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SLAB DESIGN BY MEANS OF A LOADING CURVE

CALCUL DES DALLES AU MOYEN D'UN DIAGRAMME DE CHARGE

PLATTENBERECHNUNG MIT HILFE EINES BELASTUNGSDIAGRAMMES

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The Standard Loading for Highway Bridges, which has been the minimum loading recognised by the Ministry of Transport for highway bridges in Great Britain since June, 1922, is shown in fig. 1. The average intensity of traffic on British highways is the heaviest in the world, and accordingly our standard loading is necessarily heavier than the specified bridge design loading of any other country.

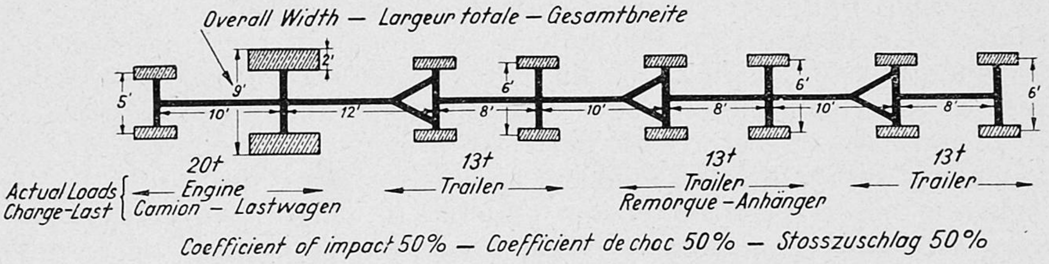


Fig. 1. (1 Ton = 1016,0 Kg. 1 Foot = 0.3048 M.)

The Bridge shall be assumed to be loaded with such Standard Trains or parts of Standard Trains as will produce the maximum stress in any bridge member provided that in any line of trains there shall not be more than one engine per 75′—0″ of the span of the bridge, and each Standard Train shall occupy a width of 10′—0″. Where the width of the carriageway exceeds a multiple of 10′—0″, such excess shall be assumed to be loaded with a fraction of the axle loads of a Standard Train. Te fraction to be used shall be the excess width in feet divided by ten.

It will be seen that the loading is made up of trains each consisting of a tractor and three trailers and weighing 90 tons, inclusive of impact at a flat rate of 50 %. For the design of any particular bridge member the carriageway is assumed to be loaded with these trains (or parts of trains) in such a way as to produce the maximum stress in the member under consideration.

Such a loading can only represent a type load and cannot, if strictly adhered to, cover all possible cases of loading that may be met with. The same average loading per square foot may result from vehicles of very different types and having wheels quite differently disposed, and the ideal bridge may be said to be one designed in such a way as to be strong enough to carry all vehicles of reasonable design having the same average load per square foot, no matter what the spacing and number of wheels. Even this requirement may not be quite sufficient as specially heavy single vehicles carrying girders or machinery parts are frequently met with.

A bridge designed strictly for the wheel and axle spacing of the Standard Train might possibly in some of its members be overstressed by vehicles of the same aggregate load but having wheels differing in number or differently arranged, or again by a heavy single vehicle such as has just been mentioned. Several wheels might be spaced closely together on one axle so that with longitudinal spans little relief to the main slab moments due to transverse bending would be experienced; or they might follow each other at short intervals, as when two rear axles are arranged only 3 or 4 feet apart, a common condition with modern six-wheelers. In this latter case with transverse slab spans the main moments would be but little relieved by longitudinal moments.

Another important factor is that of impact. In the case of a railway bridge the impact can be calculated, as it depends on the natural frequency of the bridge, the weight and arrangement of the train, the hammerblow of the locomotive, and the position of the track or tracks on which the train moves. All these are susceptible of calculation and on any well maintained railway the condition of the track itself will introduce no unknown factor of any magnitude. Moreover the number of tracks is not generally greater than two, and the tracks are definitely fixed in position.

With a road bridge the conditions are totally different. The frequency of the bridge, though it can be found by experiment, cannot so easily be calculated as it depends on the moment of inertia of the cross section and on the modulus of elasticity of the bridge material. Road bridges if not entirely of concrete generally have a considerable amount of concrete in the deck. E and I are therefore not perfectly definite, since the effective value of E depends on yield and shrinkage and that of I on the amount in which the concrete has cracked in tension. So far as weight and arrangement of vehicles are concerned there is much more variation than on a railway, hammerblow due to unbalanced moving parts is practically non existent, and there are no defined tracks on which the vehicles run, while the total width of the bridge covered by vehicles is usually greater than with a railway bridge.

There remains the condition of the roadway surface and such highway bridge impact as exists is in the main due to this factor. A corrugated surface or sett paving may produce some resonance akin to that due to the hammerblow of a locomotive, particularly on narrow bridges, but for short spans the worst effects are likely to be due to local irregularities in the surface or to a vehicle passing over a small obstruction. For such cases a steel tyred vehicle without springs may possibly on a short span produce an impact factor considerably in excess of 50 %. The modern tendency is however, towards the universal use of solid rubber or pneumatic tyres and for such tyres so high an impact factor is unlikely with heavy and slow moving vehicles.

The problem of an impact allowance for highway bridges is scarcely therefore one susceptible of any mathematical or even approximate empirical solution applicable to any special cases, and the flat rate of 50 % included in the loads of the Standard Train will not seem inappropriate for moderate spans though it would be reasonable to reduce this progressively as spans increase. The effect of resonance is likely to decrease as spans grow larger as will also that due to local irregularities or obstructions. Sudden application of loads, as on end cross girders or end slabs, might justify a higher impact factor for these members.

In slab design we have also to consider the area of dispersion and area of distribution of the wheel loads. If the width of a wheel tyre is a and the contact length in the direction of travel is b the contact area will be $a\,b$. The load is usually assumed to be dispersed through road surfacing and filling down to the slab at some angle less than 45^{0} (Fig. 2). The actual pressure

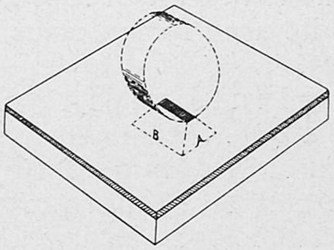


Fig. 2.

Pressure intensity — Répartition des pressions — Druckverteilung.
Slab — Dalle — Platte.
Fill — Remplissage — Auffüllung.

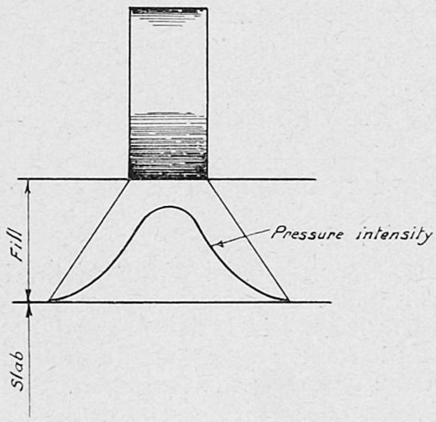


Fig. 3.

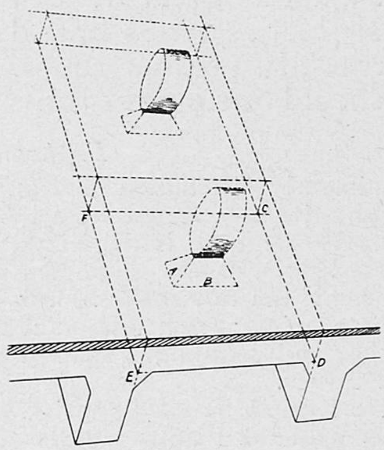


Fig. 4.

on the top of the slab within the dispersion pyramid planes will not be uniform but will vary somewhat as shown in Fig. 3, with the maximum pressure under the centre of the wheel. Some dispersion through the slab itself is usually assumed, and the final area for purposes of design is taken as AB (Fig. 2).

Whether the slab is supported on all four sides or on two sides only bending moments both longitudinal and transverse will occur beneath the point load and elsewhere. The area over which these bending moments operate may be termed the "distribution area". Thus in the case of a slab supported on two sides only and loaded in the centre with a succession of equal wheel loads each having a d i s p e r s i o n area equal to AB the d i s t r i b u t i o n area due to o n e load may be said to be $CDEF$ (Fig. 4).

In the design of slabs Pigeauds method of dealing with thin plates has been increasingly used of late years, the method being modified somewhat to give a dispersion area as indicated below.

If a = width of the wheel tyre contact in the direction of the slab span.
b = width of the wheel tyre at right angles to the slab span.
d = depth of filling and surfacing above the slab.
D = slab thickness,

then the dispersion area equals $\sqrt{(a + 2d)^2 + D^2}$ in the direction of the span and $\sqrt{(b + 2d)^2 + D^2}$ at right angles to the span.

This dispersion is somewhat more severe than an angle of 45°.

When the dispersion area has thus been found standard graphs are employed to give the maximum longitudinal and transverse bending moments. When obtained these are free moments and for continuous slabs of uniform depth a reduction factor of .8 is applied in order to allow for continuity. The method can be used for all slabs supported along the edges whether on two or four sides.

Equivalent Loading Curve. It will be seen that for any method of applying a type loading such as the Ministry of Transport Standard Load to the design of bridges the following conditions should be satisfied:

1. The method should provide not only for the type load but for all other reasonable loads having a similar average weight per unit area; and also for any heavy single loads likely to be met with.
2. It should provide for impact.
3. It should apply to all span lengths.
4. It should, as far as possible, apply to all bridge members — slabs, girders, arches, suspension cables, etc.
5. It should be simple, easily understood and capable of easy application.

Explanation of Fig. 5:

The uniformly distributed load applicable to the "loaded length" of the bridge or member in question is selected from the curve or table.

The "loaded length" is the length of member loaded in order to produce the most severe stresses. In a freely supported span the "loaded length" would thus be a) for bending moment; the full span. b) for shear at the support; the full span. c) for shear at intermediate points; from this point to the farther support.

In arches and continuous spans the "loaded length" can be taken from the influence line curves.

The live load to be used consists of two items: 1. The uniformly distributed load which varies with the loaded length, and which represents the ordinary axle loads of the M. T. standard train, perfectly distributed. 2. An invariable knife edge load of 2,700 lbs. per foot of width applied at the section where it will, when combined with the uniformly distributed load, be most effective, *i. e.* in a freely supported span: a) for bending moment at midspan; at midspan point. b) for shear at the support; at the support. c) for shear at any section; at the section.

This knife edge load represents the excess in the M. T. standard train of the heavy axle over the other axles, this excess being undistributed (except laterally as already assumed).

In spans of less than 10 ft. (*i. e.* less than the axle spacing) the concentration serves to counteract the over-dispersion of the distributed load.

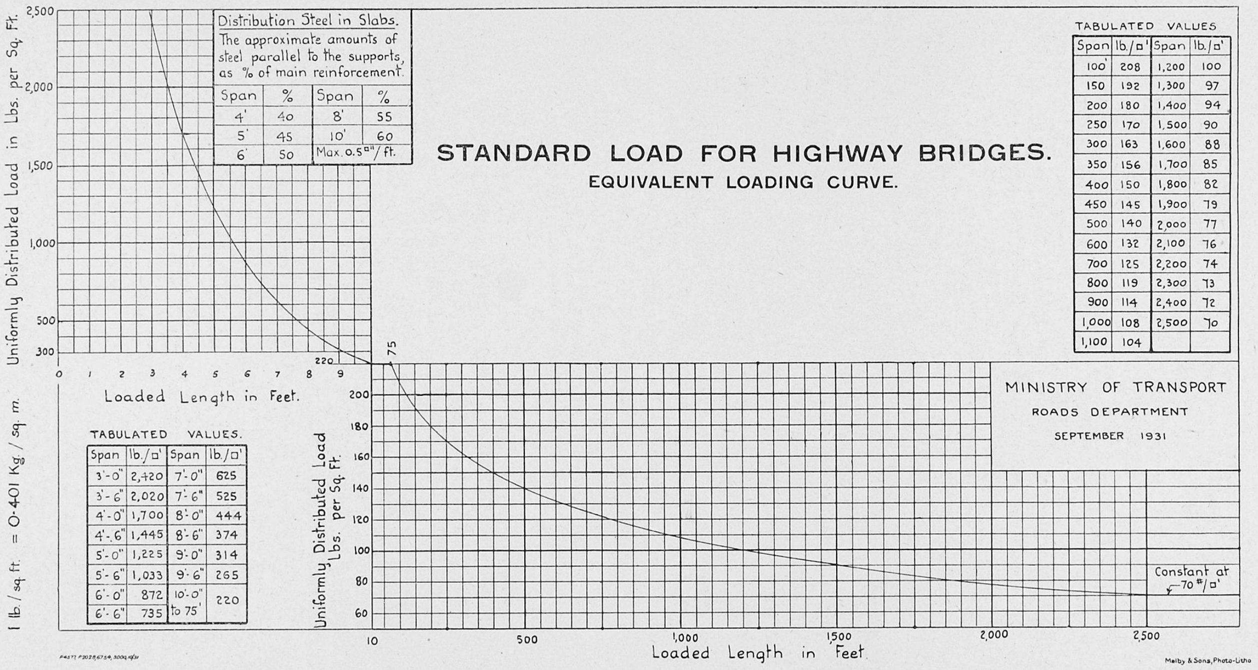
In slabs the knife edge load of 2,700 lbs. per ft. of width is taken as acting parallel to the supporting members, irrespective of the direction in which the slab spans.

In longitudinal girders, stringers, etc., this concentrated loading is taken as acting transversely to them (*i. e.* parallel with their supports).

In transverse beams the concentrated loading is taken as acting in line with them (*i. e.* 2,700 lbs. per ft. run of beam).

If longitudinal or transverse members are spaced more closely than at 5 ft. centres, the live load allocated to them shall be that calculated on a 5 ft. wide strip. With wider spacing this strip will be equal to the girder spacing.

In all cases, irrespective of span length, one knife edge load of 2,700 lbs. per foot of width is taken as acting in conjunction with the uniform distributed load appropriate to the span or "loaded length".



Distribution Steel in Slabs.
The approximate amounts of steel parallel to the supports, as % of main reinforcement.

Span	%	Span	%
4'	40	8'	55
5'	45	10'	60
6'	50	Max. 0.5□"/ft.	

TABULATED VALUES.

Span	lb./□'	Span	lb./□'
3'-0"	2,420	7'-0"	625
3'-6"	2,020	7'-6"	525
4'-0"	1,700	8'-0"	444
4'-6"	1,445	8'-6"	374
5'-0"	1,225	9'-0"	314
5'-6"	1,033	9'-6"	265
6'-0"	872	10'-0" to 75'	220
6'-6"	735		

TABULATED VALUES

Span	lb./□'	Span	lb./□'
100'	208	1,200	100
150	192	1,300	97
200	180	1,400	94
250	170	1,500	90
300	163	1,600	88
350	156	1,700	85
400	150	1,800	82
450	145	1,900	79
500	140	2,000	77
600	132	2,100	76
700	125	2,200	74
800	119	2,300	73
900	114	2,400	72
1,000	108	2,500	70
1,100	104		

1 Ft. = 0·3048 M.

Standard Load for Highway Bridges — Charges normales pour les ponts-routes — Belastungsnormen für Straßenbrücken.
Distribution steel in slabs — Armature de répartition dans les dalles — Verteilungseisen in den Platten.
Span — Portée — Stützweite.
Uniformly distributed load in lbs. per sq. ft. — Charge uniformément répartie en livres par pieds carré — Gleichmäßig verteilte Last in Pfund pro Quadratfuß.
Loaded length in feet — Longueur d'application de la charge en pieds — Belastungsländge in Fuß.
Fig. 5.

The Equivalent Loading Curve for the Ministry of Transport Standard Load for Highway Bridges (Fig. 5) may be said to satisfy all these requirements. It conforms to 1, 2 and 3. It meets 4 in that it is of general application to all members (with the exception of square or nearly square panels supported on all four sides — a type little met with in bridge design).

It also satisfies 5 and has the further advantage that it can readily be scaled up or down should circumstances so require. Since the Standard Load on which this curve is based is the minimum recognised loading in Great Britain specially heavy loading such as may occur in industrial districts must be designed for; but it should be remembered that since the curve covers the case of a 20-ton point load as well as the two 11-ton wheel loads of the Standard Train (see below) a bridge designed from the curve will carry safely a single vehicle of greater weight than the Standard Tractor, provided that the remainder of the carriageway is only lightly loaded.

The curve is reproduced in Fig. 5, together with a description of its method of application.

In the Standard Train the loading is distributed fairly evenly throughout its full length except for the excess which occurs at the heavy axle.

In applying the curve it is assumed that this uniform load is spread uniformly over the area under consideration, with the addition of a line or knife edge load equivalent to the excess load of the heavy axle. Applied to lengths between 10 feet (the average axle spacing) and 75 feet (the full train length) these loads[1]) are 220 lbs. per square foot and 2,700 lbs. per foot of width respectively. The latter is constant for all span lengths and occurs at only one position on the span. For long spans its effect is relatively unimportant and there is therefore no justification for any repetition. For lengths above 75 feet the uniform load is reduced in intensity as the greater the span the less the likelihood that the carriageway will be completely covered by Standard Trains. The impact factor also will decrease with increase of span.

For loaded lengths above 10 feet the local effect of the actual point loads is small and the Standard Load can be accurately represented by a knife edge load plus a uniform load calculated as just described. The curve above 10 feet allows in its net effect for some distribution of the point loads in the direction of the span, as though the knife edge load is undistributed the remainder is uniformly distributed.

Since this paper is more directly concerned with the design of slabs that part of the curve covering lengths below 10 feet will be of particular interest. For spans less than 10 feet account has to be taken of the fact that the full effect of the heaviest wheels will be experienced although the span is less than the average wheel spacing of 10 feet. The uniform load will therefore progressively increase as the span decreases for spans below 10 feet. For spans below this figure the local effect of the point loads must be taken more fully into account, and while the stresses are still calculated from the knife edge load and a uniform load taken from the curve this curve is in fact obtained from the consideration of actual point loads on a slab.

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[1]) Actually obtained as follows: Average axle spacing 10ft. Train width 10ft. Taking a 10 ton axle over these areas the unit load $= \frac{10 \times 2240}{10 \times 10} = 224$. Excess of heavy axle $=$ 12 tons. $\frac{12 \times 2240}{10} = 2688$. These figures are taken as 220 and 2,700 respectively.

Fig. 6 shows the maximum live load (free) bending moments in a slab spanning transversely for one 20-ton point load and for the two 11-ton wheel loads of the Standard Train, these latter being taken 12″ apart (clear). Fig. 7 gives the bending moments in a slab spanning longitudinally for the same

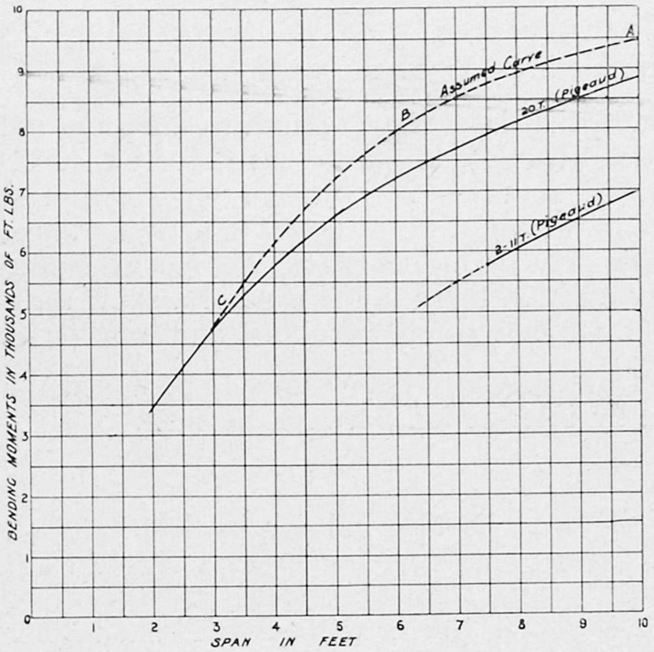

Fig. 6.

Free Bending Moments for Live Load — Moments fléchissants libres de la charge roulante — Freie Biegungsmomente für Verkehrslast.

Slabs spanning Transversely between Longitudinal Girders — Dalles transversales entre poutres longitudinales — In der Querrichtung gespannte Platten zwischen Längsträgern.

Bending moments in thousands of ft. lbs. — Moments fléchissants en 1000 pieds-livres — Biegungsmomente in 1000 Pfund-Fuß.

Span in feet — Portée en pieds — Stützweite in Fuß.

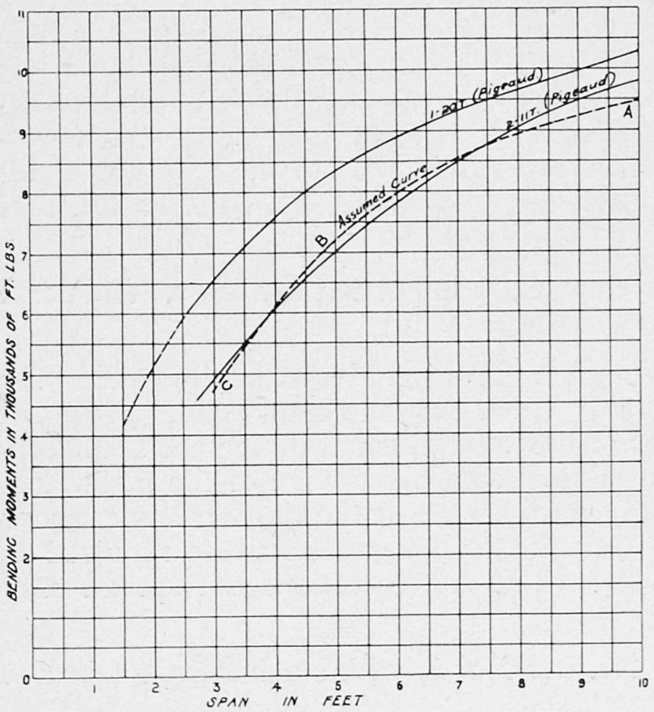

Fig. 7.

Free Bending Moments for Live Load — Moments fléchissants libres de la charge roulante — Freie Biegungsmomente für Verkehrslast.

Slabs spanning Longitudinally between Transverse Beams — Dalles longitudinales entre poutres transversales — In der Längsrichtung gespannte Platten zwischen Querträgern.

Bending moments in thousands of ft. lbs. — Moments fléchissants en 1000 pieds-livres — Biegungsmomente in 1000 Pfund-Fuß.

Span in feet — Portée en pieds — Stützweite in Fuß.

loads. In the calculation of the curves 3″ of tarmacadam on the top of the slab is assumed in each case and the slab thickness was based on a bending moment including dead load and reduced for continuity. On both figures will be seen the same dotted curve *ABC*. This curve is the one from which the equivalent loading curve (Fig. 5) is calculated for spans up to 10 feet.

For any such span the invariable undistributed knife edge load, together with the uniform load for the particular span taken from Fig. 5 will produce the bending moment for the same span obtained from the dotted curve in Figs. 6 or 7. It will be seen that this curve covers reasonably though not exactly

a) a 20-ton point load including impact,
b) the two 11-ton wheels, (including impact) of the Standard Train.

It also covers the case of a load equal in average weight to the Standard Train but having wheels differently disposed. A single curve for either direction of span has the advantage of simplicity. A further justification is that the point loads actually met with may not be disposed exactly as those of the Standard Train, that is, there may be more wheels on one axle or a number of axles close together. In addition there is the case of a skew span with a condition of loading intermediate between longitudinal and transverse.

In beam and slab construction the specially heavy load is provided for so far as the beams are concerned by inter-action between beams.

Thus requirement 1 is met.

So far as 2 is concerned 50 % impact is allowed for in the Standard Train and as it was not desired to alter the actual loads for a single train the curve was based on this figure for spans up to 75 feet, the length of one train. As has been mentioned impact is likely to be somewhat higher than this for floor systems and quite short spans and will decrease for greater spans. No increase above 50 % was actually allowed for spans below 10 feet, as slabs in practice have a somewhat greater factor of safety than beams designed to carry the same load, while in the case of beams the impact is likely to be damped to some extent before reaching the beam by transmission and dispersion through the deck. For spans above 75 feet a progressive reduction in impact was included for in the curve. This was quite empirical and was based on an impact factor of 15 % at a span of 400 feet and no impact at 2,500 feet.

These requirements 1 and 2 are specially important for slabs. The curve also satisfies 3, 4 and 5.

The equivalent loading curve given in Fig. 5 makes allowances for bending moments in both directions of which only the main bending moment need be calculated. The percentage of the main moment giving the secondary moment (or, actually the distribution steel as a percentage of the main steel obtained by Pigeauds method) is given in a table at the top left hand corner of the curve sheet. An increased amount of main steel will be required at unsupported ends of slabs where the wheel concentrations can spread only in one direction.

The knife edge load of 2,700 lbs. per foot is with slabs always taken at right angles to the slab span.

For a freely supported slab the live load bending moment is determined as follows, l being the effective span and taken as the clear span plus the effective depth of the slab, or the distance between centres of supports, whichever is less. The w appropriate to span is taken from the curve. For a 1 foot wide strip:

Bending moment due to uniformly distributed load

$$= \frac{w\,l^2}{8}$$

Bending moment due to knife edge load

$$= \frac{2{,}700}{4} \cdot l$$

Total bending moment = ______

To allow, where it applies, for continuity in the slab a reduction factor of .8 would be applied to the free bending moment at midspan and the same bending moment is assumed to operate at the support, provided that the slab is of uniform depth. Special factors to suit the particular case must be decided on if the depth of a continuous slab is greater over the supports than at centres of spans.

End spans should either be slightly reduced in length or additional reinforcement should be used.

Owing to the effect of punching shear slabs should not in general be less in thickness than 7″, or 6″ as a minimum in cases where it is specially necessary to keep down the dead load.

It will be seen that no variation is made in dispersion of loading for varying depths of fill.

With shallow fills little error is introduced by this and with deep fills the large amount of dead load makes a small amount of under-dispersion relatively unimportant. These considerations justify the gain in simplicity due to neglect of distribution of the loads through the filling.

Summary.

Starting with the British loading prescriptions for highway bridges, the author speaks first of all in general of the standard loading trains, and especially also of the question of the impact factor.

For the British loading train, a loading curve (fig. 5) has been prepared, allowing a certain equivalent load to be read off for any length of span. With this equivalent load the corresponding member of the structure can be calculated as a uniformly loaded beam. The (variable) impact allowance is already taken into consideration in the equivalent load.

The pressure distribution under single loads follows from PIGEAUD's formulae.

Résumé.

Partant des prescriptions anglaises pour les ponts-routes, l'auteur discute d'abord d'une façon générale l'importance des normes établies pour les trains de charge et plus particulièrement la question du coefficient de choc.

Pour le train de charge anglais, on a conçu un diagramme qui permet de déterminer, pour toutes sortes de portées usuelles, une certaine charge fictive, remplaçant la charge réelle. A l'aide de cette charge fictive, chaque partie de la construction peut être considérée comme poutre simple chargée uniformément. Dans cette charge fictive, le coefficient de choc est déjà compris.

La répartition des pressions sous les charges concentrées a été déterminée à l'aide des formules de PIGEAUD.

Zusammenfassung.

Ausgehend von den englischen Belastungsvorschriften für Straßenbrücken bespricht der Verfasser zunächst allgemein die Bedeutung der genormten Lastenzüge, insbesondere auch die Frage des Stoßkoeffizienten.

Für den englischen Lastenzug wurde ein Belastungsdiagramm (Fig. 5) konstruiert, das für alle möglichen Spannweiten eine gewisse Ersatzlast abzulesen gestattet. Das betreffende Konstruktionsglied kann dann wie ein mit dieser Ersatzlast gleichmäßig belasteter Balken gerechnet werden. Der (veränderliche) Stoßzuschlag ist in der Ersatzlast bereits berücksichtigt.

Die Druckverteilung unter den Einzellasten erfolgt nach den Formeln von PIGEAUD.