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SUSPENSION BRIDGES WITH CABLES FASTENED TO THE STIFFENING GIRDERS AT THE CENTRE OF THE BRIDGE

BERECHNUNG VON HÄNGEBRÜCKEN, DEREN KABEL IN DER BRÜCKENMITTE AM VERSTEIFUNGSTRÄGER BEFESTIGT IST

CALCUL DES PONTS SUSPENDUS OÙ LE CÂBLE EST FIXÉ À LA POUTRE RAIDISSEUSE AU MILIEU DU PONT

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In the usual construction of suspension bridges the cables are subjected to moving with respect to the stiffening girders. When a suspension bridge is subjected to live load, or if it is made to swing, one can see how the cables move to and fro in the direction of the bridge. If, however, the cables are fastened to the stiffening girders at the centre of the bridge, the cables will be forced to drag the bridge along in all their movements. A very effective checking of the swinging motions which may take place when subjected to strong wind, can thus be obtained. Furthermore, one will obtain certain reductions in moments and deflections in the stiffening girders.

The „locking" of the cables at the centre of the bridge is used as part of the measurements taken to check the swinging motions in a number of suspension bridges built in the U. S. A. [1]. Also in Norway this has been done in the construction of a couple of smaller suspensive bridges.

In the following the necessary theoretical foundation will be given for the designing of a suspension bridge with „locked" cables, and it will also be shown how, by means of the theory here developed, one can approximately correct for the sloping position of the suspenders in ordinary suspension bridges.

The following simplified assumptions are made:

1. The stiffnes IE of the stiffening girders is constant.
2. The cable has a parabolic form due to dead load.
3. The distance between the suspenders is short in comparison with the span l.
4. The effect of the sloped position and deformation of the suspenders and towers can be neglected.
5. The stiffening girders are straight, and the stiffening girders and the cable have the same elevation at the centre of the bridge.

Referring to Fig. 1 we can write the following equation for the equilibrium of the two halfspans [2] [3] [4].

$$(1)\quad \begin{cases} p(x) = JE\eta_l'''' - (H_s + \Delta H_s)y'' - (H_w + H_s + \Delta H_s)\dfrac{d}{dx}\left((1+y'^2)\,\eta_l'\right) + \alpha\,\Delta H_s\,\eta_l'', \\[2ex] p(x) = JE\eta_r'''' - (H_s - \Delta H_s)y'' - (H_w + H_s - \Delta H_s)\dfrac{d}{dx}\left((1+y'^2)\,\eta_r'\right) - \beta\,\Delta H_s\,\eta_r''. \end{cases}$$

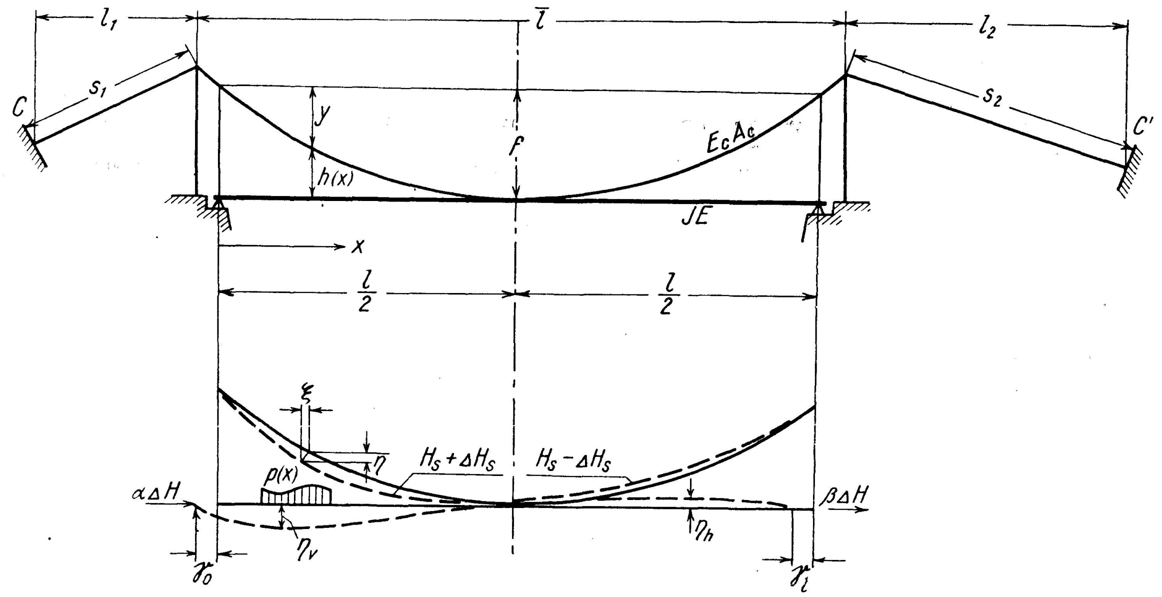

Fig. 1.

Here $p(x)$ is the loading, H_w the horizontal component of the cable force due to dead load, $H_s+\Delta H_s$ the increase in the horizontal component of the cable force in the left part of the bridge due to live load $p(x)$ and change in temperature, and $H_s-\Delta H_s$ the corresponding increase in the right hand part of the bridge. $2\Delta H_s$ is the „locking" force at the centre of the bridge; a part $\alpha\Delta H_s$ is transferred as a compression force through the stiffening girders to the left hand tower; the rest $\beta\Delta H_s=(2-\alpha)\Delta H_s$ is transferred as a tension force to the right hand tower. The forces may be taken as friction forces at both ends of the bridge ($\alpha=1$ and $\beta=1$), by hydraulic buffers or by letting the girders butt into the bridge supports ($\alpha\approx 2$; $\beta\approx 0$).

Equation (1) may be written

$$(1a)\quad \begin{cases} -M_l''+\dfrac{c^2}{l^2}M_l=(H_s+\Delta H_s)y''+p(x); & c=l\sqrt{\dfrac{H_w+H_s+(\beta-1)\Delta H_s}{JE}}=l\sqrt{\dfrac{\boldsymbol{H}}{JE}}, \\ -M_r''+\dfrac{c^2}{l^2}M_r=(H_s-\Delta H_s)y''+p(x); & c=l\sqrt{\dfrac{H_w+H_s+(\beta-1)\Delta H_s}{JE}}=l\sqrt{\dfrac{\boldsymbol{H}}{JE}}. \end{cases}$$

hereby making the approximate assumption that $(1+y'^2)\approx 1$ in order to obtain comparatively simple formulae. If desired, it may be corrected by a separate computation.

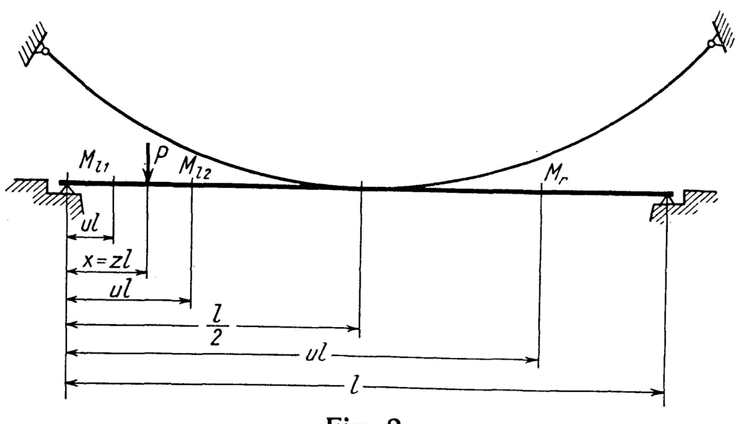

Fig. 2.

The ordinary solution of Equation (1a) for a load as shown in Fig. 2 is as follows:

$$M_{l_1} = A_1 \sinh uc + B_1 \cosh uc + \frac{l^2}{c^2} y''(H_s + \Delta H_s); \qquad 0 \leqq u \leqq z,$$

$$M_{l_2} = A_2 \sinh(1-u)c + B_2 \cosh(1-u)c + \frac{l^2}{c^2} y''(H_s + \Delta H_s); \quad z \leqq u \leqq \tfrac{1}{2},$$

$$M_r = A_3 \sinh(1-u)c + B_3 \cosh(1-u)c + \frac{l^2}{c^2} y''(H_s - \Delta H_s); \quad \tfrac{1}{2} \leqq u \leqq 1.$$

To designate the constants A and B, one has the limiting conditions at both ends of the bridge, the loading point zl and the centre point of bridge $\frac{1}{2} l$. With A and B known, one arrives at the following equation after some computation.

$$(2)\begin{cases} M_{l_1} = P\dfrac{l}{c}\dfrac{\sinh(1-z)c \sinh uc}{\sinh c} - \dfrac{16f}{c^2}\left[H_s \dfrac{\sinh(1-u)\frac{c}{2}\sinh u\frac{c}{2}}{\cosh\frac{c}{2}} + \Delta H_s \dfrac{\sinh(1-2u)\frac{c}{4}\sinh u\frac{c}{2}}{\cosh\frac{c}{4}}\right]; \\ \qquad 0 \leqq u \leqq z, \\ M_{l_2} = P\dfrac{l}{c}\dfrac{\sinh zc \sinh(1-u)c}{\sinh c} - \dfrac{16f}{c^2}\left[H_s \dfrac{\sinh(1-u)\frac{c}{2}\sinh u\frac{c}{2}}{\cosh\frac{c}{2}} + \Delta H_s \dfrac{\sinh(1-2u)\frac{c}{4}\sinh u\frac{c}{2}}{\cosh\frac{c}{4}}\right]; \\ \qquad z \leqq u \leqq \tfrac{1}{2}, \\ M_r = P\dfrac{l}{c}\dfrac{\sinh zc \sinh(1-u)c}{\sinh c} - \dfrac{16f}{c^2}\left[H_s \dfrac{\sinh(1-u)\frac{c}{2}\sinh u\frac{c}{2}}{\cosh\frac{c}{2}} - \Delta H_s \dfrac{\sinh(2u-1)\frac{c}{4}\sinh(1-u)\frac{c}{2}}{\cosh\frac{c}{4}}\right]; \\ \qquad \tfrac{1}{2} \leqq u \leqq 1. \end{cases}$$

Here $$c = l\sqrt{\frac{H_w + H_s + (\beta-1)\Delta H_s}{JE}} = l\sqrt{\frac{\boldsymbol{H}}{JE}}.$$

Equation (2) differs from the corresponding equation for an ordinary suspension bridge by the addition of the last joint, which comprises the correction for the „locking" force ΔH_s.

Returning to Equation (1)

$$p(x) = JE\,\eta_l'''' - (H_s + \Delta H_s)y'' - (H_w + H_s + (\beta - 1)\Delta H_s)\,\eta_l'',$$
$$p(x) = JE\,\eta_r'''' - (H_s - \Delta H_s)y'' - (H_w + H_s + (\beta - 1)\Delta H_s)\,\eta_r'',$$

assuming $(1 + y'^2) \approx 1$. By integrating these equation twice, and substituting $M_{l,r} = -JE\eta''$ and $M_b = -\iint p(x)\,dx^2 =$ the moment in the freely supported beam with span l and loading $p(x)$, the following formula is found

$$(3)\begin{cases} \eta_l = \dfrac{M_b - M_l - H_s y - \Delta H_s(y - 2uf)}{\boldsymbol{H}}; & 0 \leqq u \leqq \tfrac{1}{2} \\ \eta_r = \dfrac{M_b - M_r - H_s y + \Delta H_s(y - 2f(1-u))}{\boldsymbol{H}}; & \tfrac{1}{2} \leqq u \leqq 1. \end{cases}$$

Here $$\boldsymbol{H} = H_w + H_s + (\beta - 1)\,\Delta H_s.$$

M_l and M_r are substituted from Equation (2). By means of Equation (3) the movements of the cables at the centre of the bridge can be computed. The following equation is found for the longitudinal movement of the cable-element [2]) [3]) [4]).

$$d\xi = -y'\eta'\,d(ul) + \frac{H_s \pm \Delta H_s}{E_c A_c}\sec^3\varphi\,d(ul) + \varepsilon t\,d(ul).$$

Naming the movement of the „locked" cable at the centre of the bridge by ξ_m (negative by movement towards left), one gets:

$$\int_C^{l/2} d\xi = -\int_0^{l/2} y'\eta'\,d(ul) + \frac{H_s + \Delta H_s}{E_c A_c}\int_C^{l/2}\sec^3\varphi\,d(ul) + \varepsilon t\int_C^{l/2} d(ul) = \xi_m$$

and

$$\int_{l/2}^{C'} d\xi = -\int_{l/2}^{l} y'\eta'\,d(ul) + \frac{H_s - \Delta H_s}{E_c A_c}\int_{l/2}^{C'}\sec^3\varphi\,d(ul) + \varepsilon t\int_{l/2}^{C'} d(ul) = -\xi_m,$$

where

$$-\int_0^{l/2} y'\eta'\,d(ul) = \int_0^{l/2} y''\eta\,d(ul) - [y'\eta]_0^{l/2} = y''\int_0^{l/2}\eta\,d(ul),$$

$$-\int_{l/2}^{l} y'\eta'\,d(ul) = \int_{l/2}^{l} y''\eta\,d(ul) - [y'\eta]_{l/2}^{l} = y''\int_{l/2}^{l}\eta\,d(ul),$$

$$y'' = -\frac{8f}{l^2},$$

as $\eta = 0$ at the respective ends of the bridge and $y' = 0$ at the centre of the bridge. It is possible to write the equation for the movement of the cables at the centre of the bridge as follows:

$$-\frac{8f}{l^2}\left\{\int_0^{zl}\eta_{l_1}\,d(ul) + \int_{zl}^{l/2}\eta_{l_2}\,d(ul)\right\} + \frac{H_s + \Delta H_s}{E_c A_c}\left[\frac{l}{2}\left(1 - 8\left(\frac{f}{l}\right)^2 + \frac{s_1^3}{l_1^2}\right]\right.$$
$$+\,\varepsilon t\left(l_1 + \frac{l}{2}\right) = \xi_m.$$

$$-\frac{8f}{l^2}\int_{l/2}^{l}\eta_r\,d(ul) + \frac{H_s - \Delta H_s}{E_c A_c}\left[\frac{l}{2}\left(1 + 8\left(\frac{f}{l}\right)^2 + \frac{s_2^3}{l_2^2}\right] + \varepsilon t\left(\frac{l}{2} + l_2\right) = -\xi_m.\right.$$

By substituting η_l and η_r from Equation (3) the following 2 Equations are obtained:

(4a)
$$H_s\left\{\frac{\frac{s_1^3}{l_1^2} - \frac{s_2^3}{l_2^2}}{E_c A_c}\right\} + \Delta H_s\left\{\frac{L_s}{E_c A_c} + \frac{4}{3}\frac{f^2}{l\boldsymbol{H}}\left[1 - \frac{48}{c^2}\left(1 - \frac{4}{c}\tanh\frac{c}{4}\right)\right]\right\} =$$
$$2\xi_m + \varepsilon t(l_2 - l_1) + \frac{f}{\boldsymbol{H}}P\left\{2z(1-2z) - \frac{16}{c^2}\,\frac{\sinh(1-2z)\frac{c}{4}\sinh z\frac{c}{2}}{\cosh\frac{c}{4}}\right\},$$

and

$$H_s\left\{\frac{L_s}{E_c A_c}+\frac{16}{3}\frac{f^2}{l\boldsymbol{H}}\left[1-\frac{12}{c^2}\left(1-\frac{2}{c}\tanh\frac{c}{2}\right)\right]\right\}+\varDelta H_s\left\{\frac{\frac{s_1^3}{l_1^2}-\frac{s_2^3}{l_2^2}}{E_c A_c}\right\}=$$

(4b)

$$-\varepsilon t L_t+\frac{4f}{\boldsymbol{H}}P\left\{z(1-z)-\frac{4}{c^2}\frac{\sinh(1-z)\frac{c}{2}\sinh z\frac{c}{2}}{\cosh\frac{c}{2}}\right\},$$

where $\boldsymbol{H}=H_w+H_s+(\beta-1)\varDelta H_s, \quad c=l\sqrt{\frac{\boldsymbol{H}}{EJ}},$

$$L_s=\bar{l}\left[1+8\left(\frac{\bar{f}}{\bar{l}}\right)^2\right]+\frac{s_1^3}{l_1^2}+\frac{s_2^3}{l_2^2}, \quad L_t=l_1+\bar{l}+l_2.$$

Equation (4a/b) is applicable when the loading is in the interval

$$0\leqq z\leqq l/2.$$

Considering Equation (4a/b) it will be seen that one of the coefficients for H_s or $\varDelta H_s$ will always be very small. Furthermore both Equations include H_s and $\varDelta H_s$ in $\boldsymbol{H}$ and c so that the Equations must be solved by trial. Because of the form of the Equations this can be done very easily.

For the total symmetrical bridge, where $l_1=l_2$ and $s_1=s_2$ the Equations can be simplified to

(5a)
$$\varDelta H_s=\frac{3l}{4f}\,\frac{P\left\{2z(1-2z)-\frac{16}{c^2}\frac{\sinh(1-2z)\frac{c}{4}\sinh z\frac{c}{2}}{\cosh c/4}\right\}+\frac{2\boldsymbol{H}}{f}\xi_m}{1-\frac{48}{c^2}\left(1-\frac{4}{c}\tanh\frac{c}{4}\right)+\frac{3\boldsymbol{H}lL_s}{4f^2E_cA_c}},$$

(5b)
$$H_s=\frac{3l}{4f}\,\frac{P\left\{z(1-z)-\frac{4}{c^2}\frac{\sinh(1-z)\frac{c}{2}\sinh z\frac{c}{2}}{\cosh c/2}\right\}-\frac{\boldsymbol{H}}{4f}\varepsilon tL_t}{1-\frac{12}{c^2}\left(1-\frac{2}{c}\tanh\frac{c}{2}\right)+\frac{3\boldsymbol{H}lL_s}{16f^2E_cA_c}}.$$

Equation (5b) is exactly the same equation which is to be used to determine H_s in an ordinary suspension bridge.

For the movement ξ_m of the cable at the centre of the bridge, the following equation is found:

(6)
$$\begin{cases}\xi_m=\gamma_m=\gamma_0-\dfrac{\alpha\varDelta H_s}{E_bA_b}\dfrac{l}{2}+\varepsilon t\dfrac{l}{2}\\[2ex]\xi_m=\gamma_m=\gamma_l-\dfrac{\beta\varDelta H_s}{E_bA_b}\dfrac{l}{2}-\varepsilon t\dfrac{l}{2}.\end{cases}$$

$E_b A_b$ is here the modulus of elasticity and the sectional area of stiffening girders and bridge floor; γ_0; γ_2 are the movements of the stiffening girders at the respective bridge supports. ξ_m, γ_0; γ_2 are negative during movement towards the left.

For the movement of the bridge floor one has two cases:

1. The floor can move unrestricted and is hampered only by friction at the bridge supports; then $2\varDelta H_s$ is equal to the force of friction and

$\alpha = \beta = 1$. In this case $\Delta H_s \leq$ the force of friction, and Equation (4a) or (5a) can be used for computation of the movement ξ_m.

2. The movement of the floor is hampered because the construction allows only a certain movement γ_0 or γ_2. In this case $2\,\Delta H_s$ will mainly be taken by the abutment which is restricting the movement, while a smaller part, corresponding to the friction at the bridge support, is taken by the other abutment.

The influence lines [2, 3, 4] for moment, deflection etc. can now be computed by using Equations (2), (3), (4) and (5). The equations are, however, not so suitable for such computations because of the unknown „locking" force ΔH_s and the movement ξ_m. The supposed worst placement of the loads will therefore most often be found by trial, and then the desired data may be computed for all loadings by using the principle of superposition. As to the practical designing the necessary formulae for computation by the load distribution shown in Fig. 3 will be given.

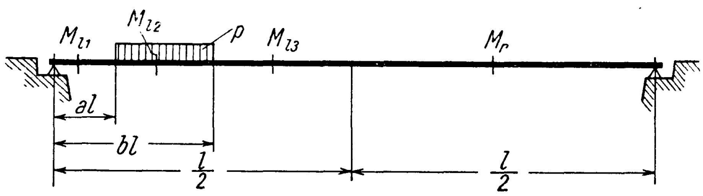

Fig. 3

$$
(7)\quad \begin{cases}
M_{l_1} = p\dfrac{l^2}{c^2}\dfrac{\sinh uc(\cosh(1-a)c - \cosh(1-b)c)}{\sinh c} \\[2mm]
\quad -\dfrac{16f}{c^2}\left[H_s\dfrac{\sinh(1-u)\frac{c}{2}\sinh u\frac{c}{2}}{\cosh\frac{c}{2}} + \Delta H_s\dfrac{\sinh(1-2u)\frac{c}{4}\sinh u\frac{c}{2}}{\cosh\frac{c}{4}}\right]; \; 0 \leq u \leq a, \\[2mm]
M_{l_2} = p\dfrac{l^2}{c^2}\dfrac{\sinh c - \sinh(1-u)c\cosh ac - \sinh uc\cosh(1-b)c}{\sinh c} \\[2mm]
\quad -\dfrac{16f}{c^2}\left[H_s\dfrac{\sinh(1-u)\frac{c}{2}\sinh u\frac{c}{2}}{\cosh\frac{c}{2}} + \Delta H_s\dfrac{\sinh(1-2u)\frac{c}{4}\sinh u\frac{c}{2}}{\cosh\frac{c}{4}}\right]; \; a \leq u \leq b, \\[2mm]
M_{l_3} = p\dfrac{l^2}{c^2}\dfrac{\sinh(1-u)c(\cosh bc - \cosh ac)}{\sinh c} \\[2mm]
\quad -\dfrac{16f}{c^2}\left[H_s\dfrac{\sinh(1-u)\frac{c}{2}\sinh u\frac{c}{2}}{\cosh\frac{c}{2}} + \Delta H_s\dfrac{\sinh(1-2u)\frac{c}{4}\sinh u\frac{c}{2}}{\cosh\frac{c}{4}}\right]; \; b \leq u \leq \tfrac{1}{2}. \\[2mm]
M_r = p\dfrac{l^2}{c^2}\dfrac{\sinh(1-u)c(\cosh bc - \cos ac)}{\sinh c} \\[2mm]
\quad -\dfrac{16f}{c^2}\left[H_s\dfrac{\sinh(1-u)\frac{c}{2}\sinh u\frac{c}{2}}{\cosh\frac{c}{2}} - \Delta H_s\dfrac{\sinh(2u-1)\frac{c}{4}\sinh(1-u)\frac{c}{2}}{\cosh\frac{c}{4}}\right]; \; \tfrac{1}{2} \leq u \leq 1.
\end{cases}
$$

Equation (3) is unchanged also for this load distribution and in the equations for H_s and ΔH_s the load joints P{ } will be changed to:

For the Equations (4a) and (5a)

$$\text{(8a)}\qquad pl\left\{(b^2-a^2)-\frac{4}{3}(b^3-a^3)-8\frac{b-a}{c^2}+\frac{8}{c^3}\,\frac{\sinh(\frac{1}{4}-a)\,c-\sinh(\frac{1}{4}-b)\,c}{\cosh c/4}\right\};\qquad 0\leqq a\leqq b\leqq\tfrac{1}{2}.$$

For the Equations (4b) and (5b)

$$\text{(8b)}\qquad p\frac{l}{2}\left\{(b^2-a^2)-\frac{2}{3}(b^3-a^3)-\frac{4}{c^2}(b-a)+\frac{4}{c^3}\,\frac{\sinh(\frac{1}{2}-a)\,c-\sinh(\frac{1}{2}-b)\,c}{\cosh c/2}\right\};\qquad 0\leqq a\leqq b\leqq\tfrac{1}{2}.$$

The other joints in Equations (4) and (5) remain unchanged.

For the special case of uniform load on the left halfspan, that is $a=0$, $b=\frac{1}{2}$, the following Equations are found:

$$\text{(9)}\qquad \begin{cases} M_l=-p\dfrac{l^2}{2c^2}\dfrac{\sinh uc}{\cosh\frac{c}{2}}\tanh\dfrac{c}{4}-\dfrac{16f}{c^2}\left[\left(H_s-p\dfrac{l^2}{8f}\right)\dfrac{\sinh(1-u)\frac{c}{2}\sinh u\frac{c}{2}}{\cosh\frac{c}{2}}+\right. \\ \qquad\qquad \left.+\,\Delta H_s\dfrac{\sinh(1-2u)\frac{c}{4}\sinh u\frac{c}{2}}{\cosh\frac{c}{4}}\right];\quad 0\leqq u\leqq\tfrac{1}{2}. \\ M_r=p\dfrac{l^2}{2c^2}\dfrac{\sinh(1-u)\,c}{\cosh\frac{c}{2}}\tanh\dfrac{c}{4}-\dfrac{16f}{c^2}\left[H_s\dfrac{\sinh(1-u)\frac{c}{2}\sinh u\frac{c}{2}}{\cosh\frac{c}{2}}-\right. \\ \qquad\qquad \left.-\,\Delta H_s\dfrac{\sinh(2u-1)\frac{c}{4}\sinh(1-u)\frac{c}{2}}{\cosh\frac{c}{4}}\right];\quad \tfrac{1}{2}\leqq u\leqq 1. \end{cases}$$

In the equations for H_s and ΔH_s the load joints will be

$$\text{(10a)}\qquad pl\left\{\frac{1}{12}-\frac{4}{c^2}\left(1-\frac{4}{c}\tanh\frac{c}{4}\right)\right\},$$

and

$$\text{(10b)}\qquad pl\left\{\frac{1}{12}-\frac{1}{c^2}\left(1-\frac{2}{c}\tanh\frac{c}{2}\right)\right\}.$$

The other joints in Equations (4) and (5) will remain unchanged.

The formulae here developed should cover all load combinations necessary for practical purposes.

From the developed formulae it is found that a „locked" suspension bridge can be designed just as an ordinary suspension bridge, as the effect of the locking force may be regarded as a correction. This effect can also, if desired, be computed from the diagrams for influence lines [2, 3] or tables [4] already developed for ordinary suspension bridges, using a span $l/2$, cable sag $f/4$ and rigidity $l/2$. This will make the computation somewhat easier but is not used here.

The „locking" of a suspension bridge will have the effect of somewhat reducing the moment in the stiffening girders. If the load is placed nearly symmetrical to the centre of the bridge, this reduction will be small or zero. The moment of the stiffening girders at the centre of the bridge will therefore as a rule decide the dimensions of the girders. In this case one gets $\Delta H_s \approx 0$ and the computation is done in the same way as for an ordinary suspension bridge.

In the following, the result is shown for a computation of a suspension bridge with these data:

$$l = 130 \text{ m}, \; l_1 = l_2 = 37{,}4 \text{ m}, \; s_1 = s_2 = 40{,}6 \text{ m}, \; f = 14{,}8 \text{ m}, \; H_w = 280 \text{ t},$$
$$JE = 21\,700 \text{ tm}^2, \; E_c A_c = 241\,000 \text{ t and } E_b A_b \approx \infty,$$

all figured for one half side (i. e. one suspension system).

The bridge has an uniformly distributed live load $p = 0{,}7$ t/m for each half width of the left half span. The following cases are computed: 1. Ordinary suspension bridge. 2. Locked bridge with moving possibility $\gamma = 0$. Locked bridge with frictional support, force of friction $2\,\Delta H_s = 5$ t. In table 1 the data of H_s, ΔH_s, $\xi_m = \gamma_0$ and the moment M and the deformation η at the ¼ points are given.

Table 1.

		$M_{1/4}$ tm	$M_{3/4}$ tm	$\eta_{1/4}$ cm	$\eta_{3/4}$ cm	H_s t	$2\Delta H_s$ t	ξ_m cm	γ_m cm
1	Ordinary suspension bridge	23,3	-21,5	55,6	-49,8	48,1	0	-14,7	—
2	Locked bridge. Fastened bridge support	4,1	- 1,6	13,6	- 1,3	48,4	86,8	0	0
3	Locked bridge. Friction bridge support	22,1	-20,5	52,6	-41,3	48,2	5,0	-11,2	-11,2

As can be seen, the locking brings about a reduction in moment and deflection which is in parts considerable.

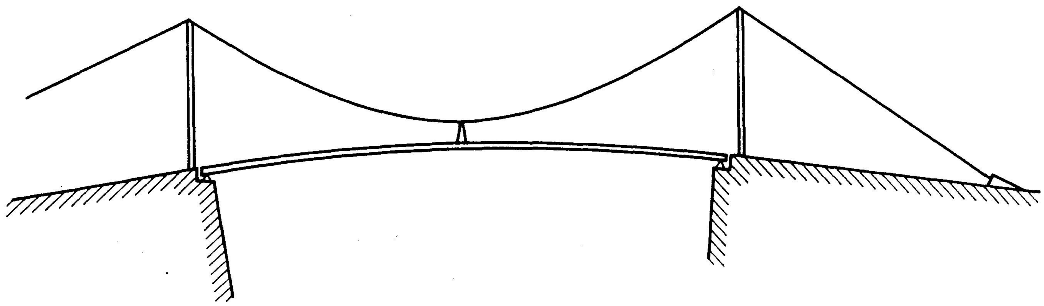

Fig. 4

This paper deals with a locked suspension bridge as shown in Fig. 1 The same method may also be used for bridges constructed as shown in Fig. 4. The formulae in this case will be a little more complicated, but the computation will not be difficult. The method may naturally also be extended to bridges with several spans; however, this is not shown here.

Approximate correction for the sloped position of the suspenders in ordinary suspension bridges

In the ordinary suspension bridges, the longitudinal movements of the cables will bring about a sloping position of the suspenders. As a result of this sloping position one gets a difference in the cable force at the right hand and left hand tower equal to

$$2\Delta H_s = -\int_0^l q\,\frac{\xi-\gamma}{h(x)}\,dx$$

Here q is the load in the hangers pr. m. of the bridge, ξ the longitudinal movement of the cable and γ the longitudinal movement of the stiffening girder. $h(x)$ is the length of the suspender.

In Fig. 5 this variation in the cable force is shown in comparison to the usually made assumption of constant cable force. Assuming that the variation $2\Delta H_s$ is due to a sort of locking at the centre of the bridge, one will get closer to the real variation of the cable force and one will attain a good correction of the sloping position of the suspenders by this assumption. For the effect of the sloping position of the suspenders the following equation can be adopted:

$$(11)\qquad 2\Delta H_s = -\int_0^l q\,\frac{\xi-\gamma}{h(x)}\,dx \approx -lq\left(\frac{(\xi_m-\gamma_m)A+\xi_m C}{B}\arctan B-\xi_m C\right),$$

where

$$A=\frac{1}{h\left(\frac{l}{2}\right)};\qquad B=\sqrt{\frac{h(o)-h\left(\frac{l}{2}\right)}{h\left(\frac{l}{2}\right)}};\qquad C=\frac{1}{h(o)-h\left(\frac{l}{2}\right)};$$

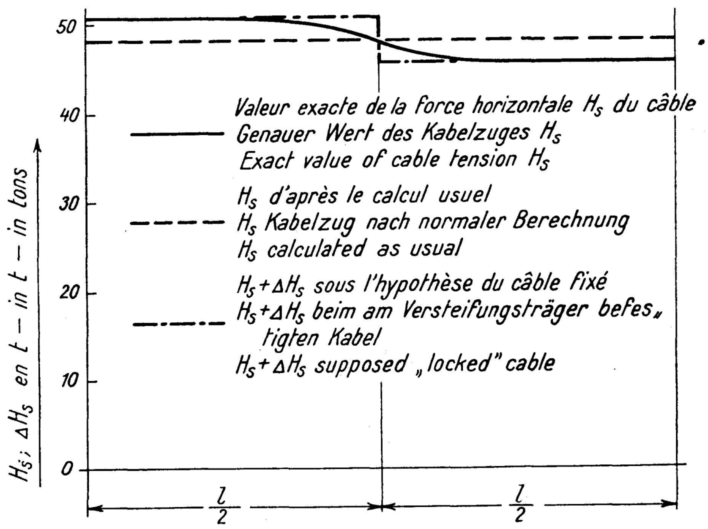

Fig. 5

ξ_m and γ_m are the longitudinal movements at the centre of the bridge for cables and bridgefloor respectively and negative for movement towards left; $h(0)$ is the length of the suspender at the end of the bridge and $h(l/2)$ at the centre of the bridge.

The movement of the stiffening girder is found by using Equations (4a), (5a), (8a), or (10a), hereby solving the equation with regard to ξ_m instead of ΔH_s. γ_m is found by Equation (6). By substituting these equations in Equation (11), one will attain an equation containing ΔH_s. This equation is best solved by trial. By friction support ΔH_s will be known and ξ_m must be solved from Equation (11). In supports with limited possibilities for movement γ_m is known and ΔH_s must be found.

By the approximate correction for the sloping position of the suspenders, the equations developed in the foregoing chapter are used.

Below are given the results for the same bridge which is computed earlier. $h(l/2) = 0{,}4$ m, $h(0) = 15{,}2$ m, $q = 1{,}8$ t/m and the movement of the bridgefloor is assumed to be: $\gamma = 0$. The computed data are compared to the usual method of computation without regard to the sloping position of the suspenders. In Fig. 5 is given the exact line of course of H_s, the line of course with supposed „locked" cable and the usual supposition that the sloping position of the suspenders may be neglected.

Table 2.

		$M_{1/4}$ tm	$M_{3/4}$ tm	$\eta_{1/4}$ cm	$\eta_{3/4}$ cm	H_s t	$2\Delta H_s$ t	ξ_m cm	γ_m cm
1	Ordinary suspension bridge	23,3	-21,5	55,6	-49,8	48,1	0	-14,7	—
2	Ordinary suspension bridge, corrected for sloping position of suspenders.	21,1	-19,2	50,3	-37,0	48,2	9,8	- 7,9	0

As can be seen, the exact computation will give a reduction in moments as well as in deflection.

References

1. Eng. News Rec., 1940, p. 736.
2. Selberg, A.: En lettvindt beregning av hengebruer. Medd. Veidir., 1942 h. 7. Norway.
3. Selberg, A.: Design of Suspension Bridges. Kgl. Norsk Videnskapselskaps skrifter 1945. Norway.
4. Asplund, S. O.: On the deflection theory of suspension bridges. Uppsala 1943. Sweden.

Summary

This paper deals with design of suspension bridges where the cables are fastened to the stiffening girder at the centre of the bridge. Suspension bridges of this type have in some cases been constructed in order to obtain a dampening effect on the swinging motions of the bridge caused by strong winds.

It appears that the design of bridges of this type for vertical loads needs only an insignificant amount of additional work compared to the usual constructional method. All necessary equations have been developed for plotting influence lines or for direct computation by any loading condition. It appears that if desirable it is possible to employ influence lines or tables that have already been developed[2, 3, 4]. For the stiffening truss the maximum moment will usually be at the middle of the span.

An example shows that fastening the cables at the centre of the bridge will cause a considerable reduction in moments and deformations compared to those obtained by the usual design.

The effect of sloping suspenders in ordinary bridges may be approximately corrected by a simple computation using the formulae developed for bridges with cables fastened to the stiffening truss. An example shows that the sloping position of the suspenders will cause a considerable reduction in moments and deflection in ordinary suspension bridges.

Zusammenfassung

Es wird die Berechnung von Hängebrücken behandelt, deren Kabel am Versteifungsträger befestigt ist. Solche Hängebrücken sind vereinzelt schon ausgeführt worden, um die bei starkem Wind auftretenden Schwingungen der Brücken zu vermindern.

Es zeigt sich, daß die Berechnung solcher Hängebrücken für vertikale Belastung im Vergleich mit Hängebrücken normaler Bauart nur unbedeutende Mehrarbeit fordert. Alle notwendigen Gleichungen für die Berechnung der Einflußlinien oder für direkte Berechnung der Schnittgrößen sind entwickelt. Dabei können Tabellen der Einflußlinien oder Diagramme für Hängebrücken normaler Bauart weitgehend verwendet werden [2, 3, 4].

Ein Berechnungsbeispiel zeigt, daß die Befestigung des Kabels am Versteifungsträger in Brückenmitte die Momente und Durchbiegungen gegenüber Hängebrücken normaler Bauart bedeutend vermindert.

Bei Hängebrücken normaler Bauart kann der Einfluß der Schrägstellung der Hängestangen durch eine einfache Zusatzberechnung ermittelt werden, wobei die hier für die Hängebrücke mit am Versteifungsträger befestigten Kabel abgeleiteten Formeln benutzt werden können. An einem Beispiel wird gezeigt, daß durch die Schrägstellung der Hängestangen Momente und Durchbiegungen erheblich verkleinert werden.

Résumé

Ce memoire traite des calculs sur les ponts suspendus dont le câble est fixé à la poutre raidisseuse. Les ponts de ce type ont été construits en définitive pour obtenir un amortissement du mouvement oscillatoire du pont par gros vent.

Il se trouve que les calculs pour une charge verticale de ce genre de ponts suspendus causent seulement un supplément négligeable de travail. Toutes les équations nécessaires au calcul des lignes d'influence ou au calcul direct sont développées. Il est donc possible de faire un large emploi des mêmes tables de lignes d'influence ou des diagrammes comme pour les ponts suspendus courants [2, 3, 4].

Un exemple calculé montre que le procédé de fixation du câble à la poutre raidisseuse au milieu du pont cause une importante réduction des moments et des flexions comparé aux ponts suspendus ordinaires.

Pour les ponts suspendus courantes, on peut corriger efficacement l'inclination des barres de suspension par un simple calcul supplémentaire où on utilise les formules qui sont développées pour les ponts dont le câble est fixé en leur milieu.

Un exemple calculé montre que l'inclinaison des barres de suspension cause une importante réduction des moments et flexions des ponts suspendus ordinaires.

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