Shear Deflections in Latticed Structures

Schubverformungen in Fachwerkkonstruktionen

Les déformations dues au cisaillement dans les ouvrages en treillis

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General Introduction

An investigation carried out by the author on the effect of shear deflections in the lattice stiffening truss of suspension bridges resulted in a general method of wide applicability to many classes of latticed structures.

In the following paper, which has been divided into two parts, part I deals with the suspension bridge stiffening truss, and part II with laced compression members.

Part I. Stiffened Suspension Bridges

Introduction

Before the spectacular failure of the Tacoma Narrows Bridge in December, 1940, focussed attention on the importance of the aerodynamic characteristics of suspension bridge structures there was a tendency for progressive designers to replace lattice stiffening trusses by relatively shallow plate girders without limit of span. This trend has now been checked, the superior aerodynamic stability of open lattice trusses having been conclusively established. It may be stated with some assurance that few if any bridges of any magnitude will be constructed in the near future with plate stiffening girders.

It has accordingly seemed advisable to look more closely into the question of the effective stiffness of lattice trusses, and how best to carry out the mathematical analysis of lattice stiffened suspension bridges.

When analysing a girder stiffened bridge, there is no uncertainty about the moment of inertia of the girder. This is not the case with the lattice truss. It is well known that for truss bridges of normal proportions, any attempt to calculate deflections from considerations of chord area alone would be wholly fallacious. Making certain assumptions as to the stress in the web and chord members, it can be shown [1] that for shallow trusses with a very small depth/span ratio, the effect of web distortions can be neglected. The lattice truss of a suspension bridge is generally regarded as coming in this class and it has been stated by one of the greatest authorities [2] that he calculates the moment of inertia very simply by multiplying the total chord area by the square of half the depth. This procedure is however by no means universally accepted. It has been suggested that an "equivalent" moment of inertia of a lattice stiffening truss be derived from a consideration of its deflection under various conditions of loading, but this results in an unsatisfactory "cut and try" procedure. In the analysis which follows it will appear that the "equivalent moment of inertia" of a lattice girder is a fictitious concept of no physical significance.

The deflection of a lattice girder can be regarded as being made up of two components, the flexural deflection, deriving from the extension of the chords, and the shear deflection arising from the stressing of the web members.

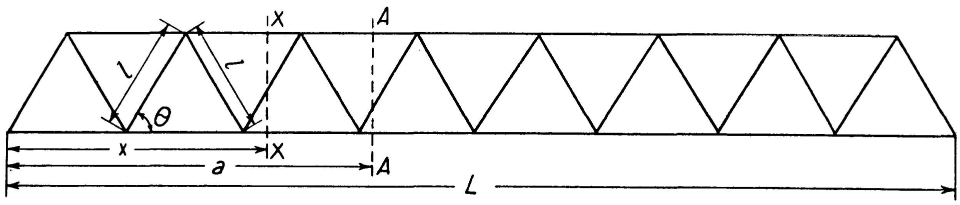

Fig. 1

In fig. 1 the simply supported truss has a web system consisting of members of equal cross-sectional area A_w, set at an angle Θ to the longitudinal axis of the truss. Any other system of truss panelling will respond to the same mathematical treatment. The assumption of uniform web members from end to end of the truss would be very near to the truth in modern suspension bridge design.

Initially an expression will be derived for the shear deflection of a truss in terms of the applied bending moment.

Let the truss be subject to any system of loading then the deflection q at any section $A-A$ from the extension of the web members only is given by the summation:

$$q = \Sigma \frac{s\,u\,l}{E\,A_w} \tag{1}$$

where s is the stress in any web member due to the external loading. u is the stress in the same member arising from a unit load at the section $A-A$, and l is the length of the member.

Now the unit load stresses in all the web members from $x=0$ to $x=a$, are given by:

$$u = \frac{1-a/L}{\sin \Theta}$$

From $x=a$ to $x=L$ the unit load stresses are

$$u = \frac{-\,a/L}{\sin \Theta}$$

At any section x, the contribution to the deflection at $A-A$ from a length dx at x is

$$\frac{F_G\,u\,dx}{E\,A_w \sin\Theta\cos\Theta}$$

where F_G is the shear at x from the external applied loads. The total shear deflection at AA is therefore

$$q = \frac{\int_0^x (1-a/L)\,F_G\,dx - a/L\int_x^L F_G\,dx}{E\,A_w \sin^2\Theta\cos\Theta}$$

$$= \frac{M_G}{E\,A_w \sin^2\Theta\cos\Theta} \tag{2}$$

This important result states that in any simply supported truss with uniform web members the shear deflection at any section is directly proportional to the applied bending moment M_G, and can be written as

$$q = \frac{M_G}{E\,A_e} \tag{3}$$

where A_e the "equivalent shear area" of the web members is $A_w \sin^2\Theta\cos\Theta$.

In a simply supported uniform beam we can write

$$-E\,I\frac{d^2 v}{d\,x^2} = M_G$$

where v is the bending deflection.

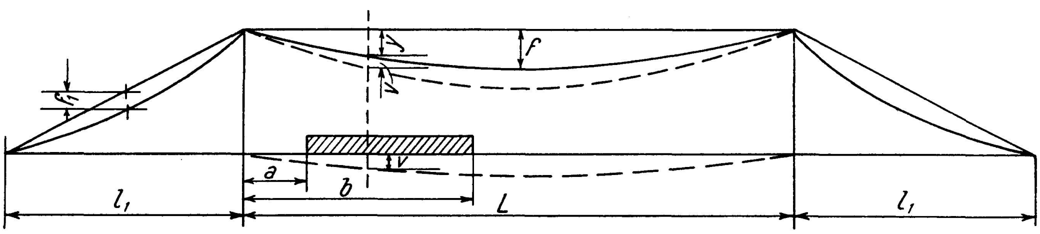

Fig. 2

Comparing this with the expression just obtained for q the shear deflection,

$$q = -\frac{I\dfrac{d^2 v}{d x^2}}{A_e} \tag{4}$$

The deflection due to shear follows therefore quite a different law to that due to bending, being proportional to the second differential of the bending deflection, and it is apparent that no "equivalent moment of inertia" can be derived for lattice girders.

The effect of shear deflection on the behaviour of the stiffened suspension bridge must now be considered.

Referring to fig. 2, making the usual assumptions in respect of non-extension and close spacing of hangers, and vertical movement of truss and cable, the fundamental equation of the suspension bridge is

$$M_G = M_L - H_L \cdot y - (H + H_L)\, v \tag{5}$$

where M_G = the bending moment in the stiffening truss at any section x.
M_L = the applied external bending moment at the same section.
H_L = the horizontal component of the live load cable tension.
H = the horizontal component of the dead load cable tension.
v = the vertical deflection of truss (and cable), due to the bending of the truss, i.e. the extension of the chord members.

H_L is unknown and is best obtained from a consideration of the kinematics of the deflected cable.

In the case of the lattice stiffened truss, the equation must be replaced by

$$M_G = M_L - H_L \cdot y - (H + H_L)\, v - (H + H_L)\, q \tag{6}$$

where q is the shear deflection of the truss. Substituting from eq. (3)

$$M_G = M_L - H_L \cdot y - (H + H_L)\, v - \frac{(H + H_L)\, M_G}{E\, A_e}$$

$$\therefore\ M_G = \frac{M_L - H_L \cdot y - (H + H_L)\, v}{1 + \dfrac{H + H_L}{E\, A_e}} \tag{7}$$

differentiating, the shear F_G is given by

$$F_G = \frac{F_L - H_L \cdot y' - (H + H_L)\, v'}{1 + \dfrac{H + H_L}{E\, A_e}} \tag{8}$$

where "dashes" denote differentiation with respect to x.

Eqs. (7) and (8) can be resolved by any of the orthodox methods, but are perhaps most readily dealt with by the series solution [3]. For those not familiar with this solution it will be expounded in its simplest form before taking into account the effect of web distortions.

The basis of the method is to express the variables M_G, M_L, y, v, in eq. (5) as sine series.

In the expression for the live load cable tension as a function of the deflection of the cable, the latter is also written as the sine series

$$v = V_1 \sin \pi x/_L + V_2 \sin 2\pi x/_L + \dots$$

In eq. (5) the coefficients of the terms of the series for M_G are then equated one by one to those on the right hand side of the equation. In the result the terms of the sine series for the deflection v are given by

$$V_\lambda = \frac{M_{L\lambda} - H_L G_\lambda}{H + H_L + \frac{\lambda^2 \pi^2}{L^2} E_t I}, \quad \lambda = 1, 2, 3. \tag{9}$$

$$\frac{H_L L_s}{E_c A_c} = \Sigma_\lambda K_\lambda V_\lambda + \Sigma_\lambda k_\lambda (W_\lambda + Z_\lambda) \tag{10}$$

In these equations:

$$M_{L\lambda} = \frac{2 p L^2}{\lambda^3 \pi^3} (\cos \lambda \pi a/L - \cos \lambda \pi b/L) \tag{11}$$

for the loading indicated in fig. 2.

$$G_\lambda = \frac{16 f}{\lambda^3 \pi^3} (1 - \cos \lambda \pi) \tag{12}$$

I is the moment of inertia of the stiffening trusses calculated simply from the chord areas and the depth.

L_s is the summation $\int \left\{ 1 + \left(\frac{d y}{d x} \right)^2 \right\}^{3/2} d x$ carried over the whole length of the cable between anchorages.

A_c is the cross sectional area of the cables.
E_t is the modulus of elasticity for the truss.
E_c is the modulus of elasticity for the cable.

$K_\lambda = \frac{16}{\lambda \pi} \cdot \frac{f}{L}$ where λ has odd values only.

$k_\lambda = \frac{16}{\lambda \pi} \cdot \frac{f_1}{l_1}$ where λ has odd values only.

W_λ and Z_λ are the deflection coefficients for the side spans corresponding to V_λ for the centre span.

These equations must now be modified to take into consideration the effect of shear deflections.

Writing B for $E_t I$, the rigidity of the stiffening truss, eqs. (9) and (10) become

$$V_\lambda = \frac{M_{L\lambda} - H_L G_\lambda}{H + H_L + \frac{\lambda^2 \pi^2 B}{L^2} \left(1 + \frac{H + H_L}{E A_e} \right)} \tag{13}$$

$$\frac{H_L L_s}{E_c A_c} = \Sigma_\lambda K_\lambda (V_\lambda + Q_\lambda) + \text{similar terms for side spans} \tag{14}$$

Where from eq. (3),

$$q = \Sigma_\lambda Q_\lambda \sin \lambda \pi x/L = \frac{M_G}{E A_e} \tag{15}$$

$$= \Sigma_\lambda \frac{\lambda^2 \pi^2 B}{L^2 E A_e} V_\lambda \sin \lambda \pi x/L \tag{15}$$

∴ in equation (14),

$$Q_\lambda = \frac{\lambda^2 \pi^2 B}{L^2 E A_e} V_\lambda \tag{16}$$

Having derived the coefficients V_λ, $v = \Sigma_\lambda V_\lambda \sin \lambda \pi x/_L$ is substituted in eq. (7), to obtain the bending moment in the stiffening truss.

Worked Examples

The series solution will now be applied to the case of a load on the central span of the bridge depicted in fig. 3 giving the maximum moment at the $^1/_4$ point of the span. The example is worked both with and without consideration of shear deflections. It will be apparent how very little additional labour is involved in taking account of web distortions.

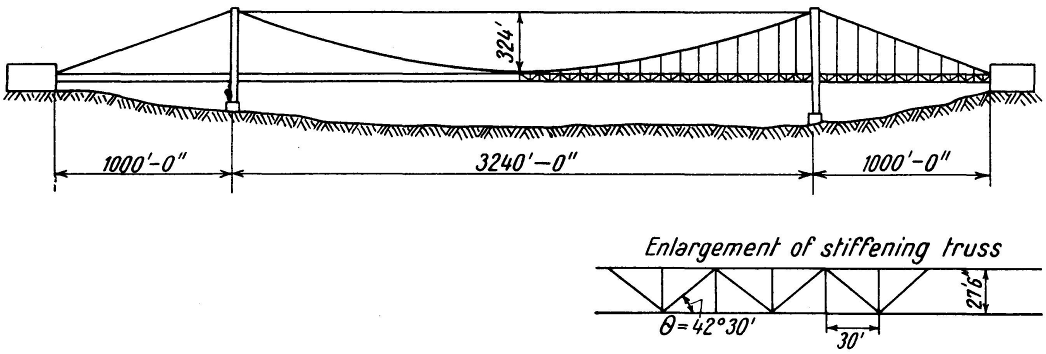

Fig. 3

For the Bridge Depicted in Fig. 3

A_w = cross-sectional area of diagonals	25 sq ins (each)
Depth of truss centre to centre of chords	27.5 ft
Chord area	130 sq in (each truss)
I	49,160 in² ft² (each truss)
$\frac{\pi^2 E_t I}{L^2} = \frac{\pi^2 B}{L^2}$	2.72×10^6 lbs (both trusses)
where E_t	29×10^6 lbs/sq in
Θ	42° 30′
Panel length	30 ft
$E_t A_w \sin^2 \Theta \cos \Theta$ (both trusses) = $E_t A_e$	493×10^6 lbs
Centre Span, "L" between bearings	3.220 ft
Side Span, "l" between bearings	990 ft
Area (both cables) A_c	960 sq in
Sag Ratio	$^1/_{10}$
Hanger Spacing	60 ft

$$L_s \begin{cases} \text{Centre Span} & 3503 \\ \text{Side Spans} & 2417 \\ \text{Anchorages} & 370 \end{cases}$$

total . . . . 6,290 ft

E_c . . . . 28×10^6 lbs/sq in

$\frac{E_c A_c}{L_s}$ . . . . 4.273×10^6 lb/ft

G_λ . . . . $\frac{331}{\lambda^3}$

g_λ . . . . $\frac{31 \cdot 22}{\lambda^3}$

K_λ 0.5063

k_λ 0.156

3240 Ft. Span Suspension Bridge. Live Load on Centre Span from $a = {}^3/_{16}\,L$ to $b = {}^5/_{16}\,L$, $p = 6050$ *lb/ft*

Calculation of H_L

$$M_{L\lambda} = \frac{2\,p\,L^2}{\lambda^3 \pi^3}\left(\cos\lambda\pi\frac{a}{L} - \cos\lambda\pi\cdot\frac{b}{L}\right)$$

$$= \frac{4046.8 \times 10^6}{\lambda^3}\left(\cos\frac{3}{16}\lambda\pi - \cos\frac{5}{16}\lambda\pi\right).$$

$H = 58.07 \times 10^6$ lbs

$H_L = 3.12 \times 10^6$ lbs (1st Approxm.)

$H + H_L = 61.19 \times 10^6$ lbs

	Centre Span						Side Span			
λ	$M_{L\lambda}$	$H_L G_\lambda$	$M_{L\lambda} - H_L G_\lambda$	$H + H_L + \frac{\lambda^2\pi^2 B}{L^2}$	V_λ	$K_\lambda V_\lambda$	$-H_L g_\lambda$	$H + H_L + \frac{\lambda^2\pi^2 B}{l_1^2}$	W_λ	$k_\lambda W_\lambda$
1	1116.51	1029.91	86.60	63.91	1.355	.686	−97.41	89.9	−1.084	−.169
3	117.698	38.15	79.55	85.63	.929	.157	−3.61	319.6	−.011	−.001
5	−37.981	8.24	−46.22	129.1	−.358	−.036	−0.78	789.0	−.001	
7	−16.367	3.00	−19.37	194.2	−.100	−.007				
9	7.767	1.41	6.36	281.0	.023	.001				
11	3.528	.77	2.76	389.6	.007	—				
13	−1.414	.47	−1.88	519.9	−.004	—				
15	−.331	.30	−.63	671.9	−.001	—				
						+.844				−.170
						−.043				−.170
						+.801				−.340
						−.340				
						+.461				

$H_L = .461 \times 4.273 \times 10^6$ lbs

$= 1.968 \times 10^6$ lbs

Calculation of H_L (contd.)

Let $\varDelta H_L = -0.2 \times 10^6$ lbs

λ	$-\varDelta H_L G_\lambda$	$H+H_L + \frac{\lambda^2\pi^2 B}{L^2}$	$\varDelta V_L$	$K_\lambda \varDelta V_\lambda$	$-\varDelta H_L g_\lambda$	$H+H_L + \frac{\lambda^2\pi^2 B}{l_1^2}$	$\varDelta W_\lambda$	$k_\lambda \varDelta W_\lambda$
1	66.02	63.71	1.036	.524	6.244	89.7	.070	.011
3	2.445	85.43	.029	.005	.231	319.4	.001	—
5	.53	128.90	.004					
				.529				.011
				.022				.011
				+.551				.022
				+.461				
				+1.012				

$3.12 - 1.968 = +1.152$

$2.92 - 4.32 \quad = -1.40$

$H_L = 1.012 \times 4.273 \times 10^6$ lbs

$\quad = 4.32 \times 10^6$ lbs

$$\frac{-.2 \times 1.152}{2.55} = \begin{array}{r} -.090 \\ 3.120 \\ \hline 3.030 \end{array}$$

$\therefore$ Final value of $H_L = 3.030 \times 10^6$ lbs

Calculation of B. M. at 1/4 Pt. for Load on Bridge 3/16 L to 5/16 L on Centre Span

$M_L \;= 1347.63 \times 10^6$ lbs ft

$H_L \;= \quad 3.030 \times 10^6$ lbs

$H_L y = \;\; 727.20 \times 10^6$ lbs ft $\qquad y = 240.00$

$$X = M_{L\lambda} - H_L G_\lambda - \frac{\lambda^2 \pi^2 B}{L^2} \cdot V_\lambda$$

λ	$M_{L\lambda}$	$H_L G_\lambda$	$M_{L\lambda} - H_L G_\lambda$	$H+H_L + \frac{\lambda^2\pi^2 B}{L^2}$	V_λ	$\frac{\lambda^2\pi^2 B}{L^2} \cdot V_\lambda$	X	$\sin \frac{\lambda \pi_x}{L}$	$X \sin \frac{\lambda \pi_x}{L}$
1	1116.510	1000.2	116.3	63.80	1.823	4.95	111.3	.7071	78.70
2	387.119		387.119	71.95	5.380	58.41	328.709	1.0000	328.709
3	117.698	37.04	80.66	85.52	.943	23.03	57.63	.7071	40.75
4	0		0	—	—	—	—	0	0
5	−37.981	7.981	−45.962	128.94	−.357	−24.189	−21.773	−.7071	15.396
6	−34.553		−34.553	158.80	−.218	−21.261	−13.292	−1.0000	13.292
7	−16.367	2.908	−19.275	194.08	−.099	−13.206	6.069	−.7071	4.291
8	0		0	—	—	—	—	0	0
9	7.767	1.366	6.401	280.93	.023	5.012	1.389	.7071	.982
10	7.391		7.391	332.50	.022	6.025	1.366	1.000	1.366
11	3.528	0.750	2.778	389.50	.007	2.332	.466	.7071	.315
12	0		0	—	—	—	—	—	0
13	−1.414	0.453	−1.867	519.77	−.004	−1.651	−.216	−.7071	.153
14	−1.150		−1.150	593.05	−.002	−1.011	−.139	−1.0000	.139
15	−0.331	0.293	−0.624	671.76	−.001	−.550	−.074	−.7071	.052
									484.14

$1347.63 - (727.2 + 484.14) = 136.29 \quad \therefore M_{1/4} = 136.29 \times 10^6$ lbs ft

Calculation of H_L for Load on Bridge from $^3/_{16}$ L to $^5/_{16}$ L of Centre Span, Including Shear Effects. p = 6050 lbs./ft.

$$M_{L\lambda} = \frac{2\,p\,L^2}{\lambda^3 \pi^3}\left(\cos\lambda\,\pi\frac{a}{L} - \cos\lambda\,\pi\frac{b}{L}\right)$$

$$= \frac{4046.8 \times 10^6}{\lambda^3}\left(\cos\frac{3}{16}\lambda\,\pi - \cos\frac{5}{16}\lambda\,\pi\right)$$

$H = 58.07 \times 10^6$ lbs

$H_L = 3.030 \times 10^6$ lbs (1st Approxm.) $\quad 1 + \frac{H+H_L}{E\,A\sin^2\Theta\cos\Theta} = 1 + \frac{61.1}{493} \doteq 1.124$

$H + H_L = 61.1 \times 10^6$ lbs $\quad \frac{E\,A}{L_s} = 4.273 \times 10^6$

Centre Span

λ	$M_{L\lambda}$	$H_L g_\lambda$	$M_{L\lambda}$ $-H_L G_\lambda$	$H+H_L$ $+1.124$ $\times \frac{\lambda^2\pi^2 B}{L^2}$	V_λ	$\frac{\pi^2\lambda^2 B}{L^2 E A_e}$	Q_λ	$(V_\lambda + Q_\lambda)$	$K_\lambda (V_\lambda+Q_\lambda)$	$\frac{\lambda^2\pi^2 B}{L^2} \times 1.124$
1	1116.510	1000.2	116.3	64.151	1.813	.006	.011	1.824	.923	3.051
3	117.698	37.04	80.66	88.556	.911	.050	.046	.957	.161	27.456
5	−37.981	7.981	45.962	137.366	−.335	.138	−.046	−.381	−.039	76.266
7	−16.367	2.908	−19.275	210.581	−.092	.270	−.025	−.117	−.008	149.481
9	7.767	1.366	−6.401	308.20	.021	.446	+.009	.030	.002	247.100
11	3.528	.750	2.778	430.233	.006	.666	.004	.010		369.133
13	−1.414	.453	−1.867	576.656	−.003	.930	−.002	−.005		515.556
15	− .331	.293	− .624	747.493	−.001				1.039	686.393

Side Span

λ	$-H_L g_\lambda$	$\frac{\lambda^2\pi^2 B}{l_1^2} \times 1.124$	$H+H_L$ $+1.124$ $\times \frac{\lambda^2\pi^2 B}{l_1^2}$	W_λ	$\frac{\pi^2\lambda^2 B}{l_1^2 E A_e}$	Q'_λ	$(W_\lambda + Q'_\lambda)$	$k_\lambda (W_\lambda + Q'_\lambda)$
1	94.597	32.270	93.370	−1.013	.058	−.059	1.072	−.167
3	3.503	290.430	351.530	−.01	.524	−.005	.015	−.001
5	.758	806.695	867.795	−.001				
								−.168
								−.168
								−.336
								1.039
								.703

$H_L = .703 \times 4.273 \times 10^6$ lbs

$= 3.0039 \times 10^6$ lbs

Let $\Delta H_L = -.01 \times 10^6$ lbs

λ	$-H_L G_\lambda$	$H+H_L + 1.124 \frac{\lambda^2\pi^2 B}{L^2}$	ΔV_λ	ΔQ_λ	$K_\lambda(V_\lambda + Q_\lambda)$	$-\Delta H_L G_\lambda$	ΔW_λ	$k_\lambda(\Delta W_\lambda + \Delta Q_\lambda)$
1	−3.3	64.141	.051	—	.026	.3	.003	—
3	−.12	88.546	.001	—	—			
					.026			
					.703			
					.729			

3.030—3.004 = .026

3.020—3.115 = −.095

$H_L = .729 \times 4.273 \times 10^6$ lbs

$= 3.115 \times 10^6$ lbs

$$\Delta H_L = \frac{.026}{.121} \times .01 = -.002$$

$$H_L = 3.030 - .002 = 3.028 \times 10^6 \text{ lbs}$$

Calculation of B.M. at 1/4 L for Load on Bridge 3/16 L to 5/16 L Centre Span, Allowing for Shear Effects

$$H_L = 3.028 \qquad H+H_L = 61.098$$

λ	$M_{L\lambda} - H_L G_\lambda$	$\frac{\lambda^2\pi^2 B}{L^2} \times 1.124$	$H+H_L + \frac{\lambda^2\pi^2 B}{L^2} \times 1.124$	V_λ	$V_\lambda \sin \frac{\lambda\pi}{4}$	λV_λ	$\lambda V_\lambda \cos \lambda\left(\frac{3\pi}{16}\right)$	$\lambda V_\lambda \cos \lambda\left(\frac{5\pi}{16}\right)$
1	116.967	3.051	64.149	1.823	1.289	1.823	1.516	1.013
2	387.119	12.202	73.300	5.280	5.280	10.560	4.041	−4.041
3	80.678	27.456	88.554	.911	.644	2.733	−.533	−2.680
4	—	—	—	—	—	—	—	—
5	−45.975	76.266	137.364	−.335	.237	−1.675	1.643	−.327
6	−34.553	109.823	170.921	−.202	.202	−1.212	1.120	−1.120
7	−19.280	149.481	210.579	−.092	.065	−.634	.352	−.527
8	—	—	—	—	—	—	—	—
9	6.398	247.100	308.198	.021	.015	.189	.105	−.157
10	7.391	305.065	366.163	.020	.020	.200	.185	−.185
11	2.777	369.133	430.241	.006	.004	.066	.065	−.013
12	—	—	—	—	—	—	—	—
13	−1.869	515.556	576.654	−.003	.002	−.039	−.008	−.038
14	−1.150	597.923	659.021	−.002	.002	−.028	.011	−.011
15	−.625	686.393	747.491	−.001	.001	−.015	.012	.008
					7.761		8.059	−8.077
							$\times \frac{\pi}{L}$	$\times \frac{\pi}{L}$
							$= 8.302 \times 10^{-3}$	$= -7.880 \times 10^{-3}$

$$M_G\left(1+\frac{H+H_L}{E\,A_w\sin^2\Theta\cos\Theta}\right)=M_L-H_L y-(H+H_L)\,v$$

$$M_G=\frac{1347.63-3.028\times240.00-61.098\times7.761}{1.124}$$

$$=\frac{1347.63-1200.901}{1.124}=\frac{146.729}{1.124}=\underline{130.521}$$

$$M_G=130.521\times10^6\text{ lbs ft}$$

$$F_G\left(1+\frac{H+H_L}{E\,A_w\sin^2\Theta\cos\Theta}\right)=F_L-H_L y'-(H+H_L)\,\Sigma\frac{\lambda\pi}{L}V_\lambda\cos\frac{\lambda\pi x}{L}$$

$$F_{G3/16}=\frac{1.8264-3.028\times.2483-61.098\times8.032\times10^{-3}}{1.124}$$

$$=\frac{.5667}{1.124}\times10^6\text{ lbs}=.5042\times10^6\text{ lbs}=\underline{225.08}\text{ tons.}$$

$$F_{G5/16}=\frac{-.6088-3.028\times.1491+61.098\times7.880\times10^{-3}}{1.124}$$

$$=-\frac{.5788}{1.124}\times10^6\text{ lbs}=-.5149\times10^6\text{ lbs}=-\underline{229.9}\text{ tons.}$$

Magnitude of Corrections for Lattice Girder Web Distortions

The following table shows the percentage error in the maximum bending moments and shears at various points along the centre span in the bridge in question resulting from neglect of the shear corrections. The maximum error in moment amounts to 6% and in shear to about 8%. These are not negligible figures, but omission of the correction is on the side of safety. The correction is so easy to apply that there is no justification for not running out its magnitude at some stage in the analysis.

It is the author's practice to analyse a suspension bridge by the simplified method given on pp. 94 and 95, and then to consider separately the various corrections to the approximate theory due to such factors as:

1. Non uniform distribution of live load.
2. Non vertical motion of cable and hangers.
3. Hanger extension.
4. Non uniform truss moment of inertia.
5. Shear deflection of stiffening truss.

These corrections can be established on a percentage basis for one particular make-up of truss and cable and can thereafter be applied throughout the inevitable changes in design, one final check up being made when this is settled.

Conclusions

The author has presented what he believes to be a satisfactory treatment of the effect of extensions of the web members of lattice stiffening trusses on the moments and shears carried by the trusses. The effect of the web extensions was found to be a relief of stress in the stiffening truss. The magnitude of the corrections in the example investigated, which is perhaps typical of modern practice in long span bridges, is of the order of 5% to 8%.

In bridges with a greater ratio of truss depth to span, the correction would be even more substantial. The additional work in the analysis to take account of the shear effect is small, and its neglect cannot be justified on structures of any magnitude, particularly when, as is frequently the case, other corrections to the simplified theory are considered at length.

Table Shewing Percentage Error in Moments and Shears on the Centre Span of a 3,240 Ft. Suspension Bridge Arising from Neglect of Shear Deflections

Position of Load	M_G without Shear Effect ft lbs $\times 10^6$	M_G including Shear Effect ft lbs $\times 10^6$	%	F_G without Shear Effect Tons	F_G including Shear Effect Tons	%
0 to $^1/_8$	119.64	112.52	6	(0) 479.3	443.4	7.5
	($^1/_{16}$)			($^1/_8$) 215.1	194.8	9.4
$^1/_{16}$ to $^3/_{16}$	148.8	142.0	4.6	($^1/_{16}$) 272.6	254.8	6.5
	($^1/_8$)			($^3/_{16}$) 246.3	226.4	8.1
$^3/_{16}$ to $^5/_{16}$	136.29	130.52	4.2	($^3/_{16}$) 244	225.1	7.8
	($^1/_4$)			($^5/_{16}$) 248.5	229.9	7.5
$^5/_{16}$ to $^7/_{16}$	125.38	120.00	4.2	($^5/_{16}$) 245.9	228.9	7
	($^3/_8$)			($^7/_{16}$) 247.1	226.6	8.3
$^7/_{16}$ to $^9/_{16}$	121.9	116.02	4.8	($^7/_{16}$) 241.3	226.9	7.9
	($^1/_2$)					

Part II. The Strength of Latticed Compression Members

Introduction

In the first part of this paper the author has established a relationship between the bending moment and the shear deflection in a simply supported lattice girder.

This relationship can be shown to be applicable to the increased deflection under load and consequent reduction in strength of laced struts as compared to struts of solid box section.

The author's method will first be used to establish the Euler Load for a pin-jointed laced column, and a correction to the Perry Strut Formula used in B.S. 449, 1948, "The Use of Structural Steel in Building", will then be derived.

Analysis

The column in fig. 4 has single or double lacing on both sides of the column, with a total cross-sectional area A_d, at an angle Θ with the longitudinal axis.

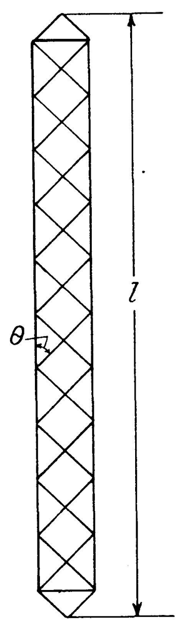

Fig. 4

Fig. 5

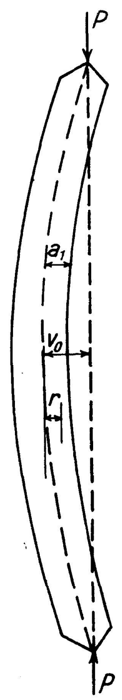

Fig. 6

Let the lateral bending deflection of the strut when subject to a load P be v at any section x (fig. 5). Then from Part I there will be an additional shear deflection of

$$q = \frac{M x}{E A_d \sin^2 \Theta \cos \Theta} = \frac{M x}{F_D} (s a y) \tag{17}$$

where M_x is the bending moment induced at x from the end loading.

If the flexural rigidity of the column in the plane of bending is $B = E I$, the differential equation of the deflection curve becomes therefore

$$-\frac{B d^2 v}{d x^2} = P (v + q) = P v + \frac{P \cdot M_x}{F_D}$$

$$P v - \frac{P B}{F_D} \frac{d^2 v}{a x^2} = -\frac{B d^2 v}{d x^2}$$

writing $v = V \sin \frac{\pi x}{l}$

$$P\left(1 + \frac{\pi^2 B}{F_D l^2}\right) = \frac{\pi^2 B}{l^2}$$

$\frac{\pi^2 B}{l^2}$ is the Euler Critical Load for a solid pin jointed strut and it will be written as F_e.

$$\text{Then } P = \frac{F_e}{1 + \frac{F_e}{F_D}} \tag{18}$$

Dividing through by A, the cross-sectional area of the strut

$$f_c = \frac{f_e}{1 + \frac{A \cdot f_e}{F_D}} \tag{19}$$

Thus the buckling load of the strut is reduced in the ratio

$$\frac{1}{1 + \frac{F_e}{F_D}} \quad \text{or} \quad \frac{1}{1 + \frac{A \cdot f_e}{F_D}}$$

This result has been obtained in a somewhat different manner by TIMOSHENKO [4].

In Great Britain the Perry strut formula (see Appendix I), has been adopted in B.S. 449, 1948, as the basis for determining the permitted axial stresses in struts; although it should be noted that for slenderness ratios l/r of less than 80, the tabulated stresses in B.S. 449 have been obtained by linear interpolation between a stress equal to 59% of the yield stress, at $l/r = 0$, and the Perry strut value at $l/r = 80$.

The Perry strut formula in the form in which it is used in B.S. 449 is (see fig. 6):

$$K_2 \cdot F_a = \frac{f_y + (\eta + 1) f_e}{2} - \sqrt{\left[\frac{f_y + (\eta + 1) f_e}{2}\right]^2 - f_y \cdot f_e} \tag{20}$$

where F_a = the permissible average stress in tons/sq in.
K_2 = load factor of 2.
f_y = guaranteed min. yield stress in tons/sq in.
f_e = Eulerian Value $= \frac{\pi^2 E}{(l^2/r^2)}$
l/r = slenderness ratio $= \frac{\text{effective length}}{\text{radius of gyration}}$
$\eta = \frac{V_0 a_1}{r^2} = .003\, l/r$.

and V_0 = hypothetical initial central deflection of the strut.
a_1 = distance from neutral axis to outside fibre on compression side of strut.

In view of eq. (19) it would be anticipated that the effect of the lacing correction on the Perry formula would be the replacement of f_e by

$$\frac{f_e}{1+\frac{A \cdot f_e}{F_D}} \qquad (20a)$$

Analysis (Appendix I) shows this to be the case.

Magnitude of Correction

Assuming that the lacing is single, at 60° to the axis of the strut, and that it is designed to resist at 4 tons/sq in a transverse shear of $2\frac{1}{2}\%$ of the axial load in the strut, il will be found that the reduction of strength of a laced strut as compared with a strut of solid box section is of the order of 3% to 4% over the whole range of practicable values of l/r.

It is probably unnecessary to reduce the stress in laced struts designed by the usual methods, but any unorthodox design, particularly if the lacing is highly stressed, should be checked by the application of the correction (20a) to the slenderness ratio, to see if its strength is substantially below that of a solid section.

Appendix I

The Perry strut formula in the form in which it is used in deriving the tables in B.S. 499 is:

$$K_2 F_A = F_y + \frac{(\eta+1) f_e}{2} - \sqrt{\left[\frac{f_y+(\eta+1) f_e}{2}\right]^2 - f_y \cdot f_e}$$

where the symbols have the values ascribed to them in this paper.

The Perry formula will now be derived for a latticed strut.

In fig. 6, let v_0, the hypothetical initial deflection of the strut, be given by

$$v_0 = V_0 \sin \pi x/l.$$

Under the axial load P the bending deflection v will also be of the sinusoidal form

$$v = V \sin \pi x/l.$$

In addition there will be the shear deflection arising from the distortion of the bracing,

$$q = \frac{M_x}{F_D}$$

The differential equation of the deflection curve becomes

$$-\frac{B\,d^2 v}{d\,x^2} = P\left(v_0 + v - \frac{B}{F_D}\frac{d^2 v}{d x^2}\right)$$

$$\therefore \frac{B\,\pi^2}{e^2} V \sin \pi x/l = P\left(V_0 \sin \pi x/l + V \sin \pi x/l + \frac{B}{F_D}\frac{\pi^2}{l^2} V \sin \pi x/l\right)$$

$$V = \frac{P\,V_0}{f_e(1 - P/F_D) - P}$$

Dividing through by the area A of the strut,

$$V = \frac{f\,V_0}{f_e\left(1 - \frac{A\cdot f}{F_D}\right) - f} \tag{21}$$

where f is the average stress in the column.

The applied bending moment at $x = l/2$ is therefore

$$\left(-\frac{B\,d^2 v}{d\,x^2}\right)_{x=l/2} = \frac{P_e\,f\,V_0}{f_e\left(1 - \frac{A\cdot f}{F_D}\right) - f} \tag{22}$$

Writing $$\frac{v_0\,a_1}{r^2} = \eta$$

where a_1 = the distance from the neutral axis to outside compression fibre of the strut. Then the total maximum conpressive stress in the strut is

$$f_1 = f\left[\frac{f_e\,\eta}{f_e\left(1 - \frac{A\cdot f}{F_D}\right) - f} + 1\right]$$

If the maximum stress f_1 is taken as the yield stress f_y,

$$f_y = f\left[\frac{f_e\cdot\eta}{f_e\left(1 - \frac{A\cdot f}{F_D}\right) - f} + 1\right]$$

Solving for f,

$$f = \frac{f_y + \dfrac{(\eta+1)\,f_e}{1 + \dfrac{A\cdot f_e}{F_D}}}{2} - \sqrt{\left\{\frac{f_y + \dfrac{f_e(\eta+1)}{1 + \dfrac{A\cdot f_e}{F_D}}}{2}\right\}^2 - \frac{f_y\cdot f_e}{1 + \dfrac{A\cdot f_e}{F_D}}} \tag{23}$$

Comparing this with (20), it is seen that as anticipated, the only change is the reduction of f_e in the ratio

$$\frac{1}{1 + \dfrac{A\cdot f_e}{F_D}}$$

Appendix II

References

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3. S. Timoshenko. "The Stiffness of Suspension Bridges". Trans American Society. Civil Engineers. Vol. 94 (1935), p. 377.
   C. D. Crosthwaite. "Corrected Theory of the Stiffened Suspension Bridge". Journal of Institute of Civil Engineers. Feb. 1947.
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5. B.S. 449, 1948, pp. 20—21, 66, 69, 71. "The Use of Structural Steel in Building".

Summary

From a critical examination of the concept of the "equivalent moment of inertia" of the lattice stiffening truss of a suspension bridge, a general treatment of the web extensions of lattice structures is derived, applicable to a wide class of structures.

The paper is in two parts. In Part 1, dealing with the lattice stiffening truss, a simple method of correction for web distortions is set out, and the magnitude of the corrections in a typical case is established. In Part II the reduced strength of laced struts, as compared with struts of solid box section, is considered.

Zusammenfassung

Ausgehend von einer kritischen Betrachtung zum Begriff des „äquivalenten Trägheitsmoments" eines fachwerkförmigen Versteifungsträgers bei Hängebrücken wird eine allgemeine Behandlung der Verformungen der Füllungsglieder in Fachwerkkonstruktionen abgeleitet, die auf mannigfache Systeme angewendet werden kann.

Der Aufsatz besteht aus zwei Teilen. Der erste behandelt den fachwerkförmigen Versteifungsträger. Es wird eine einfache Methode angegeben, um die Verformungen der Füllungsglieder zu berücksichtigen und die Größe der Korrekturen wird für einen typischen Fall berechnet. Im zweiten Teil wird die Festigkeitsverminderung bei Gitterstützen im Verhältnis zu Stützen mit Kastenquerschnitt studiert.

Résumé

Après avoir examiné du point de vue critique la conception du „moment d'inertie équivalent" de la poutre raidisseuse d'un pont suspendu, l'auteur établit un mode général d'étude de l'allongement des parties portantes des éléments en treillis, méthode qui est applicable à une très large gamme d'éléments.

Le rapport est divisé en deux parties. La première partie porte sur la poutre raidisseuse en treillis et l'auteur y expose une méthode simple pour la correction des distorsions des faces portantes; il détermine l'amplitude de ces corrections dans un cas caractéristique. Dans la deuxième partie, l'auteur étudie la réduction de résistance du treillis par rapport à la section pleine en caisson.