Application of soap-film-studies to photo-elastic stress determination

APPLICATION OF SOAP-FILM-STUDIES TO PHOTO-ELASTIC STRESS DETERMINATION.[1]

MESURE DES CONTRAINTES PAR LA MÉTHODE DE LA PHOTO-ÉLASTICITÉ À L'AIDE DE MEMBRANES DE SAVON

PHOTO-ELASTISCHE SPANNUNGSMESSUNGEN AUF GRUND VON SEIFENHAUTSTUDIEN

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Introduction.

The high photo-elastic sensitivity of transparent bakelite and phenolite makes possible fringe photographs such as those shown in Figures 6a, b, c, 14 a, b, c and 15, which are obtained quickly and which show at a glance the values of principal stress difference, $\sigma_1 - \sigma_2$, at all points in the member. Most engineering materials are considered to fail when maximum shear stress reaches a critical value, and since maximum shear stress, $\tau_{max.} = (\sigma_1 - \sigma_2)/2$, these photographs give the practical designer all the information he requires. Points of high stress are immediately evident, and since bakelite is easily machined, variations in shape of the member may be tried and the effect on stress noted. It is shown in the next paragraph that in addition to the time required for machining and annealing, only a few hours in the laboratory are necessary to obtain a $\sigma_1 - \sigma_2$ fringe photograph.

It was found by early experimenters that the samples of bakelite used could not be annealed or that residual stresses could not be removed. There are large differences in bakelite, however, and it is now possible to obtain the material in almost colorless polished plates [2]), very much like plate glass in appearance, which can be annealed almost perfectly. Annealing temperatures over 125° C should not be used and a temperature of 85° C is usually sufficient. The bakelite plate or model is placed in an electric furnace upon a horizontal plate of polished glass, with a sheet of smooth writing paper or three annealings may be necessary, and only in exceptional cases a sample lowered slowly to room temperature over a period of 16 to 20 hours [3]). Two or three annealings may be necessary, and only in exceptional cases a sample

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[1]) The writer acknowledges his indebtedness to Professor S. Timoshenko, who suggested the application of the membrane analogy described in this paper, and whose encouragement has been invaluable.

[2]) The bakelite used for the fringe photographs of this article was "Type C — 25, pale green", purchased in polished plates 15 × 30 × 1 cms. from the Apex Speciality Company, 1182 Flushing Ave., Brooklyn, N. Y.

[3]) In the experiments at the University of Michigan an automatic temperature lowering device was used, — 3—1/2° C per hour, — but most U. S. laboratories use furnaces which cool slowly enough to provide satisfactory annealing.

or three annealings may be necessary, and only in exceptional cases a sample which is usually hand sawn or filed with 1 or 2 mm. left on all edges. Before the final annealing the edges are milled leaving about 0,1 to 0,2 mm. on all edges. A slow milling feed is used and the depth of cut should not exceed 0,5 mm., being reduced for each succeeding cut, the last three cuts being only from 0,02 to 0,04 mm. deep. In this way stresses due to machining forces are not introduced. After the final annealing an edge effect is noticeable.

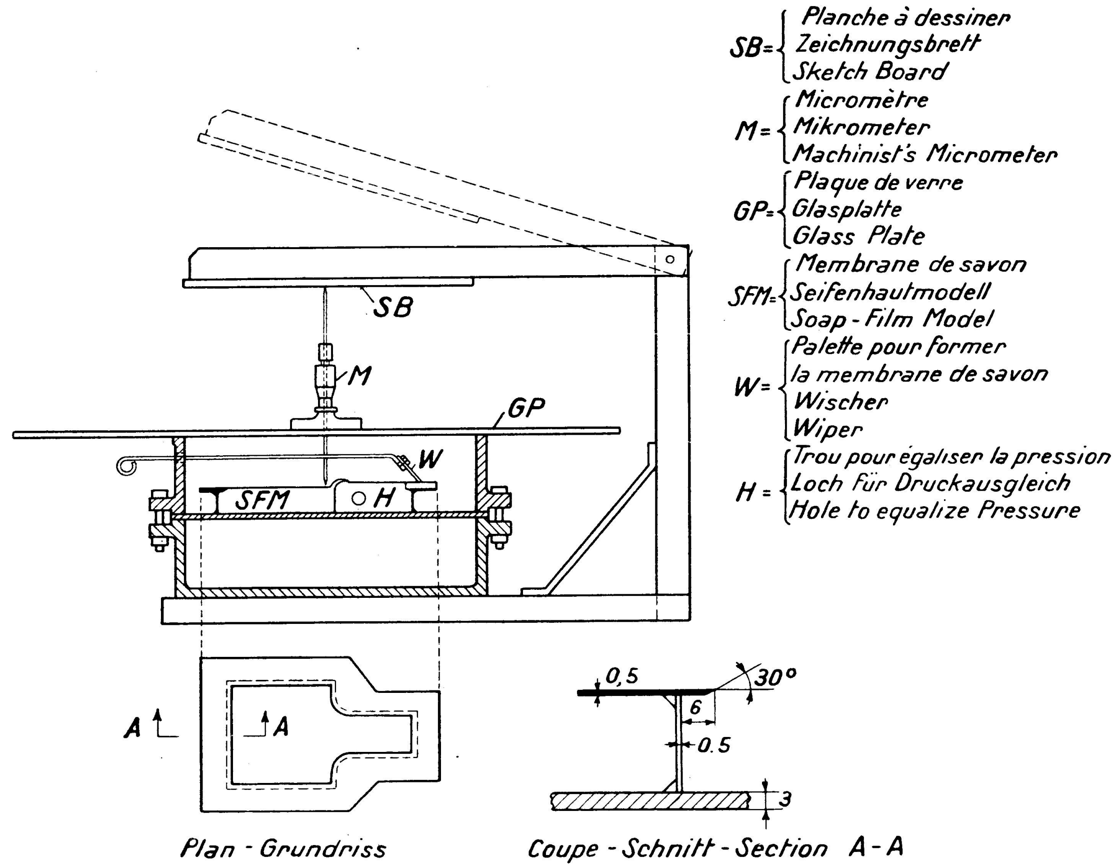


Fig. 1.

Appareil de la membrane de savon. — Seifenhautapparat. — Soap-Film Apparatus.

I. Valeurs des tensions pour la section I.
Spannungswerte für Schnitt I.
Stress Values at Section I.

II. Valeurs des tensions pour la section II.
Spannungswerte für Schnitt II.
Stress Values at Section II.

The 0,1 to 0,2 mm. of material which was left on all edges is then milled off in cuts of 0,02 to 0,04 mm. deep. The model is then entirely free from residual stress and is ready to be loaded and photographed. This is done within about four hours as the model soon shows "aging effects" around the boundaries.

When separate values of principal stresses, σ_1 und σ_2, are required the use of bakelite models may be supplemented in many problems by the membrane analogy, first suggested by J. P. Den Hartog[4]), which yields in a

[4]) J. P. Den Hartog, Z. A. M. M., Bd. 11, 1931, p. 156.

very short time curves of constant principal stress sum. A complete solution of the stress problem would be given by:

I. A set of $\sigma_1 - \sigma_2$ curves obtained from the fringe photograph, as in Figure 7,
II. A set of isoclinic curves giving the principal stress directions, as in Figure 8,
III. A set of $\sigma_1 + \sigma_2$ curves found by means of the membrane analogy, as in Figure 11.

From these curves the separate values and the directions of principal stresses at any point may be found. A description of the membrane analogy and an application to a specific problem are given below.

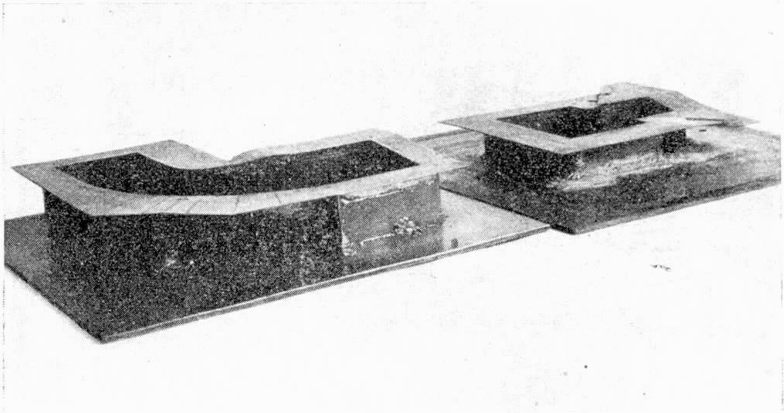


Fig. 2.
Membrane de savon. — Seifenhautmodelle. — Soap Film Models.

Methods for obtaining separate values of principal stresses.

In studying two dimensional stress problems by means of the photo-elastic method the values of principal stress difference, $\sigma_1 - \sigma_2$, and the principal stress directions, Φ, are determined readily by the use of comparatively simple apparatus. However, when the separate values of the principal stresses, σ_1 and σ_2 are required it is found that either more elaborate apparatus or more laborious procedure is necessary.

The simpler types of apparatus suitable for obtaining values of principal stress difference and principal stress directions using models of glass, celluloid and bakelite, have been described in recent articles by Messrs. Föppl [5]), Baud [6]) and Tuzi [7]).

In order to determine separately the values of σ_1 and σ_2 either the delicate interferometer method of Favre [8]) is used, by means of which these values are found directly, or one of a number of indirect methods may be used, all of which utilize values of $\sigma_1 - \sigma_2$ determined by means of the simpler types of photo-elastic apparatus referred to above.

5) L. Föppl, Z. V. D. I., Bd. 76, 1932, p. 505.
6) R. V. Baud, S. B. Z., Bd. 100, 1932, pp. 1 and 15.
7) Z. Tuzi, Sci. Papers, Inst. of Phys. and Chem. Research, Vol. 12, No. 209, 1929, pp. 21—36.
8) Favre, S. B. Z., Bd. 90, 1927, pp. 291 and 307. See also F. Tank, S. B. Z., 4 Aug. 1934, p. 45.

The extreme care and patience which must be necessary in operating the Jamin interferometer of Favre's apparatus to determine accurately an absolute retardation of a few wavelengths of light would appear to limit the practical usefulness of this method.

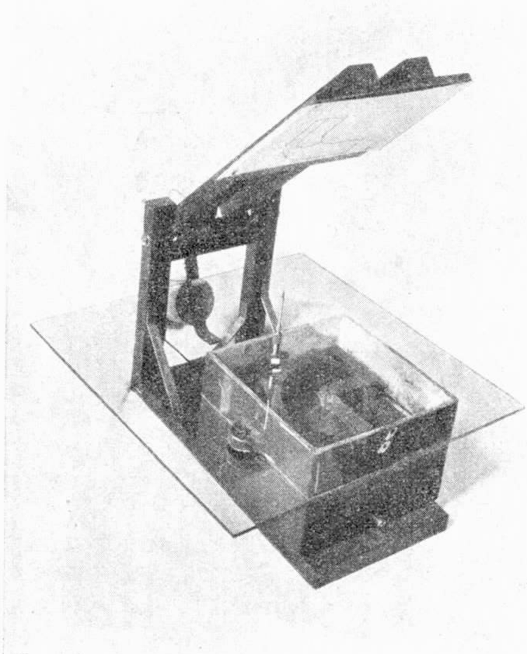


Fig. 3.
Appareil pour l'étude de la membrane de savon.
Seifenhautapparat.
Soap Film Apparatus.

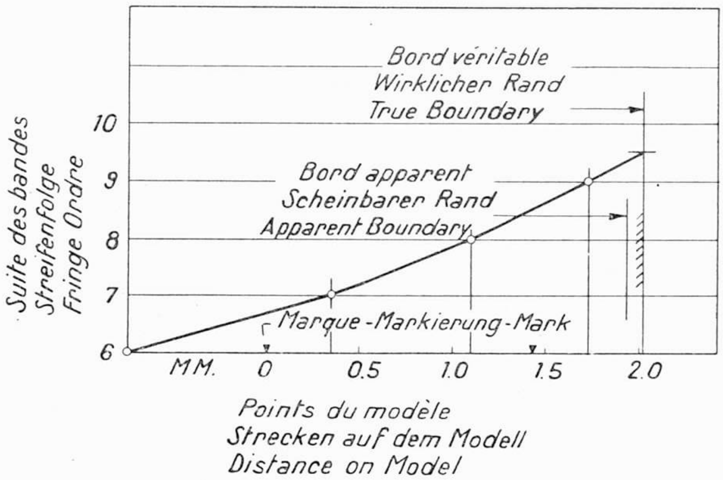


Fig. 4.

The earliest of the indirect methods to be used extensively is the lateral extensometer method developed by Coker [9]). The change in thickness of the model, which is measured by means of a sensitive micrometer, is proportional to $\sigma_1 + \sigma_2$ and consequently the method yields values of $\sigma_1 + \sigma_2$ at as many points as desired. Combining these results with the photo-elastically de-

[9]) Coker and Filon, "Photo Elasticity", Cambridge Univ. Press, 1931, p. 170.

termined values of $\sigma_1 - \sigma_2$, the separate values of σ_1 and σ_2 may be found at the points considered. The method is tedious and extreme care is necessary because of the minuteness of the quantities measured.

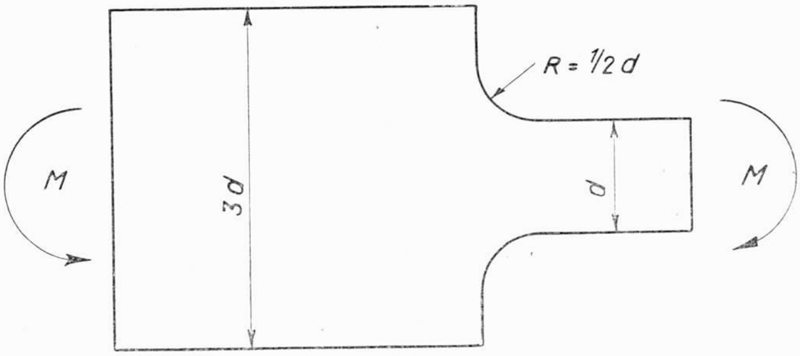
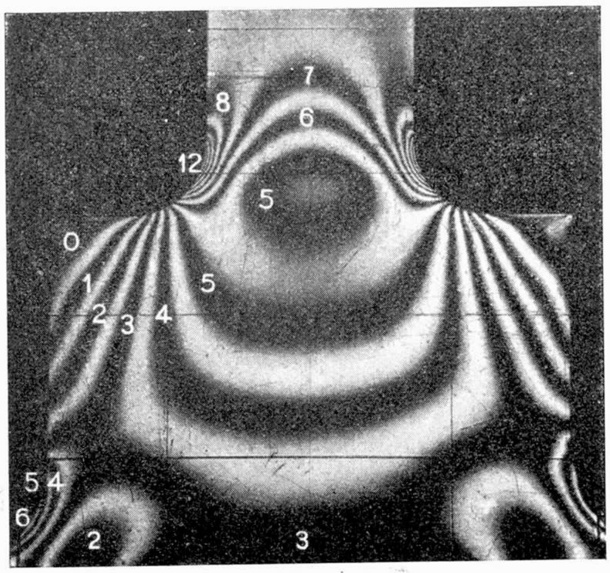


Fig. 5.

Graphical integration methods are discussed in the paper by Baud[10]). In Filon's method[11]) the principal stresses are given by two equations of the form

$$(\sigma_1)_A = (\sigma_1)_0 + \int_0^A (\sigma_1 - \sigma_2) \cot \psi \, d$$

and integration is carried out, step by step, along a principal stress trajectory. Values of $\sigma_1 - \sigma_2$ are taken from photo-elastic results. The other quantities



Tension = } $R/d = 0{,}34$
Zug =
Traction =

Fig. 6a.

Photographie des lignes de différences égales des tensions principales.
Photographie der Linien gleicher Hauptspannungsdifferenzen.
Fringe Photograph.

are determined graphically from a large scale drawing of the model, on which isoclinic lines and stress trajectories are traced. Integration is usually started

[10]) R. V. Baud, loc. cit.

[11]) L. N. G. Filon, "On the Graphical Determination of Stress from Photo-Elastic Observations", British Ass'n Report, 1923, pp. 350—357.

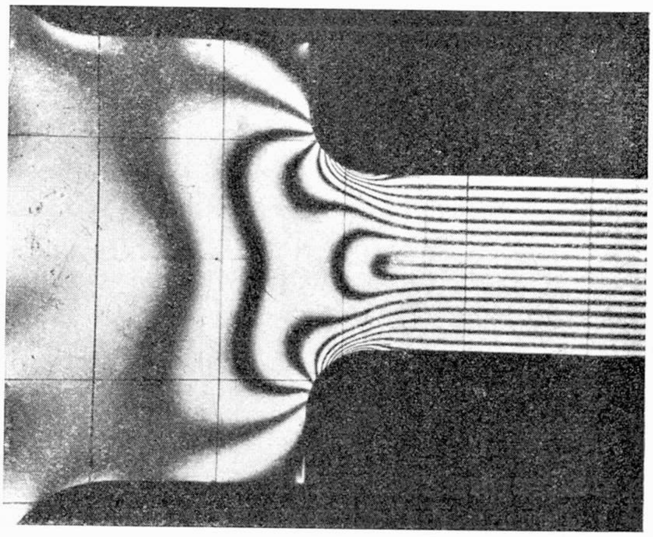


Fig. 6b.
Photographie des lignes de différences égales des tensions principales.
Photographie der Linien gleicher Hauptspannungsdifferenzen.
Fringe Photograph.
Moment fléchissant pur. — Reine Biegung. — Pure Bending.

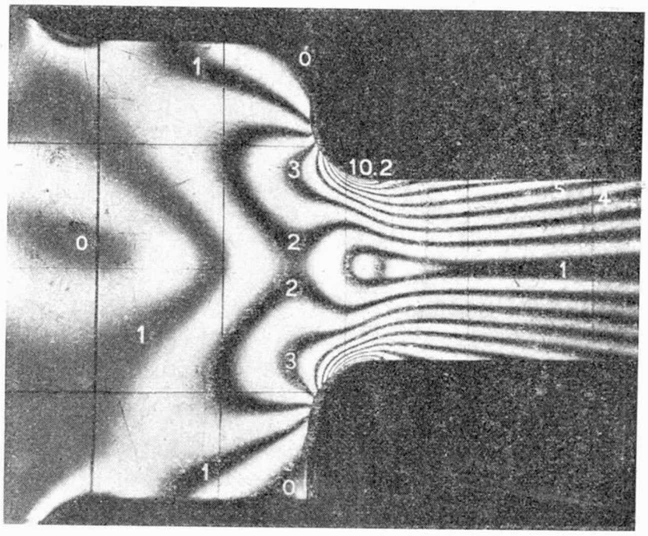


Fig. 6c.
Photographie des lignes de différences égales des tensions principales.
Photographie der Linien gleicher Hauptspannungsdifferenzen.
Fringe Photograph.
Moment fléchissant et effort tranchant. — Biegung mit Querkraft. — Cantilever Bending.

at a boundary free from external load at which the value of one of the principal stresses, such as $(\sigma_1)_0$ is zero. The process is exceedingly laborious and approximation within ten percent is not usually achieved.

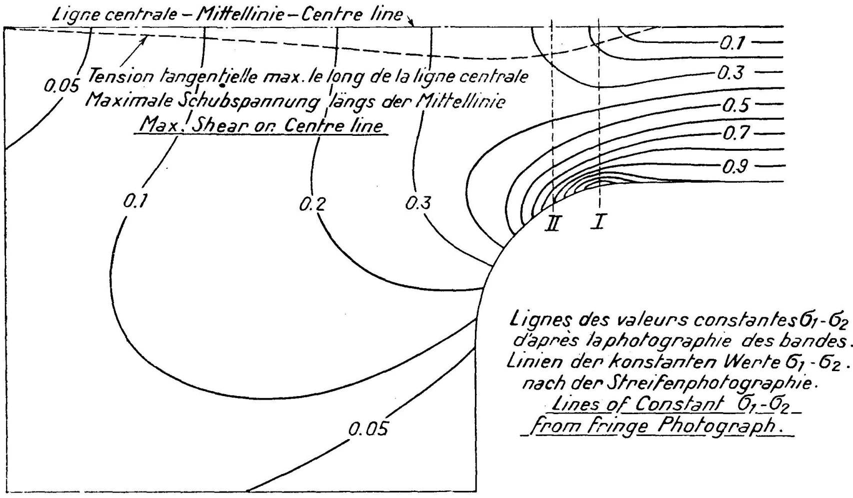


Fig. 7.

A more recent method introduced by Neuber [12]) provides a graphical construction for obtaining at any point of the member the direction of the line of constant principal stress sum, $\sigma_1 + \sigma_2$. By performing the graphical construction at a sufficiently large number of points the family of lines,

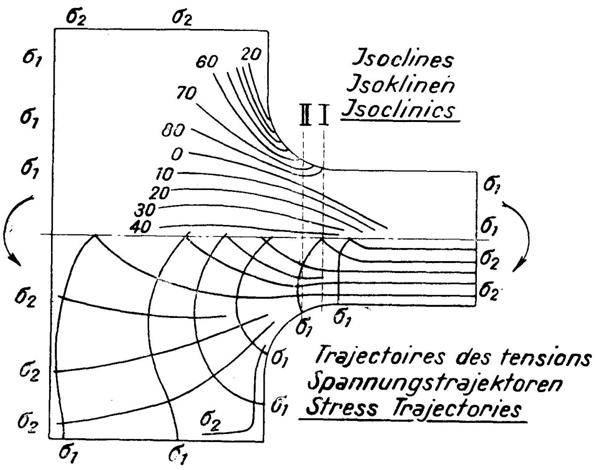


Fig. 8.

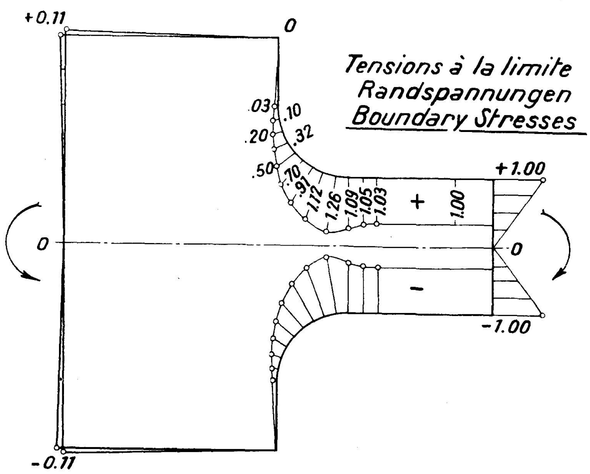


Fig. 9.

$\sigma_1 + \sigma_2$ = constant, known as isopachics, may be sketched in. Combining values of $\sigma_1 + \sigma_2$ with the values of $\sigma_1 - \sigma_2$ obtained photo-elastically, the separate values of σ_1 and σ_2 may be found. The labor involved in this method appears to be greater than that of the graphical integration method of Filon.

The methods outlined above for obtaining separate values of σ_1 and σ_2 are inconvenient because of the time and labor required. The Membrane Analogy, to be described below, provides a means of determining the values

[12]) H. Neuber, "New Method of Deriving Stresses Graphically from Photo-Elastic Observations", Proc. Roy. Soc. Lond., series A, Vol. 141, 1933, pp. 314—324.

of σ_1 and σ_2, to a satisfactory degree of accuracy, with less expenditure of time and effort than is required by any of the other methods.

The membrane analogy for two dimensional stress.

It was pointed out by J. P. Den Hartog in 1931 [13]) that it is possible to determine the principal stress sum, $\sigma_1 + \sigma_2$, by means of a membrane analogy.

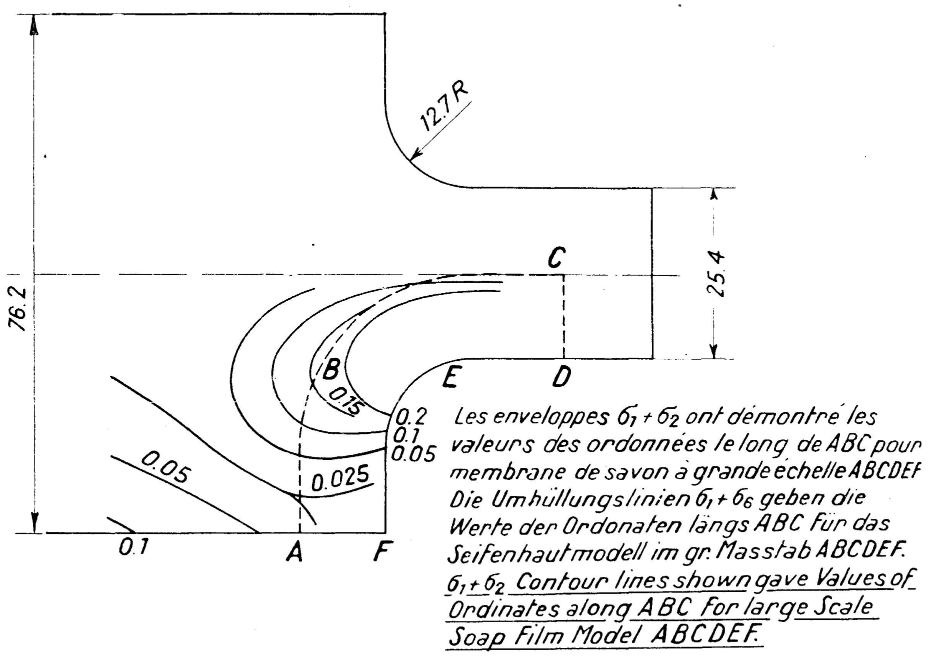


Fig. 10.

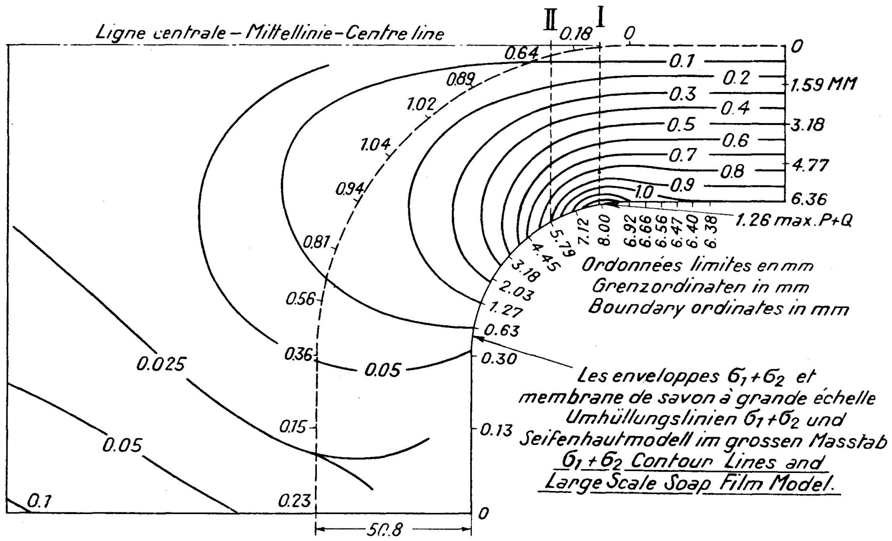


Fig. 11.

The sum of principal stresses satisfies the equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)(\sigma_1 + \sigma_2) = 0 \qquad (1)$$

which is also the differential equation for the ordinates z of a membrane. The membrane equation is obtained by assuming a membrane whose slope

[13]) J. P. Den Hartog, loc. cit.

with the xy plane is everywhere small, which is stretched by forces uniformly distributed along its boundary, and which has zero pressure difference on its two sides.

If such a membrane is stretched across an opening whose projection on the xy plane has the same shape as that of the stressed member, and whose

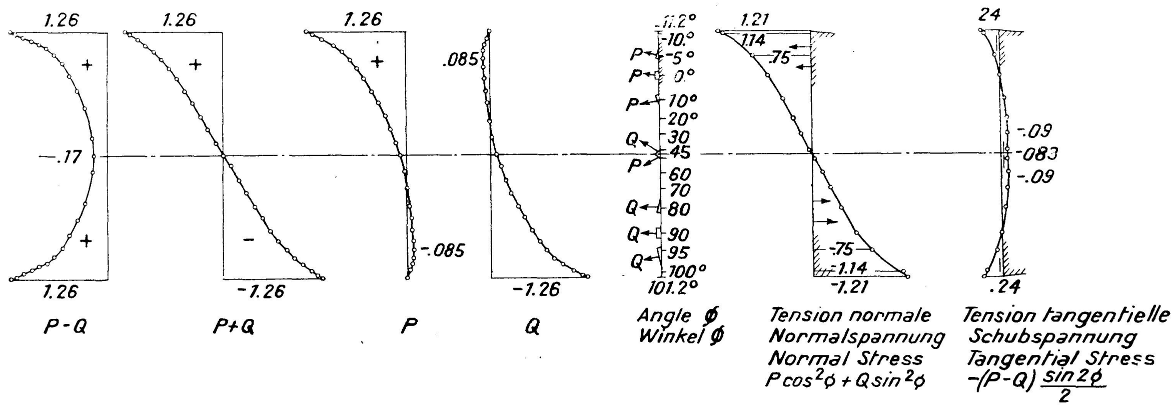


Fig. 12.

boundary ordinates z are proportional to the boundary values of $\sigma_1 + \sigma_2$, then since z and $\sigma_1 + \sigma_2$ satisfy the same differential equation, the ordinates of the membrane at interior points will also be proportional to $\sigma_1 + \sigma_2$. The boundary values of $\sigma_1 + \sigma_2$ are obtained directly from the photo-elastic results since at a boundary which is free from external load one of the principal stresses is zero and therefore $\sigma_1 + \sigma_2 = \pm\ (\sigma_1 - \sigma_2)$.

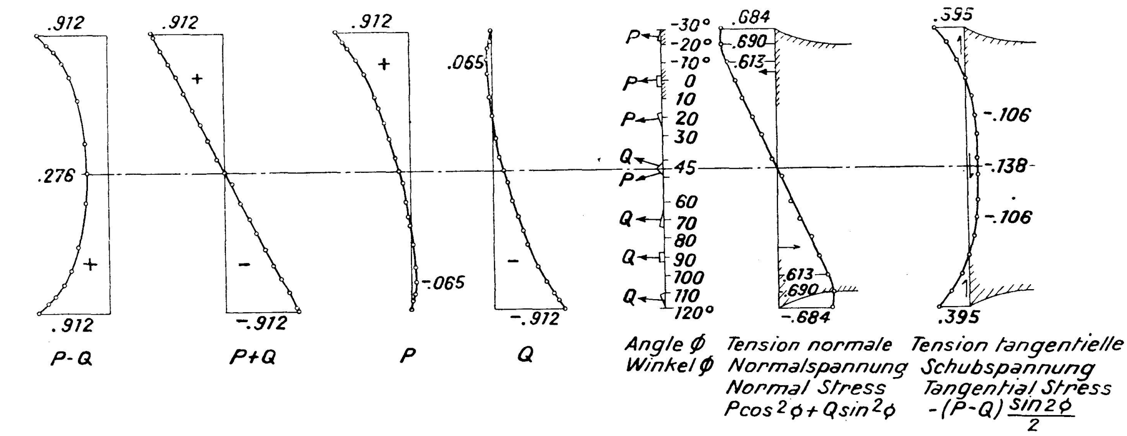


Fig. 13.

Although a membrane of stretched rubber may be used [14]), the results of the present paper were obtained by the use of a soap-film. The surface tension of the soap solution automatically provides a uniformly distributed boundary force, although offsetting this advantage the span of the soap-film is limited to approximately 8 cms because of the tendency of larger spans to break.

[14]) Mc Givern and Supper (work done under the direction of Dr. J. P. Den Hartog), "A Membrane Analogy Supplementing Photo-Elasticity", Journal Franklin Inst., Vol. 217, 1934, p. 491, also in Trans. Amer. Soc'y. Mech'l. Engrs., 1934, p. 601.

Soap-film model and apparatus.

The model, Figures 1 and 2, is made of sheet brass. The upper sheet, which is 0,5 mm. thick, has an opening of the same shape as the stressed member and is bent to give approximately the required ordinates at the boundary. It is soldered to a vertical wall of the same thickness material, which in turn is soldered to a flat base plate 3 mm. thick. The edges of the

Fig. 14.

Etude qualitative des concentrations de contraintes dans le fût sollicité à la flexion dans la zone pressée par une pièce accordante, sur la base des photographies des lignes de différences égales des tensions principales. La pièce *R* en Bakélite est pressée contre le pièce C par la même force de ressort pour tous les modèles, qui sont représentés par les photographies de la Fig. 14.

Qualitative Untersuchungen der Spannungsanhäufung im auf Biegung beanspruchten Schaft des Bereiches eines Paßstückes auf Grund von Photographien der Linien gleicher Hauptspannungsdifferenzen. Das Bakelit-Stück *R* wird gegen das Stück *C* gepreßt durch den gleichen Federdruck für alle Modelle, die in Fig. 14 photographisch dargestellt sind:

Qualitative study by fringe photographs of stress concentration in shaft in bending at region of a pressfit (photos by Timken Roller Bearing Co.). Bakelite piece *R* representing hub is pressed against piece *C* by same spring pressure for all photos. *C* is the compression edge of bakelite beam representing shaft, same pure bending moment for all photos.

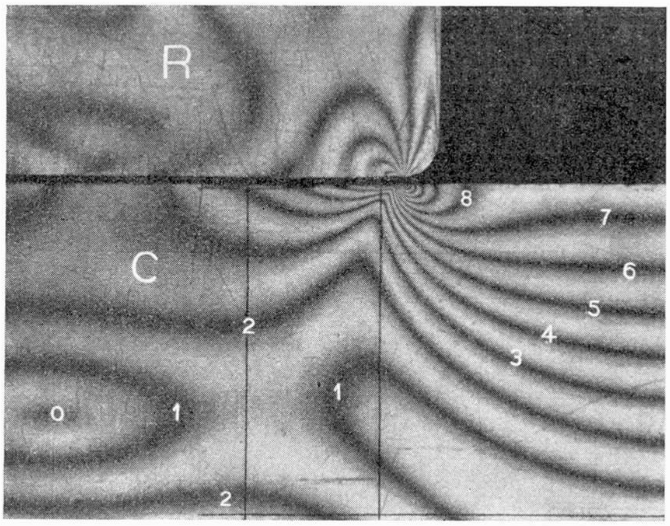


Fig. 14 a.

Concentration des contraintes pour l'exécution normale, donnant à peu près 14 bandes au maximum.

Spannungsanhäufung für gewöhnliche Ausführung, mit ungefähr 14 Streifen im Maximum.

Stress Concentration with usual design, about 14 fringes, maximum.

opening in the upper plate are filed at an angle of about 30° to a sharp edge, as in Section AA, Figure 1.

The model is clamped between the two halves of a square cast-iron box, Figures 1 and 3, and boundary ordinates are adjusted to within $\pm$ 0,03 mm. by bending the upper sheet by means of pliers or other tools. A machinist's depth micrometer, attached to a flat glass plate which is free to slide on the top of the box, is used for the final adjusting of boundary ordinates, as well as for measurements of ordinates at interior points of the soap-film surface.

A sketch board carrying a sheet of paper is hinged above the box in such manner that it may be brought down upon the upper point of the depth micrometer and thus transfer the location of a point from the soap-film surface to the paper.

The apparatus is in general the same as that described by Griffith and Taylor [15]) for the membrane analogy in torsion. The model in the present case, however, has vent holes to equalize the pressure on the two sides of the soap-film.

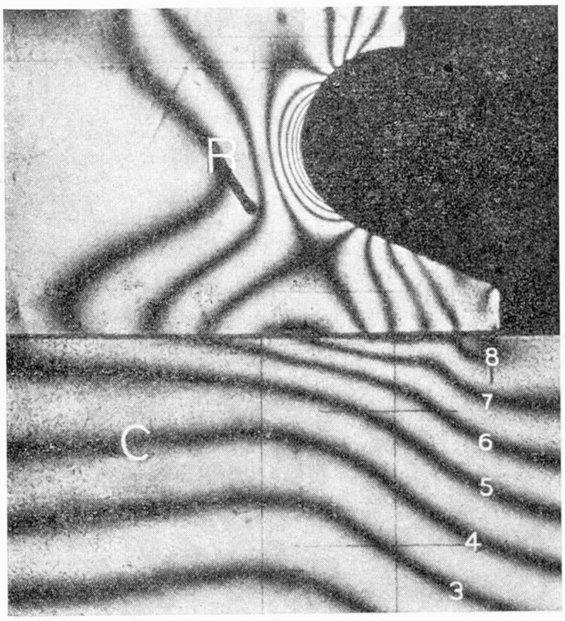


Fig. 14 b.

Exécution améliorée par une rainure circulaire. La flexibilité augmentée réduit la concentration des contraintes dans le fût à 8 bandes.

Verbesserte Ausführung mit kreisförmiger Nut. Erhöhte Biegsamkeit vermindert die Spannungsanhäufung im Schaft auf 8 Streifen.

Improved design of hub with circular groove. Increased flexibility reduces stress concentration in shart to 8 fringes.

Procedure.

The soap-film is formed from a solution of sodium oleate, distilled water and glycerine. The film is spread over the opening in the soap-film model by means of a flexible celluloid strip attached to a wire handle which passes through a hole in the box. With a suitable solution and a moist atmosphere inside the box the soap-film will last twenty or thirty hours.

Contour lines or lines of constant height on the soap-film surface represent lines of constant principal stress sum, $\sigma_1 + \sigma_2$, in the stressed member. They may be traced easily by setting the micrometer at a definite height and marking on the sketch board the points at which the lower micrometer point just touches the film. As many lines of constant $\sigma_1 + \sigma_2$ as desired may be traced in a short time.

As in previously described methods the separate values of σ_1 and σ_2 are determined by combining algebraically the values of $\sigma_1 + \sigma_2$ found from the soap-film surface with the values of $\sigma_1 - \sigma_2$ obtained photo-elastically.

[15]) Griffith and Taylor, Proceedings Inst. Mech'l. Eng'rs., 1917, p. 755.

Accuracy of the Method.

The accuracy of final results depends in the first place upon the accuracy of the boundary values of stress which are obtained photo-elastically. Messrs. Wahl and Beeuwkes [16]) have pointed out that in general the photo-elastically determined boundary stress is not the true boundary stress, but is the stress at a point slightly inside the boundary. Probably due to non-parallelism of the light beam where it passes through the specimen or to a slightly angular setting of the specimen with respect to the beam, the extreme edges do not appear in the image except as a black border of variable width. The error due to this effect may be considerable at points of stress concentration, as at fillets where the stress gradient is large. Messrs. Wahl and Beeuwkes proposed an extrapolation method for such regions. Figure 4 represents an application of their method by the present writer to determine the maximum

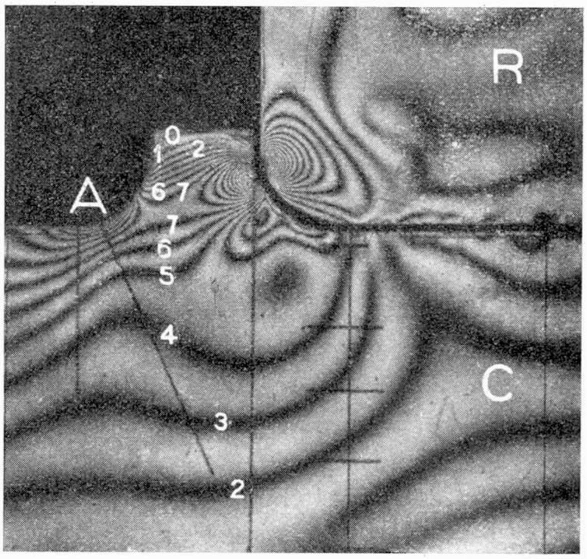


Fig. 14c.

La présence d'une dent au bord comprimé du fût déplace la concentration des contraintes au point pressé et la transfère à l'autre côté de la dent, ce qui provoque environ 15 bandes au point *A*.

Das Vorhandensein einer Nocke am gedrückten Rand des Schaftes verschiebt die Spannungsanhäufung an der Druckkante und überträgt sie auf die gegenüberliegende Seite der Nocke, wobei sich im Punkte A ungefähr 15 Streifen ergeben.

The presence of a collar on compression edge of shaft removes stress concentration under the press fit and transfers it to the opposite side of collar, giving about 15 fringes at *A*.

stress in the fillet of a model such as that shown in Figures 5 and 6. Ordinates represent values of $\sigma_1 - \sigma_2$. Abscissae are distances measured on the photo-elastic fringe photograph perpendicular to the boundary. The distance on the actual model from a scratch or mark to the boundary is measured accurately by means of a portable microscope (20 X) with travelling eyepiece. The magnification ratio from the specimen to the photograph can be measured so that the true position of the boundary on the photograph may be calculated and the true boundary stress determined by an extrapolation curve as shown in Figure 4.

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[16]) Wahl and Beeuwkes, Trans. Amer. Soc'y Mech'l. Eng'rs., 1934, p. 617.

Although the above example of extrapolation is for a bakelite model, the same type of error in boundary stress is present in experiments with glass or celluloid models and may be corrected by extrapolating in a similar manner.

Slopes of the film theoretically should be very small for equation (1) to apply, although in the example to be described slopes of 17° did not appear to introduce errors in the stress values greater than two percent.

For soap-film spans greater than about 5 cm. correction for sag due to gravity becomes necessary, a method of correction being described in the example which follows.

Inaccuracies were expected where the soap-film meets the sharp edge of the brass model, due to the increase in thickness of the film at these points. No such inaccuracies could be detected however within a distance of 1 mm. from the boundaries.

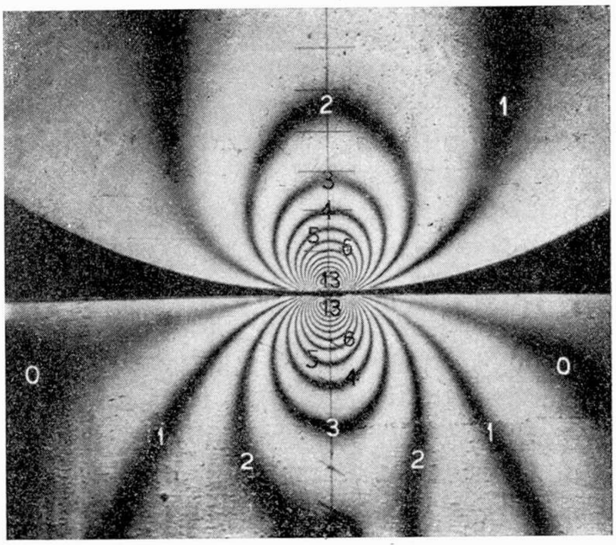

Fig. 15.

La figure montre que les tensions tangentielles maxima au voisinage des points de contact, se produisent à une distance de la surface qui est celle que donne la théorie de Hertz.

In der Berührungsfläche der Rollen tritt die maximale Schubspannung auf, wie deutlich ersichtlich ist, in einem Abstand von der Oberfläche in Übereinstimmung mit der Hertz'schen Theorie.

In contact of rollers maximum shear stress is clearly shown to occur at a distance from surface in accordance with the Hertz theory.

Illustration of the method: Flat member in pure bending.

As an example of the use of the Membrane Analogy in obtaining values of the principal stresses σ_1 and σ_2, a flat member as shown in Figure 5 was subjected to pure bending and studied by means of the Photo-Elastic Method and a Soap-Film.

For the photo-elastic work a bakelite specimen was used and fringe photographs of the type shown in Figure 6 b were obtained. From the photo-elastic results the lines of constant principal stress difference, $\sigma_1 - \sigma_2$, were drawn, Figure 7. Stress values shown on all the Figures are based upon a stress value of 1,00 at the boundary of the straight narrow portion of the member. Isoclinics and Stress Trajectories are shown in Figure 8. Boundary stresses obtained from the fringe photographs are shown in Figure 9. As previously stated these represent values of either $\pm (\sigma_1 - \sigma_2)$ or $\sigma_1 + \sigma_2$.

The first soap-film model, Figures 2 and 10, was made with ordinates antisymmetric about the centre-line and proportional to the boundary stresses of Figure 9 (3,4 mm. $\sim$ 1,00). Slopes around the boundary of the model were limited to about 18° but it was found that in a small region near the point of maximum stress the slope of the film was about twice this angle.

This large slope would introduce errors in this region, and a second soap-film model was therefore made to a larger scale to represent the region *ABCDEF* on Figure 10. The ordinates along *CDEFA* were as obtained from the photo-elastic results; those along *ABC* were taken from the soap-film surface on the first model, Figure 10, it being assumed that the error would be negligible along *ABC* due to the excessive slope in the small region of stress concentration near *E*. The ordinates in millimetres on the second model are shown on Figure 11. The width of opening was 50,8 mm. at all points. The maximum slope around the boundary of this model was about 8,5°; that of the film surface about 17°. The difference between the sine and tangent of 17° is about 5 percent. The cosine of 17° is about $4\frac{1}{2}$ percent less than unity. The error introduced by this slope is discussed later. The $\sigma_1 + \sigma_2$ contour lines, or lines of constant $\sigma_1 + \sigma_2$ as obtained from both soap-film models are shown on Figure 11.

Normal and Shear Stresses on Sections I and II.

The normal and shearing stresses across two sections of the member are determined as shown in Figures 12 and 13. Section I is taken through the point of maximum stress, about 11,2 degrees from the beginning of fillet. Section II is taken at 30 degrees from the beginning of fillet. Values of $\sigma_1 - \sigma_2$ are taken from Figure 7. It will be seen that $\sigma_1 - \sigma_2$ must be taken positive both above and below the centre line in order to agree with the direction of couples and the notation in Figure 8. Values of $\sigma_1 + \sigma_2$ are taken from Figure 11, the σ_1 and σ_2 curves are obtained by successive addition and subtraction of ordinates of the $\sigma_1 - \sigma_2$ and $\sigma_1 + \sigma_2$ curves. The principal stresses are not in general perpendicular to the sections I and II, their directions are found from the isoclinic curves of Figure 8 and are shown above the caption "Angle Ø". Some of the values of Ø shown are negative, others are greater than 90°. With the values of Ø shown the normal stress is $\sigma_1 \cos^2 \varnothing + \sigma_2 \sin^2 \varnothing$ and tangential stress is $-(\sigma_1 - \sigma_2) \dfrac{\sin 2\varnothing}{2}$.

The maximum value of stress along the boundary occurs at section I and is 1,26. Normal stresses on this section follow a linear law over the middle two-thirds of the depth of section, as in the bending of a straight beam, and rise to a peak value of 1,21 at the boundary. At section II the effect of the greater width of section is already noticeable as here the stress falls away from a straight line as the boundary is approached, and the maximum normal stress occurs away from the boundary.

Tangential components of stress are positive near the boundaries and negative in the middle portion. As net shear is zero in pure bending these components must balance. The values of tangential components along the centre line can be taken directly from the fringe photograph as has been indicated in Figure 7.

Estimation of Errors in this Example.

Errors introduced by the use of the membrane analogy as described above will not affect the values of normal or tangential components of stress at the centre line and at the boundaries. Any errors can therefore affect only intermediate values and would cause a slight distortion of the stress curves. As a check upon the accuracy, the total moment of normal forces across sections I and II was found from the curves, and in both cases was 1,9 percent greater than it should have been. Assuming the small distortion of the normal stress curve to be parabolic and symmetrical about a point halfway between the centerline and the boundary, a rough calculation shows that the maximum error in normal stress is 1,9 percent. If, as is more probable, the maximum distortion occurs near the boundary, the maximum error would be nearer to one percent.

A check of the tangential stresses for equilibrium showed an unbalance of about ten percent of the total upward or total downward forces. These stresses are small however and the chief effect of the error is a shift of the point of zero stress.

Errors due to the effect of gravity on the soap-film were corrected for in the following manner. On the small scale, antisymmetrical model, the ordinates of the film along the centre line were not zero as they should have been theoretically. In the 25,4 millimetre portion the sag was negligible, but in the 76,2 millimetre part a maximum sag of 0,150 mm. was measured. The sag (at other points) was assumed to follow a parabolic law, and was corrected for accordingly. This was checked also by the fact that $(\sigma_1 + \sigma_2)$ contour lines on opposite sides of the centre line must of necessity be symmetrical.

In the case of the large scale model it was found that due to sag in the straight portion representing the narrow part of member where stress conditions are uniform the contour lines were shifted slightly away from the edge representing the centre line and toward the edge representing the outer boundary, which was the higher edge. This shift at most was equivalent to 1,5 percent of the boundary stress, and contour lines in other parts of the film were corrected accordingly.

General comments on the method.

It is believed that the membrane analogy will find increasing use in conjunction with photo-elastic studies. For those problems to which it can be applied the errors introduced by its use can be reduced to a negligible amount by suitable proportioning of the boundaries, and by making necessary corrections for the effect of sag. The errors in the present experiments which were done under the pressure of time are probably smaller than would be present in results obtained by means of the delicate lateral extensometer method or by the graphical integration method. The amount of time required is surprisingly small, when the soap-film equipment is already available. In the present instance the large scale model was completed, with about fifty points around the boundary set to within 0,03 mm. of the desired value, in a day and a half. Tracing contour lines required only a few hours and working out of the curves in Figures 12 and 13, another day and a half.

Summary.

Because of the high photo-elastic sensitivity of transparent bakelite and phenolite, it is possible to obtain fringe photographs such as those shown in Figs. 6 a, b, c, 14 a, b, c and 15. These are obtained quickly and show clearly the distribution in a structural member of the maximum shear stresses, which are of particular interest to the practical designer who bases his calculations of the permissible stresses on the theory of maximum shear stress.

Methods of annealing the bakelite are described, and also the preparation of the models, which have no residual stresses. When the separate values of the principal stresses, σ_1 and σ_2, are required, it is recommended to use the membrane analogy to supplement the photo-elastic results obtained from fringe photographs. The membrane analogy was first proposed by J. P. Den Hartog. According to this theory the sum of the principal stresses, $\sigma_1 + \sigma_2$, satisfies an equation (1), which is also the differential equation for the ordinates of a membrane. If the membrane is stretched across an opening whose projection on the xy plane has the same shape as that of the stressed member, and whose boundary ordinates are proportional to the boundary values of $\sigma_1 + \sigma_2$, then the ordinates of the membrane at interior points will also be proportional to the corresponding values of $\sigma_1 + \sigma_2$ in the interior of the stressed member.

In illustration of the method, a description is given of a test on a member (fig. 5) subjected to pure bending. A soap film was used as membrane. The soap-film models are shown diagrammatically in fig. 1 and photographically in figs. 2 and 3. From fringe photographs of the type shown in fig. 6 b, the lines of constant stress difference, $\sigma_1 - \sigma_2$, were drawn (fig. 7). If one boundary is free from external load, one of the principal stresses is zero and therefore $\sigma_1 + \sigma_2 = \pm (\sigma_1 - \sigma_2)$. In this way the boundary stresses of $\sigma_1 + \sigma_2$ could be obtained from fig. 7. Lines of constant height on the soap-film surface represent lines of constant principal stress $\sigma_1 + \sigma_2$ and are shown in fig. 11. The stress curves of figs. 7 and 11, together with the isoclinics and stress trajectories of fig. 8, give the complete solution for the case of plane stress. Normal and shear stresses in sections I and II of fig. 7 are shown in figs. 12 and 13 respectively. The described method gives results which are sufficiently accurate, and it probably requires less time than other methods.

Zusammenfassung.

Dank der großen optischen Empfindlichkeit des Bakelites ist es möglich, Streifen-Aufnahmen, wie diese in Fig. 6 a und 6 b abgebildet sind, zu erhalten. Diese zeigen deutlich die Verteilung der maximalen Schubspannungen in einem Konstruktionsteil, die den Praktiker, der bei der Berechnung der zulässigen Beanspruchungen die Theorie der maximalen Schubspannungen zugrunde legt, hauptsächlich interessiert.

Es werden Verfahren zum Anlassen des Bakelites, sowie zur Herstellung der Modelle, die keine Restspannungen aufweisen, beschrieben. Falls die Werte σ_1 und σ_2 der Hauptspannungen getrennt verlangt werden, ist es zu empfehlen, die photo-elastischen Resultate, die durch die Streifen-Aufnahmen gegeben sind, durch die Membran-Analogie zu ergänzen. Die Membran-Analogie ist zuerst von J. P. Den Hartog vorgeschlagen worden. Die Analogie besagt, daß die Summe der Hauptspannungen $\sigma_1 + \sigma_2$ und die Ordinate

einer Membran derselben Gleichung (1) gehorchen. Der Grundriß der Membran erhält dabei die Form des beanspruchten Konstruktionsteiles; die Membran-Randordinaten müssen den Randwerten von $\sigma_1 + \sigma_2$ proportional gemacht werden. Alsdann sind die Ordinaten im Innern der Membran proportional den entsprechenden Werten von $\sigma_1 + \sigma_2$ im Innern des Konstruktionsteiles.

Der Versuch an einem Konstruktionselement (Fig. 5), das auf reine Biegung beansprucht wurde, wird beschrieben. Als Membran wurde dabei eine Seifenblase benutzt. Die Seifenblasen-Modelle und der Apparat sind schematisch in Fig. 1 und photographisch in Fig. 2 und 3 dargestellt. An Hand der Streifen-Aufnahmen, wie z. B. Fig. 6 b, sind Kurven konstanter $\sigma_1 - \sigma_2$-Werte gezeichnet worden (Fig. 7). Wenn keine äußeren Kräfte an einem Rand angreifen, so ist eine der Hauptspannungen Null; somit gilt die Beziehung $\sigma_1 + \sigma_2 = \pm (\sigma_1 - \sigma_2)$. Auf diese Weise konnten die Randwerte von $\sigma_1 + \sigma_2$ aus der Fig. 7 erhalten werden. Die Linien konstanter Höhe auf der Seifenblase entsprechen den Linien konstanter $\sigma_1 + \sigma_2$-Werte und sind in Fig. 11 dargestellt worden. Die Spannungs-Kurven der Fig. 7 und Fig. 11, zusammen mit den Isoklinen oder den sog. Richtungs-Kurven der Fig. 8, ergeben die vollständige Lösung des ebenen Spannungs-Falles. Normal- und Schubspannungen in den Querschnitten I und II der Fig. 7 sind entsprechend in Fig. 12 und 13 dargestellt worden. Die beschriebene Methode liefert Resultate von genügender Genauigkeit und sie ist wahrscheinlich weniger zeitraubend als andere Verfahren.

Résumé.

Grâce à la sensibilité photo-élastique élevée de la bakélite et de la phénolite transparentes, il est possible d'obtenir des reproductions photographiques de franges telles que celles des figures 6 (a, b, c), 14 (a, b, c) et 15. Ces clichés sont obtenus rapidement et montrent immédiatement les valeurs diverses des contraintes maxima de cisaillement dans un élément d'ouvrage, dont la répartition intéresse au plus haut point le spécialiste qui base le calcul des contraintes admissibles sur la théorie des contraintes maxima de cisaillement.

L'auteur décrit divers procédés pour le traitement de la bakélite, de même que pour la préparation des modèles qui ne doivent accuser aucune contrainte rémanente. Lorsqu'il est nécessaire de distinguer séparément les valeurs σ_1 et σ_2 des contraintes principales, il est à recommander de compléter les résultats obtenus par la photo-élasticité et qui sont fournis par les clichés de franges, en recourant à l'analogie de la membrane, qui a été proposée pour la première fois par J. P. Den Hartog. D'après cette analogie, la somme des contraintes principales $\sigma_1 + \sigma_2$ et les ordonnées d'une membrane répondent à la même relation (1). Le tracé de la membrane emprunte donc la forme de l'élément d'ouvrage chargé; les ordonnées du bord de la membrane doivent être proportionnelles aux valeurs périphériques de $\sigma_1 + \sigma_2$. Les ordonnées de l'intérieur de la membrane sont alors proportionnelles aux valeurs correspondantes de $\sigma_1 + \sigma_2$ pour l'intérieur de l'élément considéré.

Le rapport décrit ensuite l'essai sur un élément de construction (figure 5) sollicité à la flexion pure. On a utilisé comme membrane une bulle de savon. Le modèle et l'appareil sont représentés schématiquement sur la figure 1 et photographiquement sur les figures 2 et 3. En utilisant les photographies de franges comme celles de la figure 6 b par exemple, on a tracé des courbes

de valeurs constantes de $\sigma_1 - \sigma_2$, comme le montre par exemple la figure 7. Lorsqu'aucun effort extérieur ne s'applique au bord, l'une des contraintes principales est nulle; on a ainsi la relation: $\sigma_1 + \sigma_2 = \pm (\sigma_1 - \sigma_2)$. On a pu ainsi obtenir les valeurs de bordure de $\sigma_1 + \sigma_2$ à partir de la figure 7. Les courbes de niveau constant sur la bulle de savon correspondent aux courbes des valeurs constantes de $\sigma_1 + \sigma_2$ et sont représentées sur la figure 11. Les courbes de tension, des figures 7 et 11, avec les isoclines ou courbes de direction de la figure 8, donnent la solution complète du cas de la tension plane. Les contraintes normales et de cisaillement dans les sections I et II de la figure 7 ont été représentées respectivement sur les figures 12 et 13. La méthode décrite donne des résultats de précision suffisante et demande certainement moins de temps que les autres méthodes.