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Autor(en): **Young, D.H.**

Objekttyp: **Article**

Zeitschrift: **IABSE publications = Mémoires AIPC = IVBH Abhandlungen**

Band (Jahr): **2 (1933-1934)**

PDF erstellt am: **06.08.2024**

Persistenter Link: https://doi.org/10.5169/seals-3415

SHEARING STRESSES IN STEEL COLUMNS.

LE CISAILLEMENT DANS LES POTEAUX MÉTALLIQUES.

SCHUBBEANSPRUCHUNG IN STAHLSÄULEN.

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1. Introduction.

Steel columns used in structural engineering are frequently made of two or more channels laced together by diagonal bars or battens to form a composite member. Two common types of built up columns are shown in Fig. 1 a and b.

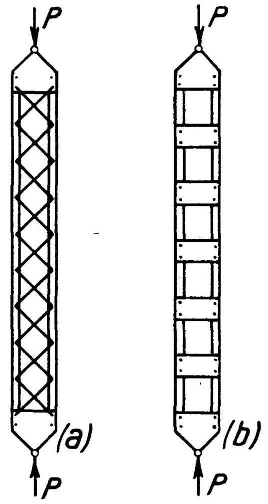

Fig. 1.

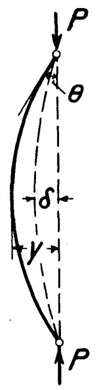

Fig. 2.

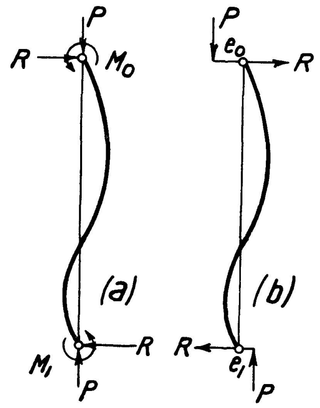

Fig. 3.

When the column deflects laterally under load, cross sections are no longer perpendicular to the line of force, and transverse shearing forces are introduced at each cross section. The diagonal bars or battens must be strong enough to resist these shearing forces and thus insure that the composite column will always act as a unit.

It is present day practice to design the lacing of such compression members by empirical rule. For example, the American Railway Engineering Association specifications require that the lacing or batten bars shall be designed to resist shearing forces not less than .025 times the total compressive force on the column, and further, that the spacing shall be such that the slenderness-ratio of the unsupported length of channel shall not be greater than 40. Such specifications are backed by a great deal of experience, and for usual proportions furnish a satisfactory basis for design.

It seems desirable, however, to have some theoretical basis for the design of these lacing bars which will take account of the dominant factors and be applicable over a large range of proportions and loading conditions. This paper is an attempt to evaluate theoretically the possible hsearing forces which may arise in compression members under various loading conditions.

2. Notations Used.

σ_{yp} = yield point stress of the steel
E = modulus of elasticity of the steel
A = cross-sectional area of the column
k = core radius of the cross section
r = radius of gyration of the cross section
I = moment of inertia of the cross section
l = length of the column
δ = center displacement of an initially curved column
e_0 = larger of two end eccentricities
e_1 = smaller of two end eccentricities
l/r = slenderness-ratio of column
e_0/k = eccentricity ratio
P = compressive load on the column
$P_e = \frac{\pi^2 EI}{l^2}$ = Euler load for the column
P/A = average compressive stress
V = transverse shearing force
V/A = average transverse shearing stress
x, y = coordinates

$$\alpha = \frac{e_1}{e_0}$$

$$q = \sqrt{\frac{P}{EI}}$$

$$\varphi = \frac{l}{r}\sqrt{\frac{P}{AE}}$$

$$\psi = \sqrt{\alpha^2 - 2\alpha\cos\varphi + 1}$$

$$\beta = 1 - \alpha\cos\varphi.$$

3. Stresses due to Bending and Thrust.

Since the shearing force which may arise at any cross section of a loaded column depends upon the deflected form of the column axis and the load on the column, it is necessary to consider briefly the general behavior of columns under load before attempting to evaluate the transverse shearing forces.

The chief factors which affect the behavior of columns under load are: imperfect elasticity of the material, initial crookedness of the axis, and non-central application of the load.

It has been shown that the effect of both initial crookedness and non-central application of the load on the behavior of a column can be well represented, either by some definite form of initial curvature or by some definite eccentricity of applied load[1]). When a column is taken with an initial curvature of the axis in the form of a half sine wave having at the center an initial displacement δ as shown in Fig. 2, there will always be a definite

[1]) See E. H. Salmon, "Columns", p. 26—32, Oxford Tech. Pub.

relation between the load and the maximum fiber stress due to combined bending and thrust which can be obtained from equation (6).

Defining the ultimate load for the column as that which first produces yielding in the most stressed fibers, the maximum fiber stress may now be set equal to the yield point stress, giving for the ultimate load on the column (to first produce yielding) the equation,

$$\frac{P}{A} = \frac{\sigma_{yp}}{1 + \frac{\delta}{k}\left(\frac{1}{1 - \frac{P}{P_e}}\right)} \tag{1}$$

It is probable that the extent of initial crookedness in a column will increase with the length of the column. Hence δ in equation (1) should be taken as some function of l. Taking, for example, $\delta = \frac{l}{400}$, it is possible to plot from equation (1) a curve showing values of $\frac{P}{A}$ against $\frac{l}{r}$ to always produce failure (by yielding). For a steel having $\sigma_{yp} = 40000$ lbs. per sq. in. such a curve is shown in Fig. 8 a. Thus if general imperfections are represented by an initial curvature the problem of determining the load to forst produce failure (by yielding) is completely solved by a curve like Fig. 8 a.

A large percentage of columns as used in structural engineering are found in rigid frame construction where definite secondary end moments arise due to the rigidity of the joints. Such members are loaded as shown in Fig. 3 a. For ordinary proportions both the axial load P and the end moments M_0 and M_1 are proportional to the external loads on the structure. This means that such loading as shown in Fig. 3 a can be considered equivalent to applying the loads P with eccentricities e_0 and e_1, as shown in Fig. 3 b, such that $P \times e_0 = M_0$ and $P \times e_1 = M_1$. An analysis of the case represented in Fig. 3 b, to determine the load first producing failure (by yielding) can now be made in the same manner as was just done for the initially curved column [2]). The shape of the elastic line of the bent column is given by equation (13). The numerically larger eccentricity is taken as e_0, and when the eccentricities are on opposite sides of the axis, e_1 must be considered as negative. Writing, as before, the relation between load and maximum fiber stress in the most stressed fiber, and setting this maximum stress equal to the yield point, gives for the ultimate load,

$$\frac{P}{A} = \frac{\sigma_{yp}}{1 + \frac{e_0}{k}} \tag{2a}$$

or

$$\frac{P}{A} = \frac{\sigma_{yp}}{1 + \frac{e_0}{k}(\psi \csc \varphi)} \tag{2b}$$

in which $\varphi = q\,l = \frac{l}{r}\sqrt{\frac{P}{AE}}$ and $\psi = \sqrt{\alpha^2 - 2\,\alpha \cos\varphi + 1}$, where $\alpha = \frac{e_1}{e_0}$

[2]) See writer's paper "Stresses in Eccentrically Loaded Steel Columns", Publications of the International Association of Bridge and Structural Engineering, Vol. 1, p. 507, Zürich, 1932.

Equations (2 a) holds as long as $\emptyset \leqq \cos^{-1}\alpha$, while for values of $\emptyset \leqq \cos^{-1}\alpha$ equation (2 b) applies[3]).

For a given value of α and various values of the ratio $\frac{e_0}{k}$ it is possible to plot from equations 2 (a) and (b) a family of curves showing $\frac{P}{A}$ as a function of $\frac{l}{r}$ to produce failure (by yielding). For a steel having $\sigma_{yp} = 40000$ lbs. per sq. in. and $\frac{e_0}{k}$ ranging from 0 to 1.0 (which includes the usual range of secondary stresses) these curves are shown in Fig. 8 b, c, d, e and f for $\alpha = 1.0$, 0.5, 0.0, -0.5 and -1.0 respectively. Thus the problem of determining the load to first produce failure (by yielding) due to any combination of end eccentricities of load, is completely solved by such curves as shown in these figures.

4. General Method of Procedure.

It will be logical to base the design of details, such as diagonal bars or battens, to resist the maximum shearing force which may arise at any cross

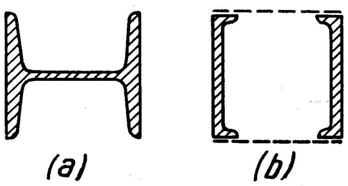

Fig. 4.

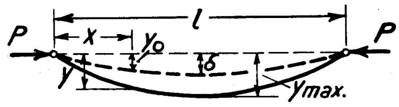

Fig. 5.

section, when the load is such as to first cause yielding in the most stressed fibers due to combined bending and thrust. In this way the strength of the column in shear will be consistent with its strength in bending and thrust.

Since in the development of equations (1) and (2) no account was taken of the additional lateral deflection due to shearing distortion of the cross section, the curves in Figs. 8 will not be rigorously applicable to columns of the built-up type. In general the effect of this additional flexibility (due to shearing distortion) will be, 1) to decrease the ultimate load for the column (below that given by the curves in Figs. 8), and 2) to increase the maximum angle of inclination of cross sections to the line of action of the force. These two factors will tend to cancel one another in evaluating the shearing force, and it seems justifiable to neglect such additional deformations when evaluating the shearing force on any cross section. Further, from a practical stand-point, most columns will be relatively short $\left(\frac{l}{r} < 80\right)$ and for such proportions the effect of deformation of the axis on the shearing force will be small. Consequently it will be on the safe side to evaluate the shearing force on the basis of the ultimate load as taken from the curves in Figs. 8.

In all cases a profile section of the type shown in Fig. 4 a, having extreme proportions such that the core radius k will be equal to the radius of gyration r, will be used. Such a type of cross section will closely approxi-

[3]) The l/r range of application of equations (2 a) and (b) is fully discussed in the writer's paper, Loc. cit.

mate to the cross section of the usual type of built-up member as shown in Fig. 4 b.

The general procedure to be followed throughout the paper may now be briefly stated: The shearing forces which may arise in columns under the loading conditions represented in Figs. 2 and 3 are to be evaluated. In all cases the problem will be treated for a column of such cross section as represented in Fig. 4 in which $k = r$ may be taken. In each case the maximum shearing force will be evaluated for the loading which first produces failure (by yielding) according to the curves in Fig. 8. Thus the lacing bars or battens will be designed to come to yielding due to shearing forces at the same time that the most stressed fibers come to yielding due to combined bending and thrust.

5. The Axially Loaded Pin-Ended Column with Initial Curvature.

Consider the pin-ended column shown in Fig. 5. The initial shape of the axis (without load) is represented by the equation,

$$y_0 = \delta \sin \frac{\pi x}{l} \tag{3}$$

Writing the relation between curvature and bending moment at any cross section gives,

$$EI\left(\frac{d^2 y}{dx^2} - \frac{d^2 y_0}{dx^2}\right) = -Py \tag{4}$$

Obtaining the value of $\frac{d^2 y_0}{dx^2}$ from equation (3) and letting the quantity $\frac{P}{EI} = q^2$ equation (4) becomes,

$$\frac{d^2 y}{dx^2} + q^2 y = -\frac{\pi^2}{l^2}\delta \sin \frac{\pi x}{l} \tag{5}$$

This represents the differential equation of the elastic line of the bent bar and its solution for the end conditions of Fig. 5 is

$$y = \frac{\delta \pi^2}{\pi^2 - q^2 l^2} \sin \frac{\pi x}{l} \tag{6}$$

The slope of the elastic line at any point will be,

$$\frac{dy}{dx} = \frac{\delta \pi^2}{\pi^2 - q^2 l^2} \cdot \frac{\pi}{l} \cos \frac{\pi x}{l} \tag{7}$$

The slope will be greatest at the end or when $x = 0$, and for this value of x equation (7) becomes,

$$\left(\frac{dy}{dx}\right)_{x=0} = \frac{\delta \pi}{l}\left(\frac{1}{1 - \frac{P}{P_e}}\right) \tag{8}$$

where $P_e = \frac{\pi^2 EI}{l^2}$ = the Euler load for the column. The maximum shearing force is [4])

$$V = P\left(\frac{dy}{dx}\right)_{x=0} \tag{9}$$

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[4]) For small angles $\sin \Theta = \tan \Theta = \frac{dy}{dx}$ may be taken.

Substituting the value of $\left(\frac{dx}{dy}\right)_{x=0}$ from equation (8) and dividing both sides of equation (9) by A gives,

$$\frac{V}{A} = \frac{P}{A}\frac{\delta\pi}{l}\left(\frac{1}{1-\frac{P}{P_e}}\right) \tag{10}$$

Equation (10) gives the value of the shearing force for any value of $\frac{P}{A}$ and δ. Since, however, $\frac{P}{A}$ is to be taken from the curve in Fig. 8 a, there is always a definite relation between $\frac{P}{A}$ and δ as given by equation (1). This equation for a fixed value of $\frac{l}{r}$ may be considered as representing this relation between $\frac{P}{A}$ and δ. Solving equation (1) for δ and substituting the value obtained into equation (10) gives,

$$\frac{V}{A} = \frac{\pi}{\frac{l}{k}}\left[\sigma_{yp} - \frac{P}{A}\right] \tag{11}$$

Equation (11) now gives the maximum value of the shearing force which can arise for the assumed curvature when $\frac{P}{A}$ is taken from the curve in Fig. 8 a. Remembering that for the cross section of a built-up column k may

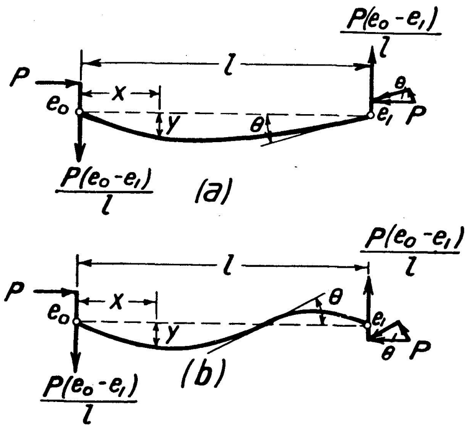

Fig. 6.

be taken equal to r, it is possible to plot a curve from equatin (11) showing the average shearing stress $\frac{V}{A}$ as a function of the slenderness-ratio $\frac{l}{r}$. Such a curve is shown in Fig. 9 a.

From an examination of this curve it is seen that the maximum shear occurs for columns having a slenderness-ratio of about 120, which is greater than the usual allowance for slenderness made by specifications. For columns of usual proportions the maximum average shearing stress varies from about 300 lbs. per sq. in. to 600 lbs. per sq. in. It will appear later that for short columns some accidental eccentricity of load at the ends may give rise to much larger shearing stresses, and that the details for short columns should be designed on a basis of such loading rather than initial curvature of the axis to represent the effect of imperfections.

6. The Eccentrically Loaded Pin-Ended Column.

Consider the general case of the eccentrically loaded column as shown in Fig. 6a and b. Writing the relationship between curvature and bending moment at any cross section gives,

$$EI\frac{d^2y}{dx^2} = -Py - Pe_0 + P\left(\frac{e_0 - e_1}{l}\right)x \tag{12}$$

The solution of this equation for the end conditions of Fig. 6 will be

$$y = e_0 \cos qx + e_1 \frac{\sin qx}{\sin ql} - e_0 \frac{\sin qx}{\tan ql} - e_0 + \frac{e_0}{l}x - \frac{e_1}{l}x \tag{13}$$

in which $q = \sqrt{\frac{P}{EI}}$ as before. The slope of the elastic line at any point is given by,

$$\frac{dy}{dx} = -qe_0 \sin qx + qe_1 \frac{\cos qx}{\sin ql} - qe_0 \frac{\cos qx}{\tan ql} + \frac{e_0}{l} - \frac{e_1}{l} \tag{14}$$

So long as e_0 and e_1 are on the same side of the axis ($\alpha = \frac{e_1}{e_0} > 0$) as represented in Fig. 6a the maximum shearing force will occur at the end of the column where $x = l$ and will be,

$$V = \frac{P(e_0 - e_1)}{l} + P\left(\frac{dy}{dx}\right)_{x=l} \tag{15}$$

Substituting $x = l$ into equation (14) gives for the absolute value of the slope,

$$\left(\frac{dy}{dx}\right)_{x=l} = \frac{e_0}{l}\left[\frac{ql(1 - \alpha \cos ql)}{\sin ql} + \alpha - 1\right] \tag{16}$$

Placing this value of $\left(\frac{dy}{dx}\right)_{x=l}$ into equation (15) gives,

$$V = \frac{Pe_0}{l}\left[\frac{ql}{\sin ql}(1 - \alpha \cos ql)\right] \tag{17}$$

Letting the quantity $(1 - \alpha \cos ql) = \beta$ and $ql = \frac{l}{r}\sqrt{\frac{P}{AE}} = \emptyset$, and dividing both sides of equation (17) by A gives,

$$\frac{V}{A} = \frac{P}{A} \cdot \frac{e_0}{l}[\emptyset \beta \csc \emptyset] \tag{18}$$

This equation is valid for all values of α from $+1.0$ to 0.0.

When e_0 and e_1 are on opposite sides of the axis ($\alpha = \frac{e_1}{e_0} < 0$), as shown in Fig. 6b, the maximum shearing force will occur at the inflection point along the column and will be,

$$V = \frac{P(e_0 - e_1)}{l} + P\left(\frac{dy}{dx}\right)_{max} \tag{19}$$

To find the maximum value of $\frac{dy}{dx}$, equation (14) may be differentiated with respect to x giving,

$$\frac{d^2y}{dx^2} = -q^2e_0 \cos qx - \frac{q^2e_1}{e_0}\frac{\sin qx}{\sin ql} + q^2e_0 \frac{\sin qx}{\tan ql} \tag{20}$$

Setting equation (20) equal to zero and solving for x gives,

$$\tan q x = \frac{\sin q l}{\cos q l - \alpha} \tag{21}$$

This equation locates the inflection point where the slope is a maximum and is seen to give $x = l$ when α is taken equal to zero.

Substituting the value of x from equation (21) into equation (14) and simplifying gives for the absolute value of the maximum slope,

$$\left(\frac{dy}{dx}\right)_{\max} = \frac{e_0}{l}\left[\frac{q l}{\sin q l}\sqrt{\alpha^2 - 2\alpha\cos q l + 1} + \alpha - 1\right] \tag{22}$$

Letting the quantity $\sqrt{\alpha^2 - 2\alpha\cos q l + 1} = \psi$ and $q l = \varnothing$ and substituting equation (22) into equation (19) and dividing both sides by A as before gives,

$$\frac{V}{A} = \frac{P}{A}\cdot\frac{e_0}{l}\,[\varnothing\,\psi\csc\varnothing] \tag{23}$$

This equation is valid for all values of α from 0 to -1.0.

Having α given, equation (18) or (23) will now give the maximum value of the average shearing stress for any given values of $\frac{P}{A}$ and e_0. Since, however, $\frac{P}{A}$ is always to be taken from the curves in Figs. 8, a definite relationship exists between $\frac{P}{A}$ and e_0 as given by equations (2). These equations for a fixed value of $\frac{l}{r}$ may be considered as representing this relation between $\frac{P}{A}$ and e_0. Solving equation (2a) for e_0 and substituting the value obtained into equations (18) and (23) gives,

$$\frac{V}{A} = \frac{1}{\frac{l}{k}}\left[\sigma_{yp} - \frac{P}{A}\right]\varnothing\,\beta\csc\varnothing \tag{18a}$$

for positive values of α and,

$$\frac{V}{A} = \frac{1}{\frac{l}{k}}\left[\sigma_{yp} - \frac{P}{A}\right]\varnothing\,\psi\csc\varnothing \tag{23a}$$

for negative values of α. Solving equation (2b) for e_0 and substituting the value obtained into equations (18) and (23) gives,

$$\frac{V}{A} = \frac{1}{\frac{l}{k}}\left[\sigma_{yp} - \frac{P}{A}\right]\frac{\varnothing\,\beta}{\psi} \tag{18b}$$

for positive values of α and,

$$\frac{V}{A} = \frac{1}{\frac{l}{k}}\left[\sigma_{yp} - \frac{P}{A}\right]\varnothing \tag{23b}$$

for negative values of α.

These last four equations now give the maximum value of the average shearing stress when $\frac{P}{A}$ in these equations is taken from the curves in Figs. 8.

It must be remembered that equations (a) hold only for values of $\frac{l}{r}$ such that $\varnothing < \cos^{-1}\alpha$ as stated previously, while for greater values of $\frac{l}{r}$ equations (b) must be used. Again taking $k = r$ it is possible from the four equations (18 a) and (b) and (23 a) and (b) to plot curves showing the average shearing stress $\frac{V}{A}$ as a function of $\frac{l}{r}$ for various values of α and $\frac{e_0}{k}$. Such curves are shown in Figs. 9 b, c, d, e and f for the same values of α and $\frac{e_0}{k}$ already considered.

These curves may now be considered as completely solving the problem of shearing forces in eccentrically loaded columns for given eccentricities of load. Comparison of the curves for various values of α shows that the most serious loading condition for short columns is equal eccentricities of load on opposite sides of the axis ($\alpha = -1.0$) as given in Fig. 9 f.

To represent the possible extent of general imperfections, the column was previously taken with an initial curvature of the axis in the form of a half sine wave having a center displacement $\delta = \frac{l}{400}$. It will now be seen that for the most serious condition of shear in short columns the column should be taken with some equal eccentricities of load on opposite sides of the axis, to represent general imperfections. Taking, for example, such eccentricities as represented by $\frac{e_0}{k} = 0.4$ this curve in Fig. 9 f may be used to represent the effect of imperfection. The $\delta = \frac{l}{400}$ curve in Fig. 9 a together with the $\frac{e_0}{k} = 0.4$ curve in Fig. 9 f may now be considered as completely solving the problem of shearing stresses in pin-ended columns due to general imperfections.

7. Critical Combination of Load and Eccentricity for Maximum Shear.

From equations (18) and (23) it is evident that there is some value of $\frac{P}{A}$ for each value of $\frac{l}{r}$ at which the average shearing stress $\frac{V}{A}$ will be a maximum. That is, for each value of $\frac{l}{r}$ there is some combination of load and eccentricity to produce failure (by yielding) which will cause a maximum shearing stress. Since the eccentricity has already been expressed in terms of the load in equations (18 a) and (b) and (23 a) and (b) these equations may now be differentiated to determine for what value of $\frac{P}{A}$ they will be a maximum. Making this differentiation and setting the value of $\frac{d\left(\frac{V}{A}\right)}{d\left(\frac{P}{A}\right)}$ equal to zero in each case it will be possible to determine $\frac{P}{A}$ as a function of $\frac{l}{r}$ to always give the maximum possible average shearing stress $\frac{V}{A}$. When this is done the equations for $\frac{P}{A}$ are as follows:

For equation (18 a),

$$\frac{\sigma_{yp} - 3\frac{P}{A}}{\sigma_{yp} - \frac{P}{A}} = \emptyset\left[\cot\emptyset - \frac{\alpha \sin\emptyset}{\beta}\right] \tag{24 a}$$

For equation (18 b),

$$\frac{3\frac{P}{A} - \sigma_{yp}}{\sigma_{yp} - \frac{P}{A}} = \alpha\,\emptyset \sin\emptyset\left[\frac{1}{\beta} - \frac{1}{\psi^2}\right] \tag{24 b}$$

For equation (23 a),

$$\frac{\sigma_{yp} - 3\frac{P}{A}}{\sigma_{yp} - \frac{P}{A}} = \emptyset\left[\cot\emptyset - \frac{\alpha \sin\emptyset}{\psi^2}\right] \tag{25 a}$$

For equation (23 b),

$$\frac{P}{A} = \frac{\sigma_{yp}}{3} \tag{25 b}$$

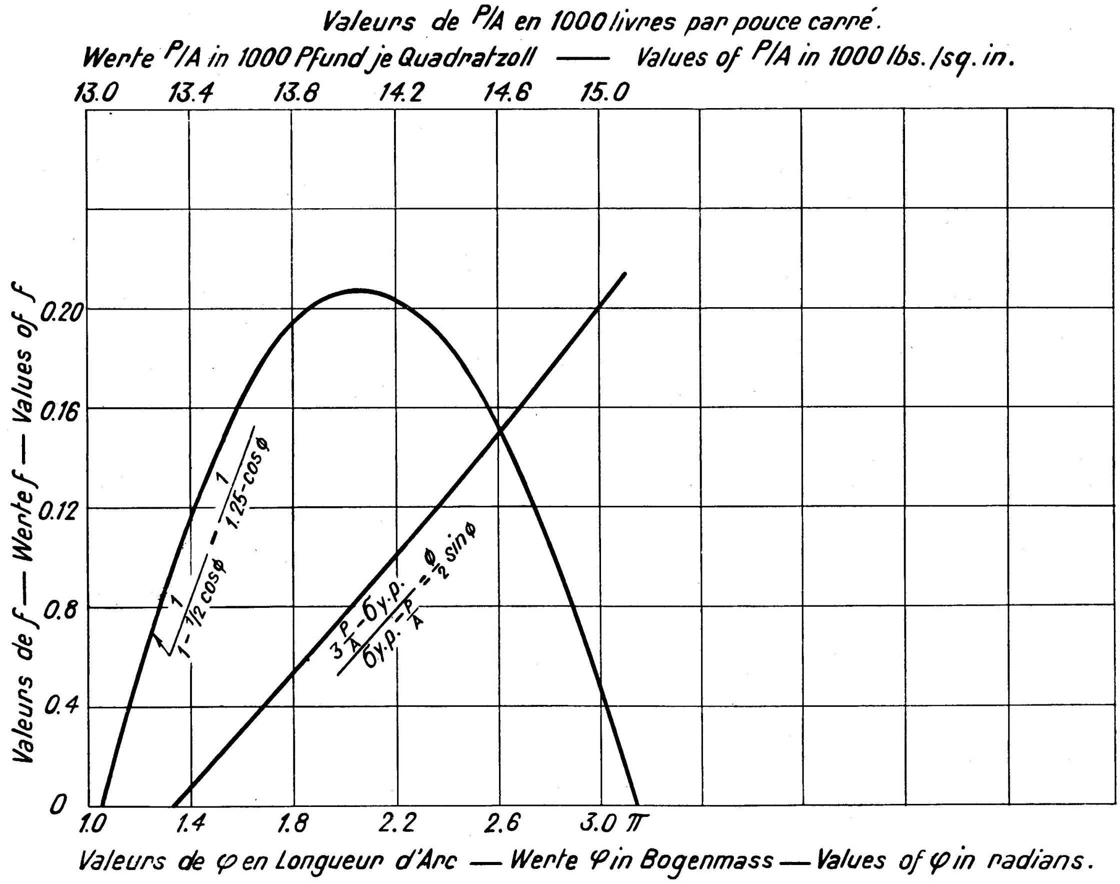

Fig. 7.

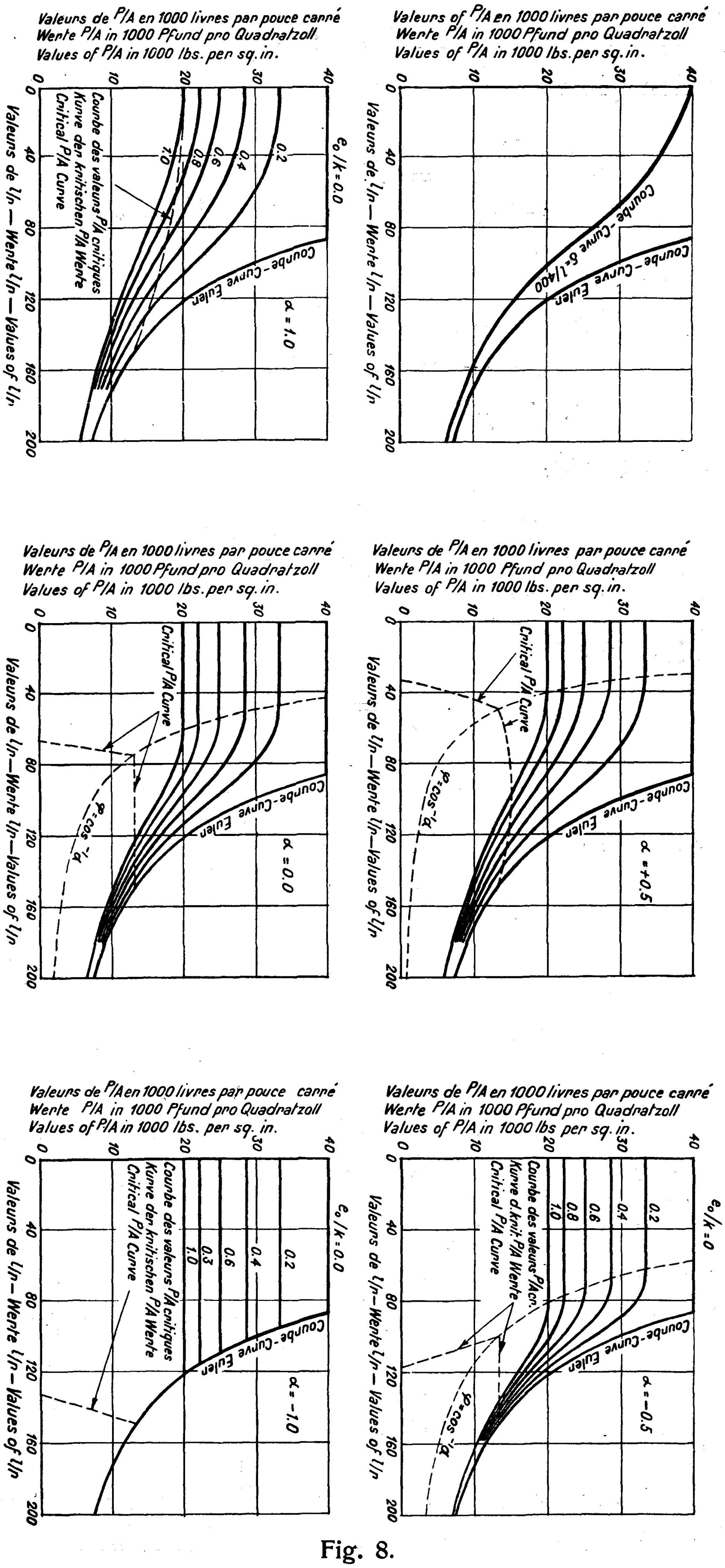
Valeurs de P/A en 1000 livres par pouce carré
Werte P/A in 1000 Pfund pro Quadratzoll
Values of P/A in 1000 lbs. per sq. in.
Valeurs de l/r — Werte l/r — Values of l/r
Courbe des valeurs P/A critiques
Kurve der kritischen P/A Werte
Critical P/A Curve
Courbe - Curve Euler
Courbe - Curve δ=l/400
Critical P/A Curve
$\varphi = \cos^{-1}\alpha$
$e_o/k = 0.0$
α = 1.0
α = 0.0
α = +0.5
α = -1.0
α = -0.5
Courbe des valeurs P/Acr.
Kurve d. krit. P/A Werte
$e_o/k = 0$

Fig. 8.

With the exception of (25 b) these equations, representing the criteria for maximum shear, must be solved graphically. Take as an example the

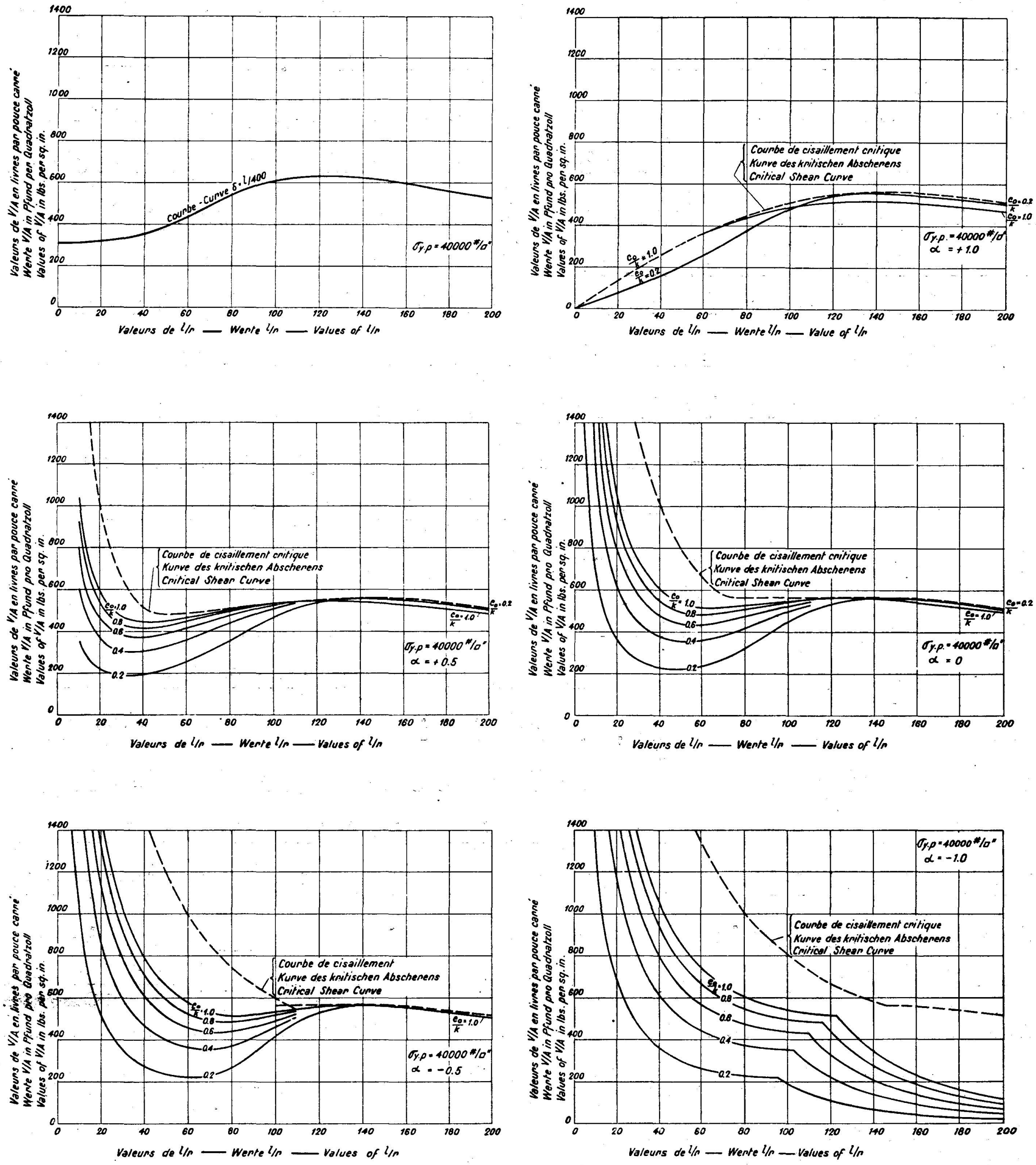

Fig. 9.

solution of equation (24 b) for $a = +0.5$. For this value of a the equation can be reduced to,

$$\frac{3\frac{P}{A} - \sigma_{yp}}{\sigma_{yp} - \frac{P}{A}} = \frac{\varnothing}{2} \sin \varnothing \left[\frac{1}{1 - \frac{1}{2} \cos \varnothing} - \frac{1}{1{,}25 - \cos \varnothing} \right]$$

Setting the left side of this equation equal to f it is possible to plot a curve showing values of f against values of $\frac{P}{A}$. In the same manner, setting the right side of the equation equal to f it is possible to plot a curve showing values of f against values of $\emptyset$. These two curves are shown in Fig. 7. Now by choosing some value of $\frac{P}{A}$ the corresponding value of f can be read from the first curve and the value of $\emptyset$ from the second [5]). Since $\emptyset = \frac{l}{r}\sqrt{\frac{P}{AE}}$, knowing $\frac{P}{A}$ and $\emptyset$, $\frac{l}{r}$ can now be computed. In this way a series of values of $\frac{l}{r}$ for various values of $\frac{P}{A}$ can be determined, and a curve plotted showing critical values of $\frac{P}{A}$ as a function of $\frac{l}{r}$ for maximum shearing stress. The curve for this particular case is shown in Fig. 8 c.

In this manner all of the criterion equations (24) and (25) were solved for critical values of $\frac{P}{A}$ as a function of $\frac{l}{r}$. For each value of a the curve of critical $\frac{P}{A}$ is shown in Fig. 8. A study of these curves shows: 1) that as a decreases from $+1.0$ to -1.0 there is a continually larger range of $\frac{l}{r}$ in which the critical value of $\frac{P}{A}$ is zero. This, of course, means a zero load at an infinite eccentricity, or causing failure by bending alone without direct thrust. That is, the shearing force always increases with eccentricity and there is practically no question of maximum shear. 2) That at a certain value of $\frac{l}{r}$ (depending upon a) the critical value of $\frac{P}{A}$ begins to increase rapidly until it becomes equal to $\frac{\sigma_{yp}}{3}$, after which it changes very slowly (for negative values of a, not at all). 3) For the steel chosen ($\sigma_{yp} = 40000$ lbs. per sq. in.) at $\frac{l}{r} = 149$ the curve of critical $\frac{P}{A}$ intersects the Euler curve, which means that for values of $\frac{l}{r}$ above 149 the column should be axially loaded to produce maximum shear.

Returning to equations (18 a) and (b) and (23 a) and (b) and now using values of $\frac{P}{A}$ in each case from the curves of critical $\frac{P}{A}$, the maximum possible shearing stress $\frac{V}{A}$ can be plotted as a function of $\frac{l}{r}$, for each value of a. This curve, for each value of a, may be considered as an envelope to the curves which might be obtained by using all possible combinations of load and eccentricity to produce failure (by yielding). The curves of critical average shearing stress are shown for each value of a in Fig. 9.

8. Practical Conclusions.

From a general study of the curves showing the average shearing stress $\frac{V}{A}$ as a function of $\frac{l}{r}$, it is evident that an axial load on an initially curved column gives rise to the greatest shearing stress for slender columns, while

[5]) In this particular case there happen tou be two values of $\emptyset$ and hence two values of l/r for each value of P/A (see Fig. 8 c).

for small values of $\frac{l}{r}$, equal eccentricities of load on opposite sides of the axis ($\alpha = -1.0$) is the most serious loading condition.

To design the details of built-up steel columns to resist a shearing stress of 650 lbs. per sq. in. would cover almost every possible case except extremely short columns with rather large end moments which tend to produce contraflexure of the axis. For these cases more careful investigation of the shearing stress on the basis of the curves presented should be made. It is of interest to note that an old form of the American Railway Engineering Association specifications for shear in columns, now abandoned, called for the lacing bars or battens of built-up steel columns to be designed to resist an average shearing stress of 600 lbs. per sq. in. for all values of $\frac{l}{r}$.

9. Summary.

A theoretical investigation of possible shearing forces in built-up steel columns is made for an axially loaded pin-ended column, in which the effect of imperfections is represented by an initial curvature of the axis in the form of a sine wave having an initial center displacement $\delta = \frac{l}{400}$. The shearing stress is evaluated for the loading which first produces failure by yielding in the most stressed fibers. The curve showing shearing stress $\frac{V}{A}$ as a function of $\frac{l}{r}$ is shown in Fig. 9 a.

A similar investigation is made for the column in rigid frame construction in which various combinations of end moments are considered in addition to the axial load. The effect of these end moments is studied by taking the compressive forces at corresponding eccentricities. Here, as before, the shearing stress is evaluated for the loading which first produces yielding due to combined bending and thrust. The shearing stress $\frac{V}{A}$ as a function of $\frac{l}{r}$, for various amounts and combinations of end eccentricities, is shown in Figs. 9 b, c, d, e and f.

Criteria are developed for the critical combination of compressive force and eccentricity to produce a maximum shearing stress for a given column. From these criteria, curves showing the critical value of $\frac{P}{A}$ as a function of $\frac{l}{r}$ to produce maximum possible shearing stress are shown in Figs. 8. The critical shears as a function of $\frac{l}{r}$, based upon these criteria are shown in Figs. 9.

From an examination of the curves in Figs. 9 it is concluded that for almost any possible case of loading the lacing bars or battens of built-up steel columns may be safely designed to resist an average shearing stress $\frac{V}{A} = 650$ lbs. per sq. in. Extremely short columns with large secondary end moments tending to produce contraflexure, may obtain greater shearing stresses than 650 lbs. per sq. in. and should be more carefully investigated on the basis of the curves presented.

Résumé.

L'auteur étudie d'une manière théorique les possibilités de mise en oeuvre d'efforts de cisaillement dans les poteaux métalliques composés, dans le cas particulier d'un poteau se terminant en pointe et soumis à une charge axiale; les imperfections dans la constitution de ce poteau étant matérialisées et représentées par une courbure initiale de son axe sous forme d'une sinusoïde accusant une déformation initiale au centre $\delta = l/400$. L'effort de cisaillement est déterminé pour la charge qui correspond à l'amorce de la fracture par écoulement dans les fibres les plus chargées. La courbe qui traduit les variations de l'effort de cisaillement $\frac{V}{A}$ en fonction de $\frac{l}{r}$ est représentée sur la figure 9 a.

La même investigation est effectuée pour le poteau qui fait partie d'un ensemble constitué par une charpente rigide dans laquelle on fait intervenir, outre la charge axiale, les moments d'extrémité, sous forme de combinaisons diverses. L'auteur étudie l'influence de ces moments d'extrémité en faisant intervenir les efforts de compression qui se manifestent pour des excentricités correspondantes. Ici, comme précédemment, il évalue l'effort de cisaillement correspondant à la charge pour laquelle se manifeste le début de l'écoulement sous l'influence de la flexion et de la poussée combinées. Les figures 9, b à f, représentent les variations de l'effort de cisaillement $\frac{V}{A}$ en fonction de $\frac{l}{r}$ pour différentes valeurs et différentes dispositions des excentricités d'extrémité.

Il établit des termes de comparaison pour la combinaison des efforts de compression et des excentricités susceptible de produire un effort de cisaillement maximum pour un poteau déterminé. A partir de ces critères, sont établies les courbes de la figure 8, qui traduisent la valeur critique de $\frac{P}{A}$ en fonction de $\frac{l}{r}$ qui correspond aux efforts maxima possibles de cisaillement. La figure 9 exprime les valeurs critiques du cisaillement en fonction de $\frac{l}{r}$ déterminées d'après les critères ci-dessus.

L'examen des courbes de la figure 9 permet d'arriver à cette conclusion que pour presque tous les cas de charge possibles, les éléments d'entretoisement et d'assemblage des poteaux métalliques composés peuvent être calculés, avec toute sécurité, pour résister à un effort de cisaillement moyen:

$$\frac{V}{A} = 4{,}5 \text{ kg/mm}^2.$$

Lorsqu'il s'agit de poteaux extrêmement courts accusant des moments d'extrémité secondaires importants tendant à produire des inflexions dans la courbure, on peut rencontrer des efforts de cisaillement supérieurs à 4,5 kg./mm²; il importe de pousser alors l'étude plus loin, sur la base des courbes indiquées.

Zusammenfassung.

Der Verfasser untersucht in theoretischer Weise die Möglichkeiten, wie die Querkräfte auf zusammengesetzte Stahlstützen wirkend, eingeführt werden können und dies für den Sonderfall einer auf Spitzen gelagerten Stütze,

die durch eine Axialkraft beansprucht ist. Die Ungenauigkeiten des Aufbaues der Stütze, dargestellt durch eine ursprüngliche Krümmung ihrer Axe in Form einer Sinusoide erzeugen eine ursprüngliche Deformation in der Mitte von $\delta = \frac{l}{400}$. Die Querkraft ist bestimmt für die Last, die dem Beginn des Bruches durch Erreichen der Fließgrenze in den stärkst beanspruchten Fasern entspricht. Die Kurve, die die Änderung der Querkraft $\frac{V}{A}$ in Funktion von $\frac{l}{r}$ darstellt, ist in Fig. 9 a gezeichnet.

Die gleiche Untersuchung ist für eine Stütze durchgeführt, die Bestandteil einer steifen Konstruktion ist, in der neben der Axialkraft die Einspannmomente in verschiedenen Kombinationen berücksichtigt werden. Der Autor untersucht den Einfluß dieser Endmomente, indem er die Druckkräfte hinzutreten läßt, die sich für die entsprechenden Exzentrizitäten ergeben. Hier wie zuvor wertet er die der Last bei Fließbeginn unter dem Einfluß von Druck und Biegung entsprechende Querkraft aus. Die Figuren 9 b bis f stellen die Änderungen der Querkraft $\frac{V}{A}$ in Funktion von $\frac{l}{r}$ dar für verschiedene Werte und Anordnungen der Endexzentrizitäten.

Er führt Vergleichsglieder ein für die Kombination der Druckkräfte und der Exzentrizitäten, die geeignet sind, für eine bestimmte Stütze eine maximale Querkraft zu erzeugen. Von diesen Kriterien ausgehend, sind die Kurven der Fig. 8 aufgestellt, die die funktionale Abhängigkeit des kritischen Wertes $\frac{P}{A}$ von $\frac{l}{r}$ zeigen, der den maximal möglichen Querkräften entspricht. Fig. 9 drückt die kritischen Werte der Querkräfte in Abhängigkeit von $\frac{l}{r}$, bestimmt auf Grund obiger Kriterien, aus.

Die Prüfung der Kurven der Fig. 9 gestattet die Folgerung, daß für beinahe alle möglichen Belastungsfälle die Verriegelungen und Verbindungsglieder für zusammengesetzte Stahlstützen mit aller Sicherheit für die Wirkung einer mittleren Querkraft von $\frac{V}{A} = 4{,}5$ kg/mm² berechnet werden können.

Falls es sich um außerordentlich kurze Stützen handelt, die bedeutende sekundäre Endmomente angeben, welche zu einer Bildung von Wendungen in der Krümmung neigen, so kann man Werten der Querkraft von mehr als 4,5 kg/mm² begegnen; man treibt alsdann das Studium an Hand der beschriebenen Kurven weiter.

Addenda.

P. S. P. } 128 Fig. 2. 3 { Panneaux à / Feldweiten zu / Panels à } 25′ = 75′.

P. S. P. } 272 et 273 Planche 8 et 9.

Dosage pour le béton armé courant:

350 kg de ciment à durcissement rapide
0,800 m³ de gravillon à l'anneau de 15 mm
0,400 m³ de sable.

Eprouvettes prismatiques de 7,1 × 7,1 × 28,4 d'abord éprouvées á la traction sous moment constant. Les morceaux ainsi obtenus sont ensuite essayés à la compression suivant une surface de 7,1 × 7,1.

Nota: Tous les résultats, sans aucune élimination, sont classés par valeur décroissante. Les courbes tracées en pointillé représentent des ensembles de résultats qui suivraient la loi de dispersion des erreurs accidentelles.

P. S. P. } 339 Fig. 30. { Fissure. / Anriß. / Crack.

In den Abhandlungen Steinman und Timoshenko wurde das englische „truss“ vielfach mit „Versteifungsfachwerk“ oder kurz „Fachwerk“ übersetzt, wodurch der allgemeine Sinn eines „Versteifungsträgers“ schon für große Ausführungen modifiziert erscheint.

Le mot anglais „truss“ dans les Mémoires de Steinman et Timoshenko fût quelquefois traduit par „poutre en treillis“, „poutre en treillis de raidissement“, „élément raidisseur continu en treillis“, ou „élément de renforcement continu en treillis“ ce qui modifie le sens général de „poutré raidisseuse“ pour le cas spécial de ponts à grandes portées.

Errata.

P. S. P. } 283 Planche 5. Lisez $m = 10$.

Diagonale	AB:	—8 kg/cm²
Montant	BC:	+9 „
Diagonale	CD:	—8,4 „