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SHEAR LAG BY RELAXATION

BERECHNUNG DER SCHUBVERFORMUNG MIT EINER ITERATIONSMETHODE

LA DÉFORMATION PAR CISAILLEMENT TRAITÉE À L'AIDE D'UNE MÉTHODE D'ITÉRATION

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1. The Problem of „shear lag" is one that arises in the design of welded gantry girders to carry heavy and fast-moving overhead cranes, more especially where they are welded together to give continuity over the supports. Lateral rigidity in such gantries is important and a modern requirement is a continuous steel plate gangway, or access platform, several feet wide, attached to the outer side of the top flange.

Considerations of economy suggest the use of the gangway as the web of a horizontal girder to resist lateral forces or „surge" from the cranes, but the maximum advantage is not obtained unless the sectional area of the plate can be incorporated with the top flange of the girder in resisting the vertical bending moments also. The weight of the wide thin gangway plate forms a large addition to that of the girder and to neglect it entirely in considering the vertical bending stress would be wasteful. On the other hand, the „shear lag" in the thin plate, even when adequately stiffened by longitudinal flats or angles, tends to lessen its effectiveness.

The object of this investigation was to assess the significance of the „shear lag" effect in practice and to determine, for a typical case, how it affects the distribution of the bending stress across the flange section as a whole.

2. The method of analysis mostly adopted is by FOURIER series which is lengthy even for uniform loading and uniformly thick plates, requiring the calculation of 18 to 30 terms of the type

$$\left(A \sinh\frac{2\pi n y}{l} + B\cosh\frac{2\pi n y}{l} + C\frac{2\pi n y}{l}\sinh\frac{2\pi n y}{l} + D\frac{2\pi n y}{l}\cosh\frac{2\pi n y}{l}\right)\cos\frac{2\pi n x}{l}$$

It would appear that this method is impracticable in this case.

It was decided to try the relaxation method, where values of stress or displacement are estimated for many points on the plate surface and are gradually improved by applying the control equations. This is a tedious process but has the advantage that it can be concentrated on any local region and is useful here since the shear lag affects the flange stress only in the region of the supports. A second advantage is the ease by which the various types of stress are found at each point, giving a graphic view of the state of stress. This leads naturally to the developing of an approximate formula described in Section 4. Approximations are well justified here, since the conditions at a support are never known exactly.

3. The nature of the problem is illustrated by the double cantilever of Fig. 1 with symmetrical loading. Considering the top flange in the region of the centre line C, shown in plan in Fig. 2, the longitudinal stress f is accompanied by shear q and transverse stress f^1. A transverse line such as AB instead of displacing to the right equally at A and B, is distorted by the shear to AB^1. When the distortions are the same for neighbouring lines, the stress f is not affected, which is the case at a distance further from the support than about $1\frac{1}{2}$ times the flange width. By symmetry the line OC does not distort and the shear stress must decrease from q at A to zero at 0. Hence the extension of CB is less than that of OA and the flange stress (f) decreases from the web to the edge in the region of the support. The amount of this reduction is the shear lag problem.

4. The solutions obtained by relaxation indicate that the flange stress (f) becomes uniform along a transverse line at a distance of about $1\frac{1}{2}$ times the flange width from the support (see Sections 9 and 10). From this an approximate formula may be obtained; its development is given in Section 7; the result is quoted here.

For constant shear force as in the cantilever of Fig. 1

$$f_m = f_0 - \frac{S}{Ah}\,\frac{5\,b}{9}$$

where f_m = average flange stress on line OC
f_0 = stress in centre of flange at O which is the maximum.
S = shear force
A = flange area (including allowance for web)
h = depth of girder
b = semi-width of flange.

The ratio f_m/f_0 is the important result giving the proportion of flange area which is effective. If $\frac{S}{Ah} = 1$, $b = 4$, $l = 18$, then $f_m = 18$ and $18 = f_0 - 2{,}22$ or $f_m/f_0 = 0{,}89$. This compares favourably with the result 0,88 obtained by relaxation in Section 10.

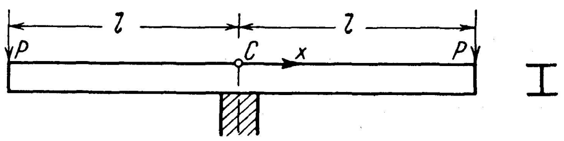

Fig. 1.
Double Cantilever — Doppelter Kragträger — Double port-à-faux.

For a continuous girder with uniform loading

$$f_m = f_0 - \frac{wb^2}{Ah}\left(\frac{5}{9}\,\frac{l}{b} - 0{,}79\right)$$

where w = load intensity
l = semi-span.

If $w = 9{,}02$ tons/ft., $b = 2{,}5$ ft., $l = 22{,}5$ ft., $A = 0{,}328$ ft.^{2}., $h = 4{,}83$ ft., then $f_m = 956$ tons/ft^2. at the support, and $956 = f_0 - 150$ or $f_m/f_0 = 0{,}865$. To compare this result with the problem solved by relaxation in Section 11 it would be nearer to take $f_0 = 956$ since the extension plate is a fraction of the total flange areas, which then gives $f_m = 956 - 150$, $f_m/f_0 = 0{,}843$. The result from Section 11 is 0,845.

5. The mathematical equations of this problem are

$\partial f/\partial x + \partial q/\partial y = 0$ (1)
$\partial f'/\partial y + \partial q/\partial x = 0$ (2)
$E\,\partial u/\partial x = f - \sigma f'$ (3)
$E\,\partial v/\partial y = f' - \sigma f$ (4)
$C(\partial u/\partial y + \partial v/\partial x) = q$ (5)

where u, v = displacements in x, y directions (see Fig. 2)
σ = Poisson's ratio
E, C = Young's, rigidity moduli

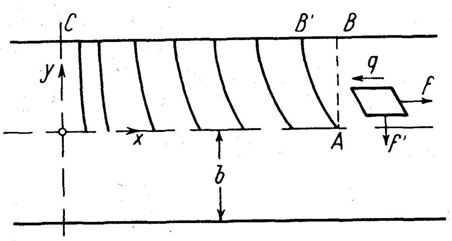

Fig. 2.
Plan View of Flange — Flanschdraufsicht — Plan de l'aile.

As will be shown later the stress f' is small and can be neglected. The boundary conditions for a uniformly thick flange are $q = 0$ on $x = 0$ and $y = b$ lines; $v = 0$ on $y = 0$ line. Since $q = 0$ on $x = 0$, $\partial q/\partial y = 0$ and hence $\partial f/\partial x = 0$ on this line. This appears to disagree with the bending moment diagram but can be explained by the effect of the width of the support rounding off the peak values. Since the amount of this rounding off cannot be found exactly and since the shear lag effects are greatest here, the problem has no exact solution and approximate formulae are justified.

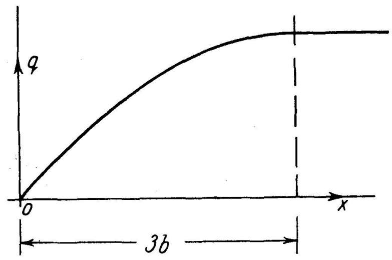

Fig. 3.
Variation of Shear near Support — Verlauf der Schubspannungen beim Auflager — Variation du cisaillement près de l'appui.

For regions where the flange stress is constant along a transverse line

$$f = \frac{M}{Ah} \quad \text{and} \quad \partial f/\partial x = \frac{dM/dx}{Ah} = \frac{-S}{Ah}$$

$\therefore\ \partial q/\partial y = \frac{S}{Ah}$ and $q = \frac{-S}{Ah}(b - y)$ for a uniformly thick flange; this is the well-known formula for shear stress in the flange.

6. To develop the approximate formula, consider first the case of constant shear force, Fig. 1. Assume that the reduction in shear stress near the support, discussed in Section 3, follows the curve shown in Fig. 3, a result

indicated by the relaxation process. Taking a parabola as the simplest form gives

$$q = -\frac{S}{Ah}(b-y)\left(\frac{2x}{3b} - \frac{x^2}{9b^2}\right)$$

$$\therefore\ C\frac{\partial u}{\partial y} = -\frac{S}{Ah}(b-y)\left(\frac{2x}{3b} - \frac{x^2}{9b^2}\right) - C\,\partial v/\partial x$$

The stress conditions on the $x=0$ line show $\partial f/\partial x = 0$ and it follows from equation (4), Section 5, if f' is neglected, that $\partial v/\partial x$ is also zero.

$$\therefore\ Cu = -\frac{S}{Ah}(by - \tfrac{1}{2}y^2)\left(\frac{2x}{3b} - \frac{x^2}{9b^2}\right) + Cu_0$$

where u_0 = displacement along $y=0$ line.

$$\therefore\ C\,\partial u/\partial x = -\frac{S}{Ah}(by - \tfrac{1}{2}y^2)\left(\frac{2}{3b} - \frac{2x}{9b^2}\right) + C\,\partial u_0/\partial x$$

and since $E\ \partial u/\partial x = f$,

$$f = -\frac{E}{C}\frac{S}{Ah}(by - \tfrac{1}{2}y^2)\left(\frac{2}{3b} - \frac{2x}{9b^2}\right) + f_0$$

which gives the reduction in stress across the flange.

To find the effective area of flange it is necessary to find the average stress f_m

$$f_m = \frac{1}{b}\int_0^b f\,dy = f_0 - \frac{E}{C}\frac{S}{Ah}\tfrac{1}{3}b\left(\tfrac{2}{3} - \frac{2x}{9b}\right)$$

The reduction is greatest at $x=0$ when, taking $E/C = 2\cdot 5$, $f_m = f_0 - \dfrac{S}{Ah}\dfrac{5b}{9}$, and the effective proportion of flange area is f_m/f_0.

7. The approximate formula is similarly calculated for a continuous girder with uniform loading. Here the shear force varies, but a parabola may be fitted to touch tangentially the sloping shear line at $x = 3b$, giving

$$q = -w(b-y)\left[\left(\frac{2l}{3b} - 1\right)x - \frac{lx^2}{9b^2}\right]$$

where w = load intensity
l = semi-span
b = semi-flange width.

Proceeding as in Section 6

$$f = f_0 - \frac{E}{C}\frac{w}{Ah}(by - \tfrac{1}{2}y^2)\left[\left(\frac{2l}{3b} - 1\right) - \frac{2lx}{9b^2}\right] + \frac{w}{Ah}\frac{\sigma y^2}{2}$$

$$f_m = f_0 - \frac{E}{C}\frac{w}{Ah}\cdot\tfrac{1}{3}b^2\left[\left(\frac{2l}{3b} - 1\right) - \frac{2lx}{9b^2}\right] + \frac{w}{Ah}\cdot\frac{\sigma b^2}{6}$$

At $x = 0$, $\quad f_m = f_0 - \dfrac{wb^2}{Ah}\left[\dfrac{5}{9}\dfrac{l}{b} - 0{,}79\right]$

8. The actual problem which gave rise to the investigation described in this paper is the crane girder illustrated in Figs. 4 and 5. The top flange had an extended thin plate to provide an access walk-way for the craneman. It was required to find the fall in stress in this plate and to estimate what proportion of its area should be included in the flange. The girders were continuous on 45 ft. spans and the maximum flange stress calculated by

the simple My/I formula was $6 \cdot 64$ tons/in² at the centre support when the two 200-ton cranes were placed as in Fig. 5. The maximum shear force was then 152 tons.

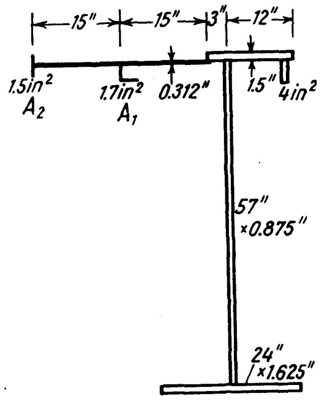

Fig. 4.

Section of Crane Girder — Querschnitt des Kranträgers — Section de la poutre de grue.

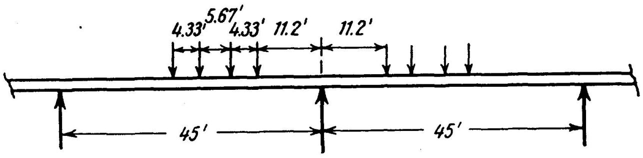

Fig. 5.

Position of the Two 200-Ton Cranes for Maximum Flange Stress. Wheel Loads each 54 Tons — Stellung der beiden 200 t Kranen für größte Flanschspannungen. Radlasten je 54 t — Position des deux grues à 200 t chacune, pour le maximum des contraintes dans les ailes. Charges roulantes de 54 t chacune.

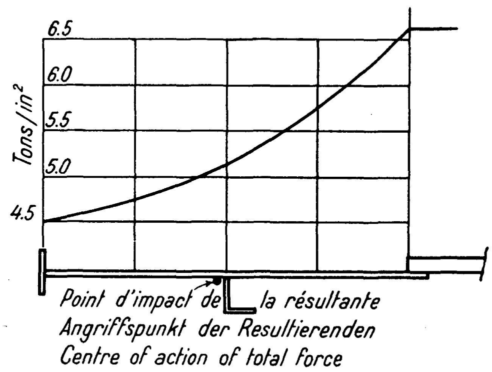

Fig. 6.

Variation of Flange Stress on Section at Centre of Support.
Verlauf der Flanschspannungen im Mittelquerschnitt des Trägers.
Variation des contraintes dans les ailes pour la section au centre de la poutre.

Proceeding as in Section 6,

$$q = -\frac{S}{Ah}\left(\frac{2x}{3b} - \frac{x^2}{9b^2}\right)\left(b - y + \frac{A_2}{t} + \left[\frac{A_1}{t}\right]\right) \quad \text{where } b = 30 \text{ in.}, \; t = 0{,}312 \text{ in.}$$

$$f = f_0 - \frac{E}{C}\frac{S}{Ah}\left(\frac{2}{3b} - \frac{2x}{9b^2}\right)\left(by - \tfrac{1}{2}y^2 + \frac{A_2 y}{t} + \frac{15\,A_1}{t}\right). \quad \left[\frac{A_1}{t}\right] \text{ from 0 to 15 in.}$$

The curve of stress at the central support ($x = 0$) is given in Fig. 6. The average value of the flange stress in the thin plate is $f_m = 5{,}20$, and the line of action of the total force is at $\bar{y} = 15{,}8$ in. The proportion of the thin plate area which is effective is $f_m/f_0 = 5{,}20/6{,}64 = 0{,}78$.

9. Relaxation process. By eliminating v from the equations (1), (3), (4) and (5) in Section 5, the following relation is found,

$$0 = \left(\frac{E}{C} - \sigma\right) \partial^2 u/\partial x^2 + \partial^2 u/\partial y^2 + \left(\frac{\sigma}{C} + \frac{1-\sigma^2}{E}\right) \partial f'/\partial x$$

or $$0 = 2{,}25\, \partial^2 u/\partial x^2 + \partial^2 u/\partial y^2 \qquad (6)$$

if $\partial f'/\partial x$ is neglected and $E/C - \sigma = 2 \cdot 25$.

The flange is then divided into rectangles and the values of u are estimated at the corners of these rectangles from the mean flange stress (f) and equation (3). The reduction of u on a transverse line is estimated from the shear strain as indicated in Section 6.

To apply equation (6) the differential coefficients are expressed by difference formulae. With the rectangles shown in Fig. 7 the values at the point (s) are

$$\partial^2 u/\partial x^2 = \frac{1}{m^2}(s_1 + s_2 - 2s)$$

$$\partial^2 u/\partial y^2 = \frac{1}{n^2}(r + t - 2s)$$

which are correct to the fourth order differential coefficients. By substituting in (6) and making $m = 1{,}5\,n$ to cancel the 2,25 in equation (6),

$$4s = s_1 + s_2 + r + t \qquad (7)$$

which holds throughout the flange; it is used to improve the estimated values of u to the required degree of accuracy, by repeated application.

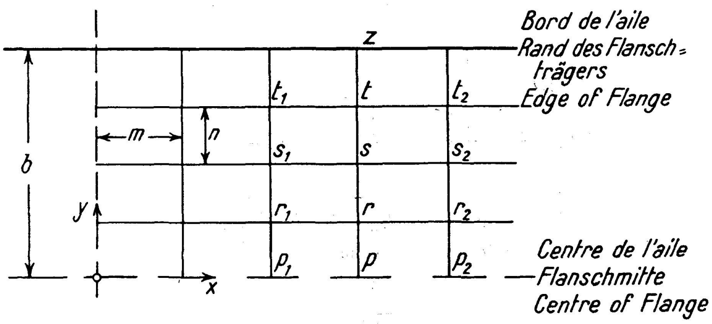

Fig. 7.
Division of Flange into Rectangles for Relaxation Process.
Mascheneinteilung des Flansches für die Iterationsmethode.
Subdivision de l'aile en rectangles pour le calcul d'itération.

The values of u on the edge of the flange are controlled by making use of the relation $q = 0$ on this edge. This becomes

$$\left.\begin{aligned} &\partial u/\partial y + \partial v/\partial x = 0 \\ &\partial u/\partial y = \frac{1}{m}\left({}^{11}/_{6}\, z - 3t + 1{,}5\, s - \tfrac{1}{3} r\right) \end{aligned}\right\} \qquad (8)$$

$$\partial v/\partial x = -\sigma \int_0^b \partial^2 u/\partial x^2 \cdot dy = -\frac{\sigma}{m^2} \int_0^b (s_1 + s_2 - 2s)\, dy$$

The integral can be evaluated by SIMPSON's Rule and thus an equation is obtained for the value of u on the edge.

It was found advantageous to deal with eight rectangles at a time by first obtaining from (7) and (8) the values of r, s, t, z in terms of p, $r_1 + r_2$, $s_1 + s_2$, $t_1 + t_2$, by solving four simultaneous algebraic equations. A table of coefficients can be formed and a routine worked out on the comptometer.

If it is decided to evaluate f^1 to find if it is in fact negligible, it is necessary first, to find v throughout by forming an equation similar to (7).

When the values of u are settled the stress f can be easily found throughout by equation (3); for example f at S in Fig. 7 $= \frac{E}{2m}(s_2 - s_1)$.

10. The example given in Figs. 1 and 2 was solved by the above process for the following dimensions $l = 18$, $b = 4$, $h = 8$, flange and web thickness $= 0{,}08$, and $S/Ah = 1$. The bending moment was rounded off over the support from $x = 0$ to $x = 1{,}5$ making $dM/dx = 0$ on $x = 0$ line.

After settling out the u values it is necessary to find if the mean stress agrees with the bending moments. Generally a small correction will be required for the u values along the $y = 0$ line which can be dealt with separately and added on.

The following values were obtained

x	1,5	3	4,5	6	7,5	9	10,5	12	13,5	15
f_0	18,8	16,5	14,5	12,5	10,7	9,09	7,43	5,88	4,41	2,94
f_m	16,5	15	13,5	12	10,5	9	7,5	6	4,5	3
f_m/f_0	0,88	0,91	0,93	0,96	0,98	0,99	1,01	1,02	1,02	1,02

Since $\partial f/\partial x$ is zero at $x = 0$ there will be little change between the $x = 0$ and $x = 1{,}5$ lines, thus 0,88 can be taken as the greatest reduction. The value of f_m/f_0 becomes constant at about $x = 12 = 3b$ which is the basis for the approximate formula.

11. A second example solved was for a section of the dimensions of Fig. 4 except that the thin stiffened extension plate was replaced by one of uniform thickness, 0,416 in., which gives the same gross area. The girder was taken as one span of a series of continuous spans carrying uniform loading $w = 9{,}02$ tons/ft.

The size of rectangle chosen was 0,625 ft. by 0,937 ft. This problem was explored thoroughly obtaining fields for u and v and calculating f, q and f^1 in the critical region. Fig. 8 gives the settled values of (Eu) and (Ev) in this region and Fig. 9 the values of f, q and f^1. Only in the very local region at the centre of the support does the value of f^1 attain the appreciable value of 5 % of the main stress. When this is multiplied by POISSON's ratio it becomes practically negligible (although it was allowed for in this case).

0		4	12	8	09	11	80	15	19	18	14	20	63	22	55	23	93	Edge of Flange / Rand d. Flanschtr. / Bord de l'aile
3	87	3	77	3	50	3	12	2	65	2	14	1	71	1	28	0	86	
0		4	25	8	34	12	11	15	51	18	47	20	90	22	81	24	19	
3	07	2	98	2	73	2	40	2	02	1	61	1	28	0	95	0	63	
0		4	46	8	73	12	64	16	10	19	04	21	46	23	34	24	67	
2	14	2	08	1	92	1	68	1	39	1	09	0	85	0	63	0	42	
0		4	90	9	53	13	67	17	26	20	23	22	60	24	45	25	79	
1	18	1	14	1	04	0	89	0	72	0	55	0	42	0	31	0	21	
0		5	61	10	82	15	29	18	98	21	97	24	31	26	14	27	42	Centre of Flange / Flanschmitte / Centre de l'aile
0		0		0		0		0		0		0		0		0		

Centre of Support
Stützenmitte
Centre de la poutre

Size of Rectangles 0,625 × 0,937 *ft.*
Größe der Rechtecke — Grandeur des rectangles

Fig. 8.

Values of Displacements u, v ($x\,E$ tons/in^2) in Region near Support
Werte der Verschiebungen u, v beim Auflager
Valeurs des déplacements u, v dans la région des supports

Upper Value / Oberer Wert / Valeur supérieure $= Eu$ — Lower Value / Unterer Wert / Valeur inférieure $= -Ev$

4	47	4	38	4	17	3	85	3	44	2	95	2	40	1	79	1	20	
0		−	01	−	02	0		+	02	−	01	+	03	+	02	0		Edge of Flange / Rand d. Flanschtr. / Bord de l'aile
	06		03		04	0			03		03		02		01	0		
4	57	4	50	4	25	3	88	3	46	2	95	2	37	1	77	1	19	
0		−	04	−	08	−	11	−	12	−	13	−	13	−	11	−	10	
−	02	−	01	0			01		03		04		02		01	0		
4	79	4	69	4	40	3	98	3	48	2	91	2	34	1	81	1	17	
0		−	16	−	29	−	38	−	43	−	44	−	44	−	43	−	42	
−	03	−	04	−	04	−	02	0		+	03	+	01	0		0		
5	38	5	07	4	68	4	14	3	64	2	91	2	31	1	73	1	15	
0		−	33	−	61	−	77	−	84	−	86	−	85	−	84	−	83	
−	08	−	14	−	13	−	10	−	05	0		0		+	01	0		
5	93	5	69	5	13	4	36	3	62	2	94	2	29	1	67	1	10	
0		−	54	−	99	−1	23	−1	27	−1	26	−1	26	−1	25	−1	16	Centre of Flange / Flanschmitte / Centre de l'aile
−	34	−	29	−	25	−	18	−	10	0		+	02	−	01	−	02	

Centre of Support
Stützenmitte
Centre de la poutre

Fig. 9.

Values of Stress f, q, f' tons/in^2 in Region near Support
Spannungswerte $f, q\, f'$ beim Auflager
Valeurs des contraintes f, q, f' dans la région des supports

Upper Value / Oberer Wert / Valeur supérieure $= f$ — Middle Value / Mittlerer Wert / Valeur moyenne $= q$ — Lower Value / Unterer Wert / Valeur inférieure $= f'$

The reduction ratio at $x = 0{,}937$ is found from the f values as $f_m/f_0 = 4{,}81/5{,}69 = 0{,}845$.

This investigation was carried out in the JAMES WATT Engineering Laboratories of the University of Glasgow under the director, Prof. G. COOK F. R. S.

Summary

The shear lag problem arises in the design of girders with very wide flanges. The problem has been studied with reference to stressed-skin aircraft structures and has been analysed by the FOURIER Series method, which is lengthy and tedious even for uniform loading. It is impracticable in most girder problems.

The analysis by relaxation, or arithmetical reiteration, is also tedious but has the advantage that it can be concentrated on any local region. It is therefore useful here since shear lag affects the stresses only in the region of the supports.

In this paper, two cases are analysed by relaxation. From these a simple formula is developed which, it is suggested, is applicable to any girder problem.

Zusammenfassung

Bei der Berechnung von Trägern mit sehr breiten Flanschen ist das Problem der Schubverformung von Bedeutung. Dieses Problem wurde untersucht bei ausgesteiften dünnwandigen Bauteilen für Flugzeuge, wobei sich diese Untersuchungen auf FOURIER'sche Reihen stützten, was zu langen und mühsamen Berechnungen auch für gleichmäßige Belastung führt. Dieses Vorgehen ist in den meisten Trägerproblemen nicht anwendbar.

Die Untersuchung mit Hilfe des Iterationsverfahrens ist zwar ebenfalls mühsam, besitzt jedoch den Vorzug, daß es auf beliebige einzelne Flanschteile angewendet werden kann. Es ist deshalb zweckmäßig, weil die Schubverformung die Spannungen in breiten Flanschen nur in Auflagernähe beeinflußt.

In der Abhandlung werden zwei Fälle durch Iteration untersucht, gestützt darauf wird eine einfache, auf beliebige Trägerprobleme anwendbare Gebrauchsformel abgeleitet.

Résumé

L'étude des poutres à très larges ailes pose le problème de la déformation par cisaillement. Ce problème a déjà été étudié à propos des surfaces portantes, en aviation; il a été analysé par la méthode des séries de FOURIER; c'est toutefois là une méthode d'application longue et pénible, même dans le cas d'une charge uniforme. Elle est impracticable dans la plupart des problèmes concernant les poutres.

L'analyse par une méthode d'itération est également fastidieuse, mais présente l'avantage de permettre une concentration des calculs sur une zone déterminée. Elle est donc intéressante ici, puisque la déformation par cisaillement n'intéresse que la région des appuis.

L'auteur analyse deux cas par la méthode d'itération. Il en déduit une formule simple, dont il propose l'application à tout problème concernant les poutres.