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Model Verification of the Classical Flutter Theory as Adapted to the Suspension Bridge

Bestätigung der klassischen Schwingungstheorie der Hängebrücken durch Modellversuche

Vérification sur modèles de l'application aux ponts suspendus de la théorie classique du battement

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Introduction

In the course of laboratory investigations, in the year 1943, of a full model of the original Tacoma Narrows Bridge, a modification of the original section was made involving a complete fairing of the form shown in Fig. 1.

When this model was subjected to horizontal winds up to the equivalent of 117 mph on the prototype in successive increments it was found to be stable in all regards[1]). At each wind increment several modes of notion, both vertical and torsional, were forced to rather substantial amplitudes only to

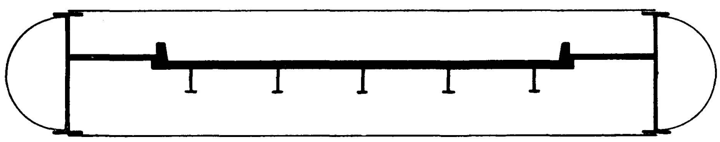

Fig. 1

decay almost instantly. At wind velocities above the equivalent of 117 mph it was found that a forced torsional oscillation beyond a certain moderate amplitude resulted in a rapid augmentation of amplitude which was almost explosive in nature whence it became necessary to secure the model by hand in order to prevent its destruction.

[1]) "Aerodynamic Stability of Suspension Bridges with Special Reference to the Tacoma Narrows Bridge", University of Washington, Bulletin No. 116. "Part III — The Investigation of Models of the Original Tacoma Narrows Bridge under the Action of Wind", by F. B. FARQUHARSON.

Although it was not recognized at the time, there was witnessed here a typical manifestation of flutter in which a vertical mode and a torsional mode of similar wave form and frequency were coupled.

This phenomenon was not remarked again until the construction of a new full model for the proposed rebuilding of the Tacoma span was complete. In its original form the design incorporated a topdeck — truss-stiffened arrangement in which the deck was essentially a thin flat plate modified by sidewalk curbs on top and stringers underneath. During early tests on this model, it was observed that the fundamental torsional motion forced by an appropriate wind developed a frequency somewhat less than the natural frequency for this mode. It was likewise noted at this time that the center of rotation in torsion was often sharply shifted up-stream. Again the significance of these typical indications of flutter remained unappreciated although the shift of the center of rotation was recognized as the result of the coupling of a vertical and a torsional mode. Subsequently similar indications of flutter were observed on section models with both one and two degrees of freedom. Dr. Friedrich Bleich was the first to recognize the significance of these experimental findings and his investigation into their meaning resulted in the rationalization of the classical flutter theory for application to the suspension bridge[2]).

Model Testing Procedure

In the course of Dr. Bleich's theoretical studies of the flutter problem on suspension bridges, certain laboratory tests were proposed for the purpose of examining the applicability of the theory. These tests were carried out at the University of Washington on a thin flat plate mounted as a section model with two degrees of freedom. Since it was desirable to investigate several variations

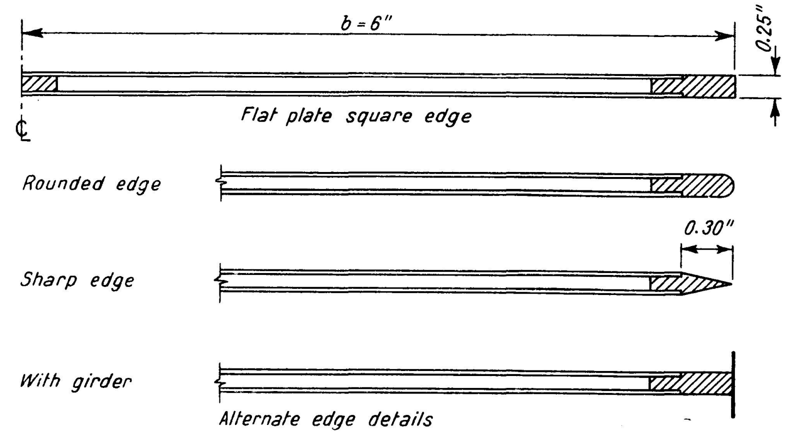

Fig. 2

[2]) "Dynamic Instability of Truss-Stiffened Suspension Bridges under Wind Action", by Friedrich Bleich, A.S.C.E. Transactions Vol. 114, 1949 p. 1177.

of the condition at the leading and trailing edges of this model a basic model was constructes from magnesium sheet and copper.

The basic model was a built-up section 12 in. wide and 60.75 in. long, and 0.25 in. thick. For model A, the edges were square, while for models B and C the edges were semi-circular and sharp respectively, to minimize vortex discharge in a horizontal wind and thus approach the theoretical conditions of an infinitely thin flat plate (see Fig. 2). For model A, with square edges, the depth-to-width ratio was 0.0208, bringing into play small vortex forces.

For models G-1 and G-2, model A was used with rudimentary girder webs of fiberboard 0.012 in. thick, attached to the edges to give depth-to-width ratios of 0.05 and 0.067 respectively. A wood model, H, 13.21 in. by 1.58 in., with semi-circular edges was available and was tested for purposes of comparison, although its depth-to-width ratio was 0.12.

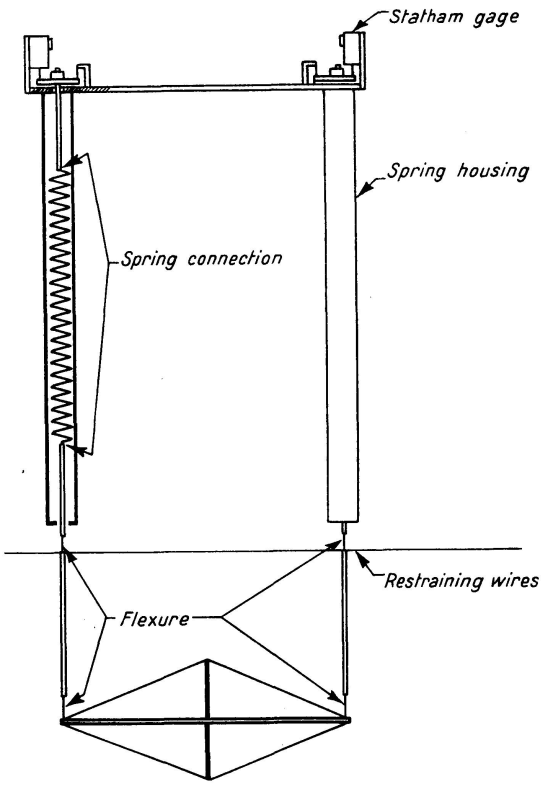

Fig. 3

The mass properties of the various models (including attachments, end plates, and one-third of the mass of the springs) are shown in Table I. These properties differed little and all models were tested on the same set of carefully matched coil springs which provided two degrees of freedom and permitted the model to move vertically and at the same time to rotate about a longitudinal axis. However, models G-1 and G-2 were also tested with added mass to alter the vertical and torsional frequencies and again with torsional spring

Table I. Comparison of experimental and theoretical results

Model[a]	Weight (lb.)	r (in.)	$\left(\frac{\omega_2}{\omega_1}\right)^2$	Flutter Frequencies ω (Radians per sec)		Critical Velocity V_c (ft per sec)		k_c (Critical Value of k)		Theoretical[b] Air Force Function, f_4	
				Observed	Theoretical	Observed	Theoretical	Observed	Theoretical	A_v	B_v
A	15.68	4.784	1.744	8.168	7.900[c]	11.33	13.58[c]	0.360	0.291[c]	0.0271	0.0360
B	15.37	4.770	1.786	8.273	8.032[c]	12.15	13.94[c]	0.340	0.288[c]	0.0164	0.0314
C	14.32	4.639	1.848	8.650	8.357[c]	12.32	14.26[c]	0.351	0.293[c]	0.0112	0.0354
H	14.54	4.314	1.971	8.671	—	11.67	—	0.409	—	0.0588	0.0251
G–1	15.68	4.784	1.744	8.273	—	8.83	—	0.468	—	0.1680	0.0297
			2.849	10.263	—	12.29	—	0.418	—	0.1707	0.0552
			3.129	10.891	—	13.16	—	0.414	—	0.1221	0.0675
			3.327	11.331	—	14.32	—	0.396	—	0.0667	0.0751
G–1	17.31	4.569	1.877	8.168	—	9.15	—	0.446	—	0.1552	0.0445
G–2	15.68	4.784	1.744	8.482	8.584[d]	6.84	6.66[d]	0.620	0.644[d]	0.1900	0.0672
			2.856	10.660	10.650[d]	8.66	8.63[d]	0.616	0.617[d]	0.2037	0.0896
			3.149	11.205	11.236[d]	9.32	9.15[d]	0.601	0.614[d]	0.2472	0.0838
			3.357	11.624	11.624[d]	9.42	9.49[d]	0.617	0.612[d]	0.2557	0.0867
G–2	17.11	5.228	1.477	7.540	7.605[d]	6.00	5.78[d]	0.628	0.658[d]	0.1712	0.0594
G–2[e]	15.68	4.784		8.376	—	7.49	—	0.563	—	0.2999	0.0914
				10.594	—	8.88	—	0.597	—	0.2913	0.0897
				11.128	—	9.33	—	0.596	—	0.3108	0.0898
				11.744	—	9.60	—	0.610	—	0.2419	0.0892

[a] Descriptions: Model A, square edges; model B, rounded edges; model C, sharp edges; model H, thick plate; model G–1, 0.6-in. girder; model G–2, 0.8-in. girder.
[b] Theoretical value of f_4 based on corresponding observed values of k_c.
[c] Theoretical values, assuming pure flutter, that is $f_4 = 0$.
[d] Theoretical values based on values of f_4 taken from Fig. 12.
[e] Model G–2 with fixed center of rotation.

action of three different values acting in parallel with the vertical springs in order to further modify the relative vertical and torsional frequencies.

Details of the coiled spring mountings are shown in Fig. 3, and the arrangement of double restraining wires (Type II), used to resist the static wind pressure and to prevent sway and interference with the spring housing, is shown in Fig. 6 of the paper "Mathematical Prediction of Suspension Bridge Behavior in Wind from Dynamic Section Model Tests" by GEORGE S. VINCENT (Page 303 of this volume). Figure 4 illustrates the restraining wire arrangement (Type I) used when it was desired to increase the torsional resistance. In both arrangements the wires were anchored so far from the model that the vertical component of their tension was small and offered negligible resistance to the vertical oscillation. However, when the single wire was attached to both supports of the model (Fig. 4) the vertical component of that portion

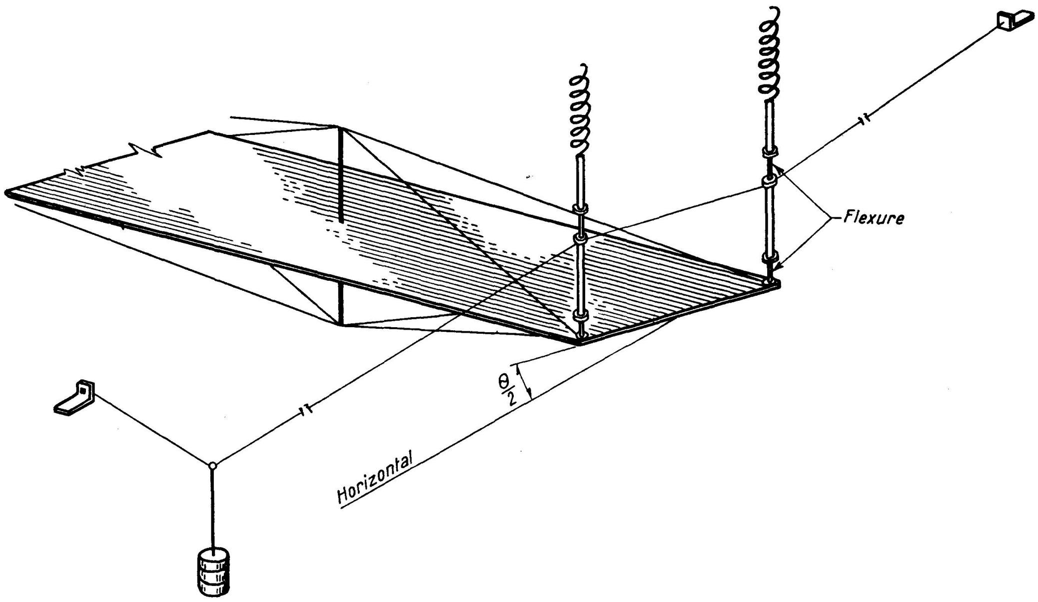

Fig. 4

between the two model supports was significant and for small angular motion, increased nearly in proportion to the angle of rotation. Alteration in the magnitude of the tensioning weight shown in Fig. 4 acted to vary the stiffness of this parallel spring action and provided a considerable range of variation in torsional frequency. This device acted as a spring which opposed the torsional motion, but not the vertical motion, corresponding to the torsional rigidity of the towers and the suspended structure of a suspension bridge.

End plates were used on all of the models as a device for providing a condition of infinite aspect ratio by preventing air circulation around the ends of the models. However, it has appeared, in connection with many comparison tests with and without end plates, that for any aspect ratio (length to width) in excess of 4.5 the end effect is negligible.

The motion of the model was recorded on a Brush magnetic oscillograph, receiving the amplified output of a Statham unbonded wire resistance strain gage incorporated in the support of one of the springs. This gage responds to the changing inertial force on the spring, and can be calibrated to record the motion.

The values of ω_1 and ω_2, the circular frequencies (radians per second) of the vertical and torsional oscillations of the models, were determined by manual excitation of these modes in still air. When the model was excited in torsion in a wind stream the frequency was less than in still air and the axis of rotation was no longer on the longitudinal axis of the model but was shifted up-stream several inches. The flutter frequency, ω, at the critical velocity, V_c (ft. per sec), is recorded in Table I, at zero degrees angle of attack, for each model, together with the observed value of $k_c = \omega b / V_c$.

The logarithmic decrements, δ_0, in still air at a double amplitude of 0.95 in. in the case of vertical oscillations and at 9° in the case of torsion, are shown for comparative purposes only in Figs. 5 to 10. Previous tests of the same springs with streamlined weights of about the same mass as these models showed δ_s, the logarithmic decrement due to structural damping, to be about 0.002. This value, when subtracted from the recorded values of δ_0, leaves δ_a, the logarithmic decrement arising from aerodynamic damping. These tests demonstrated that the aerodynamic damping, as measured by the logarithmic decrement, increases with amplitude — that is, with the velocity of vibration. This increase is an indication that the air forces opposing the oscillation vary as the square of the velocity in accordance with Froude's Law.

Analysis of Tests on Various Models

When model A was subjected to a wind at angle of attack $\beta = 0$ and manually excited as the wind velocity was increased by small increments, the center of rotation was observed to shift up-stream on a steadily steepening curve until a velocity of 11.0 fps was reached (see Fig. 5). This wind velocity corresponded very closely to the observed critical velocity V_c (the flutter velocity) in coupled motion. Further increase in wind velocity caused the center of rotation to move further up-stream but along a line of much reduced slope. The range of wind velocities used in this test was limited to 16.5 fps by the increasing violence of the motion beyond V_c where the model was self-excited and rapidly reached catastrophic amplitudes. At the maximum wind velocity reached in this test the center of rotation had moved to a position 1.4 in. ahead of the leading edge of the model.

Observations of variation in frequency with wind velocity are also recorded in Fig. 5 where the shift was from $N = 84$ cpm in still air to $N = 74.7$ cpm at the maximum velocity reached.

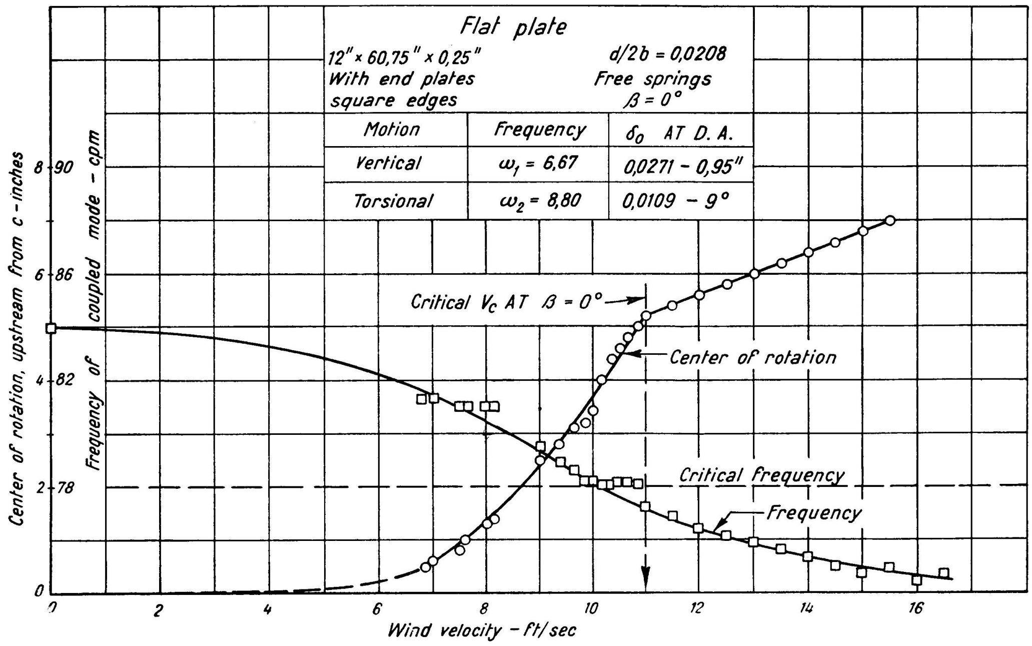

Fig. 5

Between $V = 10.1$ fps and 10.8 fps, the frequency was found to remain constant at 78 cpm ($\omega = 8.16$ radians, which is seen to lie between the observed values for ω_1 and ω_2) and the response curve characteristic of this condition shows a backward slope and is unstable — that is, there is no steady state amplitude (see curve $\beta = 0$ in Fig. 6). This condition characterizes the flutter phenomenon.

It is interesting to note that another region of constant frequency with increase in velocity exists between $V = 7.5$ fps and 8.2 fps. It is possible that a mild non-catastrophic torsional response would have been developed in this region if the damping could have been sufficiently reduced.

For the flat plate models A, B, and C, the theoretical flutter velocity, V_c, and the flutter frequency, ω, were computed according to the method of Bleich[3]). The application of this theoretical equation is simplified on a section model (with two degrees of freedom) where the frequencies ω_1 and ω_2 are associated with the same displacement form. The logarithmic decrement of structural damping $\delta_s = 0.002$ which applied in these tests had a negligible effect on V_c as was demonstrated by Bleich where it was found that a damping equivalent to $\delta = 0.05$ increased V_c by only about 6%[4]). The structural damping arising from the model suspension was therefore neglected in these tests.

The curves in Figs. 6 to 10 show amplitude plotted against wind velocity for various models. The critical velocity is that velocity at which the curve intersects the velocity axis. It will be observed that some of these curves slope

[3]) Bleich, *op. cit.*, equ. 59 (with $D = 1$), p. 1200.

[4]) Bleich, *op. cit.*, p. 1207.

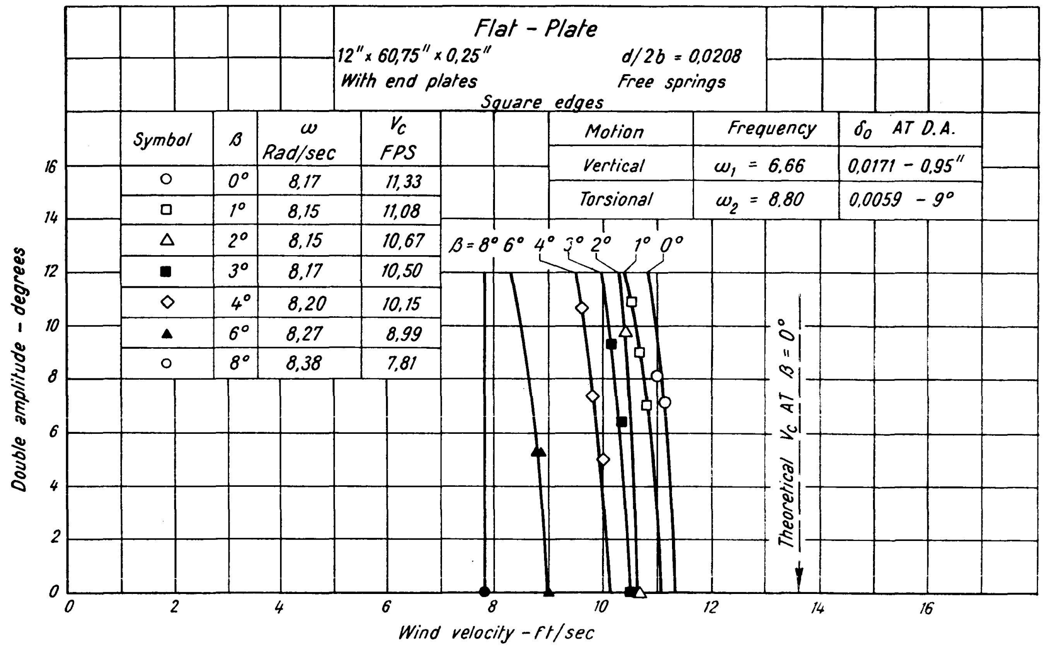

Fig. 6

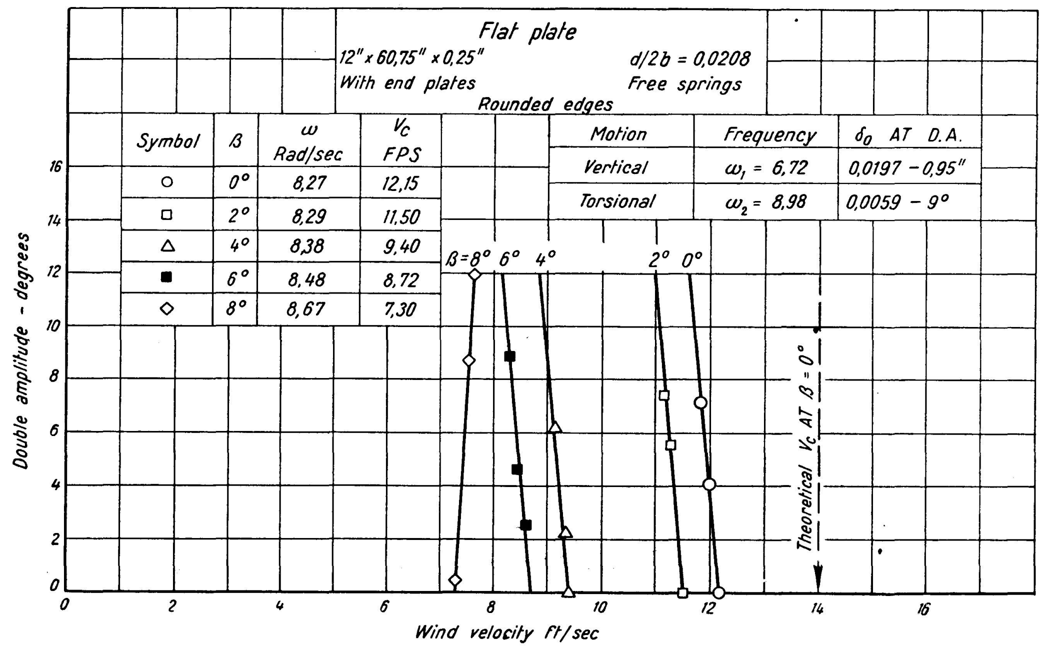

Fig. 7

backwards and others forward. The plotted points on these two types of curve are obtained by two entirely different procedures. When, for a condition which yields a backward sloping curve (for example $\beta = 0$ in Fig. 6), the wind velocity is slowly increased, no motion will be observed until the critical velocity is surpassed, whereupon motion begins spontaneously and the amplitude increases indefinitely. However if, at a velocity $V = 11.1$ fps the model is manually excited to a double amplitude of 6.8° motion will quickly decay to rest. If,

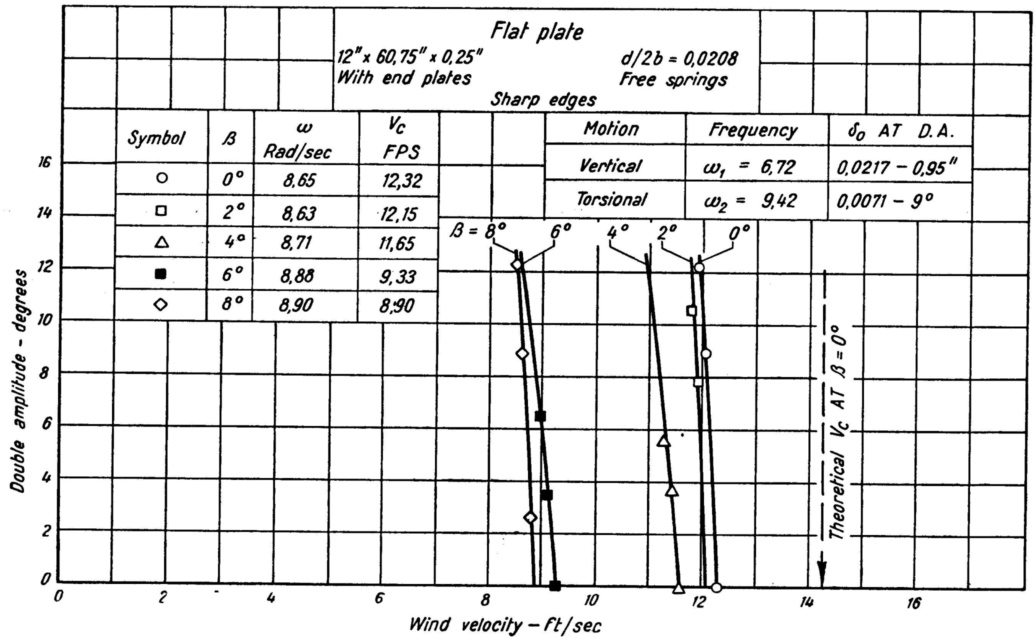

Fig. 8

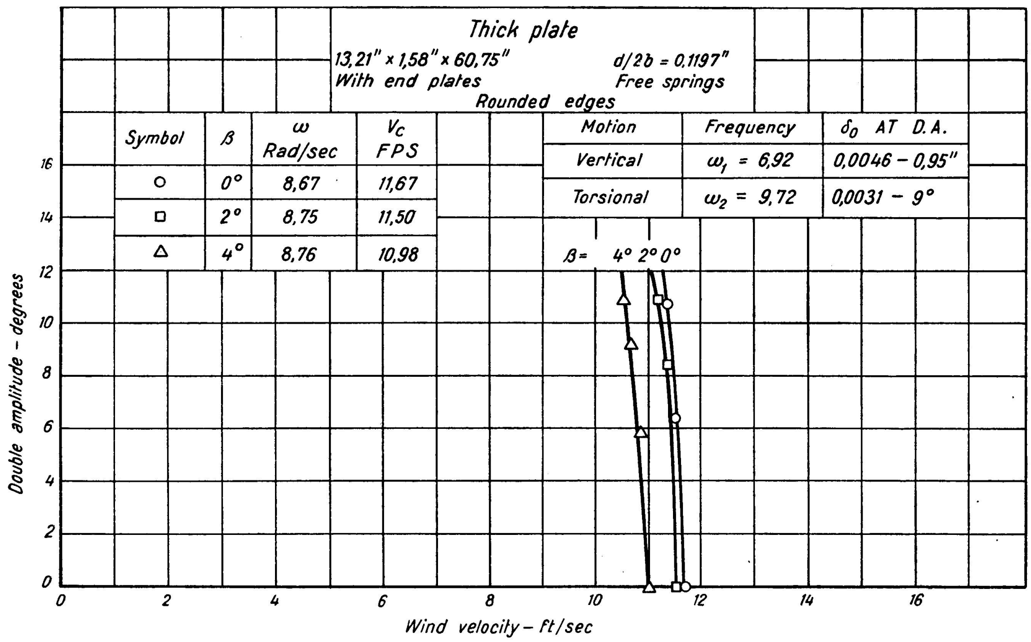

Fig. 9

while maintaining this same velocity, motion is excited beyond 7.2° of double amplitude, then the motion will increase indefinitely. The plotted points are obtained through experimentally applied amplitudes until the unstable point, below which motion will decay, is revealed.

A backward sloping curve also indicates that if, at velocities slightly below the critical value, the model is disturbed by any means so that its displacement reaches the amplitude indicated by the curve for that velocity, then the

With end plates Free springs

$\beta = 0°$

——— 0,8" Girder – $d/2b$ = 0,0667

NO		No. of restr. wires	Vertical		Torsional		ω Rad/sec	V_c FPS
			ω_1	δ_0	ω_2	δ_0		
6	○	2	6,60	0,0108	8,80	0,0048	8,48	6,84
7	□	1	6,56	0,0126	11,08	0,0058	10,66	8,66
8	△	1	6,60	0,0124	11,71	0,0029	11,21	9,32
9	●	1	6,62	0,0109	12,13	0,0060	11,62	9,42
10	◇	2 *	6,37	—	7,74	—	7,54	6,00

— — — 0,6" Girder – $d/2b$ = 0,05

NO		No. of restr. wires	Vertical		Torsional		ω Rad/sec	V_c FPS
			ω_1	δ_0	ω_2	δ_0		
1	○	2	6,66	0,0138	8,80	0,0064	8,27	8,83
2	□	1	6,58	0,0131	11,10	0,0028	10,26	12,29
3	△	1	6,62	0,0124	11,71	0,0065	10,89	13,16
4	●	1	6,66	0,0183	12,15	0,0083	11,33	14,32
5	◇	2 *	6,37	—	8,72	—	8,17	9,15

* Extra weights added on model

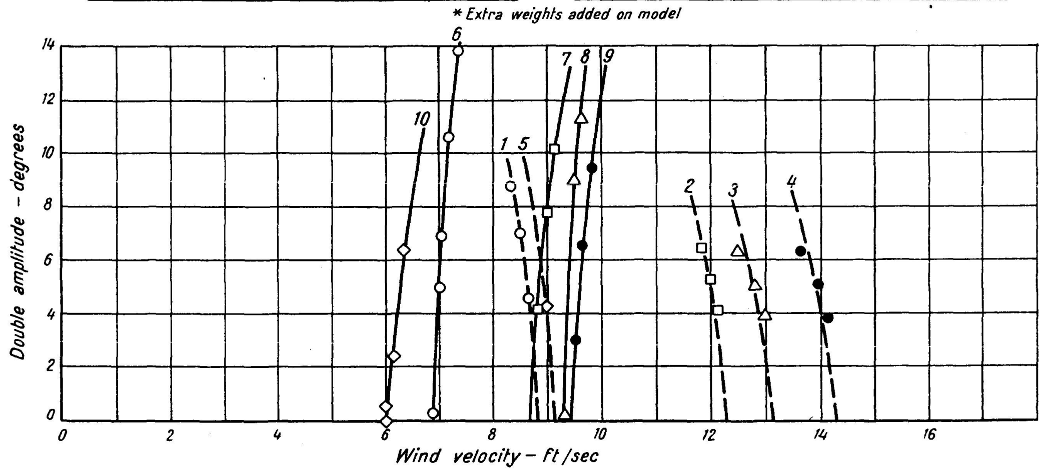

Fig. 10. Thin Flat Plate with Girders

motion will increase indefinitely. In other words, an increase in either velocity or displacement which plots to the right or above the curve precipitates instability.

A response curve which slopes forward has an entirely different meaning. At any velocity above the critical, it shows the steady state amplitude which will be built up and maintained until the velocity is increased or decreased. Though this type of response often becomes catastrophic when the velocity increases, its stability or steadiness of amplitude at a given velocity is in contrast to the instability of pure flutter. This condition is attributed to the combined action of the flutter forces and other forces induced by vortex action discharged from the leading edge. (For different shapes, the relative influence of flutter and vortex force can vary over a widerange including many shapes for which flutter is essentially non-existent.) These vortices, readily visualized by smoke stream tests on stationary and oscillating models, may at first be discharged at some multiple of the natural frequency of the model, but when vibration begins they rapidly take on the frequency of the vibration, giving rise to zones of modified pressure moving periodically across the section.

This type of response curve (with a positive slope) was found in a horizontal wind on model G-2 only, having a $d/2b$ ratio of 0.0667 (0.8 in. girder, Fig. 10). When the angle of attack was increased to $+8°$, it also appeared on model B, the round-edged plate, Fig. 7. The vortex forces influence all of the other response curves also, even though they do not reverse the slopes. A measure of their influence is indicated by the difference between the observed critical

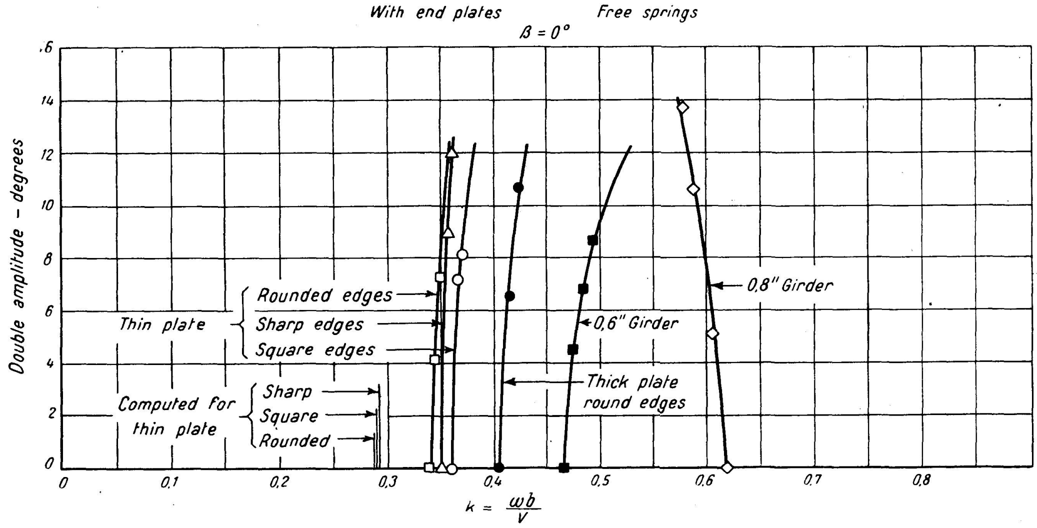

Fig. 11. Flat Plates and Shallow Girder Sections

velocities and the theoretical values, at $\beta = 0$, which are shown in Figs. 6, 7, and 8.

In Fig. 11 the amplitude response curves for the various sections in a horizontal wind are plotted against k, which places them on a common basis, except for the small effect of minor differences in mass distribution. (The slope of these curves is reversed as compared with that obtained when amplitude was plotted against V.) The presence of the vortex forces tends to reduce the critical velocity and increase k_c, this effect naturally being greatest for the blunt sections and least for the round-edged and sharp-edged ones. The marked influence of streamlining is shown by the relative position of the curve for model H (Fig. 11), which has a $d/2b$ ratio nearly twice that of the deeper girder section, yet shows a vortex effect only 12% greater than does the thin, square-edged model A. It should not, however, be concluded that streamlining affords the solution to objectionable oscillation. While it does reduce vortex forces and therefore increases the critical velocity, it does not insure a safe section, if a violent flutter, which it cannot eliminate, falls within the range of possible wind velocity. Such a streamlined section was mentioned in the opening paragraph where $d/2b = 0.17$, yet the full model predicted a violent flutter on the prototype at wind velocities above 120 mph.

Generalized Theory and Its Application

In his generalized theory of the dynamic stability of truss-stiffened suspension bridges, Dr. Bleich has extended the classical flutter theory of Theodorsen [5]), which assumed *a perfectly streamlined thin flat plate,* by taking into

[5]) "General Theory of Aerodynamic Instability and the Mechanism of Flutter", by T. Theordorsen, Technical Report No. 496, National Advisory Committee for Aeronautics, Washington, D. C., 1935, p. 419, equations XVIII and XX.

account the more or less marked effect of the particular shape of the cross-section of the suspension bridge in the vicinity of the leading edge. The resulting theory is based on three assumptions as follows:

1. The air forces acting on the flat plate, as defined by the Theodorsen equations are not unduly influenced by the periodic lift force F_v which is assumed to be of a magnitude comparable to that of the flat plate forces.
2. The frequency of the vortex shedding at the windward edge of the section — the vortices assumed to be the source of the alternating force F_v — is controlled by the frequency of the oscillating structure.
3. The lift force F_v is assumed to be a function of the wind velocity V, the torsional amplitude Φ, and the first derivative $\dot{\eta}$ of the vertical amplitude.

The periodic lift force F_v has been mathematically defined as:

$$F_v = 2\pi\rho b V^2 (A_v + B_v i)\left(\Phi + \frac{b\dot{\eta}}{V}\right)$$ [6]

n which A_v and B_v are parameters depending on the shape of the cross-section which may be determined by appropriate tests on section models of the design under consideration. In simpler form, the above equation may be written

$$F_v = 2\pi\rho b V^2 f_4 \left(\Phi + \frac{b\dot{\eta}}{V}\right)$$

$$\text{where } f_4 = A_v + B_v i$$

The magnitude and phase of the vortex force as represented by A_v and B_v was computed for model G-2 with a pivot installed on the axis of symmetry, thus eliminating vertical motion. The appropriate Bleich equation[7] was used in which the experimental values of ω, ω_2 and k_c obtained from the pivoted test were employed. Experience had demonstrated that the steadier torsional motion arising with a fixed center of rotation more than compensated for the very slight pivot friction. Values of A_v and B_v computed in this manner are plotted against K in Fig. 12.

The theoretical values of ω, V_c and k_c for the unpivoted model G-2 were computed using values of A_v and B_v taken from the curves of Fig. 12. These theoretical values shown in Table I are in good agreement with the observed values.

Although it might seem logical to compute A_v and B_v from tests on the model with two degrees of freedom using the Bleich equation 82 with equation 59[8]) substituted for Δ_0 it has been pointed out by Dr. Bleich that: "There is no method of obtaining reliable values of these parameters [A_v and B_v] other than computing them from observations on a model with fixed center of rotation . . ."

[6]) Bleich, *op. cit.*, equ. 71, p. 1209.
[7]) Bleich, *op. cit.*, equ. 89, p. 1214.
[8]) Bleich, *op. cit.*, p. 1211.

As a demonstration of the lack of reliability of values of A_v and B_v derived from a model with two degrees of freedom these parameters were computed for model G-2 with the pivot removed. Inspection of the last column in Table I for models G-1 and G-2 show these values widely scattered. Values of A_v and B_v for model G-2 have been plotted in Fig. 12 where the scattering is evident, especially for B_v.

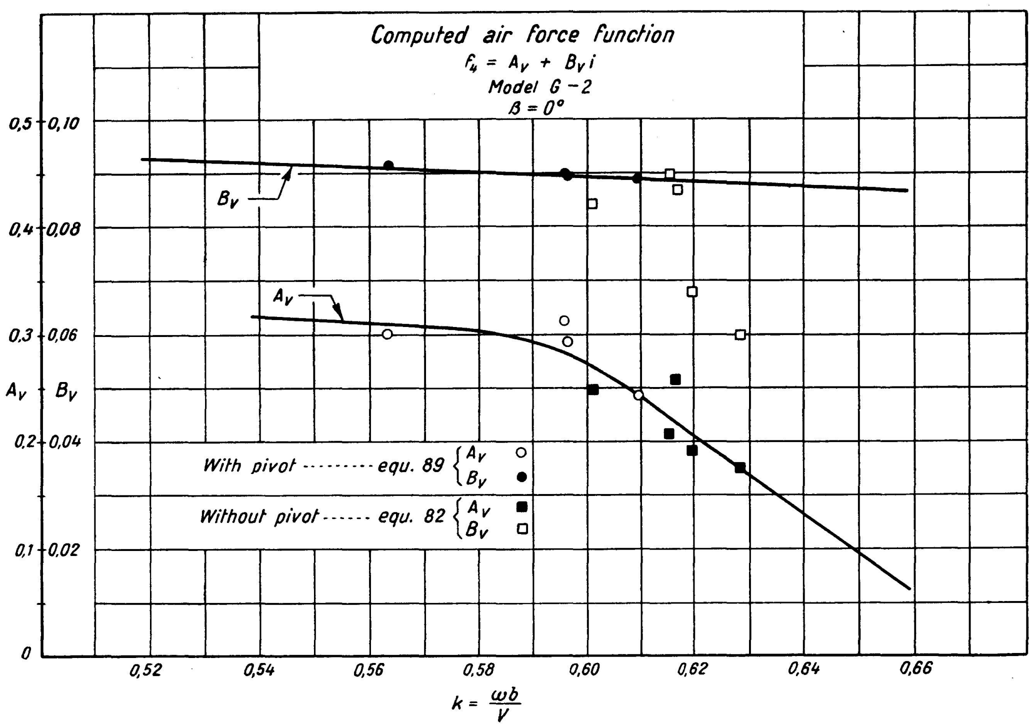

Fig. 12

Effect of Fixed Center of Rotation on Response Characteristics

Section model tests in which the vertical mode was inhibited by fixing the center of rotation were run during the design of the new Tacoma Narrows Bridge. The center of rotation was fixed at the point observed for the natural torsional mode excited in still air, with two degrees of freedom. The critical velocities obtained in these tests were in surprisingly good agreement with results obtained on the section model mounted with two degrees of freedom and also with observations on the full model. It is now quite clear that this agreement was purely fortuitous, and was brought about by an accidental relationship between the magnitude of the flutter forces and the vortex forces.

As Bleich has pointed out for an oscillating system with two degrees of freedom[9]: "The critical velocity and the flutter frequency... are controlled

[9]) Bleich, *op. cit.*, p. 1211.

by both the dynamic properties of the system and the lift force F_v, the latter depending on the particular shape of the cross section. With increasing magnitude of F_v, the critical velocity V_c decreases, whereas the flutter frequency ω increases, tending to approach the frequency of the natural torsional mode which forms one of the components of the excited motion''.

Now if the oscillating system is reduced to one degree of freedom by fixing the center of rotation on the centerline of the model, the lift force F_v, at a certain wind velocity at which the structural damping is voercome would generate excited vibrations. Again quoting Bleich regarding the effect of fixing the center of rotation[10]: "... the flat plate air forces, which are thought of as acting simultaneously on the model, are damping forces, ... and delay the excitation until a wind velocity is reached at which equilibrium exists between the exciting and damping forces. ... The air forces that act on the flat plate play the role of damping forces with the effect of delaying the start of the self-excited vibration. The frequency of the excited vibration is reduced as compared with the frequency of the motion in still air''.

It is thus evident that the vortex force is the only force exciting vibrations on a model restricted to one degree of freedom, whereas with two degrees of freedom the model is subjected to excitation growing out of the flutter forces as well as the vortex forces. There may be some value in examining the effect of fixing the center of rotation on models A and G-2 where the vortex force is much greater on model G-2 which was constructed by adding an 0.8 in. girder to model A.

In fig. 13 the response curves with two degrees of freedom are replotted from Figs. 6 and 10 and pertinent data from these tests will not be repeated. Fixing the center of rotation on model G-2 resulted in a torsional response not greatly different from that obtained with two degrees of freedom. The curves are nearly parallel and the critical velocity was increased from 9.42 fps to 9.6 fps. The reduction in frequency (from the natural frequency in still air) was about 4% in each case.

When the shallow girder was removed from the model, leaving model A, the effect of fixing the center of rotation yielded a vastly different response. With two degrees of freedom $V_c = 11.3$ fps, whereas with one degree of freedom the critical velocity was well above $V_c = 22.0$ fps, which was the maximum attainable in the tunnel. However, a few high amplitude points were obtained on an unstable response curve as shown in Fig. 13. This portion of the response curve was obtained in the same manner as that described previously for backward sloping curves.

Again the excited frequencies were below that found for the torsional mode in still air, but where the reductions on model G-2 were of the order of 4%, the removal of the girder raised the reduction from still air frequency to 11%

[10]) Bleich, *op. cit.*, p. 1213.

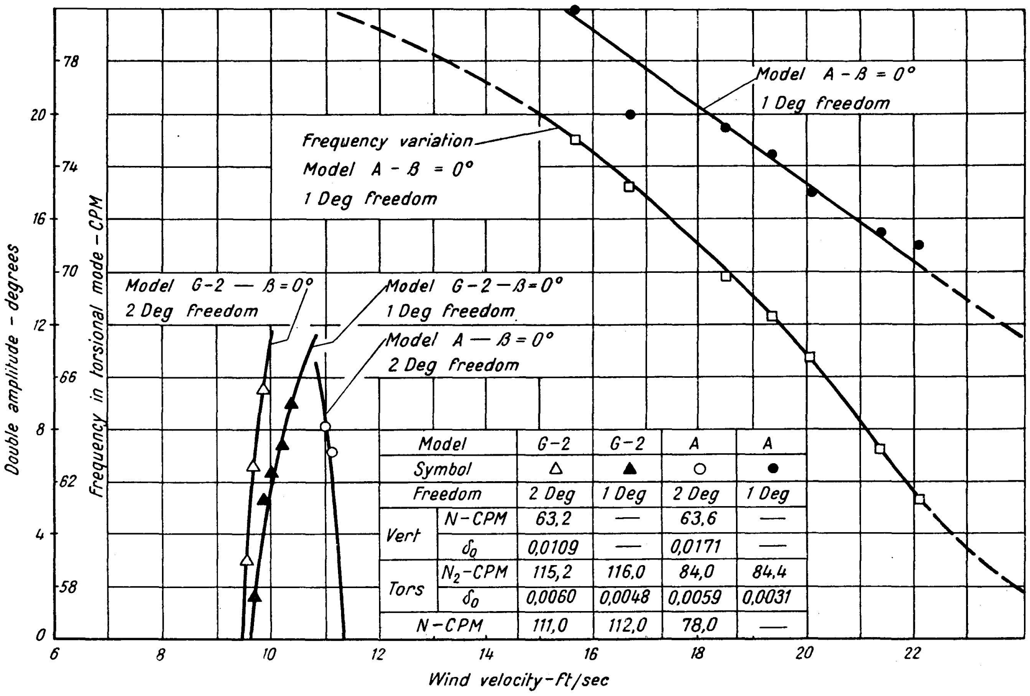

Fig. 13

at the highest amplitude recorded, and 27% at the lowest amplitude, with the curve still falling off steeply. A curve of frequency as a function of wind velocity, for model A — center fixed — has been added in Fig. 13. These frequencies were measured on the unstable side of the amplitude curve, that is, at the amplitude which just precipitated a flutter. This flutter response is so vigorous that six or seven cycles only are required for the development of a prohibitive amplitude, thus exact evaluation of frequency is difficult.

This brief investigation suggests that tests on section models with one degree of freedom in the torsional mode will yield very misleading information if the vortex force is reduced below some certain value which can only be ascertained in the wind tunnel. This lack of correspondence between tests with one and two degrees of freedom does not impair the value of the BLEICH analysis where the validity of the test with center of rotation fixed is amply sustained as a legitimate device for evaluating the vortex force, F_v[11]).

Verification of Theory. Illustrative Example

A series of tests were run on a $^1/_{75}$ scale section model of the Golden Gate Bridge and an instructive example is at ahnd in which the data recorded for the model mounted with two degrees of freedom ($\delta_s = 0.003$) is in excellent

[11]) BLEICH, *op. cit.*, p. 1218.

agreement with computations involving values of A_v and B_v derived from tests with a fixed center of rotation, that is, one degree of freedom. The results are recorded in Table II with the last column showing a conversion to prototype terms.

Table II. Golden Gate Bridge

Value	Model Observed	Model Computed	Prototype Based on Computations
ω (rad/sec)	11.40	11.17	1.29
N (cpm)	109.00	107.00	12.80
V_c (fps)	9.40	9.20	54.30 (mph)

On the model with two degrees of freedom it was observed that the center of rotation was shifted up-stream about 0.75 in. The computations based on the pivoted model suggest that the angular amplitude was about seven times as great as the vertical amplitude and the computed shift of the center of rotation was 0.67 in. The close agreement between the theoretical and observed values of the flutter characteristics indicates that the basic assumptions underlying the generalized theory are approximately correct.

Conclusion

For angles of attack up to 2° the response differs little from that in a horizontal wind. From the limited observations available, it is believed that the wind at some bridge sites may have an upward angle up to 6° or 8°. (Values of this order were measured by the writer on the Golden Gate Bridge, San Francisco Bay, California, during August, 1944.)

The conclusion seems inescapable that any truss-stiffened suspension bridge will be subject to vortex forces in addition to flutter forces and that it will be necessary to run wind tunnel tests on an oscillating model to determine the values of A_v and B_v in the air force function, $f_4 = A_v + B_v i$.

Dr. Bleich's mathematical treatment of the problem is especially valuable because it takes into account both the flutter and vortex forces in the varying proportions in which they may act on a given structure, particularly where trusses and shallow girders are present in the stiffening system. It is a distinct advantage that this method can make use of data from oscillating section model tests; it does not rely on purely theoretical considerations for determining the important and sensitive parameters representing the vortex influence. The analysis should be extended to cover vertical angularity of the wind, if stability is to be assured for bridges at some sites.

Acknowledgments

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Appendix. Notation

A_v and B_v = coefficients in the expression for the lift force F_v;
b = half bridge width;
d = depth of girder;
F_v = a periodic lift force acting at the leading edge;
f_4 = air force function ($f_4 = A_v + B_v i$);
i = imaginary unit ($= \sqrt{-1}$);
k = a dimensionless ratio $\left(= \frac{\omega b}{V}\right)$; k_c = critical value of k;
N = frequency in cycles per minute
N_1 = vertical motion;
N_2 = torsional motion;
V = wind velocity; V_c = critical wind velocity;
β = angle of attack;
Δ = a determinant;
δ = logarithmic decrement;
δ_0 = logarithmic decrement in still air;
δ_a = logarithmic decrement of aerial damping;
δ_s = logarithmic decrement of structural damping;
η = dimensionless vertical amplitude $\left(= \frac{\bar{\eta}}{b}\right)$;
$\dot{\eta}$ = first derivative with respect to time;
$\bar{\eta}$ = vertical or bending amplitude;
ρ = mass density of air (assumed 0.00238 slug);
Φ = torsional amplitude;
ω = flutter frequency;
ω_1 = frequency of the vertical component of motion;
ω_2 = frequency of the torsional component of motion.

[12]) "Mathematical Theory of Vibration in Suspension Bridges" by FRIEDRICH BLEICH, C. B. MCCULLOUGH, RICHARD ROSECRANS, and GEORGE S. VINCENT. United States Bureau of Public Roads, Government Printing Office.

Summary

In the course of experimental observations on the behavior of wind actuated suspension bridge models of girder-stiffened section similar to that of the original Tacoma Narrows Bridge, it was noted that a reduction of the $d/2b$ ratio below 0.06 disclosed aerodynamic characteristics similar to those of a thin flat plate.

The extension of this field of investigation to cover truss-stiffened sections with a closed deck revealed a response under wind action which was also characteristic of the behavior of a thin flat plate. The aerodynamic performance of this type of section is controlled by the details of the structure in close proximity to the windward edge of the deck.

Dr. Friedrich Bleich has adapted the classical theory of wing flutter to the requirements of the more complex structure of the suspension bridge and has demonstrated that the vortex discharged by portions of the structure in the vicinity of the windward edge of the deck always cause reduction of the critical velocity which would occur on a flat plate of finite thickness at the onset of flutter. In view of the wide variety of configurations at the deck level of existing structures, the determination of the aerodynamic forces acting on the oscillating structure must remain a subject for laboratory investigation.

A series of tests were made involving a number of different models on which varying degrees of vorticity were developed at the windward edge of the models by changing the form of the edge of the flat plate or by adding very shallow girders. The models with minimum vorticity showed a critical velocity approximately thirteen per cent lower than that predicted by Dr. Bleich's analysis.

An additional series of tests were also available from a section model of the Golden Gate Bridge from which the predictions are in good agreement with calculated values. This model exhibited all of the typical characteristics of a flutter phenomenon.

These verification tests have shown that the generalized theory developed by Dr. Bleich is capable of predicting the aerodynamic behavior of a certain classification of suspension bridges with an accuracy which is entirely satisfactory for evaluation of preliminary designs. A distinct advantage of the method lies in its reliance on oscillating section model tests for the determination of the very sensitive parameters associated with the shape of the bridge section.

Zusammenfassung

Im Laufe der Versuche über das Verhalten von Hängebrückenmodellen mit Versteifungsträgerquerschnitt ähnlich demjenigen der ursprünglichen Brücke über die Meerenge von Tacoma unter Windbeanspruchung zeigte sich,

daß eine Verkleinerung des Verhältnisses $d/2b$ unter den Wert 0.06 der dünnen, ebenen Platte ähnliche aerodynamische Eigenschaften aufdeckt.

Die Ausdehnung dieser Forschungen auf fachwerkförmige Versteifungsträger mit geschlossener Fahrbahndecke ergab ebenfalls ein Verhalten unter Windkräften, wie es für die ebene dünne Platte charakteristisch ist. Die Einzelheiten der Konstruktion in unmittelbarer Nähe des windseitigen Randes der Fahrbahn bestimmen die aerodynamische Wirkungsweise dieses Querschnittstyps.

Dr. F. Bleich hat die klassische Theorie der Schwingungen von Flügeln den Erfordernissen des komplizierteren Aufbaus der Hängebrücke angepaßt und gezeigt, daß der Wirbel, der sich stoßweise in der Nähe des windseitigen Randes von der Konstruktion ablöst, immer eine Reduktion derjenigen kritischen Geschwindigkeit verursacht, welche bei einer ebenen Platte endlicher Dicke beim Beginn des Schwingens auftreten würde.

In Anbetracht der großen Mannigfaltigkeit von Formen der Fahrbahndecken bestehender Konstruktionen muß die Bestimmung der aerodynamischen Kräfte, die auf eine schwingende Konstruktion wirken, Sache von Laboratoriumsversuchen bleiben.

In einer Versuchsreihe an verschiedenen Modellen wurden an der windseitigen Kante veränderliche Grade der Wirbelung entwickelt durch die Veränderung der Kantenform der ebenen Platte oder durch Anbringen sehr niedriger Träger. Die Modelle mit minimaler Wirbelbildung zeigten eine kritische Geschwindigkeit, die ungefähr 13% unter dem mit der Bleich'schen Theorie berechneten Wert liegt.

Nützlich war außerdem eine weitere Versuchsreihe mit einem Modell der Golden-Gate-Brücke, bei dem die Voraussagen gut mit den berechneten Werten übereinstimmen. Dieses Modell zeigte alle typischen Kennzeichen des Schwingungsvorganges auf.

Diese Kontroll-Versuche haben bewiesen, daß die allgemeine Theorie von Dr. Bleich geeignet ist, das aerodynamische Verhalten einer bestimmten Art von Hängebrücken mit einer für Vorprojekte durchaus befriedigenden Genauigkeit vorauszubestimmen. Ein deutlicher Vorteil der Methode liegt darin, daß sie sich für die Bestimmung der sehr empfindlichen, von der Form des Brückenquerschnittes abhängigen Kennzahlen auf Schwingungsversuche mit Querschnittsmodellen stützt.

Résumé

Au cours des observations expérimentales sur le comportement vis-à-vis du vent des modèles de ponts suspendus ayant une section renforcée par des poutres, semblable à celle du pont original de Tacoma Narrows, il a été constaté qu'un abaissement au-dessous de 0,06 du rapport $d/2b$ mettait en jeu des caractéristiques aérodynamiques analogues à celles d'une plaque mince.

L'extension de ces recherches à des sections renforcées du type à contrefiches avec tablier fermé a mis en évidence un comportement à l'égard du vent qui est également caractéristique de la plaque mince. Le comportement aérodynamique de ce type de section est conditionné par les éléments secondaires de l'ouvrage, jusqu'à proximité immédiate du bord du tablier exposé au vent.

Le Dr. Friedrich Bleich a adapté la théorie classique du battement des ailes aux exigences de la structure plus complexe que constitue le pont suspendu; il a démontré que les tourbillons mis en jeu par les parties de l'ouvrage situées au voisinage du bord sous le vent du tablier donnent toujours lieu à une réduction de la vitesse critique qui se manifesterait sur une plaque mince d'épaisseur finie au début du battement. Etant donné la large variété des configurations que l'on peut trouver au niveau du tablier sur les ouvrages existants, la détermination des efforts aérodynamiques qui agissent sur la structure oscillante doit rester dans le domaine des investigations de laboratoire.

Il a été procédé à une série d'essais impliquant un certain nombre de modèles différents sur lesquels on a fait agir des mouvements tourbillonnaires, au bord sous le vent, en changeant la forme du bord ou en ajoutant des poutres de très faible hauteur. Les modèles qui présentaient le régime tourbillonnaire le moins marqué accusaient une vitesse critique inférieure d'environ 13% à la valeur prévue d'après l'analyse du Dr. Bleich.

On a procédé également à une série d'essais supplémentaires sur un modèle de la section du Golden Gate Bridge; on a pu constater que les prévisions étaient conformes aux valeurs calculées. Ce modèle présentait toutes les caractéristiques typiques du phénomène de battement.

Ces essais de vérification ont montré que la théorie généralisée établie par le Dr. Bleich est susceptible de fournir des prévisions concernant le comportement aérodynamique d'une certaine catégorie de ponts suspendus, avec une précision entièrement satisfaisante pour les études préliminaires. Un avantage très net de cette méthode réside dans ce fait qu'elle s'appuie sur des essais d'oscillation de modèles de sections pour la détermination des paramètres très sensibles qui sont associés avec la forme de la section du pont.