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Membrane Stresses in Hyperbolic Paraboloid Shells Circular in Plan

Contraintes membranaires dans les paraboloïdes hyperboliques minces sur plan circulaire

Membranspannungen in hyperbolischen Paraboloidschalen mit kreisförmigem Grundriß

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Introduction

In recent years the behavior of thin shells in the form of hyperbolic paraboloid was studied rather extensively. However, so far problems discussed in the literature pertain to hyperbolic paraboloid shells which are rectangular or rhombic in plan. In this paper hyperbolic paraboloid thin shells circular in plan are considered, and membrane stresses for such shells are obtained. Plane polar coordinate system, being appropriate one for the problem, is employed in the solution.

The membrane state of stress is formulated in terms of Airy's stress function following the procedure first used by PUCHER [1] [1]) and since applied to the solution of a number of problems by many investigators, notably TESTER [2].

The problem of membrane displacements such as studied by FLÜGGE and GAYLING [3] for hyperbolic paraboloids rectangular in plan is not considered. However, from the established state of stress for the problem considered, discontinuities in displacements along dividing characteristics may be fully anticipated. This indicates the need for further investigations of hyperbolic paraboloids on the basis of a bending theory such as considered by HRUBAN [4] or REISSNER [5]. The latter work is of fundamental significance for this problem. It contains the derivation of the strain-displacement relations, a qualitative study of edge bending effect, and shows that the membrane equations follow by specializing the equations of the general theory.

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1) See numbered references in the Bibliography, Appendix B.

Basic Equations of the Problem

Consider a thin hyperbolic paraboloid shell having a circular plan subjected to a distributed load $p(r,\varphi)$ [2]) acting in the direction of the positive z-axis. In plane polar coordinates, the middle surface of the shell is defined by the equation:

$$z(r,\varphi) = \frac{1}{2k} r^2 \sin 2\varphi, \tag{1}$$

where k is a constant which characterizes the rise of the shell.

By considering the shell shallow, in the sense defined by Reissner [5], the governing differential equation in terms of Airy's stress function $F(r,\varphi)$ written in polar coordinates becomes:

$$\begin{aligned}\frac{1}{2}\Big[\sin 2\varphi\, F_{,rr} + \frac{2}{r}\cos 2\varphi F_{,r\varphi} - \frac{2}{r^2}\cos 2\varphi\, F_{,\varphi} \\ - \frac{1}{r}\sin 2\varphi\, F_{,r} - \frac{1}{r^2}\sin 2\varphi\, F_{,\varphi\varphi}\Big] = k\, p(r,\varphi).\end{aligned} \tag{2}$$

The horizontal projections of in-plane stress-resultants are defined in terms of the stress function by the familiar relations:

$$\begin{aligned} N_r &= \frac{1}{r}F_{,r} + \frac{1}{r^2}F_{,\varphi\varphi}, \\ N_\varphi &= F_{,rr}, \\ N_{r\varphi} &= -\frac{\partial}{\partial r}\left[\frac{1}{r}F_{,\varphi}\right] = \frac{1}{r^2}F_{,\varphi} - \frac{1}{r}F_{,r\varphi}. \end{aligned} \tag{3a, b, c}$$

Since the shell is assumed shallow, the normal in-plane stress-resultants are approximately equal to their horizontal projections.

The relations for the strain in terms of stress-resultants follow from Hooke's law and are:

$$\begin{aligned} E h \epsilon_r &= N_r - \mu N_\varphi, \\ E h \epsilon_\varphi &= -\mu N_r + N_\varphi, \\ \gamma_{r\varphi} &= \frac{2(1+\mu)}{E h} N_{r\varphi}. \end{aligned} \tag{4a, b, c}$$

The strain-displacement relations appropriate for this problem are:

$$\begin{aligned} \epsilon_r &= u_{,r} + \frac{1}{k} r \sin 2\varphi\, w_{,r}, \\ \epsilon_\varphi &= \frac{1}{r}u + \frac{1}{r}v_{,\varphi} + \frac{1}{k}\cos 2\varphi\, w_{,\varphi}, \\ \gamma_{r\varphi} &= \frac{1}{r}u_{,\varphi} + v_{,r} - \frac{1}{r}v + \frac{1}{k} r\cos 2\varphi\, w_{,r} + \frac{1}{k}\sin 2\varphi\, w_{,\varphi}. \end{aligned} \tag{5a, b, c}$$

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[2]) All symbols are defined in Appendix A. Throughout the paper, comma in subscript denotes differentiation.

The system of Eqs. (2), (3), (4) and (5) can be obtained directly from the system of equations of the general bending theory of thin shallow shells as given in polar coordinates by REISSNER [6] by assuming that the shell has no bending rigidity.

The solution of a given problem consists in exhibiting a function F such that it satisfies Eq. (2) and the appropriate boundary conditions. On physical grounds it is plausible to expect that such function not only exists, but that the membrane state of stress it defines is unique.

Boundary Conditions

For the membrane state of stress of the shell, on the boundary $r = R$ only stress-resultants N_r and $N_{r\varphi}$ may occur. Therefore, the boundary conditions should be prescribed in terms of these two quantities. The physically meaningful conditions are as follows:

a) *Free Edge.* For this case both stress-resultants must vanish, i.e.

$$N_r(R,\varphi) = 0, \qquad N_{r\varphi}(R,\varphi) = 0. \tag{6a, b}$$

The conditions (6) imply that adequate supports are provided to carry external load p either along some portions of the boundary $r = R$ and/or at the interior of the shell region.

b) *Edge Supported by Wall.* If it is assumed that a vertical wall supporting the shell possesses infinite extensional rigidity and zero bending rigidity, then the radial stress resultant N_r vanishes, but not $N_{r\varphi}$. Thus,

$$N_r(R,\varphi) = 0 \quad \text{but} \quad N_{r\varphi}(R,\varphi) \neq 0. \tag{7a, b}$$

If it is assumed further that the entire applied vertical load P is carried by the forces developed along the outer boundary, for a complete circle, we have

$$\int_0^{2\pi} N_{r\varphi}(R,\varphi)\, z,_{\varphi}(R,\varphi)\, d\varphi = P. \tag{7c}$$

In this manner two conditions (7a) and (7c) at the boundary become available for this case.

c) *Suspended Edge.* If it is assumed that along the outer boundary the shell is supported by numerous rigid elements capable of transmitting axial forces, but not shear, then $N_{r\varphi}$ vanishes but N_r differs from zero, i.e.

$$N_{r\varphi}(R,\varphi) = 0 \quad \text{but} \quad N_r(R,\varphi) \neq 0. \tag{8a, b}$$

Again, if the entire load is carried by the reactions applied at the boundary, for a complete circle, we have

$$\int_0^{2\pi} N_r(R,\varphi)\, z,_r(R,\varphi)\, R\, d\varphi = P. \tag{8c}$$

Eqs. (8a) and (8c) are the boundary conditions for this case.

d) *Fixed Edge.* This condition can be realized by supporting the edge of the shell on an infinitely rigid support. Neither N_r nor $N_{r\varphi}$ can be zero, i.e.

$$N_r(R,\varphi) \neq 0, \qquad N_{r\varphi}(R,\varphi) \neq 0. \tag{9a, b}$$

Neither one of these relations is useful as a boundary condition. However, fixed edge implies that all components of the displacement vector must vanish identically at the edge. Therefore, as may be seen from Eq. (5b), no strain in the tangential direction at the boundary can take place. Thus, according to Eq. (4b), we have

$$N_\varphi(R,\varphi) - \mu N_r(R,\varphi) = 0. \tag{9c}$$

In order to satisfy this relation for all values of φ the coefficients of all sine terms as well as cosine terms must vanish independently. In this manner two conditions become available from one relation. The condition of vertical equilibrium of boundary forces with the applied load cannot be enforced directly.

For each of the four types of boundary supports discussed above, two conditions were established. These conditions may be expressed in terms of derivatives of F on the boundary. The specification of two conditions on the curved boundary $r=R$ [7], subject to certain restrictions, is necessary and sufficient to ensure the existence of a unique solution of the problem under investigation. This is so because a hyperbolic paraboloid is a surface of negative Gaussian curvature and the governing differential equation is hyperbolic. It should be noted that these comments apply only to the problem of membrane state of stress inside one quarter of a circle; no conditions are prescribed on F or its derivatives along the lines $x=0$ or $y=0$. If the region of interest is contained within a full circle, Riemann procedure may lead to discontinuities along the dividing characteristics. Conceivably these discontinuities might be removed by recasting the problem as a boundary value problem. This would mean that one would be able to prescribe only one condition on the curved boundary $r=R$. One recalls that this is true for the case of membrane state of stress in shells of positive Gaussian curvature, where the governing differential equation is of elliptic type. Whether or not this recasting from an initial value into a boundary value problem is possible for the case of circular regions is outside the scope of this investigation.

Solution of the Problem

The problem of finding a sufficiently general solution of eq. (2) so as to satisfy the prescribed boundary conditions may be done in at least two different ways. In the one procedure, we note that the homogeneous part of eq. (2) is separable under the assumption $\bar{F}(r,\varphi) = \bar{F}_1(r\sin\varphi) + \bar{F}_2(r\cos\varphi)$. Thus the homogeneous part of the solution may be assumed formally as follows:

$$\bar{F}(r,\varphi) = \sum_n A_n r^n \sin^n\varphi + \sum_n B_n r^n \cos^n\varphi. \tag{10}$$

However, if we confine our attention to the investigation of load functions $p(r,\varphi)$ which are even functions of p with respect to lines $x = \pm y$, the function $\bar{F}$ also must be even with respect to the same lines. This leads to the conclusion that the constants A_n and B_n of the series must be equal. However, one set of free constants is not sufficient to ensure satisfaction of the two prescribed boundary conditions at $r = R$. Therefore, unlike the case of hyperbolic paraboloid shells with rectangular boundaries where this procedure is applicable [2], in the present investigation this method of solution is discarded.

An alternative general method of solving Eq. (2) is due to RIEMANN [7] and is the one used in this paper. This method of solution also was used by FLÜGGE and GAYLING [3] for determining membrane stresses and displacements of hyperbolic paraboloid shells rectangular in plan.

Riemann's method is based on the assumption that on the boundary $r = R$ for the region the value of the function $F(R,\varphi)$ and its normal derivative $F,_r(R,\varphi)$ are specified. Then the value of the function $F(r,\varphi)$ at some point (r,φ) located *inside* a region bounded by two characteristics and a curve may be uniquely determined.

The characteristics of the system are lines $x = \text{const.}$ and $y = \text{const.}$ The boundary $r = R$ defines the curve. By applying Riemann's method of integrations for the case considered, the expression for the stress function F at an

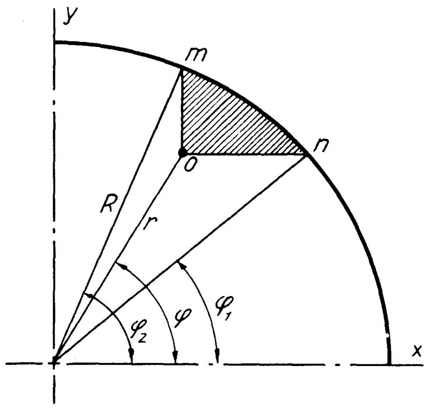

Fig. 1. Quadrant of the Shell Used in Establishing Riemann's Integration of the Equation.

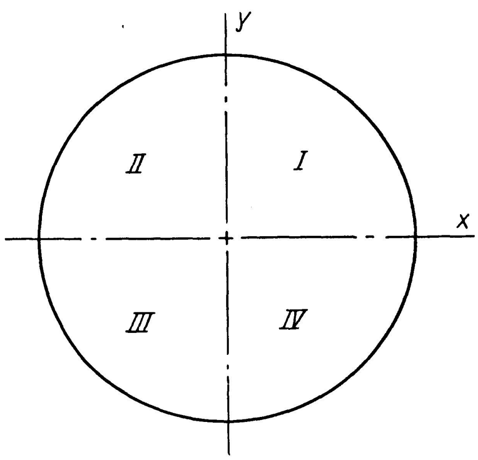

Fig. 2. Designation of the Various Regions of the Shell.

interior point (r, φ), see Fig. 1, reduces to

$$F(r,\varphi) = \tfrac{1}{2}[F(R,\varphi_1)+F(R,\varphi_2)] - \tfrac{1}{2}R\int_{\varphi_1}^{\varphi_2}\left[\sin(2\varphi)\,F_{,r}(R,\varphi)+\frac{1}{R}\cos(2\varphi)\,F_{,\varphi}(R,\varphi)\right]d\varphi + \int_{omn} k\,p(r,\varphi)\,dA\,. \tag{11}$$

Whereas the above solution is developed only for the first quadrant of a circle, the other regions II, III, and IV shown in Fig. 2 may also be included into the solution. Thus, since the load function was assumed to be even with respect to lines $x = \pm y$, $F_{I} = F_{III}$ and $F_{II} = F_{IV}$, where subscripts designate the region of stress function. Therefore, the solution inside the region III is also given by Eq. (11). For the solution inside regions II and IV the value of function F and its normal derivative F, *also* may be specified on the boundary, and F inside the regions calculated again with the aid of Eq. (11). If discontinuities in magnitude or direction of the stress-resultants along the dividing characteristics $x=0$ and $y=0$ occur, it is understood that a beam cruciform in plan view of adequate strength and rigidity must be provided.

Solution of a Particular Problem

In the remainder of the paper the investigation is confined to the particularly important problem of an uniformly distributed load $p(r,\varphi) = -p$. For this case it appears reasonable to choose the quantities on the boundary in the form of the following Fourier series:

$$F(R,\varphi) = \sum_n A_n \sin(2n\varphi), \qquad F_{,r}(R,\varphi) = \sum_n B_n \sin(2n\varphi). \tag{12a, b}$$

In order to simplify calculations we shall further assume that the distribution of the stress function F and its normal derivative at the boundary is described with sufficient accuracy by retaining only the terms corresponding to $n=1$ in the Fourier series which appear in Eq. (12). Hence,

$$F(R,\varphi) = A\sin(2\varphi); \quad \text{and} \quad F_{,r}(R,\varphi) = B\sin(2\varphi) \tag{13a, b}$$

and by direct differentiation, we obtain

$$\begin{aligned} F_{,\varphi}(R,\varphi) &= 2A\cos(2\varphi), \\ F_{,\varphi\varphi}(R,\varphi) &= -4A\sin(2\varphi), \\ F_{,r\varphi}(R,\varphi) &= 2B\cos(2\varphi). \end{aligned} \tag{14a, b, c}$$

By substituting Eqs. (13) and (14) into the governing differential Eq. (2), we obtain

$$R\sin(2\varphi)\,F,_{rr}(R,\varphi) = 2\,k\,p\,R - 4\left(B - \frac{A}{R}\right)\cos^2(2\varphi) + \left(B - 4\frac{A}{R}\right)\sin^2(2\varphi), \tag{15}$$

from which using Eq. (3b), we can obtain directly the tangential stress-resultant N_φ.

By applying Eqs. (3a) and (3c), and again using Eqs. (13) and (14), the stress-resultants N_r and $N_{r\varphi}$ along the boundary are obtained. These are

$$N_r(R,\varphi) = \frac{1}{R}\left(B - 4\frac{A}{R}\right)\sin(2\varphi),$$
$$N_{r\varphi}(R,\varphi) = \frac{2}{R}\left(-B + \frac{A}{R}\right)\cos(2\varphi). \tag{16a, b}$$

Substituting expressions (13) into (11) and carrying out the indicated integration we obtain the expression for the stress function sought:

$$\begin{aligned} F(r,\varphi) = {} & p\,k\,r^2\sin\varphi\cos\varphi \\ & + \left[p\,k\frac{R^2}{2} - \frac{A}{2} - \frac{B\,R}{4}\right]\left[\arccos\left(\frac{r}{R}\cos\varphi\right) - \arcsin\left(\frac{r}{R}\sin\varphi\right)\right] \\ & + \left[-p\,k\frac{R^2}{2} + \frac{3\,A}{2} - \frac{B\,R}{4}\right]\left[\frac{r}{R}\sin\varphi\,S_r^{1/2} + \frac{r}{R}\cos\varphi\,C_r^{1/2}\right] \\ & + \left[\frac{B\,R}{2} - A\right]\left[\frac{r^3}{R^3}\sin^3\varphi\,S_r^{1/2} + \frac{r^3}{R^3}\cos^3\varphi\,C_r^{1/2}\right], \end{aligned} \tag{17}$$

where $$S_r = 1 - \frac{r^2}{R^2}\sin^2\varphi, \qquad C_r = 1 - \frac{r^2}{R^2}\cos^2\varphi \tag{18a,b}$$

and the expressions for stress-resultants follow by applying Eqs. (3). Thus

$$\begin{aligned} N_r(r,\varphi) = {} & -2\,p\,k\sin\varphi\cos\varphi \\ & + \frac{1}{2}\left[p\,k - \frac{B}{2\,R} - \frac{A}{R^2}\right]\left[-\frac{r}{R}\sin^2\varphi\cos\varphi\,C_r^{-3/2} - \frac{r}{R}\sin\varphi\cos^2\varphi\,S_r^{-3/2}\right] \\ & + \frac{1}{2}\left[-p\,k + \frac{3\,A}{r^2} - \frac{B}{2\,R}\right]\left[-\frac{3\,r}{R}\sin\varphi\cos^2\varphi\,S_r^{-1/2} - \frac{r^3}{R^3}\sin^3\varphi\cos^2\varphi\,S_r^{-3/2}\right. \\ & \qquad \left. - \frac{3\,r}{R}\sin^2\varphi\cos\varphi\,C_r^{-1/2} - \frac{r^3}{R^3}\sin^2\varphi\cos^3\varphi\,C_r^{-3/2}\right] \\ & + \left[\frac{B}{2\,R} - \frac{A}{R^2}\right]\left[\frac{6\,r}{R}\sin^2\varphi\cos\varphi\,C_r^{1/2} - \frac{7\,r^3}{R^3}\sin^2\varphi\cos^3\varphi\,C_r^{1/2}\right. \\ & \qquad - \frac{r^5}{R^5}\sin^2\varphi\cos^5\varphi\,C_r^{-3/2} + \frac{6\,r}{R}\sin\varphi\cos^2\varphi\,S_r^{1/2} \\ & \qquad \left. - \frac{7\,r^3}{R^3}\sin^3\varphi\cos^2\varphi\,S_r^{-1/2} - \frac{r^5}{R^5}\sin^5\varphi\cos^2\varphi\,S_r^{-3/2}\right], \end{aligned} \tag{19a}$$

$$
\begin{aligned}
N_\varphi(r,\varphi) = {} & 2pk\sin\varphi\cos\varphi \\
& + \frac{1}{2}\left[pk - \frac{B}{2R} - \frac{A}{R^2}\right]\left[-\frac{r}{R}\cos^3\varphi\, C_r^{-3/2} - \frac{r}{R}\sin^3\varphi\, S_r^{-3/2}\right] \\
& + \frac{1}{2}\left[-pk - \frac{B}{2R} + \frac{3A}{R^2}\right]\left[-\frac{3r}{R}\sin^3\varphi\, S_r^{-1/2} - \frac{r^3}{R^3}\sin^5\varphi\, S_r^{-3/2}\right. \\
& \qquad \left. - \frac{3r}{R}\cos^3\varphi\, C_r^{-1/2} - \frac{r^3}{R^3}\cos^5\varphi\, C_r^{-3/2}\right] \qquad (19\text{b}) \\
& + \left[\frac{B}{2R} - \frac{A}{R^2}\right]\left[\frac{6r}{R}\cos^3\varphi\, C_r^{1/2} - \frac{7r^3}{R^3}\cos^5\varphi\, C_r^{-1/2} - \frac{r^5}{R^5}\cos^7\varphi\, C_r^{-3/2}\right. \\
& \qquad \left. + \frac{6r}{R}\sin^3\varphi\, S_r^{1/2} - \frac{7r^3}{R^3}\sin^5\varphi\, S_r^{-1/2} - \frac{r^5}{R^5}\sin^7\varphi\, S_r^{-3/2}\right],
\end{aligned}
$$

$$
\begin{aligned}
N_{r\varphi}(r,\varphi) = {} & -pk\cos 2\varphi \\
& + \frac{1}{2}\left[pk - \frac{B}{2R} - \frac{A}{R^2}\right]\left[-\frac{r}{R}\sin\varphi\cos^2\varphi\, C_r^{-3/2} + \frac{r}{R}\sin^2\varphi\cos\varphi\, S_r^{-3/2}\right] \\
& + \frac{1}{2}\left[-pk + \frac{3A}{R^2} - \frac{B}{2R}\right]\left[\frac{3r}{R}\sin^2\varphi\cos\varphi\, S_r^{-1/2} + \frac{r^3}{R^3}\sin^4\varphi\cos\varphi\, S_r^{-3/2}\right. \\
& \qquad \left. - \frac{3r}{R}\sin\varphi\cos\varphi\, C_r^{-1/2} - \frac{r^3}{R^3}\sin\varphi\cos^4 C_r^{-3/2}\right] \\
& + \left[\frac{B}{2R} - \frac{A}{R^2}\right]\left[\frac{6r}{R}\sin\varphi\cos^2\varphi\, C_r^{1/2} - \frac{7r^3}{R^3}\sin\varphi\cos^4\varphi\, C_r^{-1/2}\right. \qquad (19\text{c}) \\
& \qquad - \frac{r^5}{R^5}\sin\varphi\cos^6\varphi\, C_r^{-3/2} - \frac{6r}{R}\sin^2\varphi\cos\varphi\, S_r^{1/2} \\
& \qquad \left. + \frac{7r^3}{R^3}\sin^4\varphi\cos\varphi\, S_r^{-1/2} + \frac{r^5}{R^5}\sin^6\varphi\cos\varphi\, S_r^{-3/2}\right].
\end{aligned}
$$

On the boundary these equations reduce, as they should, to the corresponding Eqs. (16).

Note that Eqs. (19) contain two free constants A and B which can be adjusted to satisfy boundary conditions. This is done by substituting Eqs. (16) into the expressions for the appropriate boundary conditions discussed previously. Thus, using Eqs. (6a) and (6b); (7a) and (7c); (8a) and (8c); and (9c), for the four cases considered, we obtain the following results:

a) *Free Edge*

$$A = 0; \qquad B = 0. \qquad (20\text{a, b})$$

b) *Edge Supported by Wall*

$$A = +\tfrac{1}{6}pkR^2; \qquad B = +\tfrac{2}{3}pkR. \qquad (21\text{a, b})$$

c) *Suspended Edge*

$$A = +\tfrac{1}{3}pkR^2; \qquad B = +\tfrac{1}{3}pkR. \qquad (22\text{a, b})$$

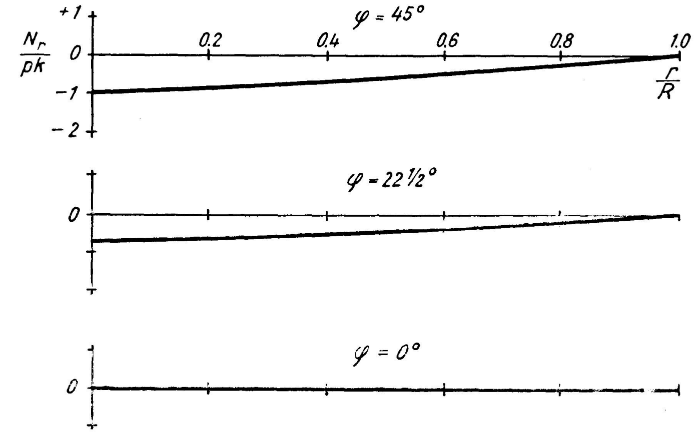

Fig. 3a. $\frac{N_r}{pk}$ For Free Edge.

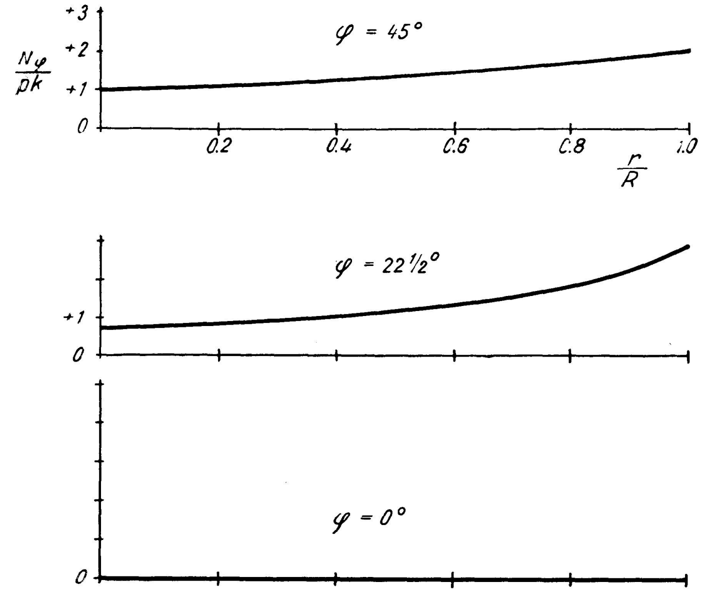

Fig. 3b. $\frac{N_\varphi}{pk}$ For Free Edge.

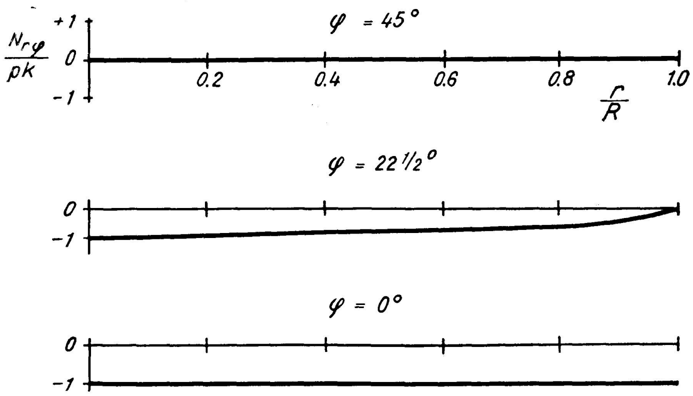

Fig. 3c. $\frac{N_{r\varphi}}{pk}$ For Free Edge.

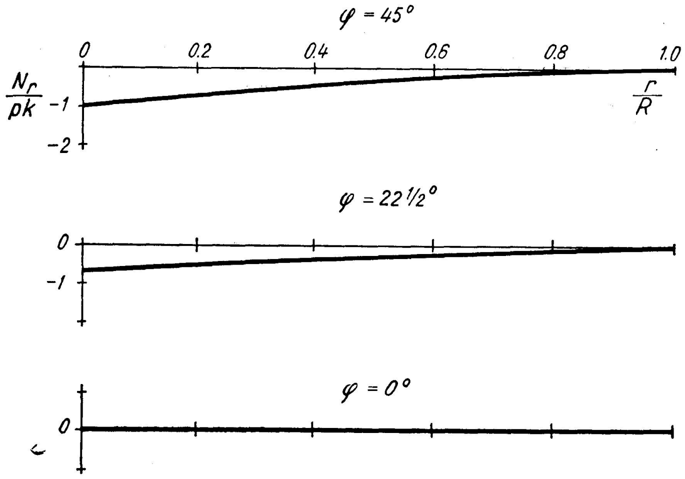

Fig. 4a. $\frac{N_r}{pk}$ For Edge Supported by Wall.

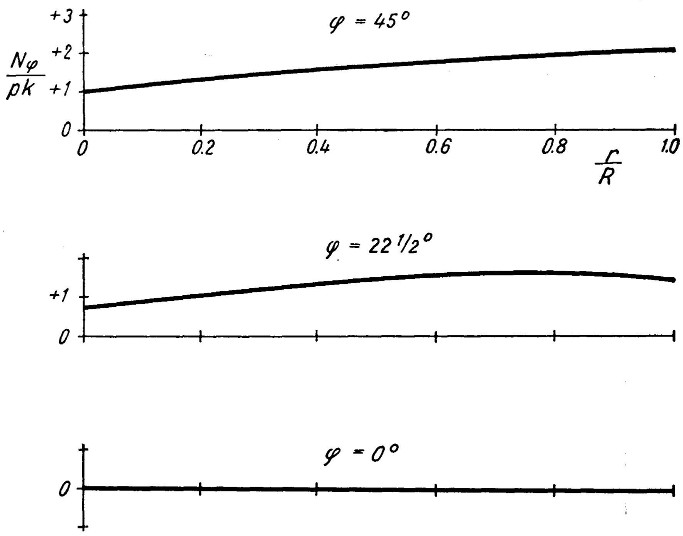

Fig. 4b. $\frac{N_\varphi}{pk}$ For Edge Supported by Wall.

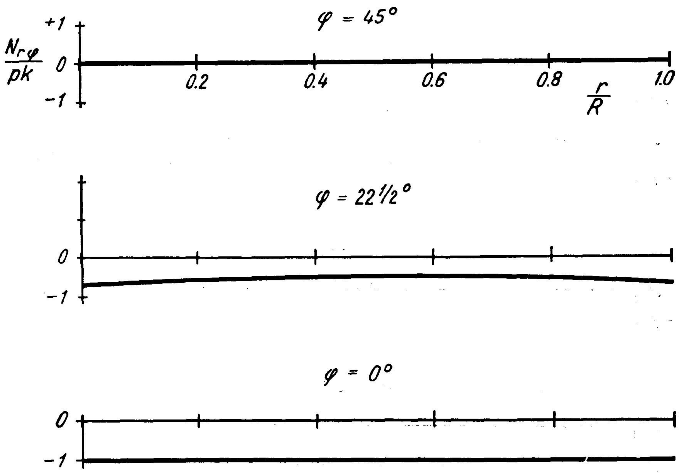

Fig. 4c. $\frac{N_{r\varphi}}{pk}$ For Edge Supported by Wall.

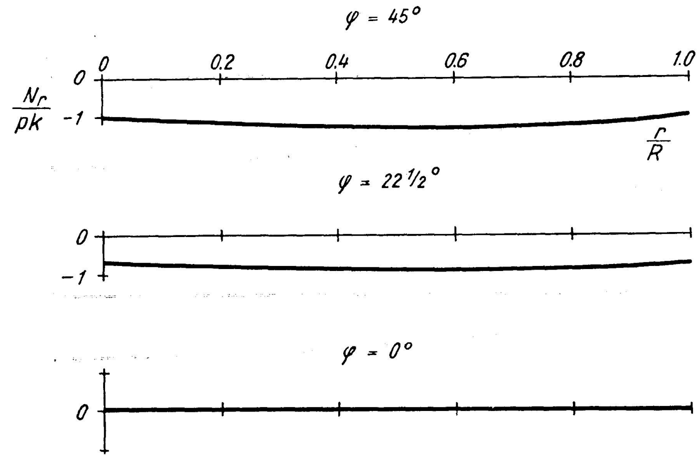

Fig. 5a. $\frac{N_r}{p\,k}$ For Suspended Edge.

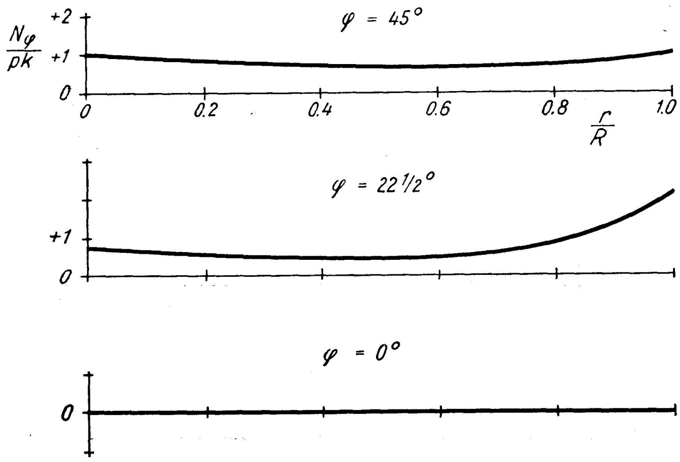

Fig. 5b. $\frac{N_\varphi}{p\,k}$ For Suspended Edge.

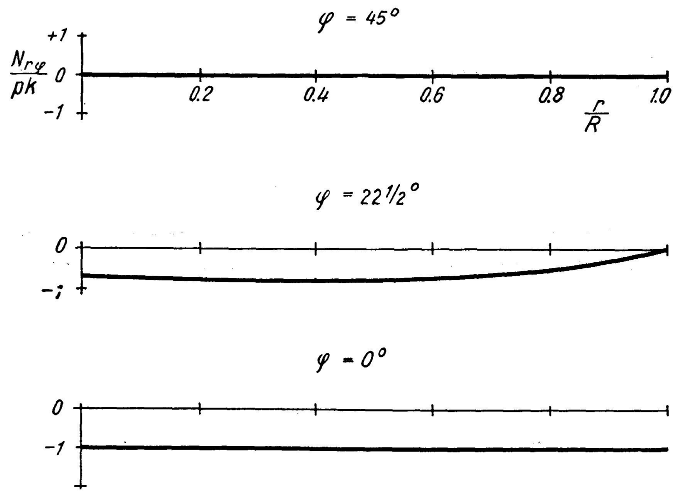

Fig. 5c. $\frac{N_{r\varphi}}{p\,k}$ For Suspended Edge.

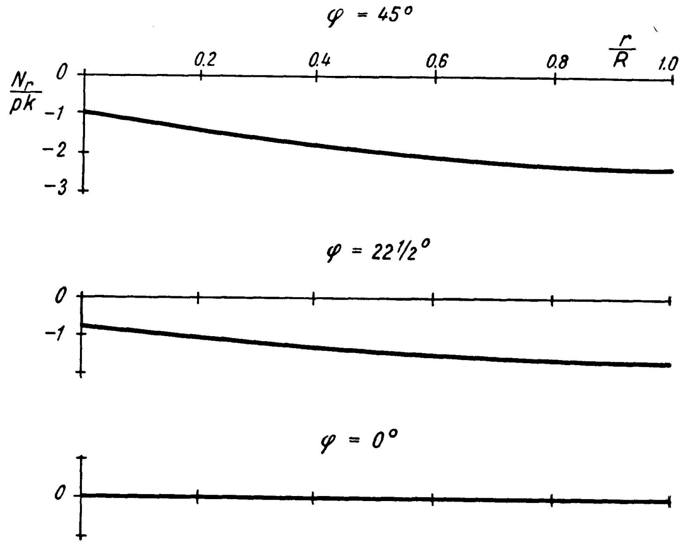

Fig. 6a. $\frac{N_r}{pk}$ For Fixed Edge.

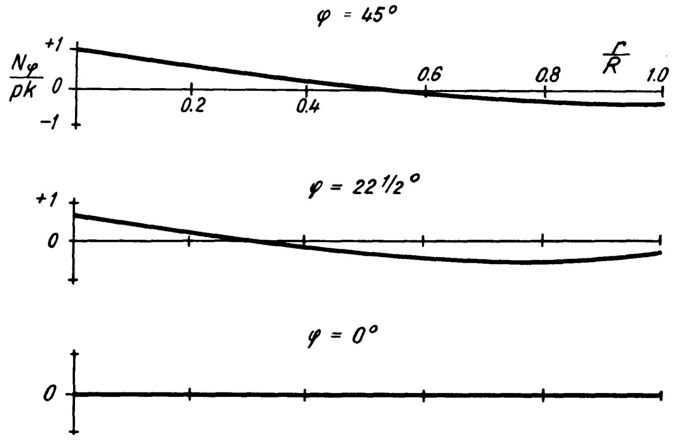

Fig. 6b. $\frac{N_\varphi}{pk}$ For Fixed Edge.

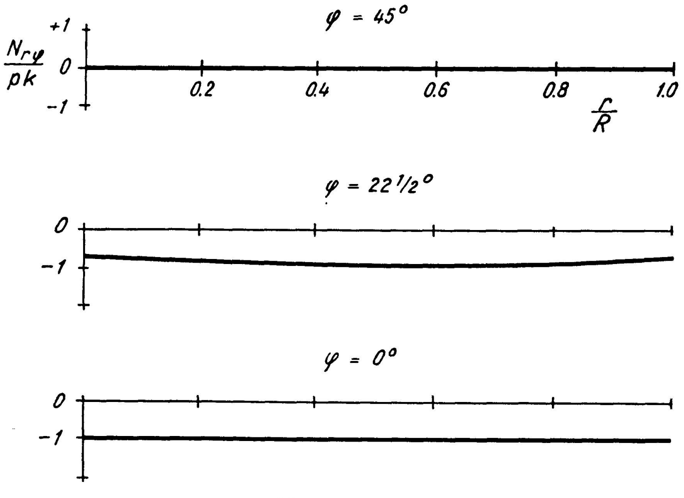

Fig. 6c. $\frac{N_{r\varphi}}{pk}$ For Fixed Edge.

d) *Fixed Edge*

$$A = +\frac{1}{6}\frac{5-\mu}{1-\mu} p k R^2, \qquad B = +\frac{2}{3}\frac{2-\mu}{1-\mu} p k R. \qquad (23\text{a, b})$$

To determine the membrane stresses N_r, N_φ, and $N_{r\varphi}$ corresponding to the various boundary conditions the above constants of integration must be substituted into Eqs. (19a, b, c). The results of evaluating such equations for $\varphi = 0°$, $22^1/_2°$, and $45°$ are plotted in Figs. 3 through 6. One may note that in all cases along $\varphi = 0°$ $N_{r\varphi} = -k p$, a constant. In all cases, at $\varphi = 0°$, $r = R$ this function exhibits a discontinuity.

Appendix A. Nomenclature

A, A_n, B, B_n	constants of integration.
$C_r = 1 - \frac{r^2}{R^2}\cos^2\varphi$.	
E	elastic modulus.
$F, \overline{F}$	stress function
h	shell thickness.
k	shell raise constant.
n	integer.
$N_r, N_{r\varphi}, N_\varphi$	stress-resultants.
p	distributed load
P	total load
r	radius vector.
R	radius at boundary.
$S_r = 1 - \frac{r^2}{R^2}\sin^2\varphi$.	
u	radial displacement.
v	tangential displacement.
w	transverse displacement.
x, y, z	co-ordinate axes.
$\epsilon_r, \epsilon_\varphi$	strain in radial and tangential directions, respectively.
$\gamma_{r\varphi}$	shearing strain.
μ	Poisson ratio.
φ	coordinate angle.

Note: Comma in subscript denotes differentiation. Functional notation is frequently used, for example, $F(R, \varphi)$ means the function of F at $r = R$ and any value of φ.

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Summary

In this paper membrane stresses in shallow hyperbolic paraboloid shells circular in plan are considered. The membrane state of stress is formulated in terms of AIRY's stress function following the procedure first used by PUCHER. The governing differential equation applicable for shallow shells is written in the plane polar coordinate system. The RIEMANN method of integrating the differential equation is employed to obtain the solution of the problem.

Four different boundary conditions-free edge, edge supported by wall, suspended edge, and fixed edge, are formulated. For these boundary conditions the problem is solved for an uniformly distributed load acting on the shell. Several graphs are prepared to indicate the resulting stress distribution. The membrane solutions found exhibit a discontinuity at $\varphi = 0°$, $r = R$.

Résumé

Les auteurs étudient le régime des contraintes de membrane dans les paraboloïdes hyperboliques minces de faible hauteur, établis sur plan circulaire. Ce régime est exprimé à l'aide de la fonction de tension d'AIRY, suivant le procédé qui a été employé pour la première fois par PUCHER. L'équation différentielle qui le contrôle et qui peut être appliquée aux voiles minces de faible hauteur est indiquée dans le système plan de coordonnées polaires. Pour obtenir la solution du problème, les auteurs appliquent la méthode de RIEMANN à l'intégration de l'équation différentielle.

Quatre conditions marginales différentes sont retenues: bords libres, bords avec appui mural, bords suspendus et bords encastrés. Pour ces conditions marginales, le problème est résolu pour une charge répartie uniformément sur le voile. Diverses courbes ont été rassemblées pour mettre en lumière la répartition des contraintes. La solution de membrane trouvée présente une discontinuité pour $\varphi = 0$.

Zusammenfassung

In dieser Abhandlung wird der Membranspannungszustand in niedrigen hyperbolischen Paraboloidschalen mit kreisförmigem Grundriß betrachtet. Der Membranspannungszustand wird mit der Airyschen Spannungsfunktion ausgedrückt, dem Verfahren folgend, das zum ersten Male von Pucher verwendet wurde. Die beherrschende Differentialgleichung, die für niedrige Schalen verwendet werden kann, wird im ebenen Polarkoordinatensystem angegeben. Um die Lösung des Problems zu erhalten, wird die Riemannsche Methode der Integration der Differentialgleichung angewendet.

Vier verschiedene Randbedingungen werden festgehalten: freier Rand, Rand mit Wandunterstützung, aufgehängter und eingespannter Rand. Für diese Randbedingungen wird das Problem für eine auf die Schale wirkende gleichmäßig verteilte Last gelöst. Verschiedene Kurven wurden zusammengestellt zur Veranschaulichung der resultierenden Spannungsverteilung. Die gefundene Membranlösung zeigt bei $\varphi = 0$ eine Diskontinuität.

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