}{E_x} \quad , \quad C_{66} = \frac{1}{G} \tag{3b}$$

The substitution of the assumed Airy stress function

$$\varphi = \int_0^\infty f(y)\cos(x\beta')\,d\beta'$$

into Eq.(2) yields

$$f(y) = A_0 e^{my} + B_0 e^{ny} + C_0 e^{py} + D_0 e^{qy} \tag{4}$$

in which

$$m = -\frac{\beta'}{D_2} \quad , \quad n = -\frac{\beta'}{D_1}$$

$$p = \frac{\beta'}{D_2} \quad , \quad q = \frac{\beta'}{D_1} \tag{4a}$$

and A_0 and B_0 are orthotropic constants to be found from the boundary conditions at y=0, while the constants C_0 and D_0 are zero because φ should be bounded as $y \to \infty$. To non-dimensionalize x and y, the following substitutions can be made.

$$\xi = \frac{x}{a} \quad , \quad \eta = \frac{y}{a} \quad , \quad \beta = a\beta' \quad , \quad d\beta = a\,d\beta' \tag{5}$$

Then, φ can be expressed in terms of the coordinates ξ and η as

$$\varphi = -\frac{2qa^2}{\pi(D_2 - D_1)} \int_0^\infty \frac{\sin\beta}{\beta^3} \left[D_1 e^{-\frac{\beta}{D_1}\eta} - D_2 e^{-\frac{\beta}{D_2}\eta} \right] \cos(\xi\beta)\,d\beta \tag{6}$$

Performing integrations, the stresses are given by the closed forms as

$$\sigma_x = -\frac{q}{\pi(D_2 - D_1)} \left[\frac{1}{D_1} \left\{ \tan^{-1}\frac{D_1(\xi+1)}{\eta} - \tan^{-1}\frac{D_1(\xi-1)}{\eta} \right\} - \frac{1}{D_2} \left\{ \tan^{-1}\frac{D_2(\xi+1)}{\eta} - \tan^{-1}\frac{D_2(\xi-1)}{\eta} \right\} \right] \tag{7}$$

$$\sigma_y = -\frac{q}{\pi(D_2 - D_1)} \left[D_2 \left\{ \tan^{-1}\frac{D_2(\xi+1)}{\eta} - \tan^{-1}\frac{D_2(\xi-1)}{\eta} \right\} - D_1 \left\{ \tan^{-1}\frac{D_1(\xi+1)}{\eta} - \tan^{-1}\frac{D_1(\xi-1)}{\eta} \right\} \right] \tag{8}$$

$$\tau_{xy} = -\frac{q}{2\pi(D_2 - D_1)} \left[-\ln\left\{ \left(\frac{\eta}{D_2}\right)^2 + (\xi-1)^2 \right\} + \ln\left\{ \left(\frac{\eta}{D_2}\right)^2 + (\xi+1)^2 \right\} + \ln\left\{ \left(\frac{\eta}{D_1}\right)^2 + (\xi-1)^2 \right\} - \ln\left\{ \left(\frac{\eta}{D_1}\right)^2 + (\xi+1)^2 \right\} \right] \tag{9}$$

3. PARTICULAR CASE OF ISOTROPY

The general case of orthotropy has been discussed in Section 2. Since isotropy is a particular case of orthotropy, all the equations presented in Section 2 must be degenerated into the expressions for isotropy when orthotropic constants are replaced by isotropic ones. In other words, when D_1 and D_2 become unity, the orthotropic equations should be reduced to isotropic expressions.

When D_1 and D_2 are replaced by unity Eqs.(7), (8), and (9) become

$$\sigma_x = -\frac{q}{\pi}\left[\tan^{-1}\frac{(\xi+1)}{\eta} - \tan^{-1}\frac{(\xi-1)}{\eta} - \frac{\eta(\xi+1)}{\eta^2+(\xi+1)^2} + \frac{\eta(\xi-1)}{\eta^2+(\xi-1)^2}\right] \qquad (10)$$

$$\sigma_y = -\frac{q}{\pi}\left[\tan^{-1}\frac{(\xi+1)}{\eta} - \tan^{-1}\frac{(\xi-1)}{\eta} + \frac{\eta(\xi+1)}{\eta^2+(\xi+1)^2} - \frac{\eta(\xi-1)}{\eta^2+(\xi-1)^2}\right] \qquad (11)$$

$$\tau_{xy} = -\frac{q}{\pi}\left[\frac{\eta^2}{\eta^2+(\xi-1)^2} - \frac{\eta^2}{\eta^2+(\xi+1)}\right] \qquad (12)$$

Eqs.(10), (11), and (12) coincide with the equation which the author have obtained independently for the case of isotropy. It should be also noted that the stress values given by Eqs.(10), (11), and (12) check quite closely with the numerical results of another reference(9).

4. NUMERICAL RESULTS FOR TIMBER

The stresses given by Eqs.(7), (8), and (9) is good for any kind of timber which exhibit physical property of orthotropy. For the sake of comparison, numerical values are calculated for two different kinds of timber and two different orientations of the grain. The kinds of timber considered are three-layered plywood and laminated delta wood. Two different orientations of the grain are strong axis in the x direction and strong axis in the y direction.

Table 1 indicates the values of elastic constants and the values of D_1 and D_2 for each case studies. The values for elastic constants, E_x, E_y, G and ν_x are as given by Lekhnitskii(3). With these constants, the values of D_1 and D_2 are computed using Eq.(2a), (3a) and (3b). In the table, the grain of the surface layers of three-layered plywood and laminated delta wood are assumed to be paralled to x axis in the case of x-strong axis.

The distribution of two normal stresses and shearing stress are shown in Fig. 2, Fig. 3 and Fig. 4 for one orientation (x-strong axis) of three layered plywood. The same stresses are shown in Fig. 5, Fig. 6 and Fig. 7 for another orientation (y-strong axis) of laminated delta wood.

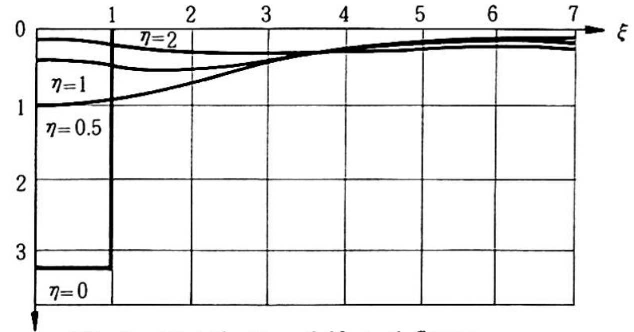

Fig. 2 Distribution of Normal Stress

$\sigma_x = (-\frac{q}{\pi})Q$

For $D_1 = 0.25$, $D_2 = 3.00$

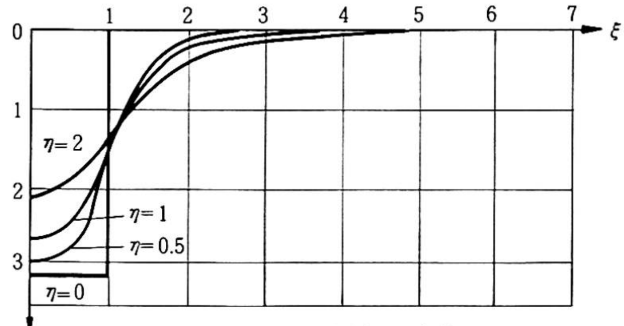

Fig. 3 Distribution of Normal Stress

$\sigma_y = (-\frac{q}{\pi})Q$

For $D_1 = 0.25$, $D_2 = 3.00$

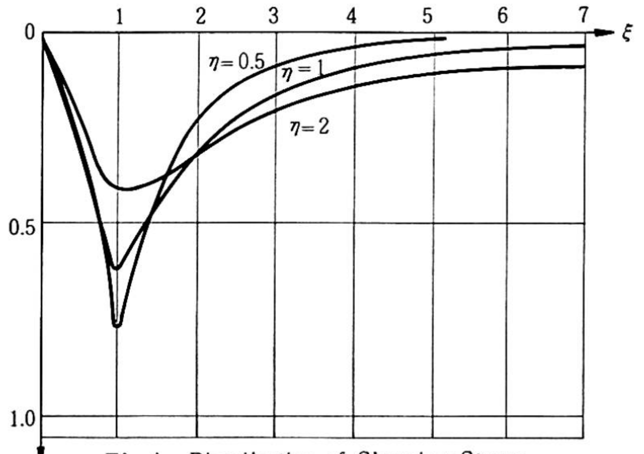

Fig. 4 Distribution of Shearing Stress

$\tau_{xy} = (-\frac{q}{\pi})Q$

For $D_1 = 0.250$, $D_2 = 3.00$

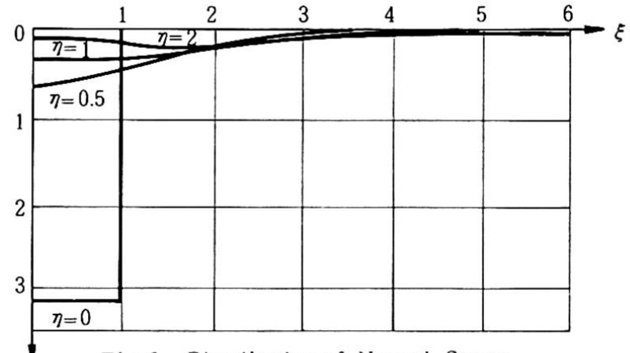

Fig. 5 Distribution of Normal Stress

$\sigma_x = (-\frac{q}{\pi})Q$

For $D_1 = 0.710$, $D_2 = 3.620$

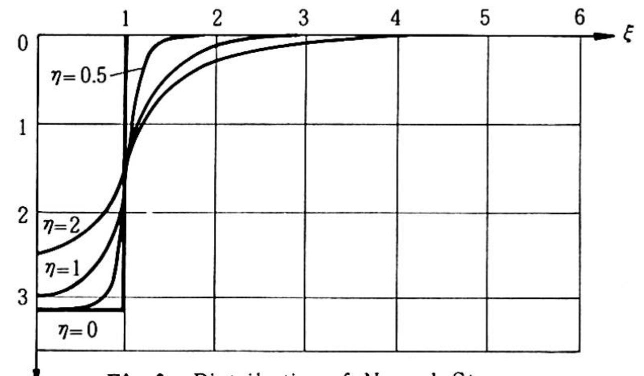

Fig. 6 Distribution of Normal Stress

$\sigma_y = (-\frac{q}{\pi})Q$

For $D_1 = 0.710$, $D_2 = 3.620$

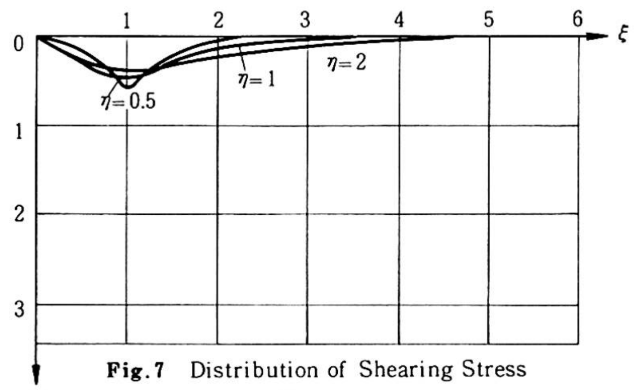

Fig. 7 Distribution of Shearing Stress

$\tau_{xy} = (-\frac{q}{\pi})Q$

For $D_1 = 0.710$, $D_2 = 3.620$

Table 1. Elastic and Orthotropic Constants

Type of Wood		Elastic Constants				Orthotropic Constants	
		E_x x 10^{-6} psi	E_y x 10^{-6} psi	ν_x	G x 10^{-6} psi	D_1	D_2
Three-layered plywood	x-strong axis	1.71	0.85	0.036	0.1	0.25	3.0
	y-strong axis	0.85	1.71	0.07	0.1	0.34	4.12
Laminated delta wood	x-strong axis	4.3	0.67	0.02	0.31	0.872	1.415
	y-strong axis	0.67	4.3	0.031	0.31	0.71	3.62

5. CONCLUSION

An analytical solution for the stresses of isotropic and orthotropic materials under a partial uniform load is presented using Fourier integral and Airy stress function. The solution of orthotropy is reduced to the solution of isotropy when the orthotropic constants are replaced by isotropic ones. Numerical values are computed and reported in figures for the cases of isotropy, three layered plywood and laminated delta wood. Two orientations of grains are considered in the timber materials. The results can be applied to the analysis and design of timber connections.

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