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Autor: Sahlin, Sven

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Transversely Loaded Compression Members Made of Materials Having No Tensile Strength

La capacité portante des pièces comprimées et fléchies transversalement, formées de matériaux sans résistance à la traction

Die Tragfähigkeit von querbelasteten, gedrückten Bauteilen, die aus Baustoffen ohne Zugfestigkeit hergestellt sind

SVEN SAHLIN
Tekn. D., Docent, Division of Building Statics and Structural Engineering, Royal Institute of Technology, Stockholm, Sweden

The present paper is a study of the load-carrying capacity of transversely loaded compression members, such as columns, walls, struts, or the like, which are made of materials having no tensile strength. Furthermore, a comparison is made with the corresponding conditions of loading in the cases where the material possesses tensile strength. In both cases, it is assumed that failure in compression is not to be taken into consideration. The transverse loading is supposed to consist of one or two symmetrically applied concentrated loads, see Fig. 1 and Fig. 4, respectively.

In dealing with the conditions of loading shown in Fig. 1, use can be made

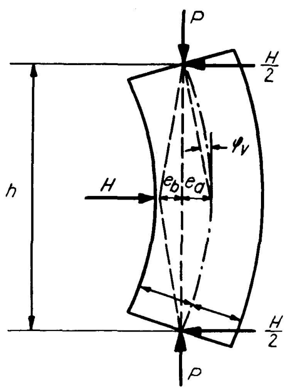

Fig. 1. Compression Member Submitted to a Transverse Load at the Centre.

of Fig. 2. Fig. 2 was taken from [1], see also [2]. This figure represents, for several values of the load and the eccentricity, the angle of rotation of the end of a wall having no tensile strength, which is submitted to an eccentric load at one end.

On the assumption that the deformations are small, that the material is elastic, and that it has no tensile strength, we obtain Eqs. (1), (2) and (3) from Fig. 1.

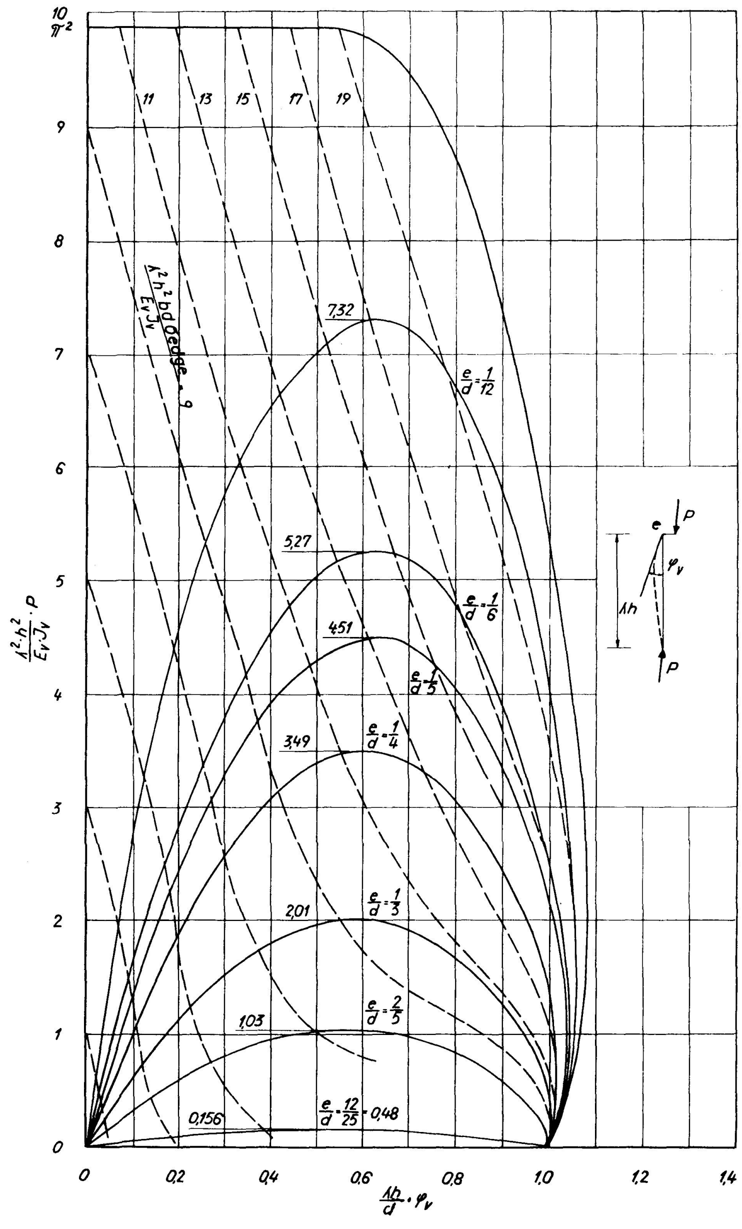

Fig. 2. Relation between the compressive load P and the angle of rotation φ_v of an end of a compression member made of a material having no tensile strength, which is submitted to an eccentric load at one end and to a central load at the other end. The full-line curves represent the relations which are obtained when the eccentricity e has certain constant values, while the dash-line curves refer to the relations which are obtained when the edge stress has certain constant values.

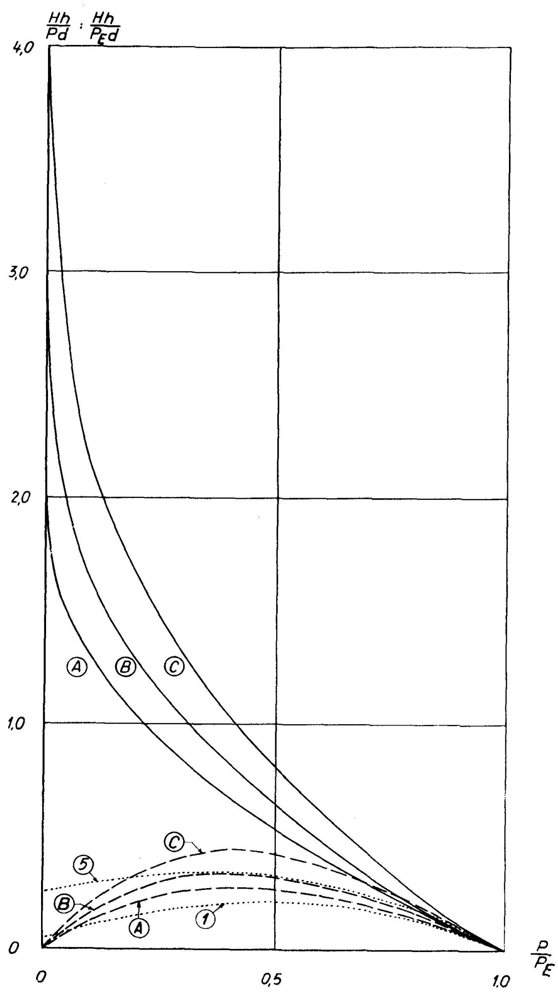

Fig. 3. Relation between the transverse load H and the compressive load P for compression members under the conditions of loading shown in Figs. 1 and 2. The abscissa represents $\frac{Hh}{Pd}$ for the full-line curves and $\frac{Hh}{P_E d}$ for the other curves. The curves are marked in accordance with the table below.

	Conditions of loading shown in Fig. 1	Conditions of loading shown in Fig. 4	$\frac{Hh}{Pd}$	$\frac{Hh}{P_E d}$
Material having no tensile strength	(A)	$\alpha=\frac{1}{3}$ (B) $\alpha=\frac{1}{4}$ (C)	——	---
Material having tensile strength*)	(1) (5)			.....

*) These curves apply only to the values assumed on p. 249.

$$\frac{2e_a}{h} = \varphi_v, \tag{1}$$

$$\frac{2e_b}{h} = \frac{H}{2P}, \tag{2}$$

$$e_a + e_b = e, \tag{3}$$

where e, P, and φ_v have the same significance as in Fig. 2, while $\lambda h = h/2$.

From Eqs. (1), (2), and (3) we get

$$\frac{\lambda h}{d}\varphi_v = \frac{e_a}{d} = \frac{e}{d} - \frac{e_b}{d} = \frac{e}{d} - \frac{H\lambda h}{2Pd} \tag{4}$$

and hence

$$\frac{H\lambda h}{2Pd} = \frac{e}{d} - \frac{\lambda h}{d}\varphi_v \tag{5}$$

and

$$\frac{Hh}{Pd} = 4\left(\frac{e}{d} - \frac{\lambda h}{d}\varphi_v\right). \tag{6}$$

For a series of constant values of P, the maximum values of the right-hand member of Eq. (6) can be found from Fig. 2 (for this purpose, $\frac{Hh}{Pd}$ may possibly be plotted as $f(e/d)$ in order that the maximum values may be determined more sharply).

After this operation has been carried out, the maximum values of $\frac{Hh}{Pd}$ for a series of values of P are known. The result of such calculations is represented by the full-line curve (A) in Fig. 3. As the Euler critical load $P_E = \frac{\pi^2 EJ}{h^2}$ has been introduced, the abscissa in this graph expresses a relative value.

In order that an idea of the absolute magnitude of H may be formed,

$$\frac{Hh}{P_E d} = f\left(\frac{P}{P_E}\right)$$

is also shown in Fig. 3 by the dash-line curve (A).

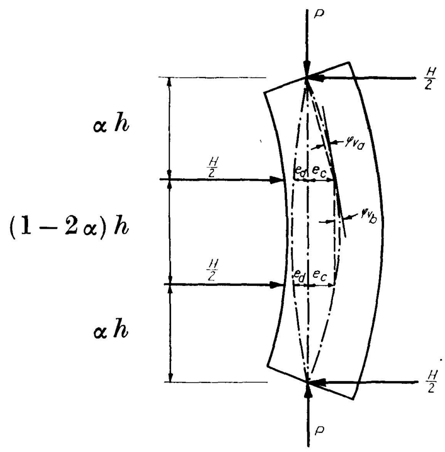

Fig. 4. Compression Member Submitted to Two Symmetrically Applied Transverse Loads $H/2$.

A direct treatment of the case of loading represented in Fig. 4 was not possible with the aid of the available data. To begin with, it was therefore necessary to make a preparatory calculation so as to determine the angle of rotation of an end of a compression member, e.g. a wall, having no tensile strength, which is subjected to loads of equal eccentricity at both ends. Fig. 5

shows the results of this calculation, which has been carried out by using those solutions of the equation of the elastic curve of members made of materials having no tensile strength which had been published in [3], [4], [5], and [1].

From Fig. 4 we get the equations (7), (8) and (9).

$$\frac{e_c}{\alpha h} - \varphi_{v_a} = \varphi_{v_b}, \tag{7}$$

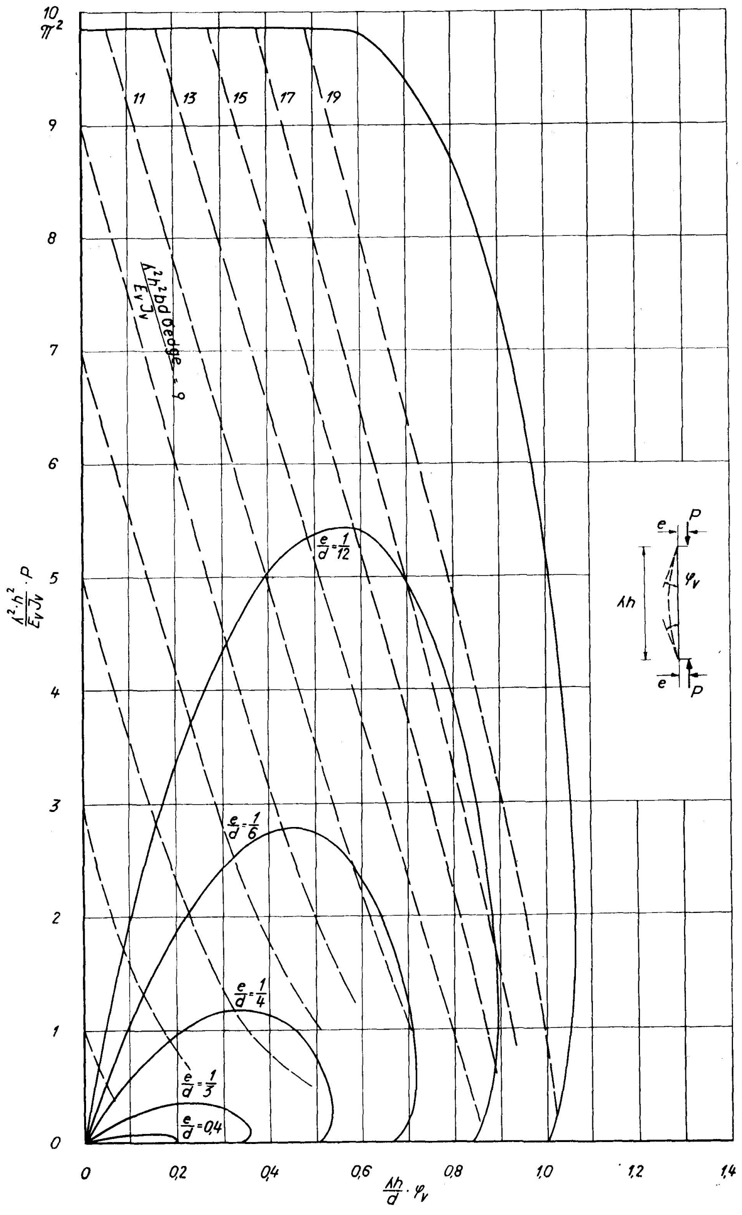

Fig. 5. Relation between the compressive load P and the angle of rotation φ_v of an end of a compression member made of a material having no tensile strength, which is submitted to loads of equal eccentricity at both ends. The full-line curves represent the relations which are obtained when the eccentricity e has certain constant values, while the dash-line curves refer to the relations which are obtained when the edge stress has certain constant values.

$$\frac{H}{2P} = \frac{e_d}{\alpha h}, \tag{8}$$

$$e_c + e_d = e, \tag{9}$$

where φ_{v_a} is the angle of rotation of an end of a compression member which is submitted to an eccentric load at one end only (Fig. 2) and φ_{v_b} is the angle of rotation of an end of a compression member which is subjected to loads of equal eccentricity at both ends (Fig. 5).

For $\lambda = \alpha = \frac{1}{3}$, Eqs. (7), (8), and (9) yield

$$\frac{H h}{P d} = 6 \left[\frac{e}{d} - \left(\frac{\lambda h}{d} \varphi_{v_a} + \frac{\lambda h}{d} \varphi_{v_b} \right) \right]. \tag{10}$$

While Eq. (6) is used for *one* transverse load, Eq. (10) is employed in a similar way for *two* transverse loads in order to find $\left(\frac{H h}{P d}\right)_{max.}$ for various values of P. In this case, $\frac{\lambda h}{d} \varphi_{v_a}$ is obtained from Fig. 2 and $\frac{\lambda h}{d} \varphi_{v_b}$ is determined from Fig. 5.

The results of this calculation are shown in Fig. 3, where the full-line curve (B) represents $\frac{H h}{P d}$ and the dash-line curve (B) represents $\frac{H h}{P_E d}$.

The curves (C) for $\alpha = \frac{1}{4}$ are obtained in an analogous manner.

If the material is assumed to be able to take certain limited tensile stresses, then the equation of the elastic curve of a member made of a material having tensile strength gives the following moment at the centre of a member submitted to combined compression and bending in accordance with Fig. 1:

$$M_{max} = \frac{H}{2k} \operatorname{tg} \left(k \frac{h}{2} \right), \tag{11}$$

where

$$k = \sqrt{\frac{P}{E J}} = \frac{\pi}{h} \sqrt{\frac{P}{P_E}} = \frac{\pi}{h} a.$$

$$M_{max} = \frac{H h \operatorname{tg}\left(\frac{\pi}{2} a\right)}{2 \pi a}. \tag{12}$$

For a rectangular cross section ($b d$), the maximum tensile stress is

$$\sigma = -\frac{P}{b d} + \frac{6 M_{max}}{b d^2}. \tag{13}$$

By substituting Eq. (12) in Eq. (13) and by transforming, we get

$$\frac{H h}{P_E d} = \left(\frac{4 \sigma h^2}{\pi E d^2} + \frac{\pi}{3} a^2 \right) a \cot \left(\frac{\pi}{2} a \right). \tag{14}$$

Thus, the maximum transverse load H is a function of both the tensile strength and the compressive load. In other words, this is a stress problem, which differs in principle from the case of a material having no tensile strength, where H was determined by the compressive load alone, that is to say, where we had to deal with a stability problem.

The following calculation shows how the results shown in Fig. 3 are modified when the material has tensile strength.

Assume $\sigma = 1\,\text{kp/cm}^2$, $E = 10\,000\,\text{kp/cm}^2$, $h/d = 25$. Then

$$\frac{H h}{P_E d} = \left(\frac{4 \cdot 625}{\pi \cdot 10\,000} + \frac{\pi}{3} a^2\right) a \cot\left(\frac{\pi}{2} a\right). \tag{15}$$

We obtain the dot-line curve (1) in Fig. 3. This curve is compared with the dash-line curve (A).

A certain tensile strength of the material causes an increase in H at very small loads P. This is further emphasised by the dot-line curve (5), which was calculated for $\sigma = 5\,\text{kp/cm}^2$, the other values being unchanged.

In the above calculations it was assumed that the transverse load H is that load at which the tensile strength of the material is exceeded, i. e. the cracking load. Now cracking does not necessarily imply that the load-carrying capacity of the member has been exhausted. A position of equilibrium which involves a partly cracked cross section and a certain tensile zone is fully conceivable. Such a possibility would permit a substantial increase in H above the cracking load in certain cases. Thus, it would be possible to raise the relevant portions of the curve (1) in Fig. 3 above the curve (A). However, it may be imagined that the stress concentration at the root of the crack would give rise to almost total cracking on the side in tension. Therefore, the case where the material has no tensile strength should for the time being be regarded as a lower limit on the safe side. Accordingly, the maximum transverse load H is determined either by the cracking load calculated from the tensile strength of the material in question, or by the ultimate load of the material having no tensile strength, whichever of these two values is the greater.

Example, See Fig. 6

$\sigma_{tensile} = 1\,\text{kp/cm}^2$
$E = 10\,000\,\text{kp/cm}^2$
$L = 6\,\text{m}$
$d = 12\,\text{cm}$
$P = 4\,\text{Mp/m}$

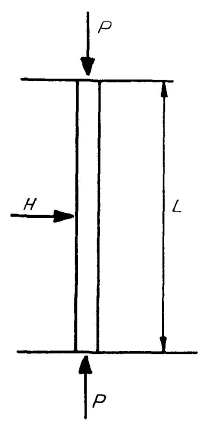

Fig. 6. Example.

$$h = \frac{L}{2} = 300 \text{ cm}, \qquad \frac{h}{d} = \frac{300}{25} = 12, \qquad \frac{\sigma}{E} = 0{,}0001,$$

$$P_E = \frac{\pi^2\, 10000 \cdot 100 \cdot 12^3}{12 \cdot 300^2} = 15\,800 \text{ kp/m},$$

$$\frac{P}{P_E} = \frac{4000}{15\,800} = 0{,}253, \qquad a = \sqrt{\frac{P}{P_E}} = 0{,}501.$$

From Eq. (14)

$$\frac{H\,12}{15\,800} = \left(\frac{4}{\pi} \cdot 0{,}0001 \cdot 12^2 + \frac{\pi}{3} \cdot 0{,}253\right) \cdot 0{,}501 \cot\left(\frac{\pi}{2} \cdot 0{,}501\right) = 0{,}141,$$

$$H = 185 \text{ kp/m } (= \text{cracking load}).$$

From Fig. 3, Curve (*A*), $\frac{H\,h}{P_E\,d} = \frac{H}{P_E} \cdot 12 = 0{,}24,$

$$H = \frac{0{,}24 \cdot 15\,800}{12} = 320 \text{ kp/m } (\leqq \text{ultimate load}).$$

Appendix

The elastic curve of a compression member made of a material having no tensile strength is assumed to have the general shape shown in Fig. 7. The total length h of the member is calculated in a way which is in principle indicated in this figure. Cf. [5] and [1].

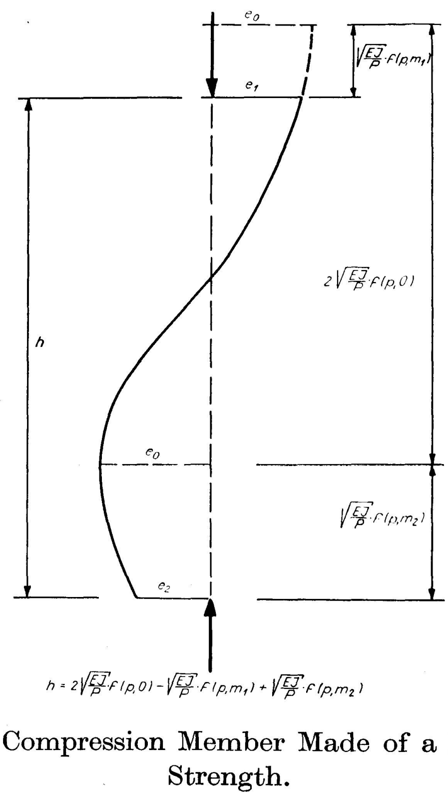

Fig. 7. Elastic Curve of a Compression Member Made of a Material Having no Tensile Strength.

The function $f = f(p, m)$, which is different in 3 intervals, is dependent on $m = 6\frac{e}{d}$ and $p = 3\frac{e_0}{d} - \frac{1}{2}$ as follows:

$$f_1(p,m) = \tfrac{1}{2}\sqrt{(1-p)(3-m)(1+2p-m)} + \tfrac{1}{2}(1-p)^{3/2}\ln\frac{2-m+p+\sqrt{(3-m)(1+2p-m)}}{1-p}, \qquad 1 \leqq m \leqq 3, \quad \frac{m-1}{2} \leqq p < 1. \tag{A 1}$$

$$f_2(p,m) = \sqrt{p(1-p)} + \tfrac{1}{2}(1-p)^{3/2}\ln\frac{1+\sqrt{p}}{1-\sqrt{p}} + \arcsin\sqrt{\frac{1-p}{1+3p}} - \arcsin\left(m\sqrt{\frac{1-p}{1+3p}}\right), \qquad 0 \leqq m < 1, \quad 0 \leqq p < 1. \tag{A 2}$$

$$f_3(p,m) = \frac{\pi}{2} - \arcsin\frac{m}{1+2p}, \qquad \frac{m-1}{2} \leqq p < 0, \quad 0 \leqq m \leqq 1. \tag{A 3}$$

where m_1 is the relative eccentricity at the top end of the member and m_2 is the relative eccentricity at the bottom end of the member.

The angle of rotation φ_{v_1} of an end of the compression member is dependent on p and m as follows

$$\frac{h}{d}\varphi_{v_1} = \frac{m_1 \pm m_2}{6} \pm h\sqrt{\frac{P}{EJ}}\,F(p, m_1), \tag{A 4}$$

where $F(p, m_1)$ is different in 3 intervals so that F_1 corresponds to f_2, etc.

$$F_1(p,m) = \frac{1}{3}\sqrt{\frac{1+2p-m_1}{(1-p)(3-m_1)}}, \tag{A 5}$$

$$F_2(p,m) = \frac{1}{6}\sqrt{\frac{1+3p}{1-p} - m_1^2} \tag{A 6}$$

$$F_3(p,m) = \tfrac{1}{6}\sqrt{(1+2p)^2 - m_1^2}. \tag{A 7}$$

We consider half the compression member in accordance with Fig. 1. We calculate the angle of rotation at the centre, observing that $m_2 = 0$ and $h = h/2$ in Eq. (A 4)

$$\frac{h}{2d}\varphi_v = \frac{m_1}{6} \pm \frac{h}{2}\sqrt{\frac{P}{EJ}}\,F(p, m_1) = \frac{e}{d} - \frac{h}{2}\sqrt{\frac{P}{EJ}}\,F(p, m_1). \tag{A 8}$$

From Eq. (5) we get

$$\frac{\lambda h}{d}\varphi_v = \frac{h}{2d}\varphi_v = \frac{e}{d} - \frac{Hh}{4Pd}. \tag{A 9}$$

Eqs. (A 8) and (A 9) yield

$$\frac{H\,h}{P\,d} = 2\,h\sqrt{\frac{P}{E\,J}}\,F\,(p, m_1). \tag{A 10}$$

From Eqs. (A 1) to (A 3) and Fig. 7 we find

$$h\sqrt{\frac{P}{E\,J}} = 2\,[f\,(p, 0) - f\,(p, m_1)], \tag{A 11}$$

which, together with Eq. (A 10), gives

$$\frac{H\,h}{P\,d} = 4\,[f\,(p, 0) - f\,(p, m_1)]\;F\,(p, m_1), \tag{A 12}$$

where $m_1 = \frac{6\,e}{d}$, cf. Eq. (4).

By substituting $P_E = \frac{\pi^2\,E\,J}{h^2}$ and by squaring Eq. (A 11), we obtain

$$\frac{P}{P_E} = \frac{4}{\pi^2}\,[f\,(p, 0) - f\,(p, m_1)]^2 \tag{A 13}$$

and multiplying by Eq. (A 12) gives

$$\frac{H\,h}{P_E\,d} = \frac{16}{\pi^2}\,[f\,(p, 0) - f\,(p, m_1)]^3\,F\,(p, m_1). \tag{A 14}$$

Eqs. (A 12), (A 13), and (A 14) give a solution to the problem of finding a maximum value of H as a function of P or P/P_E.

If the compression member is acted upon by two transverse loads, then the respective analytical expressions φ_{v_a} and φ_{v_b} are substituted in Eq. (10), and the calculation is similar to that in the case where a single transverse load is applied at the centre. An analogous procedure is used in the case of several transverse loads.

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Summary

This paper is a study of the load-carrying capacity of transversely loaded compression members which are made of materials having no tensile strength. Detailed calculations were carried out in the cases of one and two symmetrically applied, concentrated transverse loads, see Figs. 1 and 4, respectively. The former case was also compared with solutions relating to materials having tensile strength. The case of no tensile strength is a stability problem, while the case of tensile strength is a stress problem. The results are shown in Fig. 3.

Résumé

Le présent rapport est une étude de la capacité portante des pièces comprimées et fléchies transversalement, formées de matériaux sans résistance à la traction. Des calculs détaillés ont été faits en considérant une ou deux charges transversales concentrées symétriques (voir fig. 1 et 4). En outre, dans le premier cas, on a comparé la solution obtenue à celles relatives aux matériaux résistants à la traction. Le cas sans résistance à la traction est un problème de stabilité, tandis que le cas avec résistance à la traction est un problème de contraintes. Les résultats sont représentés à la fig. 3.

Zusammenfassung

Dieser Bericht behandelt die Tragfähigkeit von querbelasteten, gedrückten Bauteilen, die aus Baustoffen ohne Zugfestigkeit hergestellt sind. In den Fällen einer bzw. zweier symmetrisch angebrachten Einzelquerlasten, siehe Abb. 1 bzw. 4, wurden ausführliche Berechnungen angestellt. Der erstgenannte Fall wurde auch mit Lösungen verglichen, die sich auf Baustoffe mit Zugfestigkeit beziehen. Der Fall ohne Zugfestigkeit ist ein Stabilitätsproblem, während der Fall mit Zugfestigkeit ein Spannungsproblem ist. Die Ergebnisse sind in Abb. 3 dargestellt.

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