The vibrations of massive foundations on soil

The Vibrations of Massive Foundations on Soil

Vibrations des massifs pour machines sur le sol de fondation

Schwingungen von massiven Fundamenten auf dem Baugrund

MILOŠ NOVÁK
Praha

Introduction

One of the hitherto least explored regions of the dynamics of civil engineering structures is the vibration of foundations founded directly upon the subsoil. The study of this problem is of utmost importance in the design of foundations for imperfectly balanced machines where, supplementary to the preliminary calculation of the expected foundation vibration, an estimate must be made of the magnitude of impacts transmitted to the surroundings of the foundation during the operation of the machine. In addition, a study of this kind assists in the explanation of the behaviour of the most varied types of supports during their vibration.

The most serious obstacle to the mastering of the problem of the vibration of foundations founded on subsoil, is the fact that the foundation soil is an imperfectly elastic, in principle rheological material whose many properties moreover vary with the distance from the surface, the moisture, the manner of loading, the state of stress, the presence of subterranean water, etc. These properties of the subsoil cannot, of course, be taken into account in theoretical considerations; thus the actual subsoil is idealised and replaced by a perfectly elastic, homogeneous and isotropic half-space (or by a stratum having the same properties). The problem of the vibration of foundations on an elastic half-space is then solved with the aid of further simplifying assumptions, viz. that the solid foundation is of a cylindrical shape and that it rests on the surface of the elastic half-space. The primary importance of these theoretical considerations lies in their qualitative explanation of the problem of the vibration of foundations on subsoil. In this respect the methods of dimensional analysis can be also used with success, as reported by KOŽEŠNÍK[1]).

The idealisation of the conditions involved has the effect that the theoretical solution of the vibration of solid foundations on an elastic half-space, as well as the simple methods of calculation used predominantly in practice and usually not concerned with the mass of the subsoil, do not in certain respects correspond satisfactorily to the results of tests. The dynamic properties of the subsoil must, therefore, also be investigated experimentally under actual conditions. The nonlinearity of the vibration which renders difficult the treatment of the test results, is the cause of considerable difficulty in such investigations.

This paper outlines a simple method of calculating and evaluating the resonance curves of vertical vibration, making due allowance for its nonlinearity, and presents certain results of an experimental investigation of this problem. In this study particular attention was devoted to the hitherto not sufficiently elucidated questions of how the design characteristics, expressing the dynamic properties of the subsoil, vary with the foundation dimensions and its founding under the ground surface.

The Selection of the Theory

The characteristic feature of the vibration of foundations on subsoil is its nonlinearity manifested most markedly in vibration excited by the action of external, harmonically variable forces. Fig. 1 presents a typical set of resonance curves of a vertical forced vibration measured for various magnitudes of the exciting forces (eccentricity moment $r_0 m_0$). It can be noted that the resonance frequency decreases with the increasing magnitude of the exciting forces and that the amplitudes are not directly proportional thereto. Within the range of amplitudes occurring in the machine foundations, the reduction in the resonance frequency attains a value of 20 to 25 per cent. (In the diagrams, F is the area of the square base of foundation and G_0 is the weight of the foundation together with the vibrator.) It is obvious that the nonlinearity, in addition to the difficulties associated with the treatment of test data, considerably reduces the accuracy of the increasingly emphasized calculations of the forced vibration amplitudes. LORENZ[2]) and ROBSON[3]) have studied this phenomenon first.

It is, of course, impossible to introduce the nonlinearity into the theory of the half-space vibration; for this reason — if the nonlinearity is to be taken into account — further simplifications must be made. A suitable simplification

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[1]) J. KOŽEŠNÍK: Dynamika strojů (Prague 1958).

[2]) H. LORENZ: Dynamik im Grundbau (Grundbautaschenbuch, Berlin 1955).

[3]) J. D. ROBSON: Effects of non-linearity on the resonant frequency of a body on soil (Presented at the ninth international congress of applied mechanics, Brussels, 1956).

is offered by the theoretical results reported by Šechtěr[4]); her work proves that the vertical forced vibration of a solid foundation on an elastic half-space can be investigated with sufficient accuracy as the vibration of the same body on a massless half-space, provided the mass of the foundation is suitably increased and the damping coefficient, proportional to the velocity, appropriately selected.

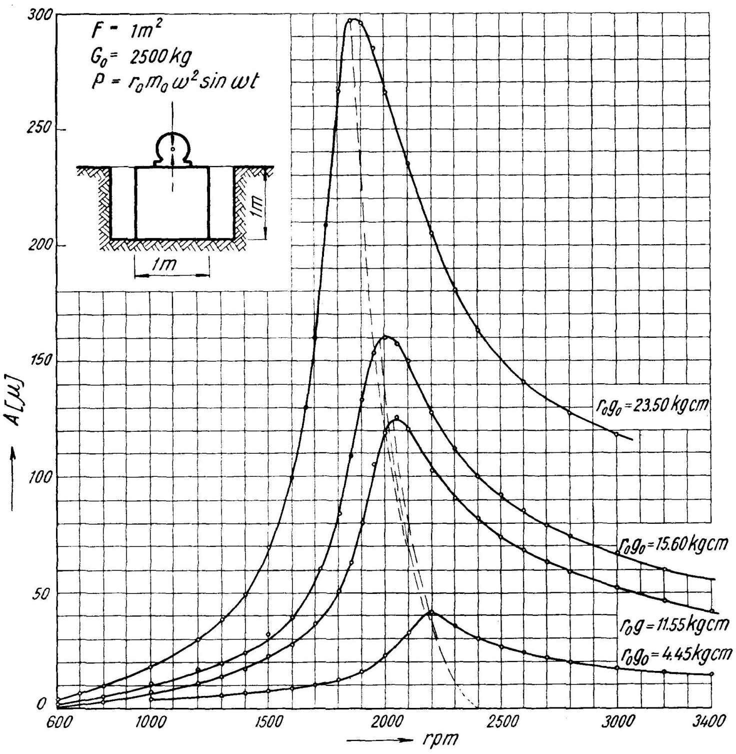

Fig. 1. A Typical Set of Resonance Curves of the Foundation Vertical Vibration.

In view of the foregoing it seems possible to define the vertical vibration of a symmetrical foundation under centrally acting external harmonically variable force whose amplitude increases with the square of the frequency ω, by the equation

$$m\,v'' + C_b\,v' + \bar{f}\,(v) = r_0\,m_0\,\omega^2 \sin \omega\, t.$$

Putting
$$C_b = 2\,m\,\omega_b\,, \qquad f\,(v) = \frac{\bar{f}\,(v)}{m}, \qquad \rho = \frac{r_0\,m_0}{m}, \tag{1}$$

we can also write

$$v'' + 2\,\omega_b\,v' + f\,(v) = \rho\,\omega^2 \sin \omega\, t. \tag{2}$$

[4]) O. J. Šechtěr: Ob učotě inercionnych svojstv gruntov pri rasčotě vertikalnych vynužděnnych kolebanij massivnych fundamentov (Trudy NII Minvojenmorstroja, No. 12, Moskva 1948).

In these equations

m is the total (theoretical) mass.
v is the displacement in the vertical direction.
C_b, ω_b is the coefficient and the circular frequency of damping, respectively.
$\bar{f}(v)$ is the nonlinear characteristic of elastic forces.
$r_0 m_0$ is the product of eccentricity and the magnitude of the rotating mass (the eccentricity moment).

Only a steady vibration will be considered in the following paragraphs. The harmonic analysis of vibration records picked up by an oscilloscope indicates that the motion is, as a rule, practically purely harmonic so that the first harmonic of the solution of Eq. (2) is satisfactory. Then for a given characteristic $f(v)$ and specified coefficients of Eq. (2) its approximate solution causes no difficulties in the case of low nonlinearity.

The Resonance Curve Equation

The nonlinear equation of motion (2) can be replaced (in the first approximation) by an equivalent linear equation

$$v'' + 2\,\omega_b\, v' + \Omega^2 v = \rho\,\omega^2 \sin\omega t, \tag{3}$$

where Ω is the function of the amplitude A of steady vibration. Denoting the nonlinear part of the reduced characteristic as $F(v)$, the characteristic can be written in the form of

$$f(v) = \frac{C}{m} v + F(v),$$

so that for a symmetrical characteristic in Eq. (3) [5])

$$\Omega^2 = \frac{C}{m} + \frac{1}{\pi A}\int_0^{2\pi} F(A\cos\tau)\cos\tau\, d\tau. \tag{4}$$

In the investigation of steady vibration of foundations on subsoil, the following assumption can also be made: It is a well-known fact that soil becomes elastic due to repeated stressing, so that the stress/strain relation becomes linear within the range of applied stresses. In view of this property of the soil, it can be assumed that in the case of steady vibration the elastic forces are directly proportional to the magnitude of the displacement and that they are a nonlinear odd function of the amplitude. This assumption is depicted in Fig. 2.

During the steady vibration the reduced elastic force is then, according to the notation of Fig. 2

[5]) Bogoljubov-Mitropolskij: Asymtotičeskije metody v těorii nělinějnych kolebanij (Moskva, 1953).

$$f(v) = f_n(A)\frac{v}{A}. \qquad (5)$$

A steady motion is then again described by Eq. (3) in which simply

$$\Omega^2 = \frac{f_n(A)}{A}. \qquad (6)$$

Otherwise, the assumption expressed by Eq. (5) can be employed for a very simple approximate analysis of forced vibration described by Eq. (2). The accuracy is, of course, lower than when using Eq. (4) but of the same order as when the approximate assumption introduced by DEN HARTOG[6]) is used.

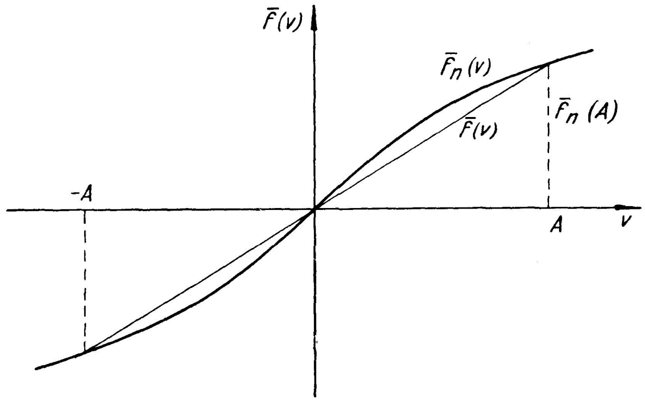

Fig. 2. The Elasticizing of the Characteristics.

When the characteristic $\bar{f}(v)$ is *asymmetric*, the equivalent linear equation can be obtained by direct linearization in which a suitably selected linear characteristic $\bar{f}_e(v)$ is substituted for the curved characteristic $\bar{f}(v)$ within the interval of steady vibration (Fig. 3).

Let us select this substituting linear characteristic in the manner similar to that used by PANOVKO[7]) in his treatment of free undamped vibrations.

In the case of an asymmetrical characteristic, the free undamped vibration takes place around the central position displaced from the origin by the abscissa Δ which can be determined from equation

$$\int_{-A_2}^{A_1} \bar{f}(v)\,dv = \theta.$$

Selecting the characteristic $f_e(v)$ so as to pass through the centre of vibration, its equation is

$$\bar{f}_e(v) = C_e(v-\Delta).$$

According to Panovko, the factor C_e is conveniently selected from the con-

[6]) DEN HARTOG: Mechanical Vibrations (New York and London, 1947).

[7]) PANOVKO: Osnovy prikladnoj těorii uprugich kolebanij (Moskva, 1957).

dition that, within the interval $(-A_2, A_1)$, the integral of quadratic deviations $\bar{f}_e(v)$ from $\bar{f}(v)$, weighed by the displacement $v_1 = v - \Delta$ should be a minimum.

Assuming that the substituting characteristic determined in this manner is satisfactory even for the approximate analysis of the steady forced damped vibration, the following linear equation can be considered for this motion relative to the centre of vibration

$$v_1'' + 2\,\omega_b\,v_1' + \Omega^2 v_1 = \rho\,\omega^2 \sin \omega\, t \tag{7}$$

where

$$\Omega^2 = \frac{5}{2\,A^5 m} \int_{-A}^{A} \bar{f}(v_1 + \Delta)\, v_1^3\, d\,v_1, \tag{8}$$

putting

$$A = \frac{A_1 + A_2}{2}, \qquad \Delta = \frac{A_1 - A_2}{2}.$$

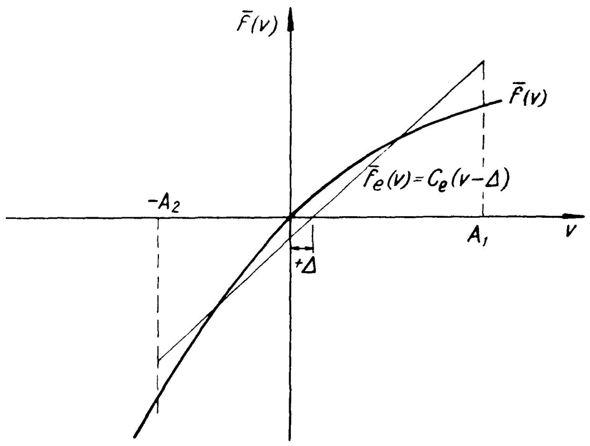

Fig 3. Direct Linearization of Asymmetrical Characteristic.

Then for both the symmetric and asymmetric characteristic, the forced steady vibration is approximately described by the linear equation

$$v'' + 2\,\omega_b\,v' + \Omega^2 v = \rho\,\omega^2 \sin \omega\, t, \tag{9}$$

in which, for the case of a symmetrical characteristic, v denotes the displacement from the equilibrium position, and for the case of an asymmetrical characteristic, the displacement from the vibration centre; Ω is given by Eqs. (4), (6) or (8).

For a steady vibration the following expression applies:

$$v = A \sin(\omega\, t + \varphi) \tag{10}$$

and the equation of the resonance curve is obtained in the form of

$$\tfrac{1}{2}\,\omega^4 (A^2 - \rho^2) - \omega^2 A^2 (\Omega^2 - 2\,\omega_b^2) + \tfrac{1}{2}\,\Omega^4 A^2 = 0 \tag{11}$$

or

$$\omega_{1,2}^2 = \frac{\Omega^2 - 2\,\omega_b^2 \pm \sqrt{(\Omega^2 - 2\,\omega_b^2)^2 - \Omega^4 [1 - (\rho/A)^2]}}{1 - (\rho/A)^2}. \tag{12}$$

The phase shift φ is given by

$$\sin\varphi = -\frac{2\,\omega_b}{\rho}\,\frac{A}{\omega}. \tag{13}$$

In accordance with the results of numerous tests the symmetrical characteristic

$$\bar{f}(v) = \bar{a}\,v + \frac{\bar{b}\,v}{d+|v|}, \tag{14}$$

is most often suitable for expressing the elastic properties of the subsoil; its curve is presented in Fig. 4 and it differs from the asymmetrical characteristic

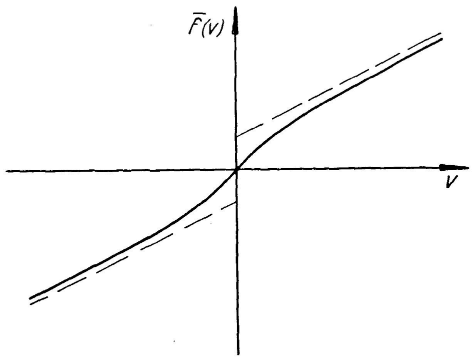

Fig. 4. Characteristic $\bar{f}(v)=\bar{a}\,v+\dfrac{\bar{b}\,v}{d+|v|}$.

derived by LORENZ [8]) in the absolute value appearing in the denominator. Putting $a=\bar{a}/m, b=\bar{b}/m$ and using the characteristic (14), Eq. (4) yields the following expression for the curve of natural frequencies

$$\Omega^2 = a + \frac{b}{A}K\left(\frac{A}{d}\right), \tag{15}$$

where the dimensionless coefficient

$$K\left(\frac{A}{d}\right) = \frac{4}{\pi}\left[1-\frac{\pi}{2}\frac{1}{\frac{A}{d}}+\frac{2}{\frac{A}{d}\sqrt{1-\left(\frac{A}{d}\right)^2}}\operatorname{arc\,tg}\sqrt{\frac{1-\frac{A}{d}}{1+\frac{A}{d}}}\right] \quad \text{for } \left(\frac{A}{d}\right)^2<1 \tag{16a}$$

and

$$K\left(\frac{A}{d}\right) = \frac{4}{\pi}\left[1-\frac{\pi}{2}\frac{1}{\frac{A}{d}}+\frac{1}{\frac{A}{d}\sqrt{\left(\frac{A}{d}\right)^2-1}}\lg\frac{\frac{A}{d}+1+\sqrt{\left(\frac{A}{d}\right)^2-1}}{\frac{A}{d}+1-\sqrt{\left(\frac{A}{d}\right)^2-1}}\right] \text{for } \left(\frac{A}{d}\right)^2>1. \tag{16b}$$

To facilitate the use of this characteristic, the curve of the coefficient $K\,(A/d)$ relative to the ratio A/d is plotted in Fig. 5. A typical set of resonance curves for the characteristic (14) is shown in Fig. 6a.

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[8]) See note 2.

Assumption (6) yields a result which, though much simpler, is qualitatively very similar, namely

$$\Omega^2 = a + \frac{b}{d+A}. \tag{17}$$

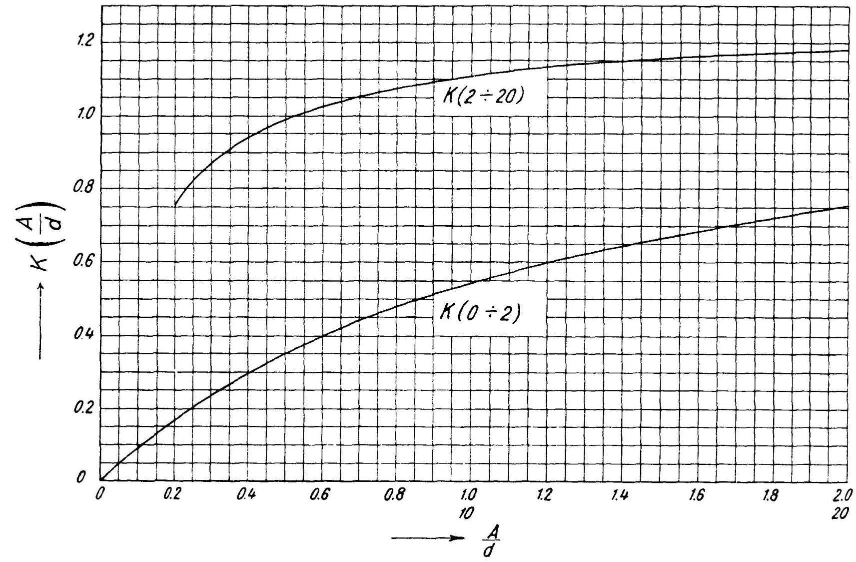

Fig. 5. Coefficient $K\,(A/d)$.

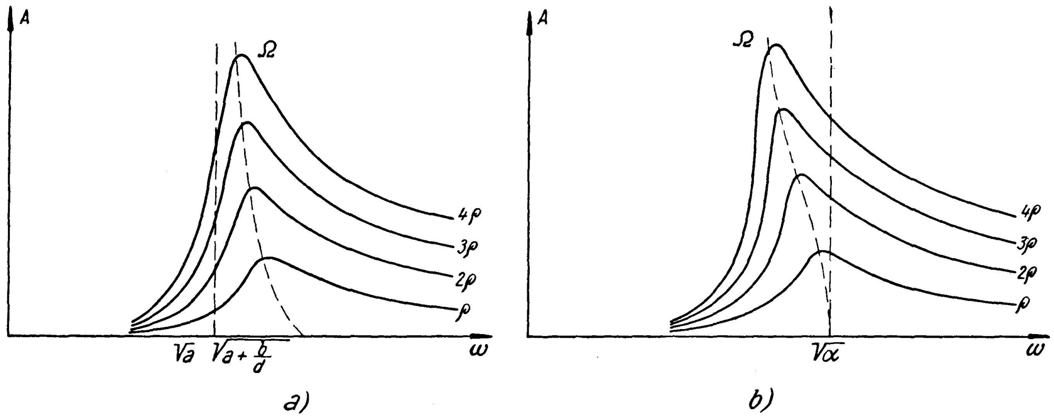

Fig. 6. The Most Frequent Types of Resonance Curves.

In certain cases it is more convenient to assume the following characteristic instead of characteristic (14):

$$\bar{f}(v) = \bar{\alpha}\, v - \bar{\beta}\, v^3 + \bar{\gamma}\, v^5 - \bar{\delta}\, v^7, \tag{18}$$

for which Eq. (4) yields [9])

$$\Omega^2 = \alpha - \frac{3}{4}\beta\, A^2 + \frac{5}{8}\gamma\, A^4 - \frac{35}{64}\delta\, A^6, \tag{19}$$

where $\alpha = \dfrac{\bar{\alpha}}{m}, \qquad \beta = \dfrac{\bar{\beta}}{m}$ etc.

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[9]) M. Novák: Über die Nichtlinearität der Vertikalschwingungen von starren Körpern auf dem Baugrunde (Acta technica, Prague 5,6/1957).

A typical set of resonance curves for the characteristic (18) is presented in Fig. 6b. Resonance curves of this type occur sometimes on imperfectly elasticized cohesive soils.

The Determination of the Nonlinear Characteristic, the Vibrating Mass and the Damping Coefficient from a Given Resonance Curve

In the experimental analysis of the dynamic properties of the subsoil (and also, of course, in other cases), there arises the problem of determining the characteristic, the vibrating mass and the damping coefficient from a given resonance curve. A simple procedure for solving this problem is as follows:

The resonance curve obtained by measurement is intersected by a pencil of lines passing through the origin (Fig. 9). It can easily be proved for a general equation of resonance curves (11) that a simple relationship exists between the frequency of the point of intersection of such a line and the Ω-curve, and the

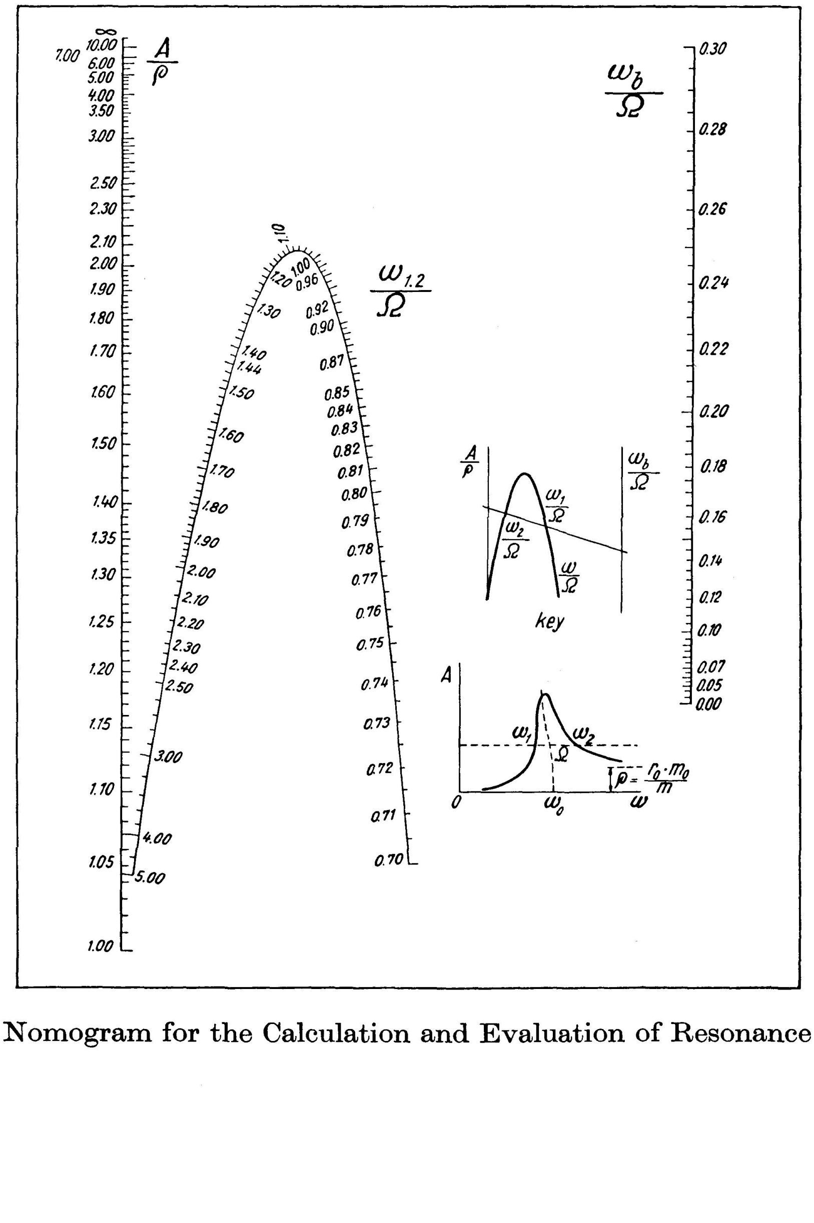

Fig. 7. Nomogram for the Calculation and Evaluation of Resonance Curves.

frequencies ω_{I}, ω_{II} of their points of intersection with the resonance curve, namely

$$\Omega = \sqrt{\omega_{I}\,\omega_{II}}\,. \tag{20}$$

Following the construction of the Ω-curve by means of Eq. (20), the quantities ρ and ω_b can be established from a nomogram (Fig. 7) indicating the relation of quantities $\omega_{1,2}/\Omega$, ω_b/Ω and A/ρ given by (12). According to (1) the determination of ρ gives the mass m. It is more advantageous, however, to calculate the damping frequency from the expression

$$\omega_b = \frac{\rho}{2\dfrac{A_T}{\omega_T}}, \tag{21}$$

where (ω_T, A_T) determine the contact point of the tangent through the origin to the resonance curve; this follows from Eq. (13) for $\varphi = -\pi/2$. Eq. (21) also gives the mutual relation of the resonance curves of systems differing only in the nonlinear member (Fig. 8).

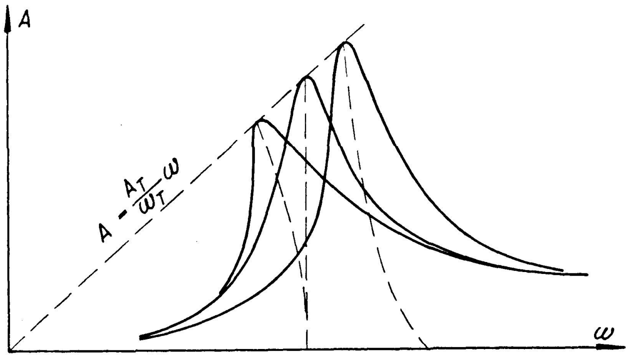

Fig. 8. Mutual Relation of Resonance Curves of Systems Differing Only in the Nonlinear Member.

The form of the characteristic can be estimated from the shape of the Ω-curve. The determined Ω-curve is compared with the theoretical Ω-curves for various characteristics according to Fig. 6, or possibly with others, and that form of characteristic is assumed whose associated Ω-curve is most similar to the experimentally determined Ω-curve. The unknown coefficients of the characteristic are then calculated from the appropriate equation of the Ω-curve written for the necessary number of points.

When the characteristic (14) is suitable, it is first of all necessary to find from three points (Ω_i, A_i), where $(i = 1, 2, 3)$ of the Ω-curve, the coefficient d by trial and error from the equation

$$K\left(\frac{A_1}{d}\right)\frac{1-K_\Omega}{A_1} - K\left(\frac{A_2}{d}\right)\frac{1}{A_2} + K\left(\frac{A_3}{d}\right)\frac{K_\Omega}{A_3} = 0, \tag{22}$$

where
$$K_\Omega = \frac{\Omega_1^2 - \Omega_2^2}{\Omega_1^2 - \Omega_3^2}.$$

For the initial estimation of d the formula $d=0{,}83\,d_n$ can be employed, d_n being calculated from the simple Eq. (17) for assumption (5). The calculation of the remaining coefficients a and b is self-evident.

An example of the above procedure is presented in Fig. 9. Full line represents the measured values, dotted line is calculated from the determined values. The curves appear to be in satisfactory agreement. The characteristic is of the type (14) and for $v \geqq \theta$ it is plotted in full line in Fig. 10. The characteristic curve, calculated for the assumption (5) from the natural frequency curve given by (6), is marked in the same diagram in dotted lines. The differences are rather small and both characteristics $\bar{f}(v)$ and $\bar{f}_n(v)$ yield, according to (15) and (17) practically the same Ω-curve and, consequently, the same resonance curve. Since the question of the form of the characteristic is more or

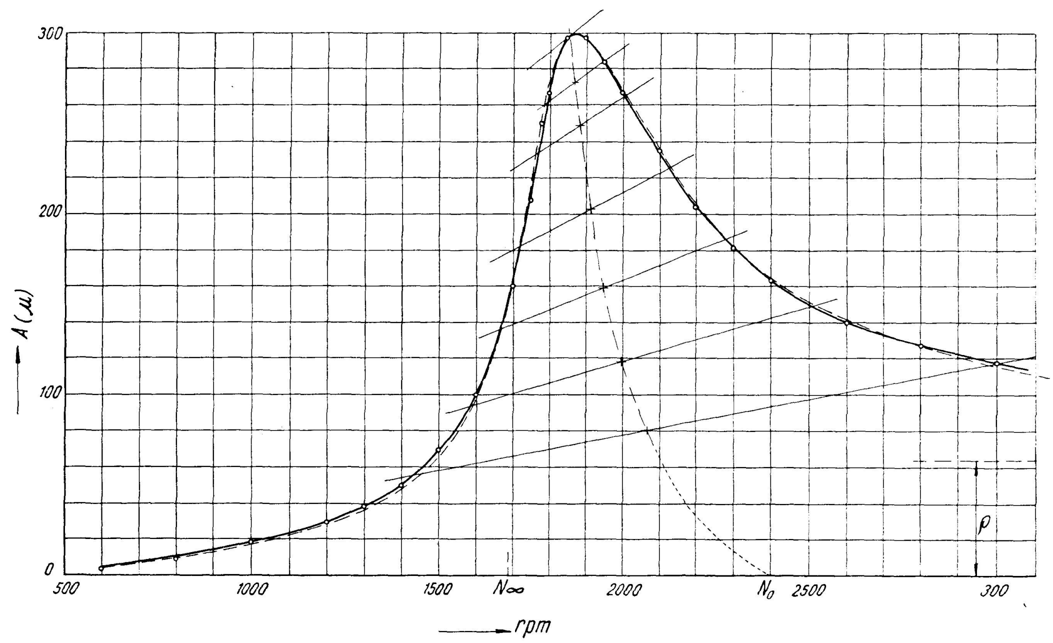

Fig. 9. Comparison of Measured and Calculated Resonance Curves.

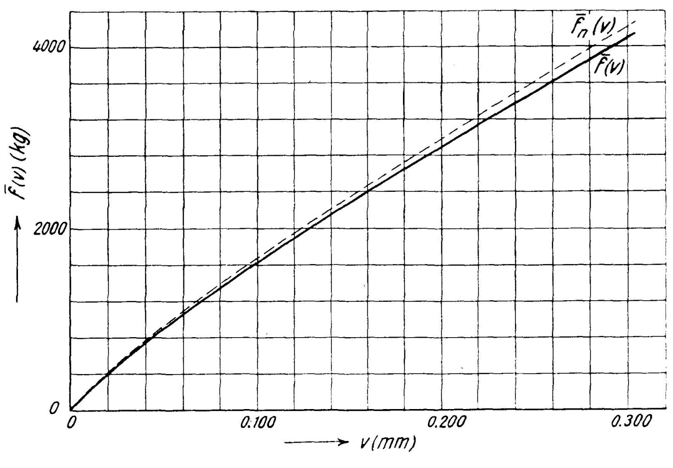

Fig. 10. Characteristic of Resonance Curve of Fig. 9.

less a matter of convention, it is obvious that for practical purposes it is possible as well as convenient either to employ for the definition of the elastic properties of the subsoil the assumption (5) or to accept as an approximation the very simple formula (6).

Some Experimental Results

The procedure outlined above has been used in the Institute for Theoretical and Applied Mechanics at Prague in the experimental investigation of the elastic, inertial and damping properties of subsoils. The majority of experiments have been carried out with a test body, screwed together from steel sheets, whose weight (including the exciter) varied between 970 and 2770 kg, amounting in special cases up to 3770 kg, and whose square bearing surface varied between 0,5 and 1,5 m². Other tests were conducted with a concrete foundation having the dimensions of $1\,m \times 1\,m \times 1\,m$. The vibration was excited by an exciter which made possible a continuous variation of the exciting force from 0 to 2000 kg in either the vertical or horizontal direction. The vibration was measured by means of an electrodynamic apparatus. The tests to be described were performed on a massive layer of loess loam. The steel foundation under test was placed on a very thin layer of plaster or rapid-hardening cement, since the experimental results are highly sensitive to contact attained between the foundation and the soil. In order to obtain straightforward results, it is also necessary to start the measurements only after a satisfactorily elastic state of the soil was reached; otherwise, for example, the dependence of the damping upon the magnitude of the exciting force can be revealed.

From the practical point of view the most interesting are the results of tests which followed the variation of the numerical characteristic of the subsoil, i. e., the elastic characteristic, the coefficient of the increase in the vibrating mass, and the damping coefficient with respect to the dimensions of the foundation and the founding of the foundation under the surface of the ground.

a) The Effect of the Foundation Dimensions Upon the Numerical Characteristic of the Subsoil

Within the range of the specific pressures attained (e. g., 0,194 to 0,482 kg/cm² for a base of 0,5 m²), the numerical characteristic of the subsoil varied rather slightly.

The effect of the size of the base of foundation upon the relative elastic characteristic $\sigma = \frac{\bar{f}(v)}{F}$ was likewise very slight within the base size of 0,5 to

1,5 m². Fig. 11 shows these curves calculated from expressions (6) and (1) when the total (theoretical) mass is considered. This fact seems to indicate that the subsoil behaves as a stratum of limited size rather than a half-space; this is obviously very advantageous for practical calculations.

The increase in the vibrating mass expressing approximately the effect of inertia depends primarily upon the size of the base of the foundation. For the coefficient α denoting the increase in the vibrating mass the following approximative expression was satisfactory:

$$\alpha = 1 + f \frac{\sqrt[3]{F}}{\sigma_{st}},$$

where F is the area of the base of foundation (m^2), σ_{st} is the static stress in the base (t/m^2) and the coefficient $f = 0{,}835$ ($tm^{-8/3}$). This relation clearly indicates that, particularly in the case of the small and light foundations normally used in the tests, the ommission of the inertia effect results in considerable errors in the test results. In the case of large and light foundations (of the plate type) the effect of inertia assists in limiting the amplitude of forced vibrations.

The increase in the damping coefficient $D = \omega_b/\omega_0$ relative to the size of the base of the foundation was remarkable. This coefficient increased roughly according to the relation $D = D_0 \sqrt[3]{F}$, where $D_0 = 0{,}106\, m^{-2/3}$; if this relation were valid even in the case of large areas, the higher degree of damping would prevent the resonance increase of the large foundation amplitude. Since the foundations founded directly on soil often operate close to resonance, the necessity of investigating the damping of large foundation vibrations is obvious. This task, however, is rendered difficult by the variability of damping. Thus, for instance, in the case of loess loam, merely the decrease in moisture

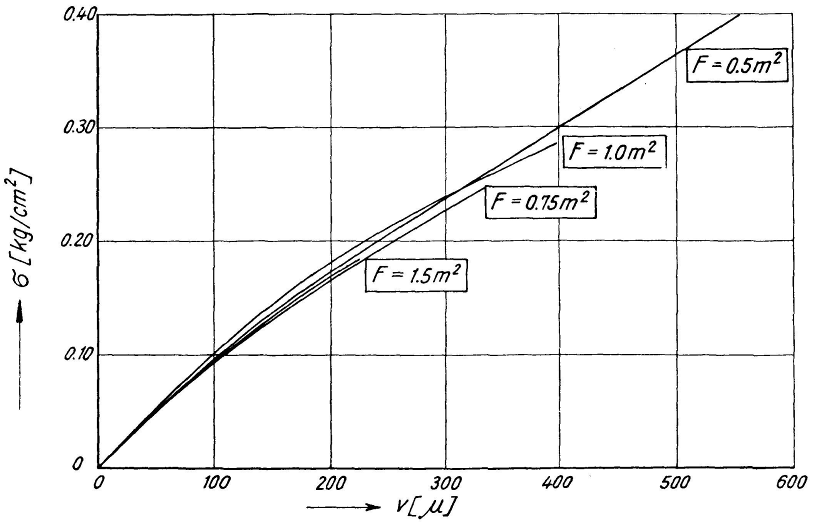

Fig. 11. Relative Characteristic for Various Sizes of the Foundation Bases.

caused by a few weeks of drought resulted in a 50 per cent increase in the damping coefficient although the increase in the subsoil rigidity was rather slight.

b) The Founding of the Foundation Under the Surface of the Ground

Resonance curves were measured on a concrete foundation laid directly in the excavation, with the air gaps caused by progressive denudation of the foundation sides filled in with a thoroughly compacted fill material. This procedure was employed in order to substitute tests on a single foundation for those conducted on several foundations laid in various depths.

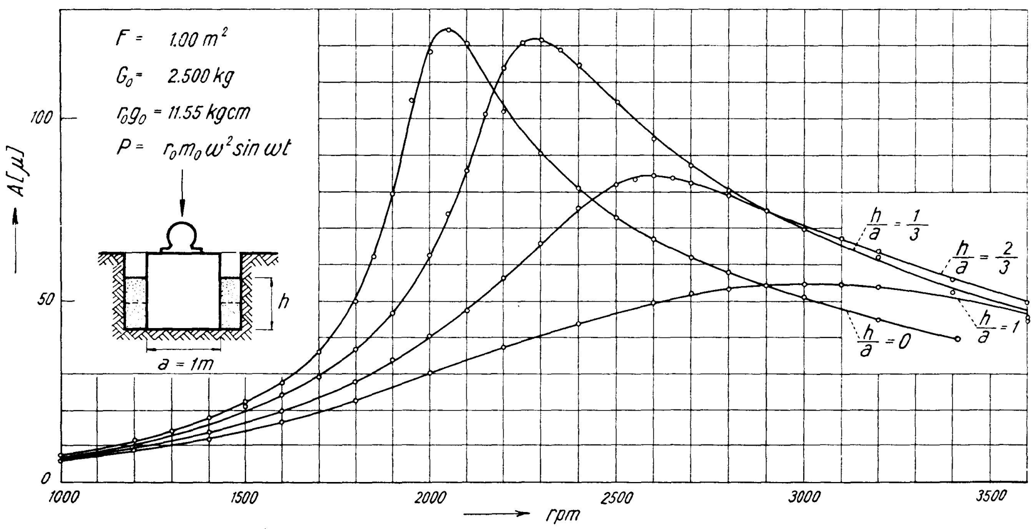

Fig. 12. Resonance Curves of Vertical Vibration Under the Action of Compacted Lateral Fill Material.

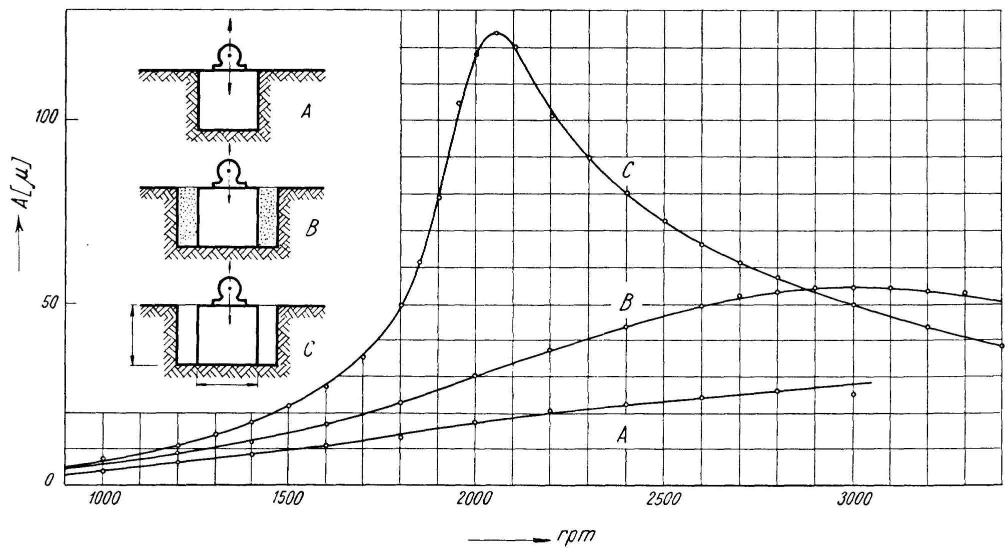

Fig. 13. Resonance Curves of Vertical Vibrations of Concrete Foundation for Various Types of Contact Between the Sides and the Surrounding Soil, at the Same Magnitude of Exciting Forces ($G_0 = 2500$ kg, $r_0 g_0 = 11{,}55$ kgcm).

Fig. 12 shows the resonance curves of the vertical vibration at various heights of the side fill, Fig. 13 indicates the extreme cases as noted, and Fig. 9 represents the evaluation of the resonance curve measured with the denudated sides of the foundation. The results of these tests can be summarized as follows:

The foundation vibration is generally affected by the depth of its founding. The type of contact between the sides and the surrounding soil, i. e., whether the foundation is surrounded by fill material, undisturbed soil or an air gap plays a significant part. For an increasing degree of compactness of the fill material, the vibrations approach those taking place in conjunction with the lateral action of the undisturbed soil; for a decreasing degree of compaction of the fill they approach those associated with an air gap (Fig. 13).

The effect of the subsoil inertia upon the foundation vibration increases with the depth of founding of the foundation, without the type of contact between the sides and the surrounding soil exerting any considerable effect. The weight of the so-called co-vibrating mass expressing approximately the effect of the subsoil inertia, was on the average (even in the case of an air gap) 40 to 60 per cent higher than during the tests on bodies having the same size as the foundation base on the surface of the same soil.

The vibration damping increases progressively with the height of the soil adjacent to the sides of the foundation, and with the degree of compactness of the fill material. Maximum damping occurs when the undisturbed soil acts laterally. The circular damping frequencies corresponding to the tests performed are plotted in Fig. 14 relative to the ratio h/a.

The rigidity of the characteristic also increases with the height of contact between the foundation sides and the surrounding soil, as well as with the intensity of compaction of the fill, as can be inferred from the increase in the resonance frequencies relative to the ratio h/a shown in Fig. 15, and from Fig. 16 plotting the characteristic. (The characteristics for $h/a=2/3$ are established approximatively from points (ω_T, A_T)). These diagrams clearly

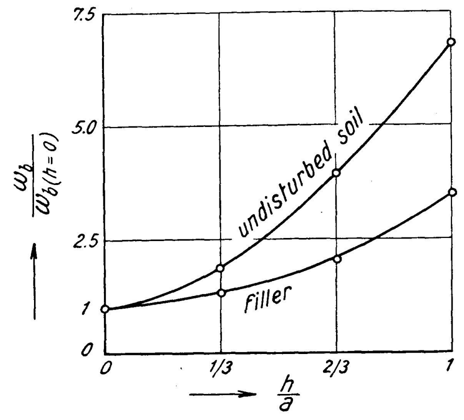

Fig. 14. Increase in Damping Circular Frequency Relative to Ratio h/a.

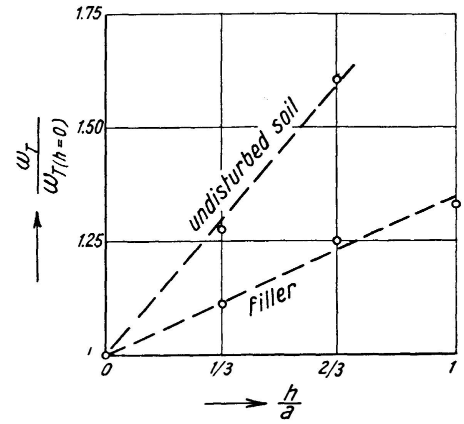

Fig. 15. Increase in Resonance Frequencies Relative to Ratio h/a.

indicate that, particularly for a low degree of compaction and for low values of the ratio h/a, the increase in the subsoil rigidity in the vertical direction due to the lateral action of the soil, is relatively small[10]) and much smaller than the increase in the vibration damping.

However, in the case of a horizontal vibration excitation, the increase in the resonance frequencies can become more pronounced (see Fig. 17).

The propagation of the vibration in the vicinity of the foundation founded

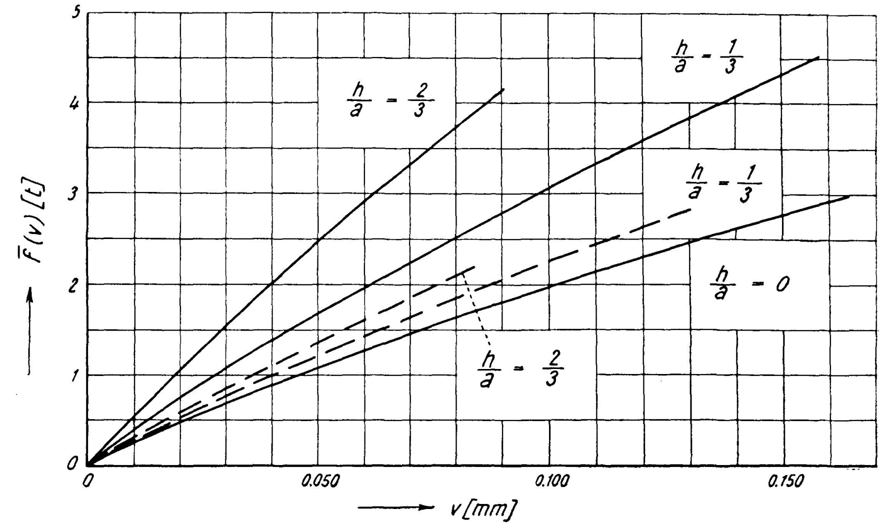

Fig. 16. Subsoil Characteristics for Various Ratios h/a (———— Undisturbed Soil, - - - - - - - Side Fill).

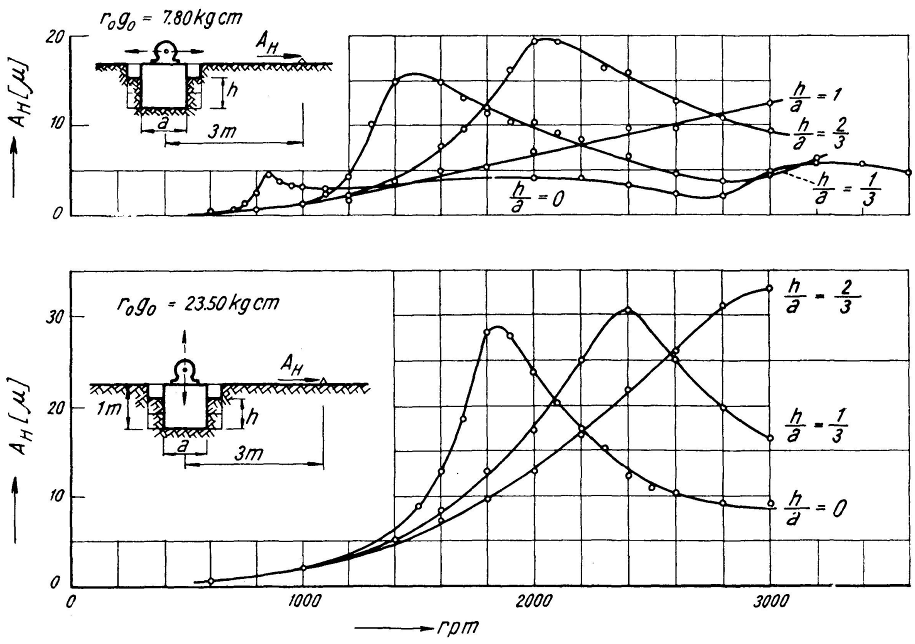

Fig. 17. Resonance Curves of Horizontal Vibration in Foundation Vicinity for Both Horizontal and Vertical Excitation.

10) O. A. Savinov: Fundamenty pod mašiny (Leningrad 1955).

under the surface of the ground is also interesting; measurements of this phenomenon together with other results of these tests can assist in the elucidation of the often asked question whether the air gaps sometimes provided around the foundations for the purpose of reducing the transmission of vibrations to the surroundings of the foundation, are as effective as they are supposed to be. This question arose as a result of certain practical experiences which have shown that the subsequent filling of the air gap resulted pratically in no change in the vibration of either the foundation or its vicinity. In agreement with previous deductions this can be explained by the fact that in these cases the air gap was only loosely filled so that the filling material was imperfectly compacted and thus ineffective.

Fig. 17 shows the transmission of horizontal vibrations to the surroundings of the foundation for both the horizontal and vertical excitation in conjunction with the lateral action of the undisturbed soil and for the case of an air gap. The vertical vibration curves are similar but the amplitudes were smaller.

According to the results of the tests described above, the expected ratio of the machine running speed and the natural frequency of the foundation should be considered when assessing the advantages and limitations of air gaps. Let us assume that the natural frequency of the foundation N_v could be successfully estimated for the case of an air gap around the foundation. Then the advantages of the air gap appear to be roughly as follows (Fig. 12, 13, 17): When the machine is to operate in the subcritical or the resonant range of N_v, it seems more advantageous to provide well compacted fill material around the foundation, or to ensure by the sequence of work the lateral action of the undisturbed soil, since a smaller amplitude of both the foundation and its surroundings can be expected with these provisions. When the machine is to operate above the resonance range N_v, a compacted lateral fill may give rise to an increase in the amplitude of the foundation and its surroundings.

The experimental investigation of the effect of founding the foundation under the surface of the ground must be supplemented by a study of the foundation during operation in order to ascertain whether thin slits causing a reduction in the lateral co-operation of the soil are formed around the foundation sides during the vibration due to the plastic deformation of the soil.

The results of tests described very concisely in this paper cannot, of course, be considered as absolutely valid; they may, however, provide a qualitative picture of some of the hitherto not satisfactorily explained properties of the vibration of foundations on subsoil.

Conclusion

Eq. (2) can be employed conveniently for the calculation of forced vibration in the vertical direction since it enables us approximately to take into account the nonlinear elastic, inertial and damping properties of the subsoil.

Its application in the tests is simple and may yield fresh knowledge of the dependence of numerical characteristics employed in practical calculations upon various important circumstances, such as the size and the shape of the base of foundation, the depth of founding of the foundation, etc. The tests conducted indicate that in experimental investigation attention must be paid not only to the elastic, but also to the inertial and damping properties, since it appears that the two last-mentioned properties could in certain cases be used to advantage in limiting the amplitude of forced vibration.

Summary

One of the main properties of the vibrations of machine foundations on soil are their non-linearity, which manifests itself in the fact that there is no direct proportionality between the exciting force and the amplitude of the vibration, and that the resonance frequency is dependent on the amplitude. This non-linearity causes considerable difficulties not only in the calculation of the vibrations of machine foundations, but also in the interpretation of tests. The experimental investigation of the elasticity, inertia and damping properties of soil with the aid of test foundations in which the non-linearity was taken into account has helped to elucidate the problem as to how the influence of the above-mentioned properties of the soil on the vibrations of a foundation vary with the size and the depth of the foundation. The tests have shown that the damping coefficient and the inertia influence of the soil increase mainly with the size of the foundation base; on the other hand, the elasticity of the soil per unit area depends only to a very slight extent on the size of this base. The vibration of the foundations and the transmission of the vibrations to the surroundings are dependent not only on the foundation depth, but also on the nature of the contact between the sides of the foundation and the surrounding soil.

Résumé

Parmi les caractéristiques essentielles que présentent les vibrations des massifs pour machines sur le sol de fondation, il faut mentionner leur non-linéarité, qui s'exprime par l'absence de proportionnalité nettement définie entre l'effort qui provoque les vibrations et l'amplitude de ces dernières. D'autre part, la fréquence de résonance dépend de l'amplitude. La non-linéarité donne lieu à de notables difficultés, non seulement pour le calcul des vibrations des massifs de fondation, mais aussi pour l'interprétation des résultats des essais. Les recherches expérimentales qui ont été effectuées sur les caractéristiques d'élasticité, d'inertie et d'amortissement du sol de fondation, à l'aide de massifs de fondation d'essai, recherches au cours desquelles il a été

tenu compte de cette nonlinéarité, ont contribué à élucider la question de l'influence des caractéristiques ci-dessus du sol sur les vibrations des massifs de fondation, en fonction des dimensions de ces massifs et de leur profondeur d'encastrement. De ces essais, il résulte que le coefficient d'amortissement et l'influence de l'inertie du sol augmentent essentiellement avec les dimensions de base des massifs; en revanche, l'élasticité du sol rapportée à l'unité de surface ne dépend que très peu de ces dimensions. Les vibrations du sol de fondation et leur transmission dans le voisinage dépendent non seulement de la profondeur d'encastrement du massif, mais aussi des conditions suivant lesquelles les parois latérales du massif sont en contact avec le sol.

Zusammenfassung

Zu den Haupteigenschaften der Schwingungen von Maschinenfundamenten auf dem Baugrund gehört dessen Nichtlinearität, die dadurch zum Ausdruck kommt, daß keine gerade Proportionalität zwischen der Erregungskraft und Schwingungsamplitude besteht und daß die Resonanzfrequenz von der Amplitude abhängig ist. Die Nichtlinearität verursacht namhafte Schwierigkeiten nicht nur bei der Berechnung der Schwingungen von Maschinenfundamenten, sondern auch bei der Auswertung von Prüfungen. Die experimentelle Forschung von Elastizitäts-, Trägheits- und Dämpfungseigenschaften des Baugrundes mit Hilfe von Prüfungsfundamenten, bei der die Nichtlinearität berücksichtigt wurde, hat zur Klärung der Frage, wie sich der Einfluß der obenerwähnten Baugrundeigenschaften auf die Fundamentschwingungen mit der Fundamentgröße und Gründungstiefe ändert, beigetragen. Aus den Prüfungen ergibt sich, daß der Dämpfungskoeffizient und Trägheitseinfluß des Baugrundes hauptsächlich mit der Größe der Fundamentsohle wächst; die auf die Flächeneinheit bezogene Baugrundelastizität hängt dagegen von der Größe dieser Sohle nur sehr wenig ab. Dabei ist die Fundamentschwingung und die Schwingungsübertragung in die Umgebung nicht nur von der Gründungstiefe, sondern auch davon abhängig, auf welche Weise die Seitenwände des Fundamentes mit dem umliegenden Boden aneinandergrenzen.

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