CALCULATION OF LATERAL TRUSS IN SUSPENSION BRIDGES.

DIE BERECHNUNG VON WINDVERBÄNDEN BEI HÄNGEBRÜCKEN.

CALCUL DU TREILLIS LATÉRAL DES PONTS SUSPENDUS.

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During the last years several treatises have been published dealing with the subject of suspension bridges under lateral loadings. All of these treatises, with exception of the one written by L. MOISSEIFF and F. LIENHARD [3], deal only with suspension spans symmetrical about the center, and simply supported at the towers. The method presented by L. MOISSEIFF and F. LIENHARD is not restricted to this special class, but depends on a very laborious numerical solution by successive approximation.

In the present paper it is intended to give a method permitting direct stress computation of suspension bridges of any form, with the lateral truss continuous or simply supported over any number of spans, the only condition being constant stiffness IE of the lateral truss within each span. The method presents an improvement and extension of a previously published work [7] dealing with symmetrical suspension bridges of one span with symmetrical lateral loads.

Deduction of Fundamental Equations.

The following equations are based on the assumption that the spacing of the hangers is small compared with the length of the span. A further assumption is that the deformations are so small that the vertical distance between cable and roadway may be replaced by the length of the hangers $h(x)$.

We indicate wind pressure against truss, lower half of hangers and any live load by $W_t(x)$, the same against upper half of hangers and cable by $W_c(x)$, combined dead and live load per unit length of roadway by $q(x)$, and the combined cable tension due to dead and live load by H.

Using notations from Figs. 1 and 2 the following equation for the lateral deflection of the cables $\Delta(x)$ is obtained:

$$W_c(x)\,dx + q(x)\,\frac{\delta(x) - \Delta(x)}{h(x)}\,dx = -\,H\Delta''(x)\,dx,$$

or:

$$H\Delta''(x) = -\,W_c(x) - q(x)\,\frac{\delta(x) - \Delta(x)}{h(x)} \tag{1}$$

For the lateral deflection of the lateral truss:

$$W_t(x)\,dx - q(x)\,\frac{\delta(x) - \Delta(x)}{h(x)}\,dx = -\,d\,V = -\,M''(x)\,dx = (IE\,\delta''(x))''dx,$$

or: $$(IE\delta''(x))'' = -M''(x) = W_t(x) - q(x)\frac{\delta(x) - \Delta(x)}{h(x)}. \qquad (2)$$

In these equations $\Delta(x)$ indicates the lateral deflection of the cable and $\delta(x)$ the same for the lateral truss.

The wind forces $W_t(x)$ and $W_c(x)$ and the corresponding deformations will produce some change in the cable tension $H = H_w + H_s$ to $H = H_w + H_s + H_l$, where $H_w + H_s$ is the cable tension due to dead and live load and H_l is the additional tension due to lateral loads.

The length of the cable element dL under dead and live load is $dL^2 = dx^2 + dy^2$, dx and dy being projections in horizontal and vertical directions respectively. Due to the lateral forces as for instance wind pressure, the length of the cable element will change to $dL + \varepsilon dL$ with projections $dx + d\xi$ along the span, $dy + d\eta$ and $d\Delta$ in vertical and lateral directions respectively. We have:

$$(dL + \varepsilon dL)^2 = (dx + d\xi)^2 + (dy + d\eta)^2 + d\Delta^2.$$

Assuming $d\xi^2$ to be a small quantity of higher order we obtain the following equations:

$$\frac{H_l}{E_c A_c} L_s = \frac{1}{2}\int (\Delta'(x))^2 dx + \int y'\eta' dx + \frac{1}{2}\int (\eta')^2 dx,$$

or:

$$\frac{H_l}{E_c A_c} L_s = \frac{1}{2}\int (\Delta'(x))^2 dx - y''\int \eta dx + \frac{1}{2}\int (\eta')^2 dx. \qquad (3)$$

For the vertical deformation η the following expression may be obtained:

$$\eta = \frac{H_l 16 f}{H c^2}\,\frac{\sinh\frac{cx}{2l}\sinh\frac{c}{2}\left(1 - \frac{x}{l}\right)}{\cosh\frac{c}{2}} - \frac{H_l}{H}y, \quad \text{where} \quad c = l\sqrt{\frac{H}{I_t E}}.$$

I_t is the moment of inertia of the stiffening girder. Inserting this expression for η in equation (3) and neglecting the therm $\frac{1}{2}\int(\eta')^2 dx$ as a small quantity of higher order, we obtain:

$$H_l = \frac{1}{2}\,\frac{\int (\Delta'(x))^2 dx}{\frac{L_s}{E_c A_c} + \frac{16 f^2}{3 l H}\left[\frac{12}{c^2}\left(\frac{2}{c}\tanh\frac{c}{2} - 1\right) + 1\right]} \approx \frac{3 l H}{32 f^2}\int(\Delta'(x))^2 dx. \qquad (4)$$

Equation (4) determines H_l and thus $H = H_w + H_s + H_l$.

It may be shown that the additional cable tension H_l is small, usually less than 1 $^0/_{00}$ of H and thus of no importance in comparison with H_w and H_s. We get for instance in the calculated example: For lateral truss hinged at the towers $H_l = 42{,}6$ tons or 0,95 $^0/_{00}$ of H which is 44700 tons. For continuous lateral truss we get $H_l = 13{,}3$ tons or 0,3 $^0/_{00}$ of H.

The increase in cable force in this exceptionally long suspension bridge is insignificant and as H_l will be comparatively less in smaller bridges we may always put $H_l = 0$ and $H = H_w + H_s$ without making any error noticeable in the results of the computations.

By putting $H_l = 0$ and $H = H_w + H_s =$ constant for a given vertical loading of the bridge, the principle of superposition will be valid for all lateral loadings. In other words we may divide the lateral loadings into symmetrical and asymmetrical parts and treat these independently of each other. This will in many cases simplify the computations considerably, but is otherwise of no importance for the deductions.

We return to equations (1) and (2). These can only be solved by laborious numerical computations by successive approximations [3]. Especially in the case of unsymmetrical bridges or complicated distribution of the lateral forces $W_t(x)$ these computations become very troublesome.

If we can put the stiffness of the lateral truss $IE =$ constant within each span, the equations (1) and (2) may be solved to any degree of accuracy. We rewrite the equations:

$$H\Delta''(x) = -W_c(x) - q(x)\,\frac{\delta(x) - \Delta(x)}{h(x)}, \tag{5}$$

and:

$$IE\delta''''(x) = -M''(x) = W_t(x) - q(x)\,\frac{\delta(x) - \Delta(x)}{h(x)}. \tag{6}$$

We introduce the following substitution into equations (5) and (6):

$$q(x)\,\frac{\delta(x) - \Delta(x)}{h(x)} = \sum A_r \sin \pi r\,\frac{x}{l} - B_0 - B_l\,\frac{x}{l}. \tag{7}$$

By this substitution equations (5) and (6) become directly integrable and we have only to take into account the different boundary conditions at the ends of each span.

Solution for Bridges with Continuous or Discontinuous Lateral Truss.

From as suspension bridge with an arbitrary number of spans we cut out one span, as shown in Figs. 1 and 2, for closer examination.

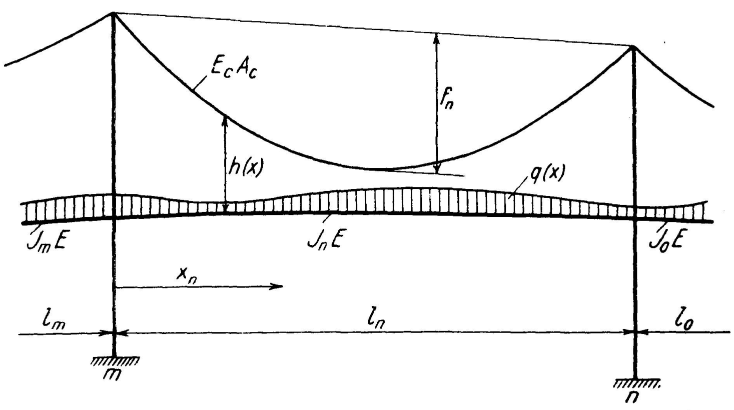

Fig. 1

In the investigation we at first consider the moments at the supports M_m and M_n as known, and indicate their deformations of the simply supported lateral truss with $M_m\delta_m(x)$ and $M_n\delta_n(x)$, where $\delta_m(x)$ and $\delta_n(x)$ indicate the deformations due to moments $M_m = 1$ and $M_n = 1$ respectively. For a constant stiffness of the lateral truss we have:

$$M_m \delta_m(x) = M_m \frac{l^2}{6EI} \frac{l-x}{l}\left(1 - \left(\frac{l-x}{l}\right)^2\right);$$

$$M_n \delta_n(x) = M_n \frac{l^2}{6EI} \frac{x}{l}\left(1 - \left(\frac{x}{l}\right)^2\right).$$

Due to the lateral forces on the span under consideration and on the neighbour spans, the towers will suffer certain deflections, see Fig. 3. These deformations of the towers called η, ν and ξ, must be assumed given or determined by a preliminary computation.

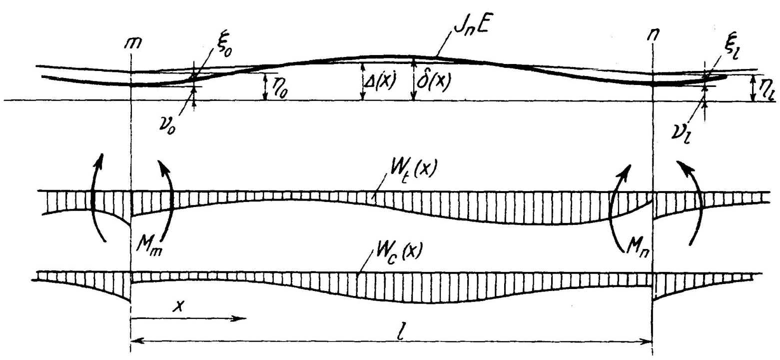

Fig. 2

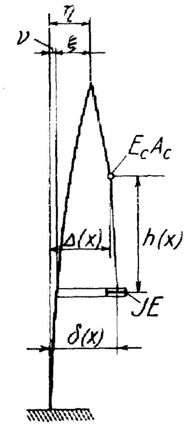

Fig. 3

Introducing substitution (7) into equation (5) and integrating twice we obtain:

$$\Delta(x) = -\frac{1}{H}\iint W_c(x)dx + \frac{l^2}{\pi^2 H}\sum \frac{A_r}{r^2}\sin \pi r \frac{x}{l} + \frac{1}{2H}B_0 x^2 + \frac{1}{6Hl}B_l x^3 + Cx + D.$$

By introducing: $-\frac{1}{H}\iint W_c(x)dx = \frac{M_{wc}}{H} = \Delta_0(x)$, which is equal to the lateral deflection of a free hanging cable under the lateral load $W_c(x)$, and the boundary conditions: $x=0$; $\Delta(x)=\eta_0$ and $x=l$; $\Delta(x)=\eta_l$, we obtain $D=\eta_0$; $C=-\frac{B_0}{2H}l - \frac{B_l}{6H}l + \frac{\eta_l - \eta_0}{l}$, and with all this introduced in the equation:

$$\Delta(x) = \Delta_0(x) + \frac{l^2}{\pi^2 H}\sum \frac{A_r}{r^2}\sin \pi r \frac{x}{l} + \frac{B_0}{2H}(x^2 - lx) + \frac{B_l}{6Hl}(x^3 - l^2 x)$$
$$+ \eta_0 \frac{l-x}{l} + \eta_l \frac{x}{l}. \tag{8}$$

By integrating equation (6) twice in the same way as done above, and taking into account the boundary conditions at the supports we get:

$$M(x) = M_0(x) + M_m \frac{l-x}{l} + M_n \frac{x}{l} - \frac{l^2}{\pi^2}\sum \frac{A_r}{r^2}\sin \pi r \frac{x}{l} - \frac{B_0}{2}(x^2 - lx)$$
$$-\frac{B_l}{6l}(x^3 - l^2 x), \tag{9}$$

where $M_0(x)$ is the moment on the simply supported truss under the load $W_t(x)$. M_m and M_n are the moments at the supports and are temporarily considered to be known.

By integrating the equation (9) twice we obtain the following equation:

$$\delta(x) = \delta_0(x) + M_m \delta_m(x) + M_n \delta_n(x) - \frac{l^4}{\pi^4 IE} \sum \frac{A_r}{r^4} \sin \pi r \frac{x}{l}$$

$$+ \frac{B_0}{24 EI}(x^4 - 2lx^3 + l^3 x) + \frac{B_l}{360 IE}(3x^5 - 10l^2 x^3 + 7l^4 x)$$

$$+ \nu_0 \frac{l-x}{l} + \nu_l \frac{x}{l}, \tag{10}$$

where $\delta_0(x)$ is the deflection of the simply supported truss under the load $W_c(x)$, the deflection due to the moments at supports $M_m\delta_m(x)$ and $M_n\delta_n(x)$ and the boundary conditions $\delta(0) = \nu_0$; $\delta(l) = \nu_l$.

We have thus found the necessary expressions for computing $\delta(x)$ and $\Delta(x)$. It remains to determine the constants A_r; B_0 and B_l. The moments M_m and M_n are still considered to be known.

To determine the coefficients A_r, B_0 and B_l we produce another set of equations by demanding the substitution (7) to be satisfied when we introduce $\delta(x)$ and $\Delta(x)$ from equations (8) and (10) into equation (7), or:

$$\delta(x) - \Delta(x) = \frac{h(x)}{q(x)} \sum A_r \sin \pi r \frac{x}{l} - \frac{h(x)}{q(x)} B_0 - \frac{h(x)}{q(x)} B_l \frac{x}{l} = \delta_0(x)$$

$$+ M_m \delta_m(x) + M_n \delta_n(x) - \frac{l^4}{\pi^4 IE} \sum \frac{A_r}{r^4} \sin \pi r \frac{x}{l} + \frac{B_0}{24 IE}(x^4 - 2lx^3 + l^3 x)$$

$$+ \frac{B_l}{360 IE}(3x^5 - 10l^2 x^3 + 7l^4 x) + \nu_0 \frac{l-x}{l} + \nu_l \frac{x}{l} - \Delta_0(x)$$

$$- \frac{l^2}{\pi^2 H} \sum \frac{A_r}{r^2} \sin \pi r \frac{x}{l} - \frac{B_0}{2H}(x^2 - lx) - \frac{B_l}{6Hl}(x^3 - l^2 x) - \eta_0 \frac{l-x}{l} - \eta_l \frac{x}{l}.$$

Arranging this yields the following equation:

$$\sum A_r \left(\frac{h(x)}{q(x)} + \frac{l^4}{\pi^4 IE r^4} + \frac{l^2}{\pi^2 H r^2}\right) \sin \pi r \frac{x}{l} = B_0 \left(\frac{h(x)}{q(x)} + \frac{x^4 - 2lx^3 + l^3 x}{24 IE}\right.$$

$$\left. - \frac{x^2 - lx}{2H}\right) + B_l \left(\frac{h(x)}{q(x)} \frac{x}{l} + \frac{3x^5 - 10l^2 x^3 + 7l^4 x}{360 IEl} - \frac{x^3 - l^2 x}{6Hl}\right) + \delta_0(x)$$

$$+ M_m \delta_m(x) + M_n \delta_n(x) - \Delta_0(x) - \xi_0 \frac{l-x}{l} - \xi_l \frac{x}{l}. \tag{11}$$

As we have $\eta - \nu = \xi$.

To determine the series $\sum A_r$ with k terms we must demand equation (11) to be satisfied for $k+2$ different values of x. Two of these values naturally will be for $x = 0$ and $x = l$. For these values we get:

$$B_0 = \xi_0 \frac{q(0)}{h(0)} \quad \text{and} \quad B_l = \xi_l \frac{q(l)}{h(l)} - \xi_0 \frac{q(0)}{h(0)}. \tag{12}$$

The two constants B_0 and B_l may thus be computed directly when the deflections of the towers are known.

In equation (11) remains as unknown coefficients only A_r, as M_m and M_n are considered to be known. To determine the series with k terms the necessary equations are produced by demanding equation (11) to be satisfied for the values $x_1, x_2, \ldots x_k$. For all other values of x the equation (11) will be approximately satisfied.

How many terms $A_1, A_2, A_3, \ldots$ must enter into a computation depends on the size of the bridge and the loadings. For smaller bridges 1—2 terms and for larger ones 3—5 terms will be sufficient.

As mentioned before, we may assume the principle of superposition valid without restriction. For bridges symmetrical about the centre the lateral loadings may be divided into a symmetrical and an asymmetrical part and we may determine the coefficients A_r by a set of equations corresponding to the symmetrical loading for r odd, and one corresponding to an asymmetrical loading for r even. Especially for very big bridges where the series have to be determined by many terms this will mean a considerable reduction of the computations.

Equation (11) gives an expression for A_r of the form:

$$A_r = \alpha_r + \beta_r M_m + \gamma_r M_n.$$

The first term α_r is the value we get for A_r when the lateral truss is hinged at the towers.

It now remains to determine the moments at the supports M_m and M_n. This is obtained by putting $\delta'(l)_m = \delta'(0)_n$ at the towers, the index m and n refers to the number of the span under consideration. For the slope of the lateral truss we have:

$$\delta'(x) = \delta_0'(x) + M_m \delta_m'(x) + M_n \delta_n'(x) - \frac{l^3}{\pi^3 IE} \sum \frac{A_r}{r^3} \cos \pi r \frac{x}{l}$$
$$+ B_0 \frac{4x^3 - 6lx^2 + l^3}{24 IE} + B_l \frac{15x^4 - 30l^2x^2 + 7l^4}{360 l IE} - (\nu_0 - \nu_l)\frac{1}{l}. \tag{13}$$

For the ends of the span we get:

$$\left.\begin{aligned}
\delta'(0) &= \delta_0'(0) + M_m \delta_m'(0) + M_n \delta_n'(0) - \frac{l^3}{\pi^3 IE} \sum \frac{A_r}{r^3} + B_0 \frac{l^3}{24 IE} \\
&\quad + B_l \frac{7 l^3}{360 IE} + \frac{\nu_l - \nu_0}{l}. \\
\delta'(l) &= \delta_0'(l) + M_m \delta_m'(l) + M_n \delta_n'(l) - \frac{l^3}{\pi^3 IE} \sum \frac{A_r}{r^3}(-1)^r \\
&\quad - B_0 \frac{l^3}{24 IE} - B_l \frac{8 l^3}{360 IE} + \frac{\nu_l - \nu_0}{l}
\end{aligned}\right\} \tag{14}$$

where M_m and M_n also enter into the coefficients A_r.

As we assume the stiffness of the lateral truss to be constant within each span, we get directly:

$$\delta_m'(0) = \frac{l}{3IE}; \qquad \delta_m'(l) = -\frac{l}{6IE} \quad \text{and} \quad \delta_n'(0) = \frac{l}{6IE}; \qquad \delta_n'(l) = -\frac{l}{3IE}.$$

By putting the slope of the ends of two neighbour spans meeting at the tower m equal, and arranging the equations in regard to the unknown quantities M_l, M_m and M_n we obtain an equation of the form:

$$a_m M_l + b_m M_m + c_m M_n = d_m. \tag{15}$$

For each tower we get an equation of this form, and the moments at the towers are determined by solving a set of linear equations of the type of equation (15).

When the moments at the towers have been found, these are introduced into the formula for the coefficients A_r which are of the form:

$$A_r = \alpha_r + \beta_r M_m + \gamma_r M_n,$$

and thus all data necessary for computing load transfers, shearing forces, moments and deformations in a continuous lateral truss have been found.

In order to carry out a complete calculation of the lateral truss and the towers we have in addition to equations (8), (9) and (10) also to deduce formulas for the shearing forces V and T in lateral truss and cables respectively. For the shear V we have:

$$V(x) = \frac{dM(x)}{dx} = V_0(x) - \frac{M_m - M_n}{l} - \frac{l}{\pi}\sum\frac{A_r}{r}\cos\pi r\frac{x}{l} - B_0\frac{2x-l}{2} - B_l\frac{3x^2-l^2}{6l}, \tag{16}$$

where $V_0(x)$ is the shear in the simply supported lateral truss under the loading $W_t(x)$.

The cables will transmit a shearing force to the towers equal to:

$$T(x) = H\frac{d\Delta(x)}{dx} = V_c(x) + \frac{l}{\pi}\sum\frac{A_r}{r}\cos\pi r\frac{x}{l} + B_0\frac{2x-l}{2} + B_l\frac{3x^2-l^2}{6l} + \frac{\eta_l-\eta_0}{l}H$$

where $V_c(x)$ is the shear due to the loading $W_c(x)$ acting on the cable as a simply supported beam.

At the towers with $x=0$ and $x=l$ respectively, we get the reactions on the towers:

$$\left.\begin{aligned} V(0) &= V_0(0) - \frac{M_m - M_n}{l} - \frac{l}{\pi}\sum\frac{A_r}{r} + B_0\frac{l}{2} + B_l\frac{l}{6},\\ &\text{and:}\\ V(l) &= V_0(l) - \frac{M_m - M_n}{l} - \frac{l}{\pi}\sum\frac{A_r}{r}(-1)^r - B_0\frac{l}{2} - B_l\frac{l}{3}.\end{aligned}\right\} \tag{17}$$

$$\left.\begin{aligned} T(0) &= V_c(0) + \frac{l}{\pi}\sum\frac{A_r}{r} - B_0\frac{l}{2} - B_l\frac{l}{6} + \frac{\eta_l-\eta_0}{l}H\\ &\text{and:}\\ T(l) &= V_c(l) + \frac{l}{\pi}\sum\frac{A_r}{r}(-1)^r + B_0\frac{l}{2} + B_l\frac{l}{3} + \frac{\eta_l-\eta_0}{l}H.\end{aligned}\right\} \tag{18}$$

Thus we have determined all the equations required for the computation of the lateral truss and the towers of a continuous or discontinuous suspension bridge under any lateral loading $W_t(x)$ and $W_c(x)$. For a discontinuous bridge we have only to introduce M_m and M_n as zero.

When we compute bridges where the deflections of the towers may be expected to be of some importance, a preliminary design must be made where the towers are assumed to be undeformable. In this computation the constants B_0 and B_l will equal zero, and it will be sufficient to determine the series A_r with two or three terms. With the results of this preliminary computation the lateral forces acting on the towers are determined from the equations given above. Besides this we have an axial force in the towers. With all forces acting on the towers given we may calculate the tower deflections.

Usually the tower deflections obtained from the preliminary and the exact calculations will differ but little.

If the moment of inertia I of the lateral truss is varying, for instance as shown in Fig. 4, we may approximately take into account the effect of this variation by introducing the actual deflections $\delta_m(x)$ and $\delta_n(x)$ which the moments $M_m = 1$ and $M_n = 1$ will produce in the truss acting as a simply supported beam. This will not be quite correct but it will be a very good and easy approximation, which will give a sufficient degree of accuracy.

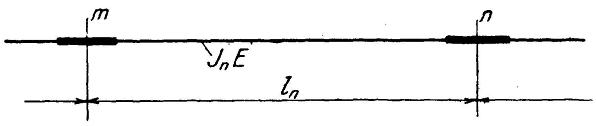

Fig. 4

The method for the calculation given above may be used on suspension bridges of quite an arbitrary form and for any combination of loads. We may for instance work out the calculation for a moving concentrated load and use the results to design the influence lines.

Example.

In the following we shall employ the method presented on the bridge shown in Fig. 5[8].

As shown in Figs. 5 and 6 the bridge will, due to unsymmetrical live load, $p(x) = 9$ t/m., have to be considered as an unsymmetrical bridge loaded with an unsymmetrical live load.

We have the following data:

$$l_1 = 300 \text{ m.}, \; l_2 = 1200 \text{ m.}, \; l_3 = 300 \text{ m.}, \; I_1E = I_2E = I_3E = 2 \times 10^9 \text{ tm}^2.$$

$H = 44700$ tons. The remaining dimensions may be seen in Fig. 5. Weight of cable $q_c = 9$ t/m. Weight of lateral truss and roadway $q_t = 23{,}75$ t/m.

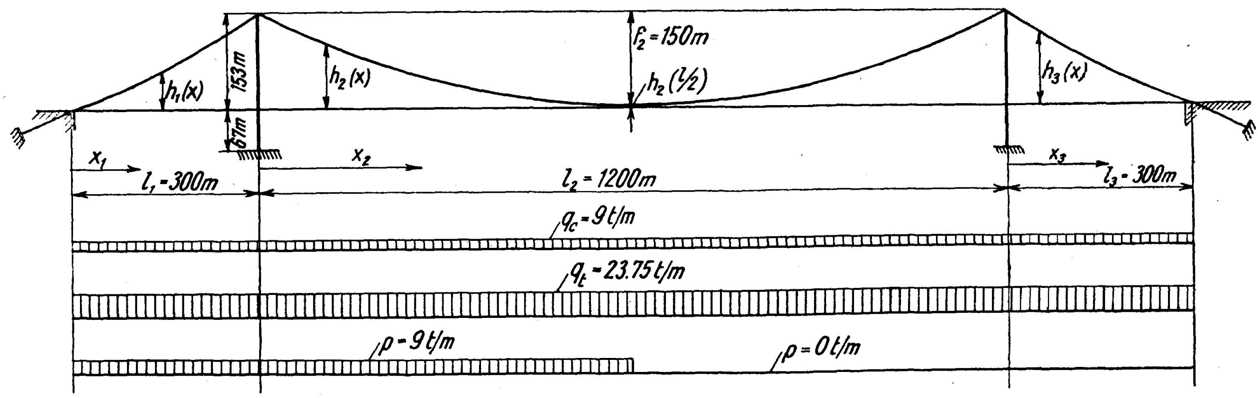

Fig. 5

Live load $p = 9$ t/m. load carried by the hangers, owing to dead load $q(x) = q_t = 23{,}75$ t/m., owing to combined dead and live load $q(x) = q_t + p = 32{,}75$ t/m. We assume the stiffening girder to be too flexible for distribution of the load on the hangers. At the centre of the bridge we assume $q(x) = q_t + \frac{1}{2} p = 28{,}25$ t/m. Wind forces on the cables: $W_c(x) = 0{,}2$ t/m. Wind forces on the unloaded part of bridge: $W_t(x) = 1{,}8$ t/m., and on the loaded part: $W_t(x) = 2{,}3$ t/m.

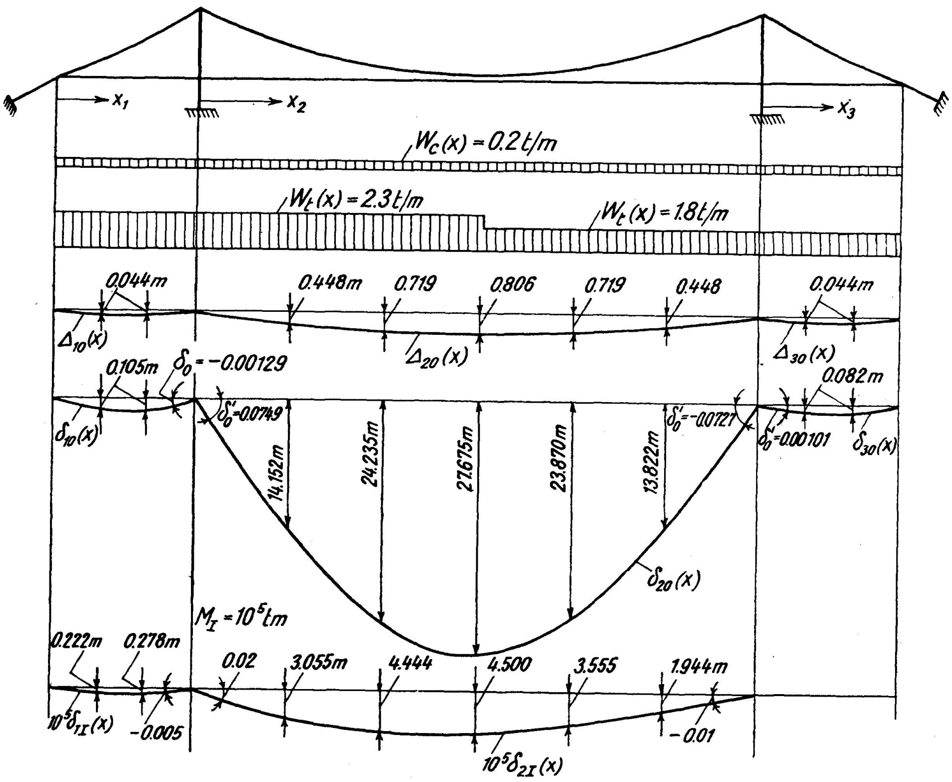

Fig. 6

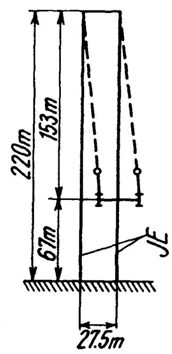

Fig. 7

The wind loadings $W_c(x)$ and $W_t(x)$ together with the functions $\delta_0(x)$; $\Delta_0(x)$ and $\delta_I(x)$ are shown in Fig. 6, the slope $\delta_0'(x)$ and $\delta_I'(x)$ at the supports are also shown. Fig. 7 shows dimensions of the towers. The lateral stiffness of the towers is: $IE = 3{,}5 \times 10^9$ t/m². As towers with this stiffness suffer considerable deformations these will have to be taken into consideration.

To distinguish the three spans from each other all coefficients and data carry a double subscript, for instance A_{25} is the coefficient A_5 in span no. 2.

Preliminary calculations of tower reactions with the series A_r determined by three terms for main span and by two terms for side spans and with the towers considered to be nondeformable and the lateral truss to be hinged, yield loadings on the towers corresponding to the following deformations:

For tower I: $\eta_I = 0{,}807$ m; $\nu_I = 0{,}101$ m and $\xi_I = 0{,}708$ m.
For tower II: $\eta_{II} = 0{,}773$ m; $\nu_{II} = 0{,}095$ m and $\xi_{II} = 0{,}678$ m.

These deformations will enter into the following calculations and will here be used both for the case of hinged and continuous lateral trusses.

Determination of the coefficients B_0 and B_1 (Eq. 12):

$$B_{10}=0; \quad B_{1l}=0{,}708\,\frac{32{,}75}{153}=0{,}1518\,\text{t/m}; \quad B_{20}=0{,}708\,\frac{32{,}75}{153}=0{,}1518\,\text{t/m};$$

$$B_{2l}=0{,}678\,\frac{23{,}75}{153}-0{,}1518=-0{,}0465\,\text{t/m}; \quad B_{30}=0{,}678\,\frac{23{,}75}{153}=0{,}1054\,\text{t/m};$$

$$B_{3l}=-B_{30}=-0{,}1054\,\text{t/m}.$$

Thus all external forces required for computing the coefficient A_r are given. We determine the series A_r with 2 terms for the side spans and with 5 terms for the main span as this yields sufficiently accurate results.

For span No. 1 we have at point $x=\frac{1}{3}\,l$ equation (11):

$$A_{11}\left(\frac{42{,}16}{32{,}75}+\frac{300^4}{\pi^4\cdot 2\cdot 10^9}+\frac{300^2}{\pi^2\cdot 44700}\right)\sin\frac{\pi}{3}+A_{12}\left(\frac{42{,}16}{32{,}75}+\frac{300^4}{\pi^4\cdot 2\cdot 10^9\cdot 2^4}\right.$$

$$\left.+\frac{300^2}{\pi^2\cdot 44700\cdot 2^2}\right)\sin\frac{2\pi}{3}=0{,}1518\left(\frac{42{,}16}{32{,}75}\,\frac{1}{3}+\frac{3\left(\frac{1}{3}\right)^5-10\left(\frac{1}{3}\right)^3+7\,\frac{1}{3}}{360\cdot 2\cdot 10^9}\,300^4\right.$$

$$\left.-\frac{\left(\frac{1}{3}\right)^3-\frac{1}{3}}{6\cdot 44700}\,300^2\right)+0{,}105+M_l\,2{,}22\cdot 10^{-6}-0{,}044-0{,}708\,\frac{1}{3}.$$

For span No. 2 we get at point $x=\frac{1}{6}\,l$ from equation (11):

$$A_{21}\left(\frac{69{,}7}{32{,}75}+\frac{1200^4}{\pi^4 2\cdot 10^9}+\frac{1200^2}{\pi^2 44700}\right)\sin\frac{\pi}{6}+A_{22}\left(\frac{69{,}7}{32{,}75}+\frac{1200^4}{\pi^4 2\cdot 10^9 2^4}\right.$$

$$\left.+\frac{1200^2}{\pi^2 44700\cdot 2^2}\right)\sin\frac{\pi}{3}+A_{23}\left(\frac{69{,}7}{32{,}75}+\frac{1200^4}{\pi^4 2\cdot 10^9 3^4}+\frac{1200^2}{\pi^2 44700\cdot 3^2}\right)\sin\frac{\pi}{2}$$

$$+A_{24}\left(\frac{69{,}7}{32{,}75}+\frac{1200^4}{\pi^4 2\cdot 10^9 4^4}+\frac{1200^2}{\pi^2 44700\cdot 4^2}\right)\sin\frac{2\pi}{3}+A_{25}\left(\frac{69{,}7}{32{,}75}\right.$$

$$\left.+\frac{1200^4}{\pi^4 2\cdot 10^9\cdot 5^4}+\frac{1200^2}{\pi^2 44700\cdot 5^2}\right)\sin\frac{5\pi}{6}$$

$$=0{,}1518\left(\frac{69{,}7}{32{,}75}+\frac{\left(\frac{1}{6}\right)^4-2\left(\frac{1}{6}\right)^3+\frac{1}{6}}{24\cdot 2\cdot 10^9}\,1200^4-\frac{\left(\frac{1}{6}\right)^2-\left(\frac{1}{6}\right)}{2\cdot 44700}\,1200^2\right)$$

$$-0{,}0465\left(\frac{69{,}7}{32{,}75}\,\frac{1}{6}+\frac{3\left(\frac{1}{6}\right)^5-10\left(\frac{1}{6}\right)^3+7\,\frac{1}{6}}{360\cdot 2\cdot 10^9}\,1200^4-\frac{\left(\frac{1}{6}\right)^3-\frac{1}{6}}{6\cdot 44700}\,1200^2\right)$$

$$+14{,}152+M_l 3{,}055\cdot 10^{-5}+M_{II} 1{,}944\cdot 10^{-5}-0{,}448-0{,}708\,\frac{5}{6}-0{,}678\,\frac{1}{6}.$$

And for span No. 3 equation (11) will become for $x=\frac{1}{3}\,l$:

$$A_{31}\left(\frac{93{,}16}{23{,}75}+\frac{300^4}{\pi^4 2\cdot 10^9}+\frac{300^2}{\pi^2 44700}\right)\sin\frac{\pi}{3}+A_{32}\left(\frac{93{,}16}{23{,}75}+\frac{300^4}{\pi^4 2\cdot 10^9\cdot 2^4}\right.$$

$$\left.+\frac{300^2}{\pi^2 44700\cdot 2^2}\right)\sin\frac{2\pi}{3}=0{,}1054\left(\frac{93{,}16}{23{,}75}+\frac{\left(\frac{1}{3}\right)^4-2\left(\frac{1}{3}\right)^2+\frac{1}{3}}{24\cdot 2\cdot 10^9}300^4\right.$$

$$\left.-\frac{\left(\frac{1}{3}\right)^2-\frac{1}{3}}{2\cdot 44700}300^2\right)-0{,}1054\left(\frac{93{,}16}{23{,}75}\,\frac{1}{3}+\frac{3\left(\frac{1}{3}\right)^5-10\left(\frac{1}{3}\right)^3+7\frac{1}{3}}{360\cdot 2\cdot 10^9}300^4\right.$$

$$\left.-\frac{\left(\frac{1}{3}\right)^3-\frac{1}{3}}{6\cdot 44700}300^2\right)+0{,}082+M_{II}\,2{,}78\cdot 10^{-6}-0{,}044-0{,}678\,\frac{2}{3}.$$

The remaining equations for the three spans are found in just the same way. The equations are given in the tables below.

Span No. 1.

$x_1 =$	A_{11}	A_{12}	$N_1 =$	$M_I 10^{-6}$
$^1/_3\, l_1$	1,329	1,161	-0,091	2,22
$^2/_3\, l_1$	2,633	-2,510	-0,104	2,78

Span No. 2.

$x_2 =$	A_{21}	A_{22}	A_{23}	A_{24}	A_{25}	$N_2 =$	$M_I 10^{-5}$	$M_{II} 10^{-5}$
$^1/_6\, l_2$	8,014	3,126	2,622	2,056	1,136	14,490	3,055	1,944
$^1/_3\, l_2$	12,431	1,786	0,	-0,727	-0,643	24,631	4,444	3,556
$^1/_2\, l_2$	14,006	0	-0,600	0	0,254	28,425	4,500	4,500
$^2/_3\, l_2$	12,624	-1,980	0	0,921	-0,837	24,098	3,556	4,444
$^5/_6\, l_2$	8,418	-3,825	3,424	-2,755	1,542	14,169	1,944	3,055

Span No. 3.

$x_3 =$	A_{31}	A_{32}	$N_3 =$	$M_{II} 10^{-6}$
$^1/_3\, l_3$	3,610	3,440	-0,089	2,78
$^2/_3\, l_3$	1,750	-1,583	-0,106	2,22

In the tables the sign N is used for the terms:

$$N=B_0\left(\frac{h(x)}{q(x)}+\frac{x^4-2lx^3+l^3x}{24IE}-\frac{x^2-lx}{2H}\right)+B_l\left(\frac{h(x)}{q(x)}\,\frac{x}{l}+\frac{3x^5-10l^2x^3+7l^4x}{360lIE}\right.$$

$$\left.-\frac{x^3-l^2x}{6Hl}\right)+\delta_0(x)-\Delta_0(x)-\xi_0\frac{l-x}{l}-\xi_l\frac{x}{l},\quad \text{see equation} \qquad (11)$$

The solving of the above equations for A_r give the following results:
For span No. 1:

$A_{11} = -0{,}0546 + 1{,}38\ M_I\ 10^{-6}$.
$A_{12} = -0{,}0158 + 0{,}34\ M_I\ 10^{-6}$.

For span No. 2:

$$A_{21} = 1{,}9812244 + 3{,}208163\ M_{\mathrm{I}}\ 10^{-6} + 3{,}183674\ M_{\mathrm{II}}\ 10^{-6}.$$
$$A_{22} = 0{,}1753247 + 2{,}245826\ M_{\mathrm{I}}\ 10^{-6} - 1{,}922078\ M_{\mathrm{II}}\ 10^{-6}.$$
$$A_{23} = -0{,}8877938 - 0{,}19105\ M_{\mathrm{I}}\ 10^{-6} - 0{,}60728\ M_{\mathrm{II}}\ 10^{-6}.$$
$$A_{24} = -0{,}1217391 - 0{,}70870\ M_{\mathrm{I}}\ 10^{-6} + 0{,}64783\ M_{\mathrm{II}}\ 10^{-6}.$$
$$A_{25} = 0{,}5642023 - 0{,}17445\ M_{\mathrm{I}}\ 10^{-6} + 0{,}17575\ M_{\mathrm{II}}\ 10^{-6}.$$

For span No. 3:

$$A_{31} = -0{,}0431 + 1{,}02\ M_{\mathrm{II}}\ 10^{-6}.$$
$$A_{32} = 0{,}0194 - 0{,}264\ M_{\mathrm{II}}\ 10^{-6}.$$

Lateral truss hinged at the towers.

For this case we have only to introduce $M_{\mathrm{I}} = M_{\mathrm{II}} = 0$ in the calculations above. All the coefficients A_r are then directly given and thus all necessary data for determining transfer of forces, shear, moment and deflections have been found. The results of these calculations are shown in Fig. 8. As may be seen, the transfer of force from lateral truss to cables will reduce the moment at the centre to about 23% of what it would have been in a simply supported truss.

Checking the deformations of the towers by the final values of the coefficients we get:

For tower I: $\eta_I = 0{,}835\,\mathrm{m}$; $\nu_I = 0{,}101\,\mathrm{m}$; $\xi_I = 0{,}734\,\mathrm{m}$.
For tower II: $\eta_{II} = 0{,}784\,\mathrm{m}$; $\nu_{II} = 0{,}095\,\mathrm{m}$; $\xi_{II} = 0{,}689\,\mathrm{m}$.

These values agree so well with those of the preliminary calculation that any correction is unnecessary.

Continuous lateral truss.

All the coefficients A_r are calculated above, it remains now to find the constraint moments M_{I} and M_{II}.

Introducing the calculated coefficients A_r into equations (14) and (15) and arranging we get the two following equations for the moments at the supports M_{I} and M_{II}:

Continuity at tower No. I gives:

$$0{,}15275\,M_{\mathrm{I}}\ 10^{-6} + 0{,}01829\ M_{\mathrm{II}}\ 10^{-6} = -0{,}025730.$$

Continuity at tower No. II gives:

$$0{,}01835\ M_{\mathrm{I}}\ 10^{-6} + 0{,}15503\ M_{\mathrm{II}}\ 10^{-6} = -0{,}024252.$$

The solving of this equations yields:

$$M_{\mathrm{I}} = -1515860\ \mathrm{tm.};\quad M_{\mathrm{II}} = -138460\ \mathrm{tm.}$$

We have now all data required to determine the final coefficients A_r, force transferred from lateral truss to cables, shear, moments, and lateral deflections of truss and cables. The results of these calculations are plotted in Fig. 9.

Checking the deformations of the towers we get:

For tower I: $\eta_I = 0{,}558\,\mathrm{m}$; $\nu_I = 0{,}083\,\mathrm{m}$; $\xi_I = 0{,}475\,\mathrm{m}$.
For tower II: $\eta_{II} = 0{,}506\,\mathrm{m}$; $\nu_{II} = 0{,}074\,\mathrm{m}$; $\xi_{II} = 0{,}432\,\mathrm{m}$.

In this case the deformations of the towers are considerably less than given by the preliminary calculation. However, another calculation with improved tower deflections would only change the results to a slight degree, and it will not be made here.

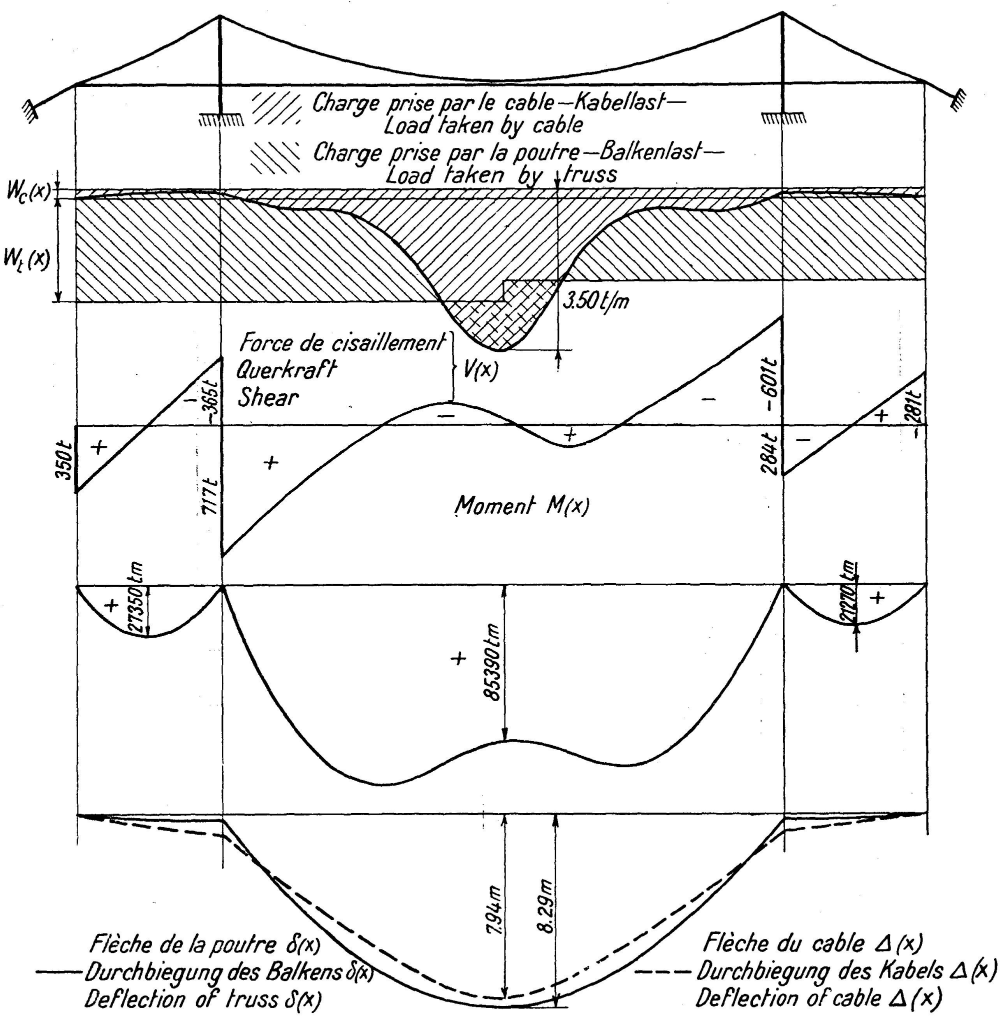

Fig. 8

As will be seen A_{25} turns out comparatively large in the calculations. This indicates that more terms should have been included, for instance A_{27}, in order to get a more correct representation of the force transferred from the lateral truss to the cables, see Fig. 8 and 9. In an unsymmetrical case like the present one this would increase the work of calculation very much. We have, however, for shearing force: $\frac{1}{5} A_{25}$, and for the moment: $\frac{1}{5^2} A_{25}$, so that in plotting diagrams for shear and moment the term A_{25} will yield a sufficient degree of accuracy, see Figs. 8 and 9.

By comparing the diagrams for moments and deflections in cases of continuous and hinged lateral trusses it will be seen that the advantage of making the truss continuous is the reduction of the lateral deflections. The maximum positive moment will of course be reduced, but on the other hand large moments will appear at the supports by far surpassing the positive moments at the middle of the span.

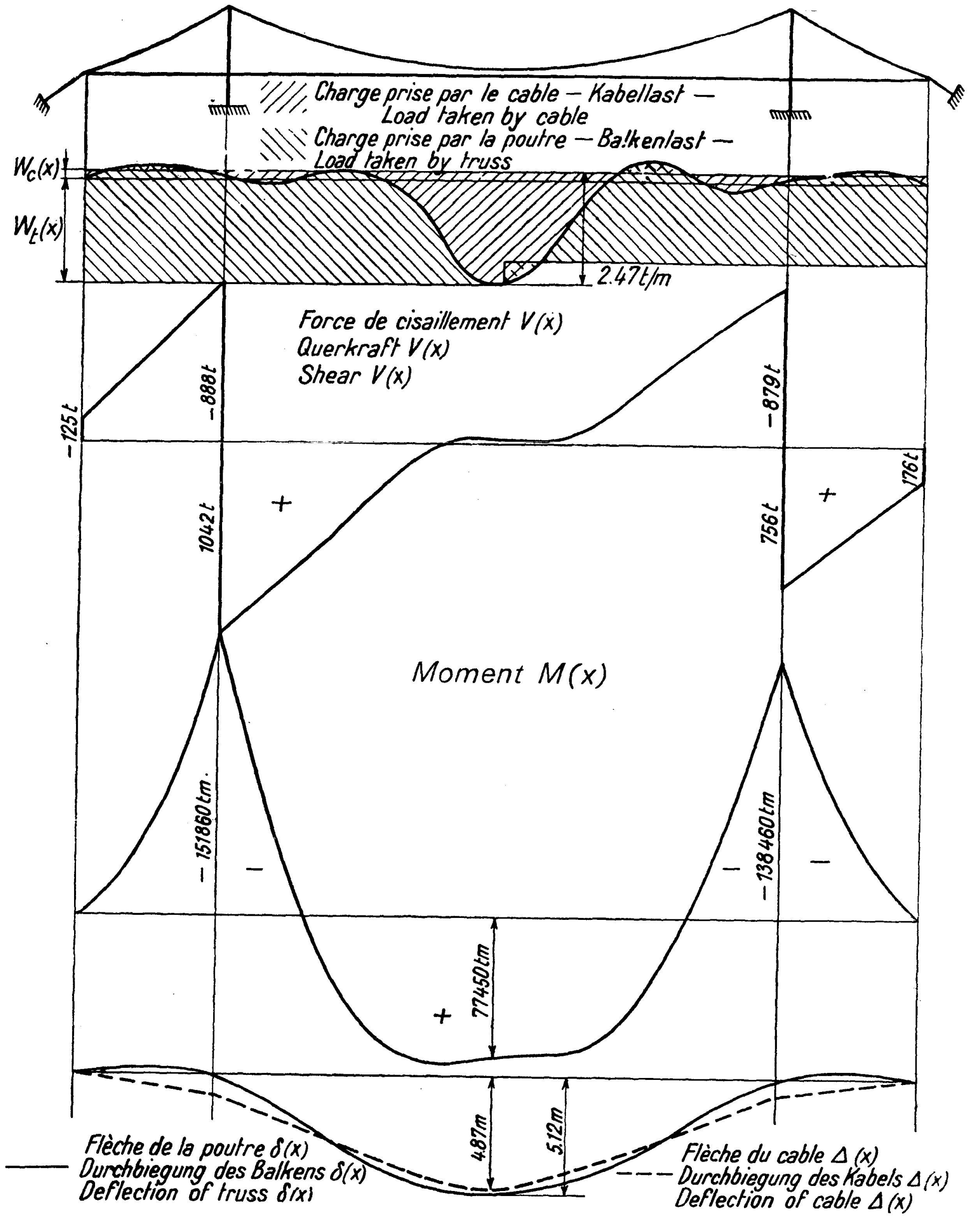

Fig. 9

As the chords of the lateral truss consist of the stiffening girders it will be easily seen that an increase in the cross sectional area of these, needed to take account of the large lateral moments at the towers, also gives an opportunity to increase the moment of inertia of the stiffening girder, which in turn will take into account the large moments in the stiffening girder at the towers, owing to vertical loadings. The recently brought-out constructions with continuous stiffening girders without supports at the towers (floating truss) should accordingly hardly be justified. In general we should bear in mind that for larger suspension bridges it will be the action as cross section in the lateral truss that gives the lower limit for the dimensions of the stiffening girder.

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Summary.

The calculation of the lateral truss of suspension bridges is achieved by expressing the cables relieving of the lateral truss as a FOURIER series. By this substitution all the equations become directly integrable, and we may find equations for the calculation of a suspension bridge of a completely arbitrary form across an arbitrary number of spans, and with continuous or hinged lateral truss.

Résumé.

Le calcul du treillis latéral des ponts suspendus est établi de la façon suivante: on exprime la décharge du treillis latéral due aux câbles par une série de FOURIER. Ceci permet d'intégrer toutes les équations du problème. Il est aussi possible d'établir les équations pour un pont suspendu de forme arbitraire à un nombre quelconque d'ouvertures, dont le treillis peut être continu ou interrompu.

Zusammenfassung.

Die Berechnung von Windverbänden der Hängebrücken wird so ausgeführt, daß man die Entlastung des Windverbandes durch die Kabel als eine Fouriersche Reihe ausdrückt. Durch diesen Ansatz werden alle Gleichungen direkt integrierbar. Es ist auch möglich, Gleichungen einer Hängebrücke mit willkürlicher Form und beliebiger Anzahl Felder, mit durchlaufendem oder unterbrochenem Windverband aufzustellen.

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