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BUCKLING OF WEBS IN DEEP STEEL I GIRDERS

BEULUNG DER STEGBLECHE HOHER VOLLWANDTRÄGER AUS STAHL

LE VOILEMENT DE L'ÂME DES POUTRES D'ACIER EN DOUBLE *T* DE GRANDE HAUTEUR

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I. Introduction

In the design of deep steel I girders, the possibility of web buckling is frequently a decisive factor in determining the appropriate web thickness. Theoretical solutions are now-adays available for most of those types of loading which may occur in webs of deep girders, and which may cause buckling of webs [1, 2, 3, 4]. However, these theoretical solutions are based on idealized assumptions that seldom or never are complied with in practical structures. Moreover, as a rule, the theoretical solutions do not pay any attention to the fact that the load-bearing capacity of slender webs is under certain conditions considerably higher than the theoretical critical load [3, 5, 6].

In order to elucidate the behaviour of web plates in actual welded steel I girders, an investigation has been made during the last few years at the Institution of Structural Engineering and Bridge Building, Royal Institute of Technology, Stockholm. An extensive treatise covering, among other things, the records and measurements, discussing the results and giving the conclusions has recently been published in English [7]. Many tests were made including buckling tests on webs submitted to shearing stresses, normal stresses (bending moments), and combined shearing and normal stresses. In addition to the tests, extensive studies were made of the literature on empirical and theoretical results obtained by other investigators who have dealt with buckling of plates.

The purpose of this paper is to give a brief account of the principal test results, and the conclusions which may be drawn from these results and test results obtained by other investigators regarding the load-bearing capacity and the behaviour of thin webs submitted to edge loads in the plane of the web. Appropriate factors of safety providing against buckling of webs in steel I girders are briefly discussed, and, finally, a tentative general design procedure, together with nominal factors of safety, is advanced.

II. Tests and test results

All tests were made on rectangular web plates welded to the flanges of the respective test girders or test specimens. The tests comprised three test series, viz., test series A in which the webs were submitted to shearing forces along all four edges, test series B in which the webs were submitted to bending moments acting on two opposite sides, and test series C in which the webs were subjected to shearing forces in conjunction with bending moments. These three types of loading corresponded approximately to the conditions met with in the web of a continuous I girder at various points of its spans, see fig. 1. All test specimens are listed in Table 1 which shows the dimensions of the webs tested. The table also gives the initial deflection observed in each test on the respective web plates. The measurements of the initial deflections are referred to a middle plane passing through the edges of the respective web plates.

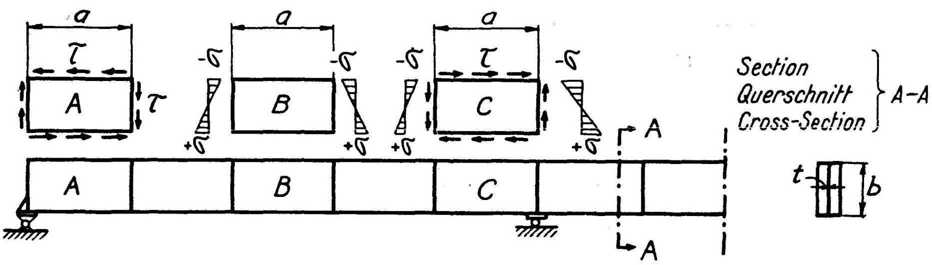

Fig. 1.

Various types of edge loading of web panels in continous I girder.

Verschiedene Randbelastungen des Stegbleches eines durchlaufenden vollwandigen Trägers.

Divers types des charges appliquées aux bords des panneaux d'âme des poutres en double *T* à âme pleine.

The buckling tests made on the various types of web plates included mainly the determination of the lateral deflection at various points of the webs as a function of the load, and strain-gauge measurements of the elongation at some points of the webs. The deflections were measured with the aid of gauge bars or, in most cases, special gauge frames which were fastened to the flanges of the test specimens, and which were used as reference bases for a movable Zeiss dial indicator. The accuracy of this method of measurement was found to vary within the limits of about $\pm$ 0,02 mm. The elongation measurements were made by means of Huggenberger tensometers attached to the webs by means of electromagnets.

Table 1

Test series	Type of load	Test No.	Web plate material complied with specification for standard steel	Thickness of web plate mm	Face dimensions of web plate mm	Number of web stiffeners	Side ratio α ($\alpha \geqq 1$)	Maximum initial deflection mm
A	Shearing forces uniformly distributed along four sides	A 1	St 44	5,1	1000 × 1000	None	1,00	2
		A 2	St 37	3,5	1000 × 2000	None	2,00	3
		A 3	St 37	3,5	1000 × 2000	One	1,00	4
		A 4	St 37	3,5	1000 × 2000	Three	2,00	4
		A 5	St 37	4,0	700 × 2400	None	3,43	2
		A 6	< St 37	3,2	700 × 2400	None	3,43	6
B	Bending moments acting on two opposite sides	B 1	St 37	3,8	1286 × 800	None	1,61	5
		B 2	St 37	3,8	1286 × 800	None	1,61	4
C	Distributed shearing forces in conjunction with bending moments	C 1	St 44	5,1	1000 × 1000	None	1,00	5
		C 2	St 44	5,1	1000 × 1000	None	1,00	13*
		C 3	St 37	3,8	1286 × 800	None	1,61	4

* Intentional initial deflection.

Fig. 2 shows the dimensions and design of test specimen A 2 in test series A. The external forces applied to the test specimen are also shown in Fig. 2. The test specimens A 1, A 5 and A 6 were similar in design to the test specimen A 2, although the webs had other dimensions, see table 1. For the tests A 3 and A 4, the same test specimen as that employed in test A 2 was used, but in these tests, the web was provided with one and three stiffeners respectively, see table 1.

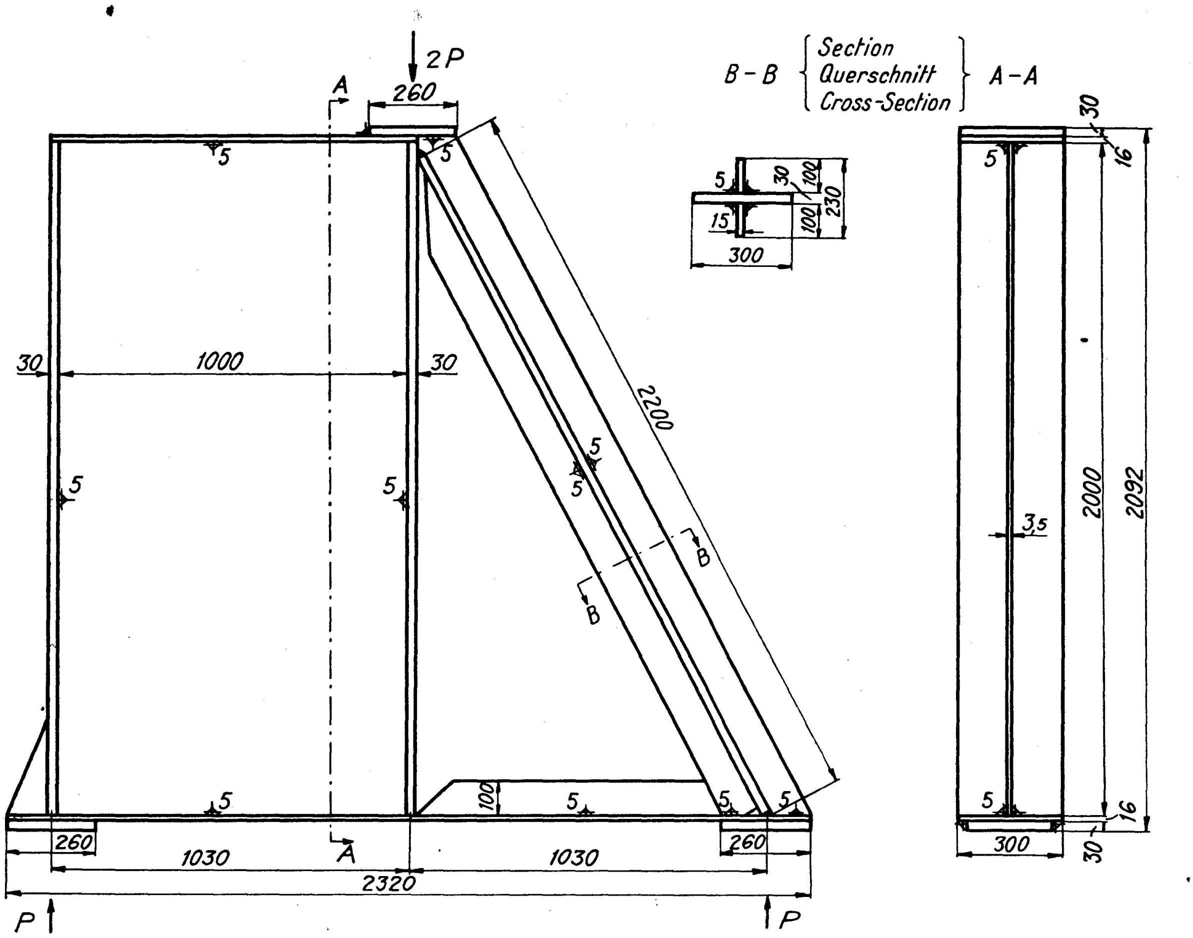

Fig. 2.
Test specimen A2 — Versuchsbalken A 2 — Poutre d'essai A 2.

The design and dimensions of the test specimen used for the tests B 1, B 2 and C 3 are shown in Fig. 3. The web of the test specimen was divided into three panels, and the tests were made on the middle panel.

The design and dimensions of the test specimen employed in the tests C 1 and C 2 are shown in Fig. 4. In this case, the web of the test specimen was divided by thick stiffeners into two panels of equal size, and both web panels were tested simultaneously.

In those cases where the same test specimen was used in more than one test, the residual deflections of the respective webs produced by foregoing tests were removed as far as possible by straightening.

In all tests it was found that the respective webs underwent lateral deflections directly the load had been applied to the test specimens. In most cases the increase in deflection was almost constant from zero load up to

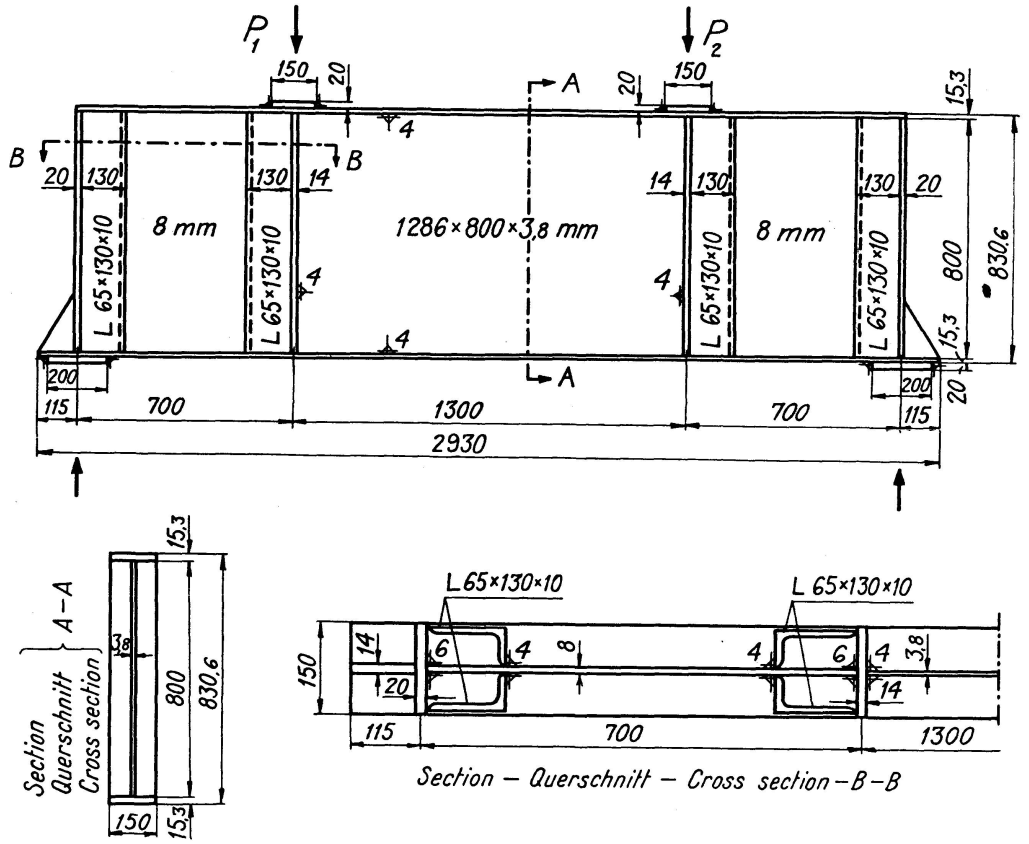

Fig. 3.

Design and dimensions of the test specimens B 1, B 2 and C 3. In the tests B 1 and B 2 the loads P_1 and P_2 were of equal magnitude $= P$, whereas in the test C 3 the load P_1 was equal to Q, i. e. $\neq 0$, and $P_2 = 0$.

Versuchsbalken B 1, B 2 und C 3. Bei den Versuchen B 1 und B 2 waren die Lasten P_1 und P_2 von derselben Größe und $= P$. Beim Versuch C 3 war $P_1 = Q$, d. h. $\neq 0$, und $P_2 = 0$.

Poutres d'essai B 1, B 2 et C 3. Dans les essais B 1 et B 2, les charges P_1 et P_2 étaient égales à P. Dans l'essai C 3, la charge P_1 était égale à Q, c'est-à-dire $\neq 0$, alors que $P_2 = 0$.

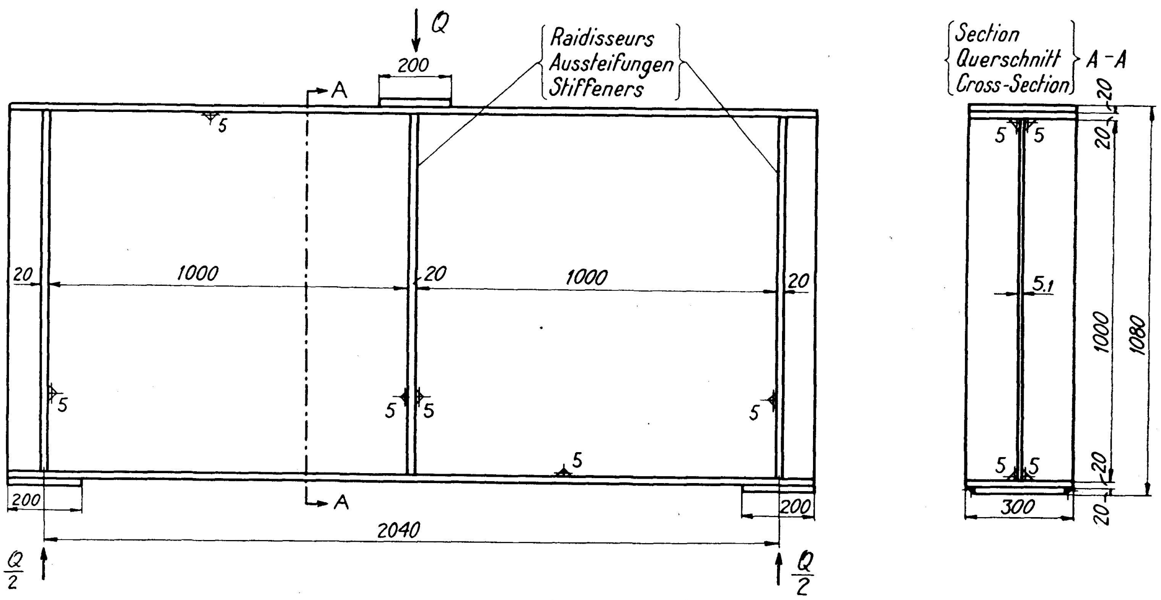

Fig. 4.

Test specimens C 1 and C 2 — Versuchsbalken C 1 und C 2 — Poutres d'essai C 1 et C 2.

the loads that caused yielding in the respective webs, and, hence, no elastic stability phenomena corresponding to the theoretical critical loads of plane webs were observed. Fig. 5 shows typical examples of the load-deflection curves obtained in most cases. In two of the tests, however, viz., the tests A 2 and A 6, the rate of increase in deflection rose rather rapidly at loads which were about 10 to 20 per cent above the theoretical critical loads calculated on the assumption of simply supported edges [1], thus indicating the occurrence of elastic stability phenomena, see Fig. 6. Nevertheless, in both these cases the rate of increase in deflection decreased when the load was further increased, and the ultimate loads of the webs were far beyond the theoretical critical loads.

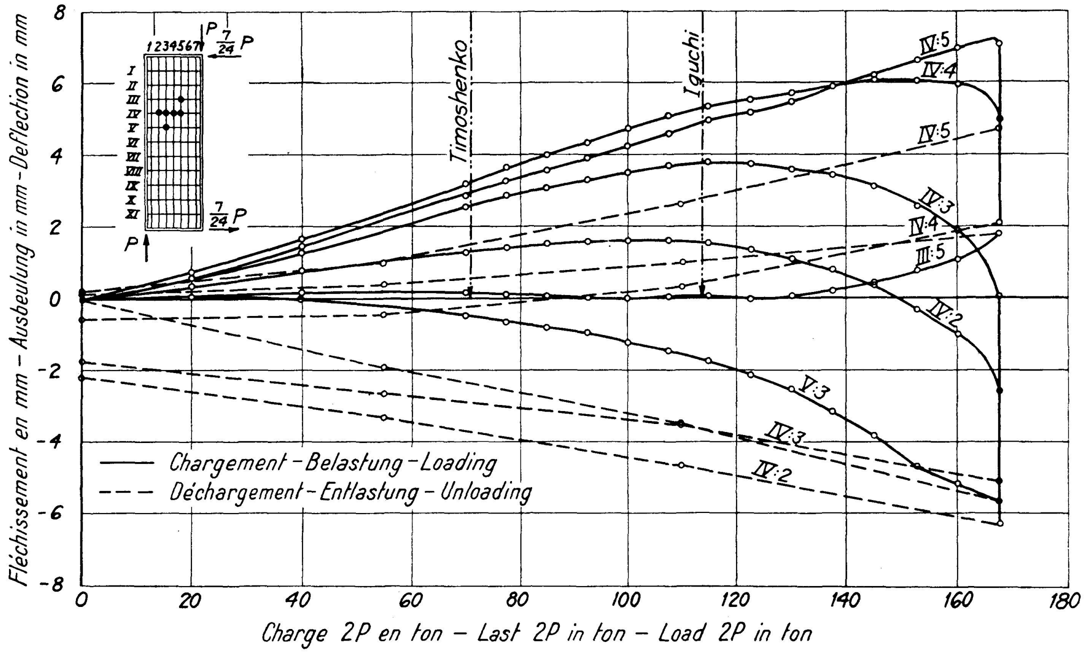

Fig. 5.

Test specimen A 5. Relative deflections at several points of the web plotted as a function of the load. The theoretical critical loads computed according to TIMOSHENKO (web simply supported along all edges) and IGUCHI (web clamped along the long edges and simply supported along the short edges) are marked in the graph.

Versuchsbalken A 5. Die Ausbeulung in verschiedenen Punkten des Stegbleches in Abhängigkeit von der Last dargestellt. Die theoretischen Beulspannungen nach TIMOSHENKO (einspannungsfreie Lagerung an allen Rändern) und IGUCHI (das Stegblech ist an den langen Rändern vollkommen eingespannt und an den kurzen Rändern einspannungsfrei gelagert) sind im Bild angegeben.

Poutre d'essai A 5. Flèches relatives mesurées en quelques points de l'âme et représentées en fonction de la charge. Les charges critiques théoriques calculées selon TIMOSHENKO (âme simplement appuyée sur ses quatre bords) et selon IGUCHI (âme encastrée aux bords longs et simplement appuyée sur les bords courts) sont indiquées au diagramme.

The shapes of the deflection surfaces obtained in the test series A and C showed fairly close agreement with those obtained from theoretical calculations [1, 2, 3]. In test series B the agreement was not so satisfactory [1, 4]. Figs. 7, 8 and 9 show contour lines of equal deflection of some of the tested webs.

The strain-gauge measurements made on the webs showed that, even at small loads, the states of stress in the webs differed greatly from those calculated on the assumption of plane webs. The differences became more and more strongly marked as the loads, and hence the deflections, increased. This was due to the fact that membrane stresses were set up in the webs when the deflections became of the same order of magnitude as the web thickness.

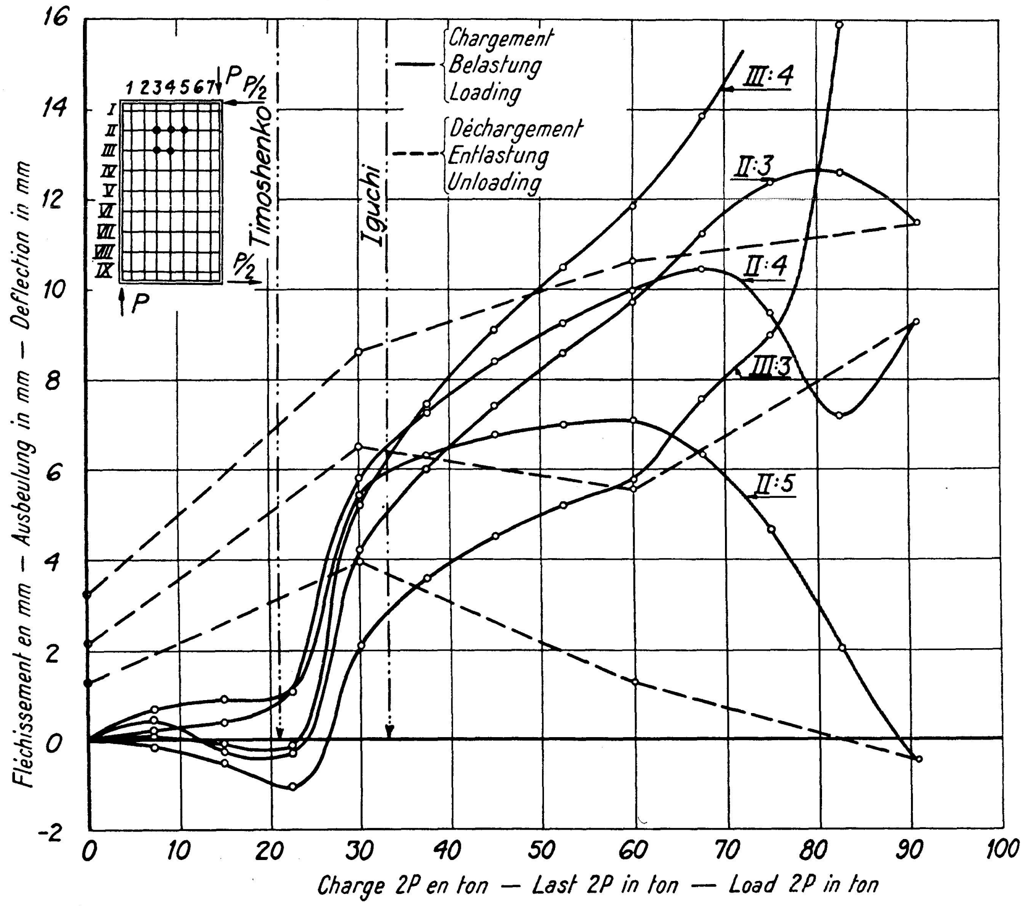

Fig. 6.

Test results obtained from test specimen A 2, see further caption of Fig. 5.
Versuchsergebnisse für Versuchsbalken A 2, siehe Bildunterschrift Abb. 5.
Résultats des essais effectués sur la poutre A 2, voir aussi la légende de la fig. 5.

Strain-gauge measurements were also made on the flanges of some test specimens in test series A and B. The results obtained from test series A showed that the shearing stresses were not uniformly distributed along the edges of the webs, and that they varied to a considerable extent. However, the webs were so slender that these variations in the shearing edge stresses had no notable effect upon the general behaviour of the webs in buckling. In test series B the strain-gauge measurements indicated that web buckling due to normal stresses (bending moments) might cause a transfer of stresses from the compression area of the web plate to the flange in compression.

In the test C 2 a comparison was made between two web panels equal in dimensions and loaded in the same manner. One of the web panels, panel B in Fig. 9, had a comparatively small initial deflection of about 1 mm, whereas the other web panel A was intentionally provided with a large initial deflection of 13 mm. The test showed that the relative deflections under load of the panel A were much smaller than those of panel B. This was due to the fact that the membrane stresses in the plate having large initial deflections were larger than in the other plate which was relatively flat at the outset.

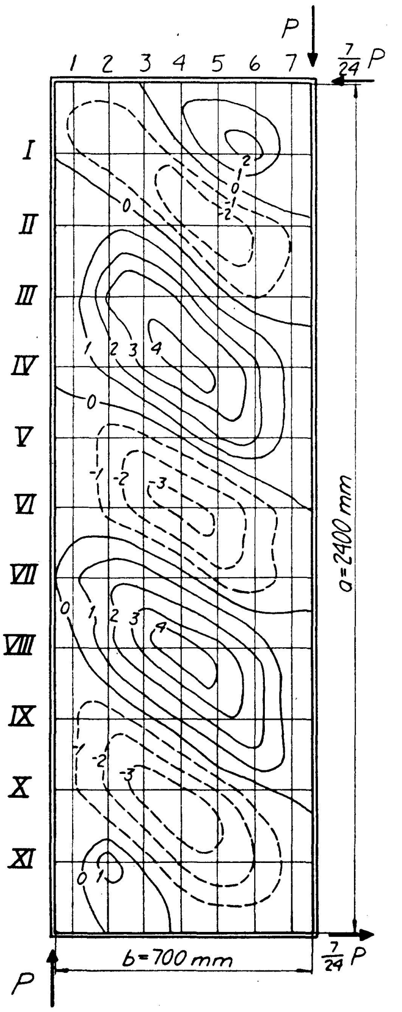

Fig. 7.

Test specimen A 5. Contour lines of equal relative deflection of the web at the load 2 P = 100 ton (= 1,4 times the theoretical critical load for simply supported edges). Difference in level between consecutive contour lines = 1 mm.

Versuchsbalken A 5. Schichtlinienplan der Ausbeulfläche bei der Last 2 P = 100 t (= 1,4 mal die theoretische Beullast für einspannungsfreie Ränder). Schichtlinienabstand = 1 mm.

Poutre d'essai A 5. Lignes de niveau d'égale flèche relative de l'âme soumise à la charge 2 P = 100 tonnes (= 1,4 fois la charge critique théorique de l'âme simplement appuyée sur ses quatre bords). La différence de niveau entre les lignes de niveau voisines est de 1 mm.

Yield phenomena in the web plates were observed in several tests at loads which were considerably lower than the ultimate loads. These yield phenomena were entirely local in character, and it seems that all these phenomena were due to high initial stresses set up during welding in the immediate neighbourhood of the seams. No influence of these local yielding phenomena on the load-bearing capacity of the web plates was observed in any of the tests. In some cases, these phenomena gave rise to small permanent plastic deflections, but no further permanent deformations were produced on repeated loading. In other words, the local yielding phenomena, which occurred at comparatively small loads, resulted in local stress equalization, and had subsequently little or no effect on the elastic behaviour of the web plates.

Table 2 gives for each test the load which was regarded as the limit of elastic behaviour of the web plate, judging from the deflection measurements, and the ultimate (yield point) load. Moreover, the table includes the theoretical critical loads calculated on the assumption of simply supported edges, and the ratios of the previously mentioned loads to these theoretical critical loads.

Table 2.

Test series	Test specimen No.	Side ratio α	Theoretical critical load of web plate with simply supported edges ton	Estimated elastic behaviour limit		Ultimate load	
				Load ton	Ratio of elastic behaviour limit to theoretical critical load	Load ton	Ratio of ultimate load to theoretical critical load
A	A 1	1,0	47,5	80 — 100	1,7 — 2,1	158	3,3
	A 2	2,0	21,4	45 — 60	2,1 — 2,8	91	4,2
	A 3	1,0	31,5	60 — 70	1,9 — 2,2	110	3,5 [1]
	A 4	2,0	86,0	110 — 120 [2]	1,3 — 1,4	170	2,0 [1]
	A 5	3,4	71,5	100 — 110	1,4 — 1,5	167,5	2,3
	A 6	3,4	36,6	60 — 70	1,6 — 1,9	105	2,9
B	B 1	1,6	69,7	70 — 85	1,0 — 1,2	> 135	> 1,9
	B 2	1,6	69,7	80 — 110	1,2 — 1,6	150	2,2
C	C 1	1,0	44,8	80 — 90	1,8 — 2,0	125	2,8
	C 2	1,0	44,8	70 — 80	1,6 — 1,8	125	2,8
	C 3	1,6	31,0	80 — 90	2,6 — 2,9	130	4,2

[1]) The web stiffeners used in these tests were of considerably greater strength than those commonly employed in practice. In the case of the test specimen A 3, which was provided with o n e web stiffener, the ultimate load might therefore have been higher than in the case of a web fitted with a stiffener of normal strength. In the case of the test specimen A 4, which was provided with t h r e e web stiffeners, the primary failure was caused by direct yielding due to shearing stresses, and not by stresses due to buckling. In this case, it is therefore not probable that the ample dimensions of the web stiffeners have resulted in any considerable increase of the ultimate load.

[2]) These values, which were not checked by any unloading, are probable values estimated by means of the relative deflection-load curves.

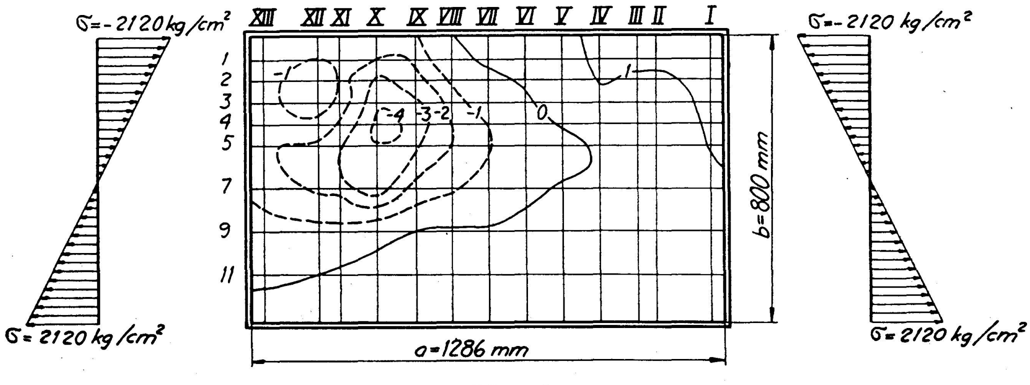

Fig. 8.

Test specimen B 2. Contour lines of equal relative deflection of the web at the load 2 P = 140 ton (= 2,0 times the theoretical critical load for simply supported edges). Difference in level between consecutive contour lines = 1 mm.

Versuchsbalken B 2. Schichtlinienplan der Ausbeulfläche bei der Last 2 P = 140 t (= 2,0 mal die theoretische Beullast für einspannungsfreie Ränder). Schichtlinienabstand = 1 mm.

Poutre d'essai B 2. Lignes de niveau d'égale flèche relative de l'âme soumise à la charge 2 P = 140 tonnes (= 2,0 fois la charge critique théorique de l'âme simplement appuyée sur ses quatre bords). La différence de niveau entre les lignes de niveau voisines est de 1 mm.

Table 2 shows that all ultimate loads observed in the tests are much higher than the theoretical critical loads for simply supported edges. In comparison with the ultimate loads, the loads determining the elastic behaviour limit of the web plates are remarkably low. This is probably due to two circumstances, viz., first, the above-mentioned local yielding phenomena which occurred in the weld seams and in their immediate neighbourhood, and second, the stress transfer which is likely to occur in a web plate submitted to buckling when the proportional limit or the yield point is exceeded in a limited portion of the plate. Thus, if the test specimens had been annealed prior to the tests so as to remove initial stresses, the elastic behaviour limits would certainly have been considerably higher in many cases.

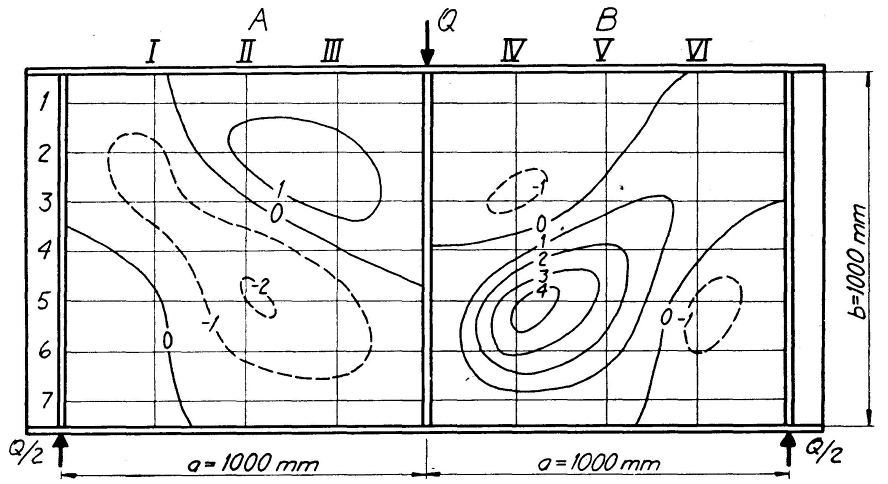

Fig. 9.

Test specimen C 2. Contour lines of equal relative deflection of the web panels at the load Q = 60 ton (= 1,35 times the theoretical critical load for simply supported edges). Difference in level between consecutive contour lines = 1 mm.

Versuchsbalken C 2. Schichtlinienplan der Ausbeulfläche bei der Last Q = 60 t (= 1,35 mal die theoretische Beullast für einspannungsfreie Ränder). Schichtlinienabstand = 1 mm.

Poutre d'essai C 2. Lignes de niveau d'égale flèche relative de l'âme soumise à la charge Q = 60 tonnes (= 1,35 fois la charge critique théorique de l'âme simplement appuyé sur ses quatre bords). La différence de niveau entre les lignes de niveau voisines est de 1 mm.

All failures of the test specimens were due to the fact that the yield points of the respective web materials were exceeded. However, in view of the extremely complicated stress conditions which were produced in the web plates under the action of middle surface stresses (including membrane stresses) and bending stresses due to buckling, the test results obtained in most cases do not allow to decide where that yielding was produced which determined the ultimate load.

III. Load-bearing capacity and behaviour of webs submitted to edge loads acting in plane of web

In the course of the past few decades a great number of investigators have treated the problem of buckling of webs theoretically and empirically, among others, TIMOSHENKO [1], CHWALLA [12], MOHEIT [3], GABER [5], KROMM and

Marguerre [6], Lyse and Godfrey [8], Seydel [9], Lahde and Wagner [10], and Moore [11]. From the results thus available from other investigations as well as from our own investigation certain conclusions may be drawn regarding the load-bearing capacity and behaviour of webs in actual structures submitted to edge loads in the plane of the web.

Thus, all tests show convincingly that some of the assumptions which afford foundations for the linear theory of plates and for the solutions of the buckling problem according to this theory cannot be complied with in practice. This applies first and foremost to the assumption that the web is perfectly plane and to the collateral condition that the deflections of the web shall be small in comparison with the web thickness. Since these assumptions are not fulfilled in reality, it follows that the character of the buckling phenomenon will in most cases be quite different from that indicated by the linear theory of plates. Other idealized assumptions that cannot be strictly conformed to in reality are, for example, those relating to the load distribution along the edges, the central load, and the boundary conditions at the edges.

The authors characterize the behaviour of thin webs subjected to „buckling" loads applied at the edges by three stages according to the magnitude of deflections.

Stage I. Plane wall (disc) stage. No deflections.

Stage II. Plate stage without membrane effect. Deflections occur, but they remain so small in comparison with the web thickness that the effect of the displacement components parallel to the web is inconsiderable, and the membrane stresses are therefore nil or negligible.

Stage III. Plate stage with membrane effect. The deflections are so large that membrane stresses are set up in the plate. These membrane stresses impede the increase in deflections.

In actual structures, stage I, which would be met with in the case of plane webs, will never occur, since the webs are never exactly plane or centrally loaded. As soon as an edge load is applied to a thin web in an actual structure, deflections will take place.

In most cases, the process of buckling is characterized by deflections which increase continuously and approximately linearly from zero load up to the instant when the proportional limit or the yield point is exceeded in comparatively large portions of the plates. In some cases, however, the deflections are relatively small at the outset, but increase at a comparatively high rate, when the load exceeds the theoretical critical load corresponding to the boundary conditions, thus indicating elastic stability phenomena. After the deflections have reached a certain magnitude, the increase in deflection becomes considerably lower, and the subsequent increase in deflection is most frequently approximately linear up to the instant when the proportional limit or the yield point is reached.

Now, the tests made [7, 11] indicate that in actual structures, in which initial deflections of the same order of magnitude as the thickness of the web may be taken to be unavoidable, elastic stability phenomena are liable to occur rather seldom. Under these circumstances, the linear theory of plates is nothing but a satisfactory method for determining the loads at which the deflections of a web *may* become large on account of a rapid increase in deflection. On the other hand, the linear theory does *not* provide any basis for determining the load-bearing capacity of slender webs. For calculating the load-bearing capacity of slender webs recourse must be had to the non-linear theory of plates [6].

The test results show that the boundary conditions at the edges are largely dependent on the method of connecting the web to the flanges. In the case of welded edges, the boundary conditions seem to be very difficult to define, and are subject to wide variations. On account of initial stresses in and about the weld joints, even relatively low loads can give rise to local yielding at the welds. Some tests indicate that these initial stresses may cause the boundary conditions at the edges of the web to become approximately similar to simple supports. On the other hand, in the case of riveted or bolted edges, it appears that the clamping effect at the edges may be regarded as rather close in character to completely clamped edges [3, 11].

It has been found from the tests that the first permanent plastic deformations of buckled webs are due to the fact that the proportional limit or the yield point is exceeded in localized areas. Thus, in webs with welded edges, permanent plastic deformations (deflections) can occur even at low loads. These deformations seem to be due to local yielding in the weld seams or in their vicinity. However, when the same load is repeatedly applied to the web, the material in the areas of local yielding is subjected to strain hardening, and no further permanent plastic deformations are produced. With the exception of yield phenomena in weld seams, the first yielding in buckled slender webs occurs in the middle of the waves due to buckling, and this happens irrespective of the boundary conditions of the webs [10]. These yield phenomena are also local in character on account of the wide variations in the bending stresses, and result only in stress equalization and stress transfer, but have little effect on the load-bearing capacity of the webs. This capacity is not exhausted until yielding begins in comparatively large portions of the web.

As regards the causes of failure of webs submitted to buckling, two general cases are to be distinguished. In the case of thick webs, in which buckling occurs within the plastic behaviour range, the deflections at the instant of failure are very small. Hence, the stresses in the web can be calculated with a fair degree of approximation on the assumption that the web is plane, stage I. The ultimate load of the web can then be estimated by using HUBER-v. MISES-HENCKY's condition for yielding [13].

On the other hand, in the case of slender webs, in which buckling occurs within the elastic behaviour range, the deflections at the instant of failure can be, and generally are, large, stage III. The stresses are then made up of middle surface stresses (including membrane stresses) and bending stresses due to buckling. The bending stresses, however, seem to play a comparatively unimportant part in connection with failure because these stresses vary to a considerable extent from point to point of the web. Since no theoretical solutions have so far been evolved for the stage III in the case of plates subjected to edge loads of the types under consideration, it is not yet possible to estimate the ultimate loads of slender plates by means of calculation. At all events, it has been found that the theoretical critical load of plane web plates bears no direct relation to the ultimate load, and that the ratio of the ultimate load to the theoretical critical load increases with the slenderness of the web.

IV. Discussion of factors of safety

The factors of safety against buckling of webs stipulated by standard specifications or sanctioned by customary practice vary considerably from one country to another. This applies first and foremost to buckling within the elastic range. Thus, for instance, in the United States and Sweden the safety factors used within the elastic range are as high as about 1,8 to 2,2 referred to the theoretical critical loads of simply supported plates. It seems probable that these high factors of safety may be attributed to the fact that the insight into the general character of the process of buckling is incomplete, and, hence, that the danger involved in buckling is over-estimated. On the other hand, the safety factors proposed in Germany (1,4 to 1,6) rest upon more correct understanding of the nature of buckling [14].

The following discussion of factors of safety is based on the experience resulting from tests. The discussion is confined to rectangular webs with edges possessing a certain flexural rigidity. The flexural rigidity is assumed to comply with the following conditions, viz., first, no notable deflections at right angles to the plane of the web can occur at the edges, and second, the edges are able to take membrane stresses.

To begin with, it may be stated that buckling, whether in the elastic or inelastic behaviour range, does not result in any sudden collapse of the web. For this reason, buckling of webs of the type defined in the above is to be regarded as equivalent in respect of danger to such types of loading as bending moments, shearing forces, etc., which do not involve any risk of elastic stability phenomena. Accordingly, if strength considerations alone were decisive, the factor of safety against buckling should be about 1,8 referred to the ultimate load (yield point load) of the web.

In fact, this principle is applied in aircraft design to the extremely slender plate and shell structures used in that field. For several reasons, however, this principle is not fully sufficient in the case of webs of deep girders. Thus, for instance, no methods of calculation are available at the present time for determining the ultimate loads of buckled webs in deep girders when buckling occurs within the elastic behaviour range. Furthermore, in the case of deep girders it is required that the deflections of the web due to exceptional loads should be elastic so as to disappear after removal of the load, and that they should not be excessively large.

It thus follows that the safety against buckling should be chosen so as to comply with the following requirements:

a) A sufficient margin of safety shall be provided to ensure that the ultimate load (yield point load) of the web is not exceeded.

b) The deflections due to exceptional loads shall be elastic so as to disappear after removal of load, and shall not be too large.

Now, in the case of buckling within the plastic behaviour range, the condition a) in the above is the decisive factor in determining appropriate safety factors. Thus, within this range the web may approximately be taken to comply with the idealized assumptions regarding flatness and centricity of load, and the load or stress corresponding to HUBER-v. MISES-HENCKY's condition for yielding [13] can be calculated on these assumptions. The working stress is then obtained by dividing this stress by the normal factor of safety of about 1,8.

On the other hand, when buckling occurs within the purely elastic behaviour range the condition b) in the above is of importance in the choice of appropriate safety factors against buckling. An examination of all buckling tests of the types under consideration that are known to the Authors [3, 5, 7, 9, 10, 11] shows that the maximum deflections observed at loads corresponding to the theoretical critical loads of plates simply supported along the edges were not in a single case as large as the thickness of the respective webs. Nevertheless, the tests examined include a great variety of specimens, such as webs with welded [7], bolted [3], riveted [5, 10, 11], and simply supported edges [9]. Thus, the test specimens may be taken to represent nearly all types of webs used in practice.

For steel webs, the minimum slenderness ratio b/t (b = depth of web, t = web thickness) which is required in order that buckling may occur in the elastic behaviour range is about 70. This slenderness ratio corresponds to a web of St 52 subjected to buckling due to shearing stresses, assuming the web to be simply supported along the edges. Even at this minimum slenderness ratio, a deflection of the same order of magnitude as the thickness of the web can scarcely be perceived by the eye, and can by no means be considered to be objectionable on aesthetic grounds.

In the case of buckling within the elastic behaviour range it is therefore justified to use the critical load calculated according to the linear theory on the assumption that the web is simply supported along the edges as a fair value of a load at which the magnitude of the lateral deflections of the web is not yet objectionable. It is to be observed, however, that this value represents a lower limit since, in many cases, for instance, when the web is riveted or bolted at the edges, and most frequently in the case of welded edges, a web may be subjected to still heavier loads without undergoing any objectionable deflection.

Now, the critical loads calculated according to the linear theory of plates bear no direct relation to the practical ultimate loads, which are always larger than the theoretical critical loads when buckling occurs within the elastic behaviour range. The difference between these loads increases as the slenderness ratio of the web becomes greater, and decreases when the slenderness ratio becomes so small that buckling takes place in the inelastic behaviour range. Consequently, if the theoretical critical load is exceeded within the purely elastic behaviour range, the web does not undergo any permanent deformations. Webs with welded edges constitute an exception in this respect because small permanent deformations can occur in these webs even at slight loads on account of local initial stresses set up in the welds. However, these permanent deformations are small, and occur, on the whole, only on first application of the loads at which the local stresses are equalized by yielding. Apart from these small deformations, which have practically no effect on the load-bearing capacity and behaviour of the web, webs with welded edges can also be subjected to loads exceeding the critical load, and their behaviour may nevertheless continue to be completely elastic.

In the case of buckling within the elastic range, there are therefore no strength considerations requiring the use of high factors of safety referred to the theoretical critical loads in web design. Since deflection will always occur in actual structures, it is sufficient to limit the magnitude of these deflections so as to ensure that they are not objectionable from an aesthetic point of view. On the other hand, it matters little or nothing whether these deflections amount to, say, $^1/_3$ or $^2/_3$ of the thickness of the web. As the tests have shown that the critical loads calculated from the linear theory of plates on the assumption that the web is simply supported along the edges produce relatively small deflections, these loads may therefore be used as practically acceptable values for design purposes. With a view to rational utilization of webs, the factors of safety referred to the theoretical critical loads should be low.

In some tests, particularly those tests where buckling was due to shearing stresses [7, 9], it has been observed that slender webs having large side ratios undergo larger deflections and are therefore more heavily stressed than webs having small side ratios. This is probably due to the circumstance that the flexural rigidity of the long supports (flanges) of webs having large side ratios is not sufficient to permit full development of the membrane stresses in the webs. For this reason, the safety factors used for webs having large side ratios should be slightly higher than in the case of small side ratios.

V. Tentative general design procedure and nominal factors of safety providing against buckling of webs in deep steel girders

This tentative proposal applies exclusively to rectangular web panels, whose all edges possess a certain amount of flexural rigidity in the plane of the web plate. For instance, this condition may be considered to be complied with by web panels of an ordinary bridge plate girder of I section provided with vertical web stiffeners.

The proposal is confined to the following types of loads applied to the edges of the web panels, viz., shearing stresses only, normal stresses (bending moments) only, and combined shearing and normal stresses. The proposal is based on the assumption that a critical stress curve for bars in buckling is available for the structural steel grade used as web material. These curves represent the critical stress σ_{cr} as a function of the slenderness ratio of the bar $\lambda = l/i$.

The proposal is confined to the general design procedure to be used in calculating web plates so as to provide against buckling only. Accordingly, it does not touch upon practical details, such as the minimum thickness of web or increase in web thickness required in order to allow for corrosion. Due regard must of course be paid to these circumstances in drawing up actual design rules.

The web material and the actual edge stresses τ_a (pure shearing stress, see Fig. 1), σ_a (pure normal stress due to bending moment, see Figs. 1 and 10), or σ_a and τ_a (combined shearing and normal stresses, see Fig. 1) are assumed to be given.

The nominal factor of safety against buckling of a web plate is calculated in four stages. If buckling occurs within the elastic behaviour range, i.e. if the real purpose of the design is to limit the deflections of the web plate, use shall be made of the stages Nos. 1, 2, 3 and 4 specified below.

If buckling occurs in the inelastic behaviour range, i.e. if the actual purpose of the calculation is to provide against yielding or permanent plastic deformations, use shall be made of the stages Nos. 1, 2, 3 and 5.

Stage 1. The Euler critical stress σ_e is determined from the equation

$$\sigma_e = \frac{\pi^2 E}{12(1-\nu^2)} \cdot \left(\frac{t}{b}\right)^2 \tag{1}$$

where E = Modulus of elasticity
ν = POISSON's ratio
t = Thickness of web plate
b = Depth (width) of web plate

Stage 2. The critical stress, or critical stresses, as the case may be, is determined as follows on the assumption that the web panel is simply supported along all edges.

a) When the web panel is submitted to shearing stresses only, the critical stress $\overset{0}{\tau}_{cr}$ is determined from

$$\overset{0}{\tau}_{cr} = k_s\, \sigma_e \tag{2}$$

where k_s is a function of the side ratio $\alpha = a/b$ of the web plate (a = length of web plate)

$$\left.\begin{aligned} k_s &= 4{,}00 + \frac{5{,}34}{\alpha^2} \quad \text{for } \alpha < 1 \\ k_s &= 5{,}34 + \frac{4{,}00}{\alpha^2} \quad \text{for } \alpha > 1 \end{aligned}\right\} \tag{3}$$

b) When the web panel is submitted to normal stresses due to bending moments only, the critical maximum edge stress $\overset{0}{\sigma}_{cr}$ is determined from

$$\overset{0}{\sigma}_{cr} = k_b\, \sigma_e \tag{4}$$

where k_b is a function of the side ratio α of the web panel as given by the curve in Fig. 10.

c) When the web panel is submitted to combined shearing and normal stresses, the critical stresses σ_{cr} and τ_{cr}, as shown by CHWALLA [12], are approximately determined from

$$\left.\begin{aligned} \sigma_{cr} &= n_{sb}\, \sigma_a \\ \tau_{cr} &= n_{sb}\, \tau_a \end{aligned}\right\} \tag{5}$$

where
$$n_{sb} = \frac{1}{\sqrt{\left(\frac{\tau_a}{\overset{0}{\tau}_{cr}}\right)^2 + \left(\frac{\sigma_a}{\overset{0}{\sigma}_{cr}}\right)^2}} \tag{6}$$

In Eq. (6) $\overset{0}{\sigma}_{cr}$ and $\overset{0}{\tau}_{cr}$ are the critical bending edge stress and the critical shearing stress respectively for the web panel submitted to bending moments alone according to Eq. (4), or to shearing stresses alone according to Eq. (2).

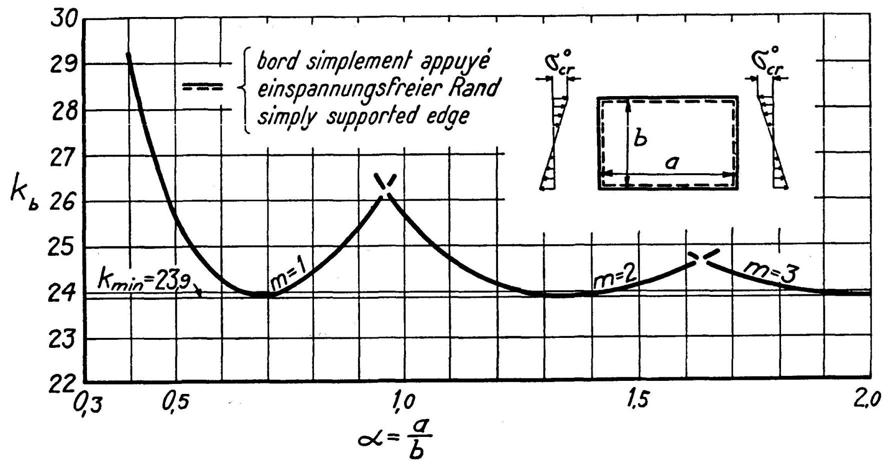

Fig. 10.
Values of k_b plotted as function of side ratio of web panel α.
Werte k_b in Abhängigkeit vom Seitenverhältnis α des Stegbleches.
Les valeurs de k_b représentées en fonction du rapport des bords α du panneau d'âme.

S t a g e 3. The idealized comparison stress σ_{icr}^{el} is determined as follows:

a) When the web panel is submitted to s h e a r i n g s t r e s s e s only:

$$\sigma_{icr}^{el} = \overset{0}{\tau}_{cr}\sqrt{3} \quad (\overset{0}{\tau}_{cr} \text{ according to Eq. (2)}) \tag{7}$$

b) When the web panel is submitted to n o r m a l s t r e s s e s d u e t o b e n d i n g m o m e n t s only:

$$\sigma_{icr}^{el} = \overset{0}{\sigma}_{cr} \quad (\overset{0}{\sigma}_{cr} \text{ according to Eq. (4)}) \tag{8}$$

c) When the web panel is submitted to c o m b i n e d s h e a r i n g a n d n o r m a l s t r e s s e s:

$$\sigma_{icr}^{el} = \sqrt{\sigma_{cr}^2 + 3\,\tau_{cr}^2} = n_{sb}\sqrt{\sigma_a^2 + 3\,\tau_a^2} \tag{9}$$

$$(\sigma_{cr} \text{ and } \tau_{cr} \text{ according to Eq. (5)})$$

S t a g e 4. If the idealized comparison stress calculated from Eqs. (7), (8), or (9)

$$\sigma_{icr}^{el} \geqq \sigma_P$$

where σ_P is the proportional limit of the web material, or, more correctly, the upper stress limit for elastic buckling, determined from the critical stress curve for bars, buckling occurs in the e l a s t i c b e h a v i o u r r a n g e. In this case, the factors of safety are as follows:

a) When the web panel is submitted to shearing stresses only:

$$n_s = \frac{\tau_{cr}^0}{\tau_a} \; (\tau_{cr}^0 \text{ according to Eq. (2)}) \tag{10}$$

b) When the web panel is submitted to normal stresses due to bending moments only:

$$n_b = \frac{\sigma_{cr}^0}{\sigma_a} \; (\sigma_{cr}^0 \text{ according to Eq. (4)}) \tag{11}$$

c) When the web panel is submitted to combined shearing and normal stresses, the safety factor is n_{sb} as determined by Eq. (6).

The safety factors n_s and n_{sb} calculated from Eq. (10) or Eq. (6) shall be, under normal conditions of loading,

$$\left.\begin{aligned} n &\geqq 1{,}2 && \text{for } \alpha < 1{,}5 \\ n &\geqq 1{,}5 - \frac{0{,}225}{\alpha - 0{,}75} && \text{for } \alpha > 1{,}5 \end{aligned}\right\} \tag{12}$$

and, under exceptional conditions of loading,

$$\left.\begin{aligned} n &\geqq 1{,}0 && \text{for } \alpha < 1{,}5 \\ n &\geqq 1{,}3 - \frac{0{,}225}{\alpha - 0{,}75} && \text{for } \alpha > 1{,}5 \end{aligned}\right\} \tag{13}$$

The safety factor n_b calculated from Eq. (11) shall always be $\geqq 1{,}5$ under normal conditions of loading, and $\geqq 1{,}3$ under exceptional conditions of loading.

In addition to the requirements in respect of the factors of safety n_s, n_b and n_{sb} specified in the above, it is necessary to ascertain in each individual case that the following requirements are fulfilled, viz., under normal conditions of loading,

$$n \geqq \frac{1{,}8\, \sigma_{icr}^{el}}{\sigma_P} \tag{14}$$

and, under exceptional conditions of loading,

$$n \geqq \frac{1{,}6\, \sigma_{icr}^{el}}{\sigma_P} \tag{15}$$

where σ_{icr}^{el} corresponding to the respective types of loading is determined from Eqs. (7), (8) and (9).

Stage 5. If the idealized comparison stress calculated from Eqs. (7), (8) or (9)

$$\sigma_{icr}^{el} > \sigma_P$$

buckling occurs within the inelastic behaviour range.

In this case, the factor of safety is determined as follows.

To begin with, calculate the „comparison slenderness ratio"

$$\lambda = \pi \sqrt{\frac{E}{\sigma_{icr}^{el}}} \tag{16}$$

After λ has been computed, the critical stress σ_{icr}^{pl} in the case of buckling within the inelastic (plastic) behaviour range corresponding to the slenderness ratio λ is determined with the aid of the critical stress curve of the web material.

The factors of safety are as follows:

a) When the web panel is submitted to shearing stresses only:

$$n_s = \frac{\sigma_{icr}^{pl}}{\sqrt{3}\,\tau_a} \tag{17}$$

b) When the web panel is submitted to normal stresses due to bending moments only:

$$n_b = \frac{\sigma_{icr}^{pl}}{\sigma_a} \tag{18}$$

c) When the web panel is submitted to combined shearing and normal stresses

$$n_{sb} = \frac{\sigma_{icr}^{pl}}{\sqrt{\sigma_a^2 + 3\,\tau_a^2}} \tag{19}$$

The factor of safety n calculated from Eqs. (17), (18) or (19) shall be, under normal conditions of loading,

$$n \geqq 1{,}8 \tag{20}$$

and, under exceptional conditions of loading,

$$n \geqq 1{,}6 \tag{21}$$

VI. Conclusions and economical considerations

The design procedure advanced in the preceding section has been applied to a great many tests made by various investigators [3, 5, 7, 8, 9, 11]. In all cases it has turned out that the proposed design procedure complies with very severe requirements in respect of both lateral deflections and actual factors of safety referred to the ultimate loads of the tested webs.

It is obvious that the low nominal factors of safety proposed in the above for buckling within the purely elastic behaviour range will affect the expenditure of materials used for webs as compared with the design rules used at the present time. Thus, for instance, the application of the proposed nominal factors of safety, as compared with the nominal factor of safety $n = 1{,}8$, will bring about a saving in material for all types of loading considered. This saving amounts to about 10 to 15 per cent of the weight of web in the case of buckling due to shearing stresses, and to about 15 per cent in the case of buckling due to normal stresses. Since the cross-sectional area of the web of deep plate I girders is approximately equal to the total flange area, the total saving will be 5 to 7,5 per cent and 7,5 per cent of the weight of girder respectively.

Summary

A brief account is given of some tests regarding buckling of webs in welded deep girders. On the basis of the results obtained from these tests and from tests made by other investigators, the general behaviour of webs in buckling and appropriate factors of safety providing against buckling are discussed. A tentative general design procedure, together with nominal factors of safety providing against buckling, is advanced. The design procedure thus advanced will result in a considerable saving in web material required for deep girders as compared with the design rules used at present, for instance, in Sweden and the United States.

Zusammenfassung

Der Aufsatz enthält einen kurzen Bericht über die Ergebnisse einer Versuchsreihe, bei der die Beulung der Stegbleche hoher Vollwandträger untersucht wurde. Auf der Grundlage dieser Versuchsergebnisse sowie der Ergebnisse, die von anderen Forschern erzielt worden sind, besprechen die Verfasser das Verhalten dünner Stegbleche unter Randbelastung in der Blechebene und die erforderlichen formalen Beulsicherheitszahlen. Ein allgemeines Bemessungsverfahren nebst formalen Beulsicherheitszahlen wird in Vorschlag gebracht. Im Vergleich zu den beispielsweise in Schweden und USA gegenwärtig üblichen Verfahren ergibt das empfohlene Bemessungsverfahren eine erhebliche Werkstoffersparnis bei der Konstruktion von Stegblechen für hohe Vollwandträger.

Résumé

Le présent article contient une description sommaire d'une série d'essais portant sur le voilement de l'âme des poutres en acier soudé de grande hauteur. En se basant sur les résultats de ces essais et des essais effectués par d'autres investigateurs, les auteurs examinent la tenue générale des âmes voilées et les coefficients de sécurité convenables contre le voilement de l'âme. En outre, les auteurs proposent un procédé de calcul général pour le dimensionnement des âmes et présentent des coefficients de sécurité nominaux contre le voilement. Quand on compare ce procédé à ceux qu'on emploie à présent, par exemple en Suède et aux Etats-Unis, on trouve qu'il résulte en une économie considérable des matériaux utilisés pour les âmes des poutres de grande hauteur.

Bibliography

1. Timoshenko, St.: Theory of Elastic Stability. New York 1936.
2. Iguchi, S.: Die Knickung der rechteckigen Platte durch Schubkräfte. Ingenieur-Archiv, vol. 9 (1938), p. 1.
3. Moheit, W.: Schubbeulung rechteckiger Platten mit eingespannten Rändern. Thesis Techn. Hochschule Darmstadt. Leipzig 1939.
4. Nölke, K.: Biegungsbeulung der Rechteckplatte. Ingenieur-Archiv, vol. 8 (1937), p. 403 (Thesis).
5. Gaber, E.: Beulversuche an Modellträgern aus Stahl. Bautechnik, vol. 22 (1944), p. 6.
6. Kromm, A. and Marguerre, K.: Verhalten eines von Schub- und Druckkräften beanspruchten Plattenstreifens oberhalb der Beulgrenze. Luftfahrtforschung, vol. 14 (1937), p. 627.
7. Wästlund, G. and Bergman, S. G. A.: Buckling of Webs in Deep Steel I Girders. Meddelande nr. 8, Statens Kommitté för Byggnadsforskning. Stockholm 1947.

8. Lyse, I. and Godfrey, H. J.: Investigation of Web Buckling in Steel Beams. Trans. Am. Soc. Civ. Eng., vol. 100 (1935), p. 675.
9. Seydel, E.: Ausbeul-Schublast rechteckiger Platten (Zahlenbeispiele und Versuchsergebnisse). Zeitschr. f. Flugtechnik und Motorluftschiffahrt, vol. 24 (1933), p. 78.
10. Lahde, R. and Wagner, H.: Versuche zur Ermittlung des Spannungszustandes in Zugfeldern. Luftfahrtforschung, vol. 13 (1936), p. 262.
11. Moore, R. L.: Observations on the Behaviour of Aluminum Alloy Test Girders. Proc. Am. Soc. Civ. Eng., vol. 72 (1946), p. 729.
12. Chwalla, E.: Beitrag zur Stabilitätstheorie des Stegbleches vollwandiger Träger. Der Stahlbau, vol. 9 (1936), p. 161.
13. Lode, W.: Der Einfluß der mittleren Hauptspannung auf das Fließen der Metalle. Forschungsarb. aus d. Gebiete d. Ingenieurwesens V.D.I. Berlin 1928.
14. Chwalla, E.: Erläuterungen zu Knick- und Beulvorschriften für Baustahl, DIN E 4114. 1940, Supplement to Bautechnik, vol. 17 (1939), No. 51/52, and vol. 18 (1940), No. 2/3.