Non-linear analysis of reinforced concrete structures

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Non-Linear Analysis of Reinforced Concrete Structures

Analyse non linéaire des ossatures en béton armé

Untersuchung des nichtlinearen Verhaltens von Tragwerken aus Stahlbeton

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1. Introduction

The need for general methods of non-linear analysis of structures has been felt for a long time.

Due to the inherent difficulties of non-linear analysis simplifying assumptions must always be used, the validity of which it is impossible to judge. Therefore, methods based on very general hypotheses are not only useful for a particularly accurate design of important structures, but mainly for judging the validity of the simplifying assumptions used in current practice.

The enormous volume of computation involved in the non-linear analysis of even simple structures makes it necessary to use electronic computers. On the other hand, given the small number of basic different computations and the possibility of using the general matric theory of structural analysis, comparatively simple programs can be established.

Plane structures made up of linear elements under the action of a monotonic loading are studied in the present paper. The change of geometry during loading and shake-down problems are not considered. Although these effects could also be taken into account, the corresponding programs would become considerably more involved.

As only those structures are considered, whose deformations depend mainly on rotations, the analysis below is based on moment-curvature diagrams, which are also taken in function of the normal acting forces. In order to simplify the program, the variation of the normal forces and the effects of the longitudinal strain are disregarded.

The moment-curvature diagrams are obtained from stress-strain relations, and special attention is paid to the behaviour of reinforced concrete cross sections. It must be emphasized that the program of structural analysis is completely general and can be applied to any other material if adequate moment-curvature diagrams are used.

2. Basic Assumptions

The computation of the moment-curvature diagrams is based on Bernouilli's hypotesis and on the stress-strain diagrams for the concrete and the steel. As is well known, there is always a randomness in the mechanical properties of these materials. In the present paper this randomness is not considered and the computations are based on mean diagrams. The problem of deciding whether it is better to consider the mean diagrams or the safe diagrams corresponding to values with a small probability of not being attained is not discussed here. This problem was analysed in a previous paper [1] and it is intended to study it in more detail in the near future.

Both concrete and steel diagrams depend on time, mainly for large strains. For concrete this effect was very carefully studied in a recent paper by RÜSCH [2]. The diagram proposed by RÜSCH, that was adopted by the European Committee for Concrete, is also used in the present paper. This diagram (fig. 1) consists of a parabolic curve up to a $2\,^0/_{00}$ strain followed by a straight up to

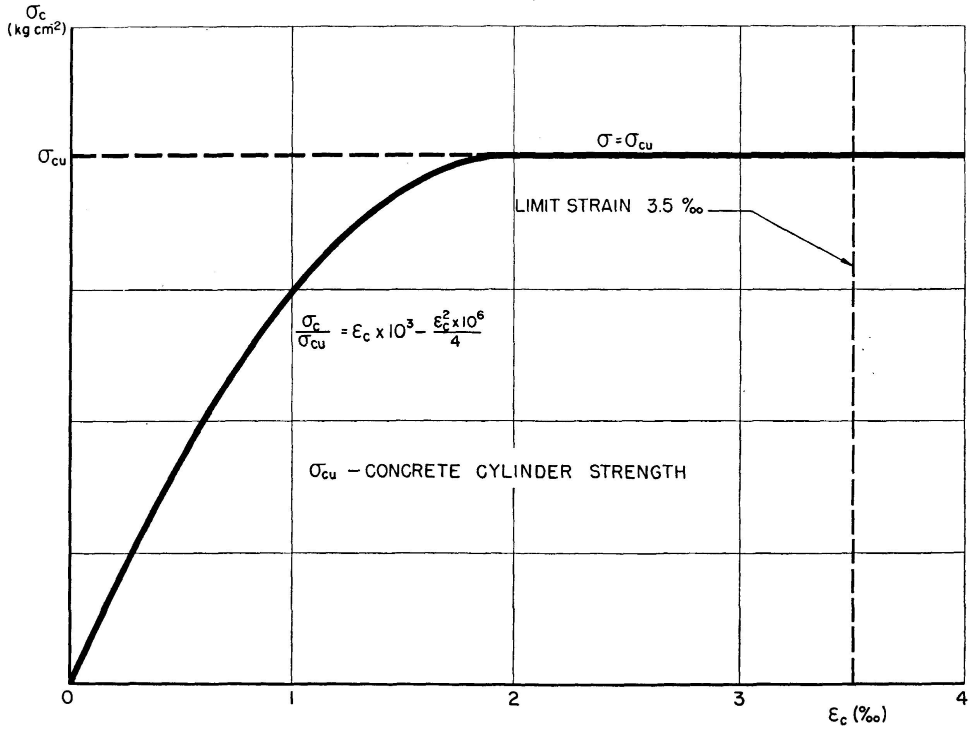


Fig. 1.

3.5 ‰ and can be considered to correspond to a loading time from a few hours to several days. For small strains the modulus of elasticity given by this diagram is correct only for concrete with a cylinder strength of about 300 kgcm^{-2}, but although it would be easy to correct it for other types of concrete this was not deemed necessary. As indicated, the value of σ_{cu} (fig. 1) corresponds to the mean strength measured on cylinder tests.

The behaviour of the reinforced concrete sections is studied disregarding the tensile strength of concrete, although the influence of the concrete around the bars under tension is taken into account. In fact for beams with a small percentage of reinforcement the mean strain along the bars is but a small fraction of the strain computed by the usual reinforced concrete theory. This influence was taken into account in accordance with theoretical and experimental results published [3].

In order to make possible to use the same assumptions even before the first cracks appear, the corresponding part of the stress-strain diagram for steel was obtained by considering a fictitious modulus of elasticity, in such a

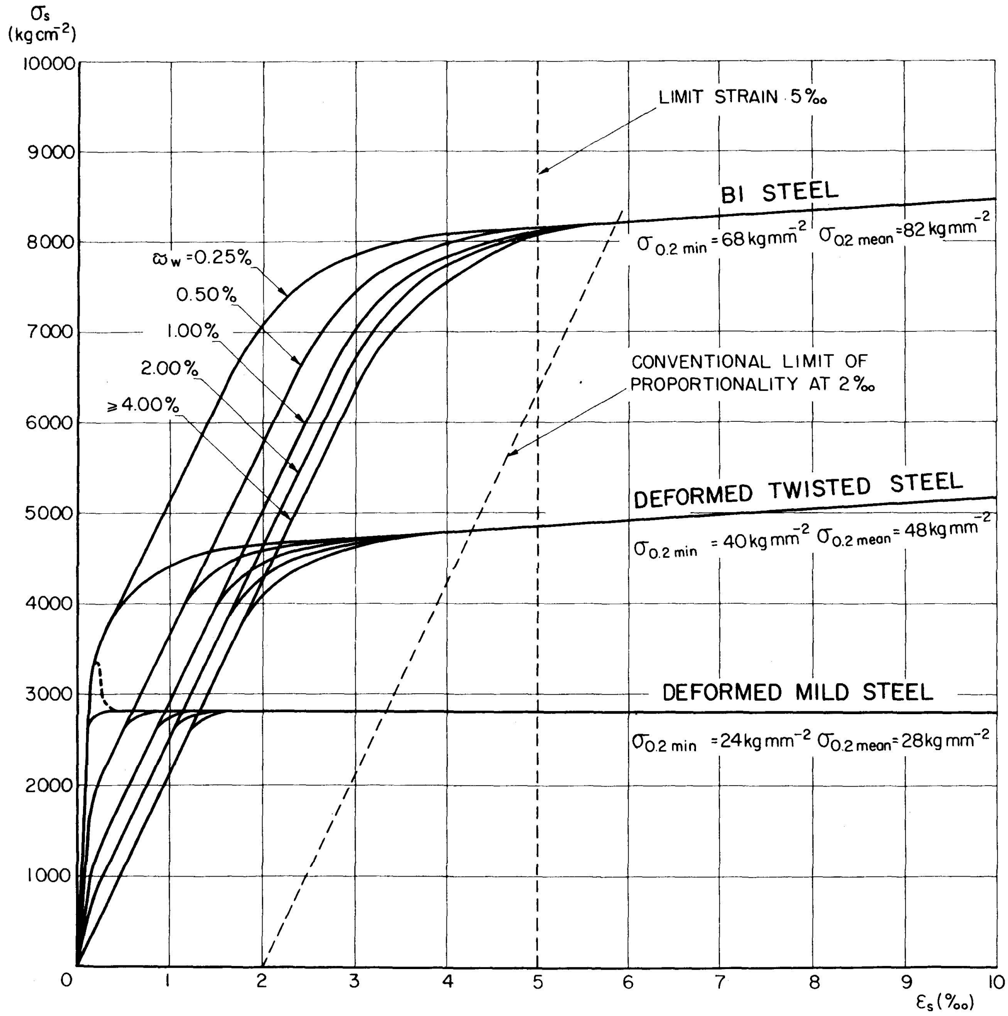


Fig. 2.

way that, for a given moment, the curvatures of the fictitious cross-section equal those in the actual cross-section.

From these principles, the diagrams presented in Fig. 2 were obtained. These diagrams correspond to deformed mild steel, deformed twisted bars and bi steel special bars, made up of two parallel rods 2 cm apart connected, each 10 cm, by a small welded transverse bar.

For determining the ultimate bending moments, limits were ascribed both to concrete and to steel strains, the limit for the former being $3.5\,^0/_{00}$ and for the latter $5\,^0/_{00}$ [2]. The ultimate bending moment is considered to be attained when one of these strains is reached.

3. Determination of the Moment-Curvature Diagrams

A first program was established to compute the moment-curvature diagrams for rectangular or tee-sections under simple bending or bending combined with a compression sufficiently low not to eliminate the tensile stresses. For tee-beams, only the cases of uniform width compression zones were considered.

Moment-curvature diagrams for positive and negative moments are computed in separate.

The reduced normal forces, n, and bending moments, m, are defined as

$$n = \frac{N}{b\,h\,\sigma_{cu}} \quad \text{and} \quad m = \frac{M}{b\,h^2\,\sigma_{cu}}$$

according to Fig. 3. Two percentages of reinforcement are considered, one $\tilde{\omega}_0$, taking as reference the width of the compression zone and the other, $\tilde{\omega}_w$, the width of the web. The moments are computed with respect to an axis at the middle depth of the bar.

The stress-strain diagrams are defined by the ordinates at 13 points for

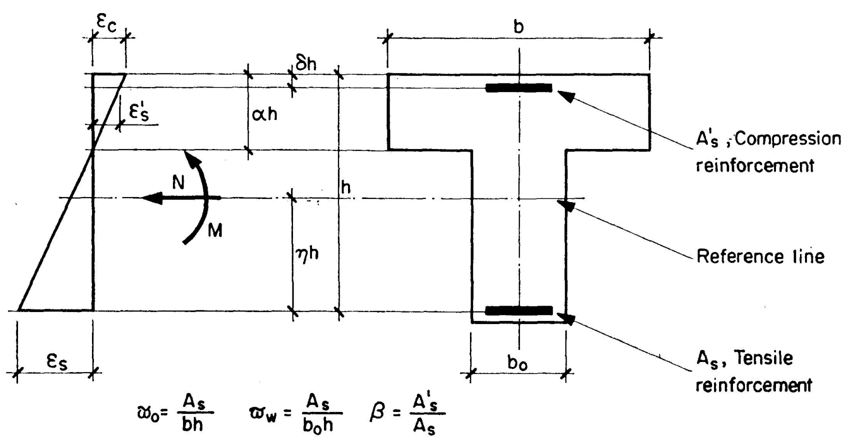
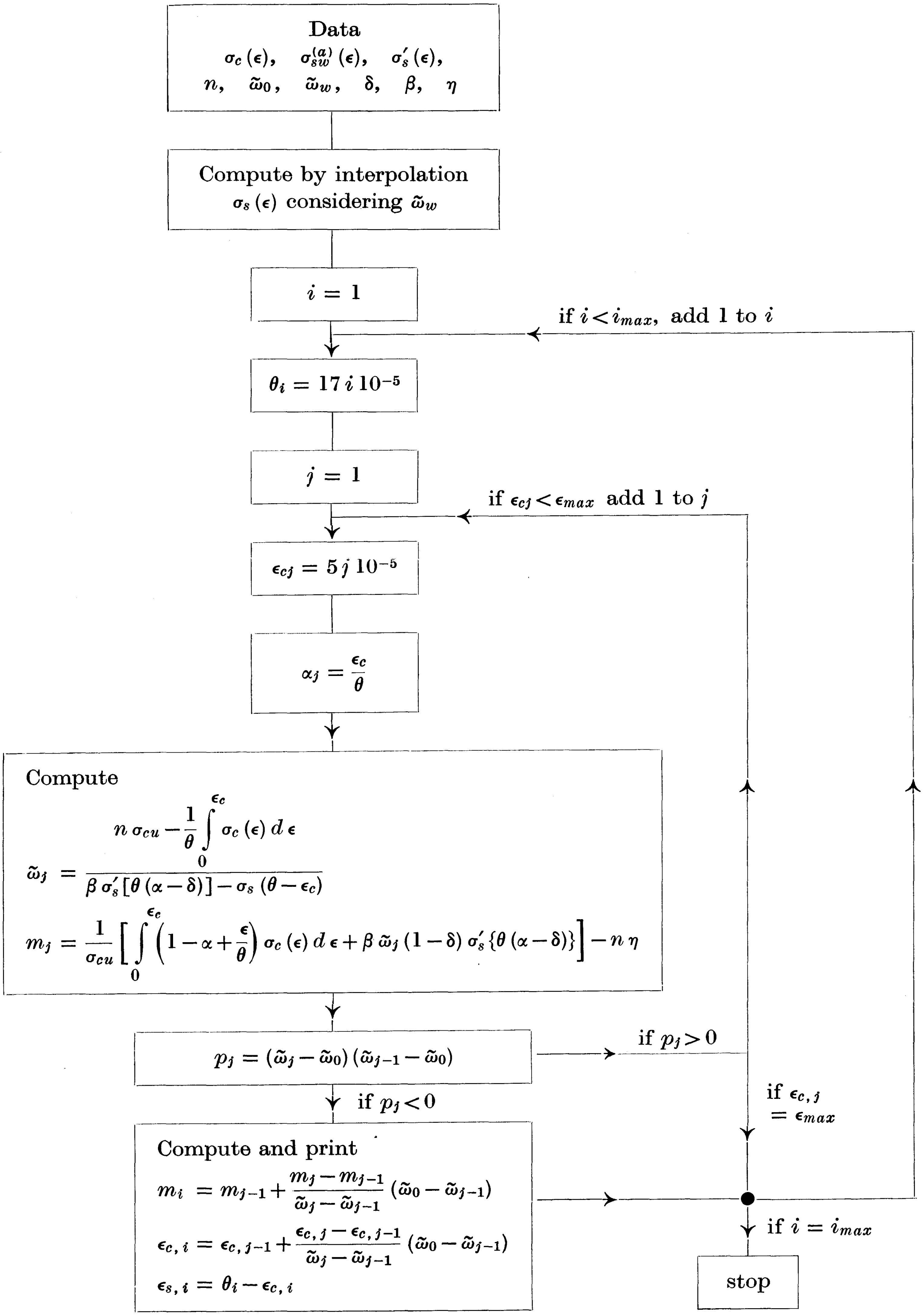


Fig. 3.



(a) σ_{sw} — set of diagrams $\sigma_s(\epsilon)$ corresponding to different $\tilde{\omega}_w$.

Fig. 4. Flow-chart for the computation of the moment-curvature diagrams.

steel and at 23 points for concrete. Intermediate values are obtained by linear interpolation.

For given values of the reduced normal force, n, and of the reduced curvature, $\theta = \epsilon_s + \epsilon_c$, the program automatically computes the correspondent values of the reduced moment, m, and the maximal concrete and steel strains. The flow chart for this computation is presented in Fig. 4.

The values considered for θ correspond to 0.02 (0.02) 0.1 (0.1) 1.2 θ_{max}, where $\theta_{max} = (3.5 + 5.0)\,10^{-3} = 8.5 \times 10^{-3}$. For each value of θ, the percentage of reinforcement that would correspond to a given ratio between steel and concrete strains, $\tilde{\omega}_j$, is compared with the actual percentage of tensile reinforcement, $\tilde{\omega}_0$, the concrete strains being successively increased. When the difference $\tilde{\omega}_j - \tilde{\omega}_0$ changes of sign, ϵ_s and ϵ_c, and the corresponding value of m

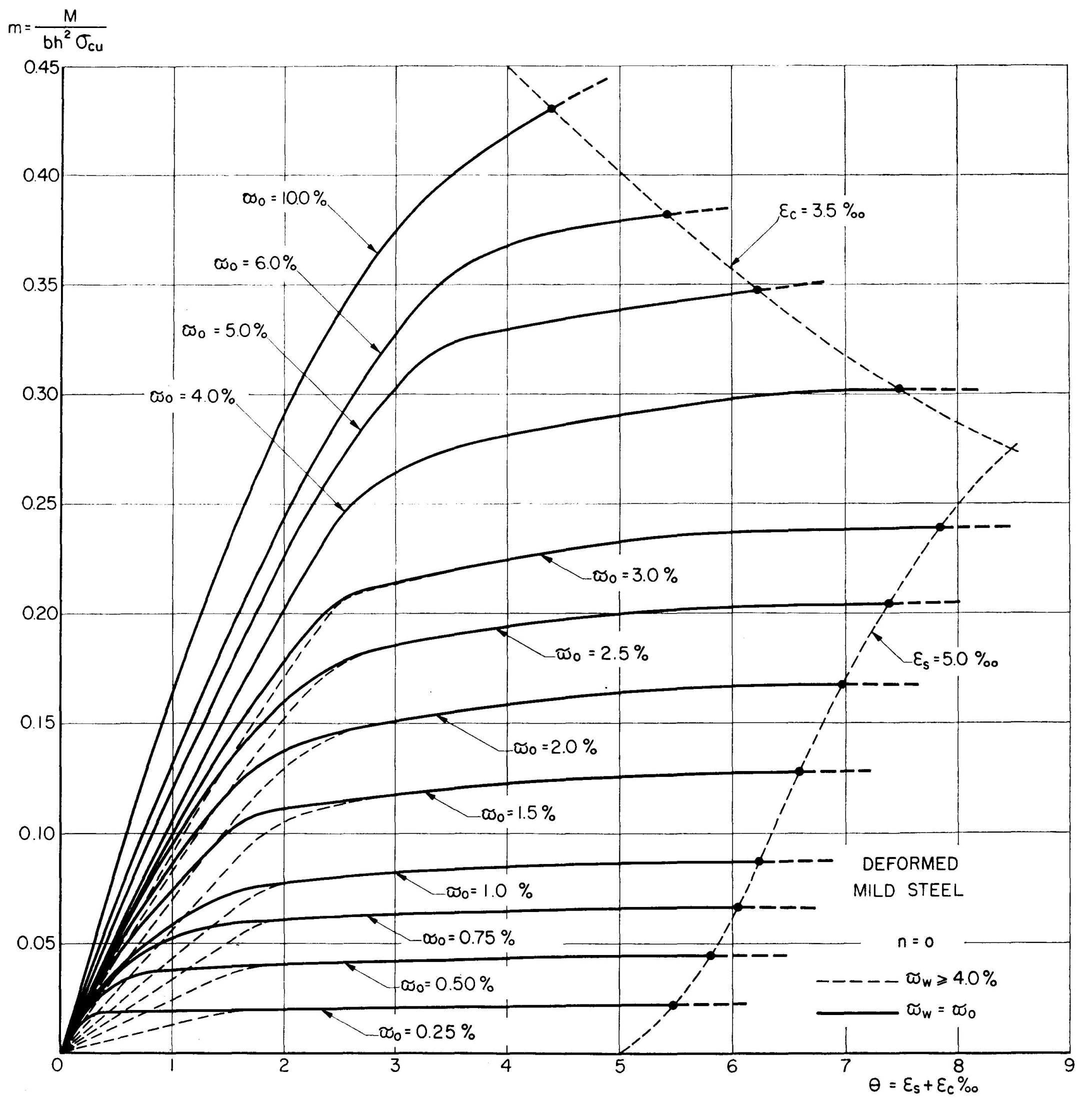


Fig. 5.

are computed by interpolation. These computations are based on a program prepared by ARGA E LIMA [4] used as a subroutine.

Fig. 5 to 7 present the moment-curvature diagrams for three types of steel, assuming $n=0$. The continuous lines correspond to rectangular beams and the dashed lines to tee beams having percentage of reinforcement with respect to the web above 4%. So the dashed line diagrams would still correspond to the behaviour of rectangular beams but disregarding the effect of the concrete enveloping the bars. Attention is called to the important difference between the two types of diagrams for small percentages of reinforcement.

The ultimate bending moment m_u and the corresponding rotation, θ_u, are defined by the conditions $\epsilon_s = 5\,^0/_{00}$ or $\epsilon_c = 3.5\,^0/_{00}$. The lines defining these conditions are independent of the concrete and steel qualities.

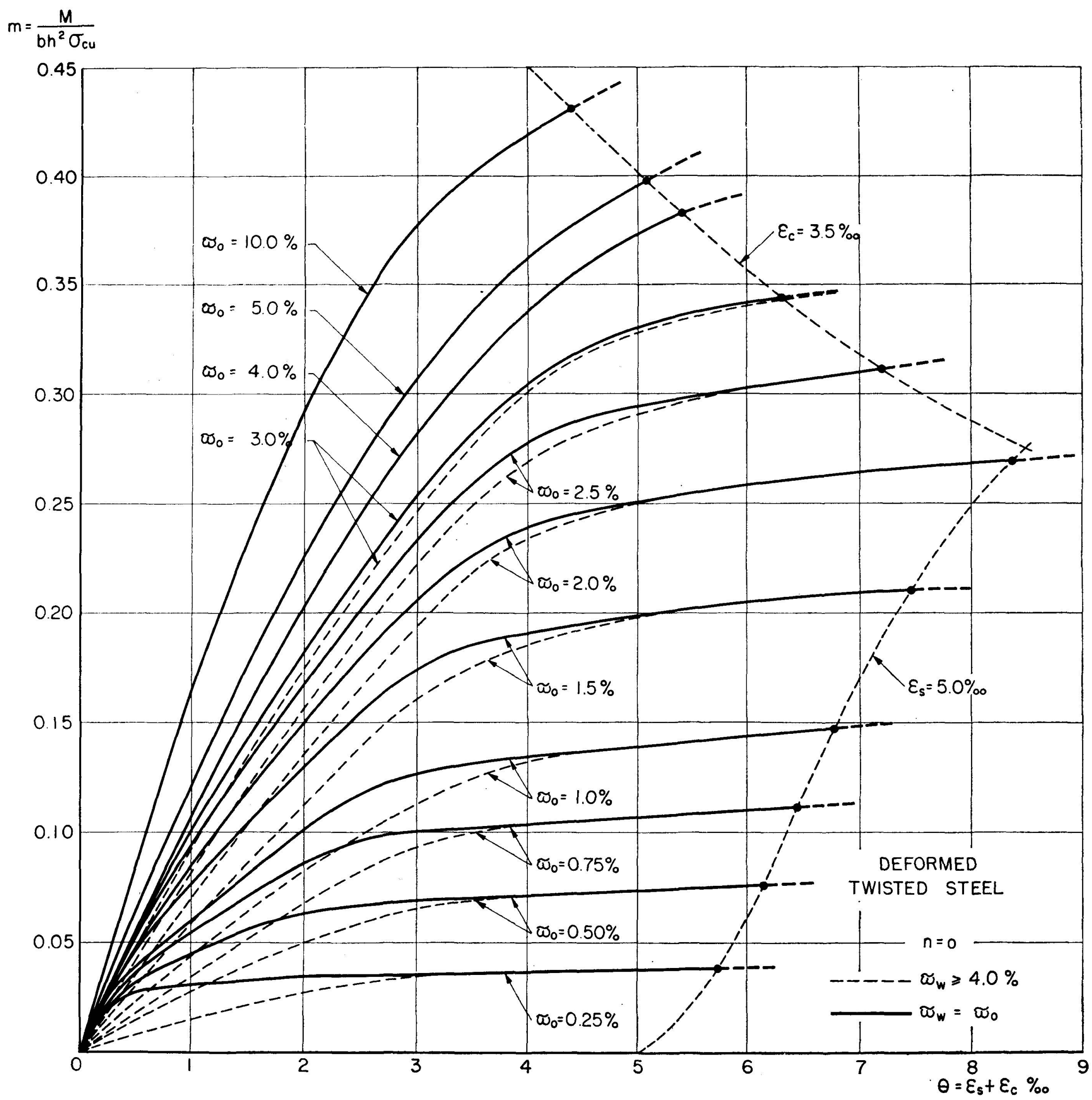


Fig. 6.

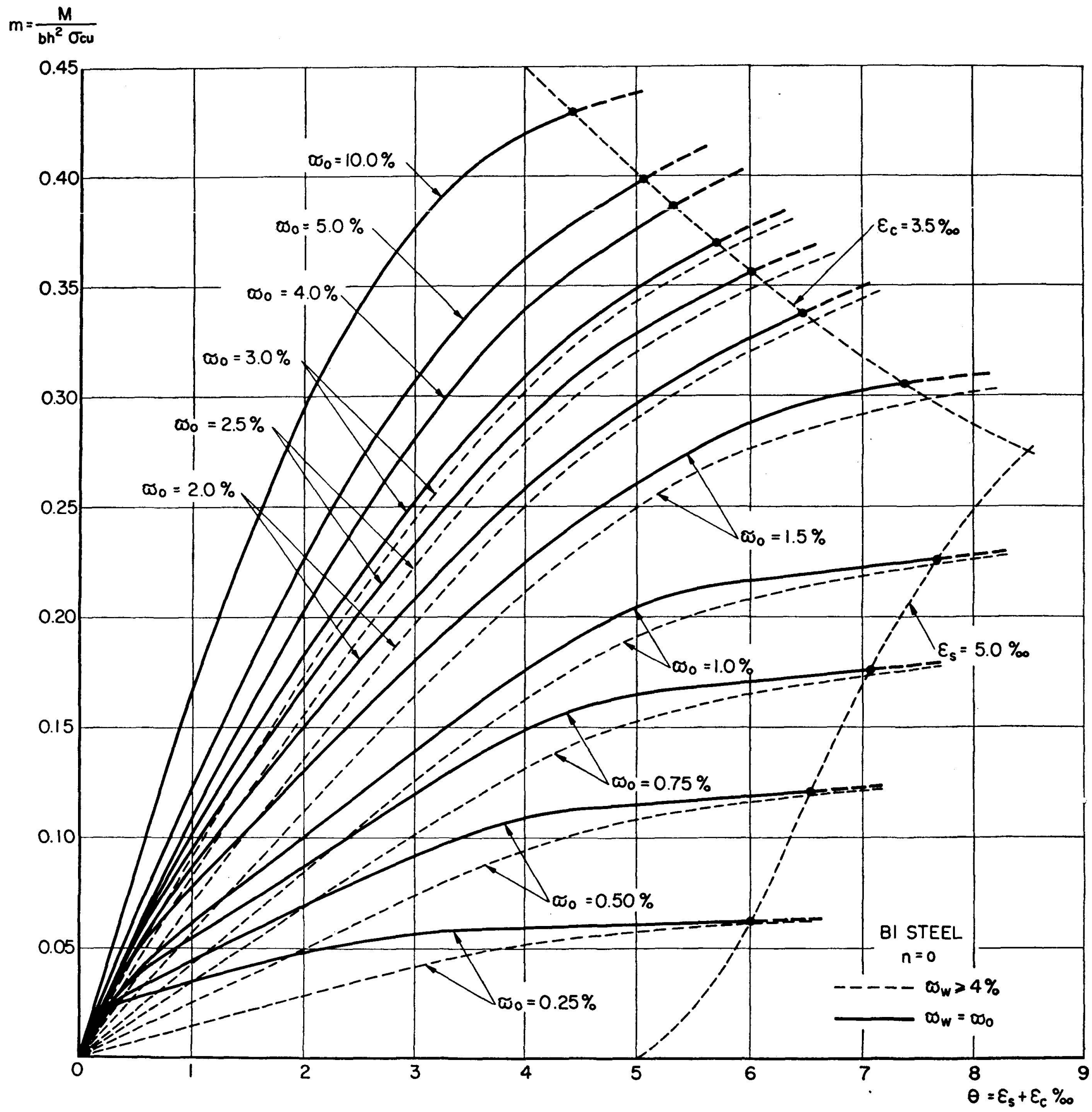


Fig. 7.

4. Determination of the Generalized Stiffness Matrix for a Straight Bar

As is well known, the elastic stiffness matrix for a straight bar of constant inertia, when only rotations are considered, is

$$\begin{pmatrix} k_1 & k_3 \\ k_3 & k_2 \end{pmatrix} = \frac{2\,E\,I}{L} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}.$$

The elements of this matrix are the values of the bending moments that it is necessary to apply at the ends of the bar to obtain the rotations $\theta_0 = 1$, $\theta_{10} = 0$ and $\theta_0 = 0$, $\theta_{10} = 1$.

For non-linear behaviour, the stiffness matrix can also be generalized but its elements are functions of the bending moments along the bar.

These moments being known, the corresponding end rotations are obtained by simple integrations along the bar. For this computation the mechanical properties of the bar are defined by the moment-curvature diagrams at eleven equally spaced cross sections (fig. 8).

The bending moment at section i is given by

$$M_i = M_0 + (\mu - M_0 + M_{10}) \frac{i}{10} - \mu \frac{i^2}{100},$$

where $\mu = p \frac{L^2}{2}$ being p the uniformly distributed load.

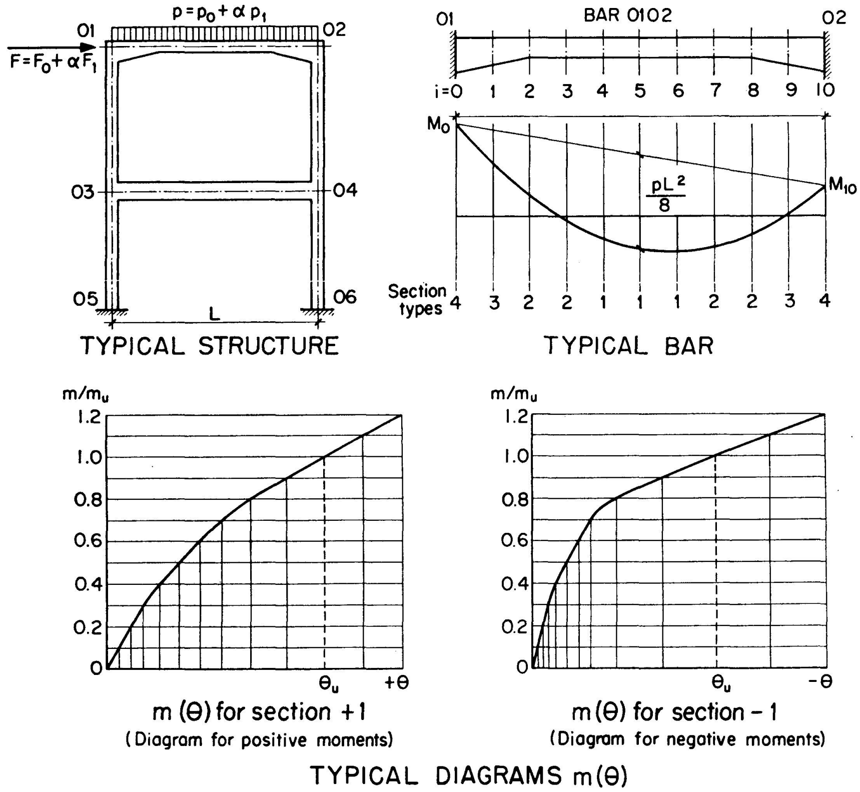


Fig. 8.

From the moment M_i it is easy to compute the corresponding reduced curvature θ, by means of the $m\ (\theta)$ diagrams. The values of k_1, k_2 and k_3 are given by

$$k_1 = \frac{I_3}{I_1 I_3 - I_2^2}, \qquad k_2 = \frac{I_1 L^2 + I_3 - 2 I_2 L}{I_1 I_3 - I_2^2}, \qquad k_3 = \frac{I_2 L - I_3}{I_1 I_3 - I_2^2},$$

were $$I_1 = \sum_{i=0}^{10} \frac{\theta_i a_i}{M_i h_i}, \qquad I_2 = \frac{L}{10} \sum_{i=0}^{10} \frac{i\, \theta_i a_i}{M_i h_i}, \qquad I_3 = \frac{L^2}{100} \sum_{i=0}^{10} \frac{i^2 \theta_i a_i}{M_i h_i}$$

for $$a_0 = a_{10} = \frac{L}{20} \quad \text{and} \quad a_1 \ldots a_9 = \frac{L}{10}.$$

In a given computing cycle the stiffness matrix is obtained by considering extrapolated values of the bending moments. For an increasing load cycle the bending moments determined for the next two lower loads are taken into account for the computation of the extrapolated values. For iterative cycles a linear expression of the initial and final moments of the preceding cycle is adopted, in which a suitable relaxation factor is introduced. It is important to emphasize that convergence will not be obtained if unsuitable values of the relaxation factor are chosen.

5. Structural Analysis

The established program concerns plane structures made up of straight bars, Fig. 8, assumed under the action of bending moments M and forces F, applied at the ends of the bars, and of uniform loads p distributed along the bars.

All these loadings are assumed to depend on a parameter α, in such a way that the behaviour of the structure is studied for successive increments of this parameter.

The load vectors considered are

$$\{F\} = \{F_0\} + \alpha\{F_1\},$$
$$\{p\} = \{p_0\} + \alpha\{p_1\}.$$

For structures with linear behaviour (5), the stiffness matrix for the complete structure can be defined in function of the stiffness matrices of the component bars (k) by the expression

$$(K) = (a)^T (k)(a),$$

where (a) is the displacement transformation matrix. Matrix (a) depends solely on the geometry of the structure and relates the displacements of the bars with those of the structure.

If the stiffness changes with the acting loads, the same expression can be used but the elements of (k) are then functions of these loads.

The displacements $\{r\}$ can be easily computed by the expression

$$\{r\} = (K)^{-1}\{R\},$$

where $\{R\}$ is the applied load system.

The moments and the shear forces corresponding to these displacements $\{r\}$ can also be computed by the simple expressions

$$\{M\} = (k)(a)\{r\},$$

$$V_0 = \frac{pL}{2} - \frac{M_0 - M_{10}}{L},$$

$$V_{10} = -\frac{pL}{2} - \frac{M_0 - M_{10}}{L}.$$

V_0 and V_{10} being the shear forces to the left and to the right of a bar and M_0 and M_{10} the bending moments at the same cross sections.

6. Program

The computation data consist of the elements indicated in Fig. 9 and 10. These include values of α, table A, giving cross-section types, matrix (a), load vectors, and tables B and C, giving moment-curvature diagrams for the different cross sections types.

The flow-chart is presented in Fig. 11.

There are three types of cycles. The first considers successive values of α. Each value of α corresponds to a given load. α can increase from cycle to cycle or remain constant, which means that an iteration is performed. The second type of cycle corresponds to the values of β, each value concerning a bar. Finally the third type of cycle corresponds to the values of i, which designates the order of the cross sections along each bar.

For a value of α, the moments at the ends of the bars are extrapolated by means of the preceding expressions and the elements of the stiffness matrix and load vector of each bar are computed by performing eleven i cycles. The stiffness matrix and the load vector of the complete structure are formed after each value of β has been considered and the rotations, the translations, and the corresponding moments and shear forces are computed by the given matric expressions. The displacements, the number of the bars and the corresponding moments and shear forces are printed before proceeding to another cycle.

For the Stantec-Zebra computer, for which this program was prepared, the limits are the following: 8 bars, 19 typical cross sections and 8 unknown displacements (rotations or translations). The computing time for a structure made up of 2 bars amounts to about 2 minutes per load cycle.

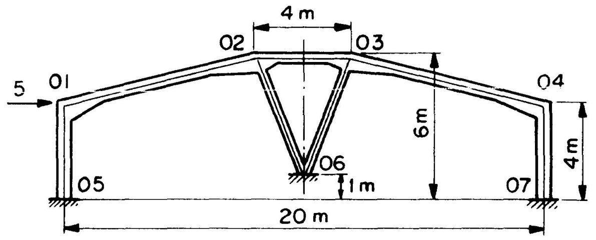
It was deemed preferable not to automatize the iterative cycles but to indicate the number of these cycles in the data, thus making possible to analyse the convergence of the results.

An existing program [6] for the computer analysis of rectangular frames could also have been adopted for the study of non-linear behaviour but it seemed preferable to introduce the displacement transfer matrix instead of automatically forming the complete stiffness matrix, so as not to occupy too many memories and to secure more freedom in the choice of structural types.

It should also be emphasized that it is easy to consider the influence of the

Number of bars	7
Number of rotations	4
Number of translations	1
Concrete cylinder strength σ_{cu} (kg cm^{-2})	300
Relaxation factor	0.2

Sketch:



Values of α	0.1	0.2	0.4	0.6	0.6	0.8	0.8	0.9	0.9	1.0	1.0	1.0	1.1	1.1
1.2	1.2	1.4	1.4	1.4	1.6	1.6	1.8	1.8	2.0	2.0	2.0	Number of values of α	26

Table A. Cross-section types

Bars	L (cm)	Abscissae (XL)											Distributed load (kg cm^{-1})	
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	p_0	p_1
0102	826	1	2	3	4	4	4	4	4	3	2	1	20	0
0203	400	1	2	3	5	5	5	5	5	3	2	1	20	0
0304	826	1	2	3	4	4	4	4	4	3	2	1	20	0
0501	400	6	6	6	6	7	7	7	6	6	11	11	0	0
0602	539	10	10	10	9	9	9	9	9	9	8	8	0	0
0603	539	10	10	10	9	9	9	9	9	9	8	8	0	0
0704	400	6	6	6	6	7	7	7	6	6	11	11	0	0

Table B. Matrix (a)

Bars	Disp.					Initial moments M' (kg cm)	Applied forces (kg)		
	1	2	3	4	5		Transl.	F_0	F_1
0102	1	0	0	0	+ 4.7	− 1 000 000	5	0	2000
	0	1	0	0	+ 4.7	− 1 000 000			
0203	0	1	0	0	− 20	− 1 000 000			
	0	0	1	0	− 20	− 1 000 000			
0304	0	0	1	0	+ 4.7	− 1 000 000			
	0	0	0	1	+ 4.7	− 1 000 000			
0501	0	0	0	0	− 25	500 000			
	1	0	0	0	− 25	− 1 000 000			
0602	0	0	0	0	− 20	− 50 000			
	0	1	0	0	− 20	50 000			
0603	0	0	0	0	− 20	50 000			
	0	0	1	0	− 20	− 50 000			
0704	0	0	0	0	− 25	− 500 000			
	0	0	0	1	− 25	1 000 000			
					↑ $\times 10^{-4}$				

Fig. 9. Data for the structural analysis program.

Table C. Positive Part of m (θ)

Section type	b (cm)	h (cm)	m_u (+)	m/m_u $\times 10^{-5}$												
				0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0	1,1	1,2
+ 1	40	80	0.038	0	6	11	16	22	28	36	54	114	240	574	1274	1974
+ 2	40	66	0.076	0	10	20	32	48	72	102	136	178	270	615	975	1335
+ 3	40	52	0.147	0	14	28	46	72	104	136	172	210	280	645	945	1245
+ 4	40	50	0.076	0	10	20	32	48	72	102	136	178	270	615	975	1335
+ 5	40	40	0.076	0	10	20	32	48	72	102	136	178	270	615	975	1335
+ 6	40	60	0.076	0	10	20	32	48	72	102	136	178	270	615	975	1335
+ 7	40	60	0.038	0	6	11	16	22	28	36	54	114	240	574	1274	1974
+ 8	40	150	0.038	0	6	11	16	22	28	36	54	114	240	574	1274	1974
+ 9	40	20	0.038	0	6	11	16	22	28	36	54	114	240	574	1274	1974
+10	40	20	0.147	0	14	28	46	72	104	136	172	210	280	645	945	1245
+11	40	80	0.038	0	6	11	16	22	28	36	54	114	240	574	1274	1974

Table D. Negative Part of m (θ)

Section type	b (cm)	h (cm)	m_u (−)	m/m_u $\times(-1)\times 10^{-5}$												
				0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0	1,1	1,2
− 1	40	80	0.038	0	6	11	16	22	28	36	54	114	240	574	1274	1974
− 2	40	66	0.076	0	10	20	32	48	72	102	136	178	270	615	975	1335
− 3	40	52	0.147	0	14	28	46	72	104	136	172	210	280	645	945	1245
− 4	40	50	0.076	0	10	20	32	48	72	102	136	178	270	615	975	1335
− 5	40	40	0.076	0	10	20	32	48	72	102	136	178	270	615	975	1335
− 6	40	60	0.076	0	10	20	32	48	72	102	136	178	270	615	975	1335
− 7	40	60	0.038	0	6	11	16	22	28	36	54	114	240	574	1274	1974
− 8	40	150	0.038	0	6	11	16	22	28	36	54	114	240	574	1274	1974
− 9	40	20	0.038	0	6	11	16	22	28	36	54	114	240	574	1274	1974
−10	40	20	0.147	0	14	28	46	72	104	136	172	210	280	645	945	1245
−11	40	80	0.038	0	6	11	16	22	28	36	54	114	240	574	1274	1974

Fig. 10. Data for the structural analysis program.

changes in geometry during deformation, as these only affect the displacement transformation matrix the elements of which can be expressed in function of the displacements.

The present paper as well as other ones [7 and 8] show that matric methods are not only useful for studying the linear but also the non-linear behaviour of structures.

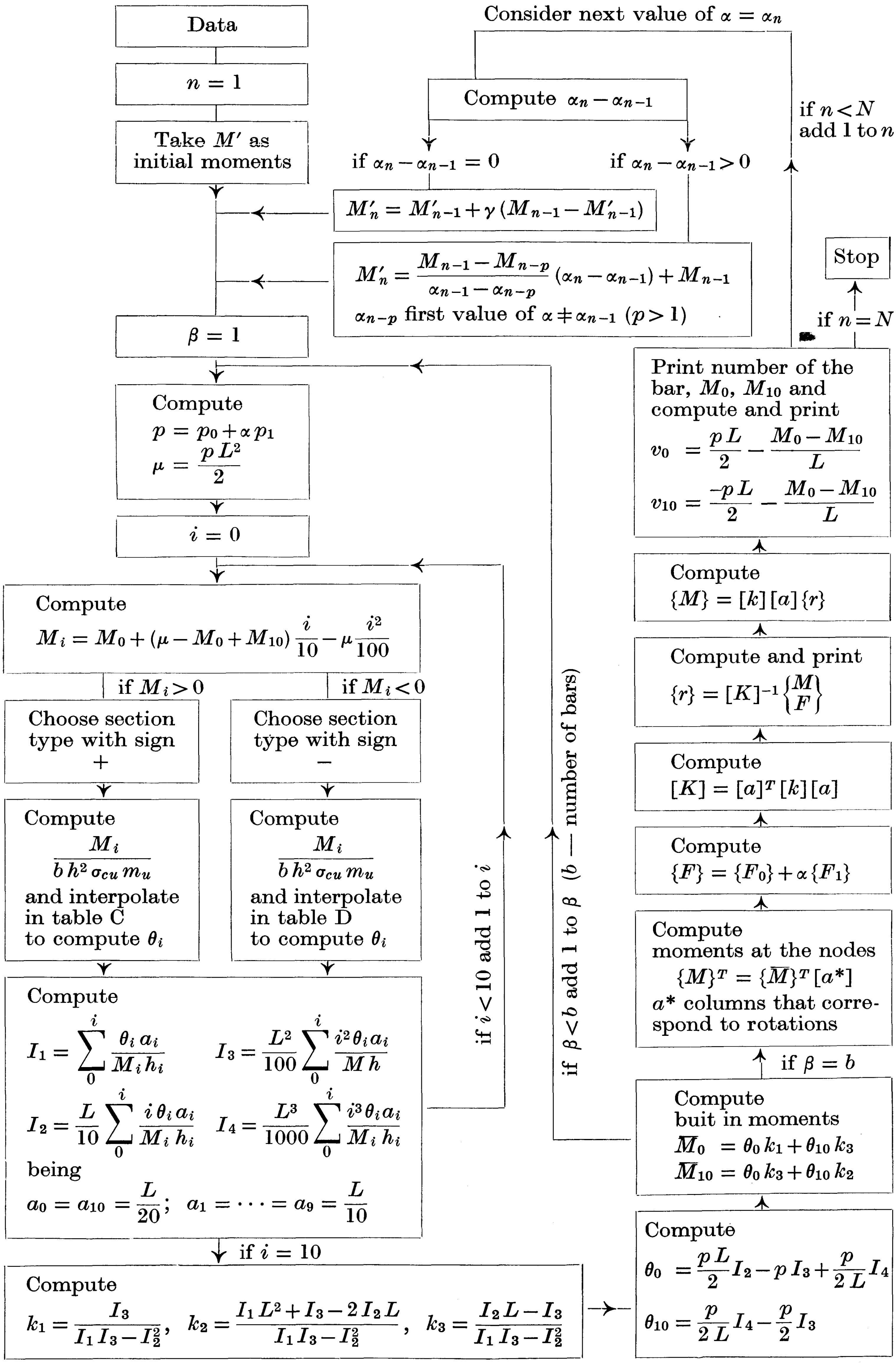


Fig. 11. Flow-chart for non-linear structural analysis.

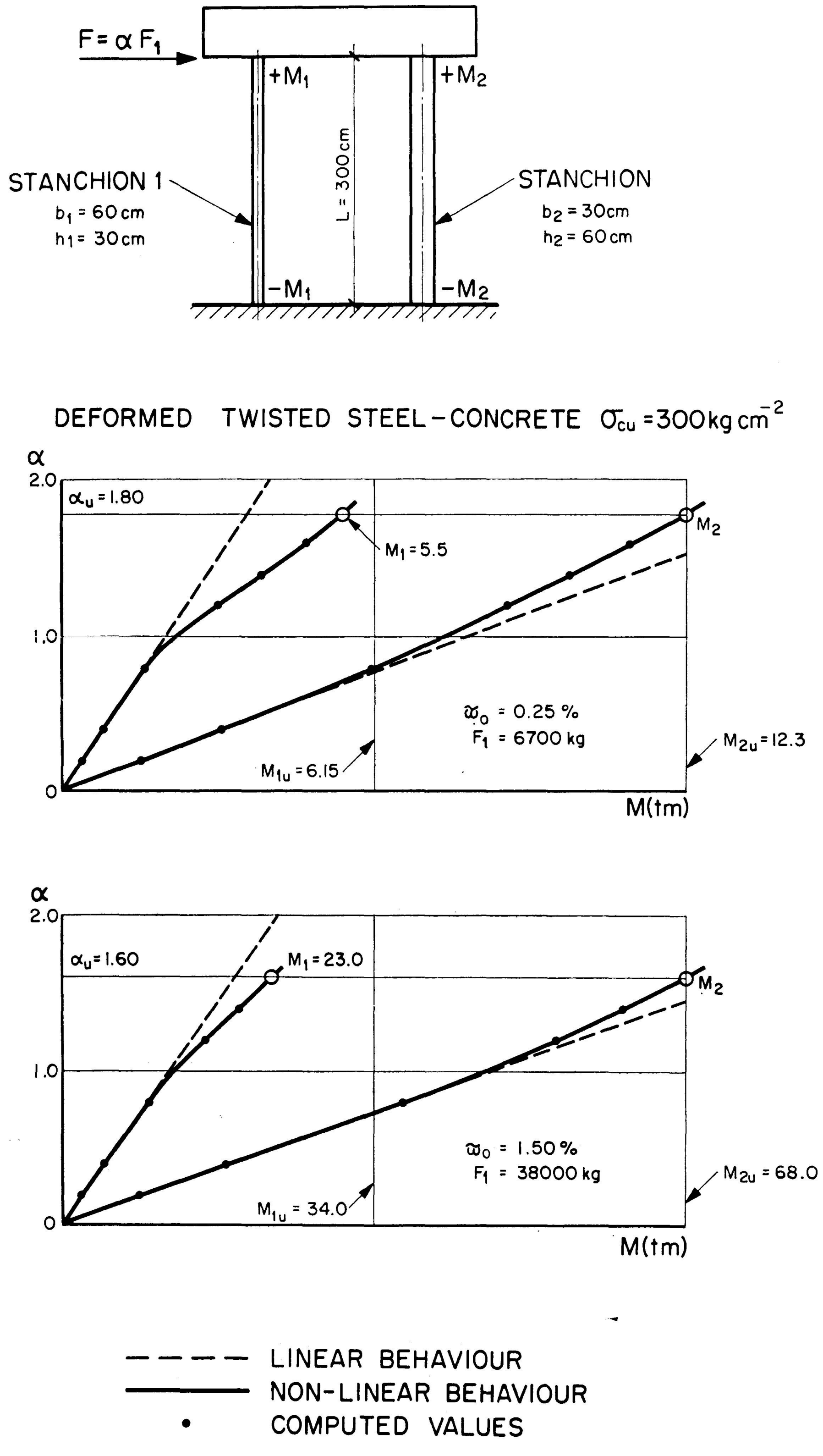

F = α F1
+M1
+M2
L = 300 cm
STANCHION 1
b1 = 60 cm
h1 = 30 cm
STANCHION
b2 = 30cm
h2 = 60cm
-M1
-M2
DEFORMED TWISTED STEEL-CONCRETE σcu = 300 kg cm-2
α
2.0
αu = 1.80
M1 = 5.5
M2
1.0
ϖ0 = 0.25 %
F1 = 6700 kg
M2u = 12.3
M1u = 6.15
0
M(tm)
α
2.0
αu = 1.60
M1 = 23.0
M2
1.0
ϖ0 = 1.50 %
F1 = 38000 kg
M2u = 68.0
M1u = 34.0
0
M(tm)
LINEAR BEHAVIOUR
NON-LINEAR BEHAVIOUR
COMPUTED VALUES

Fig. 12.

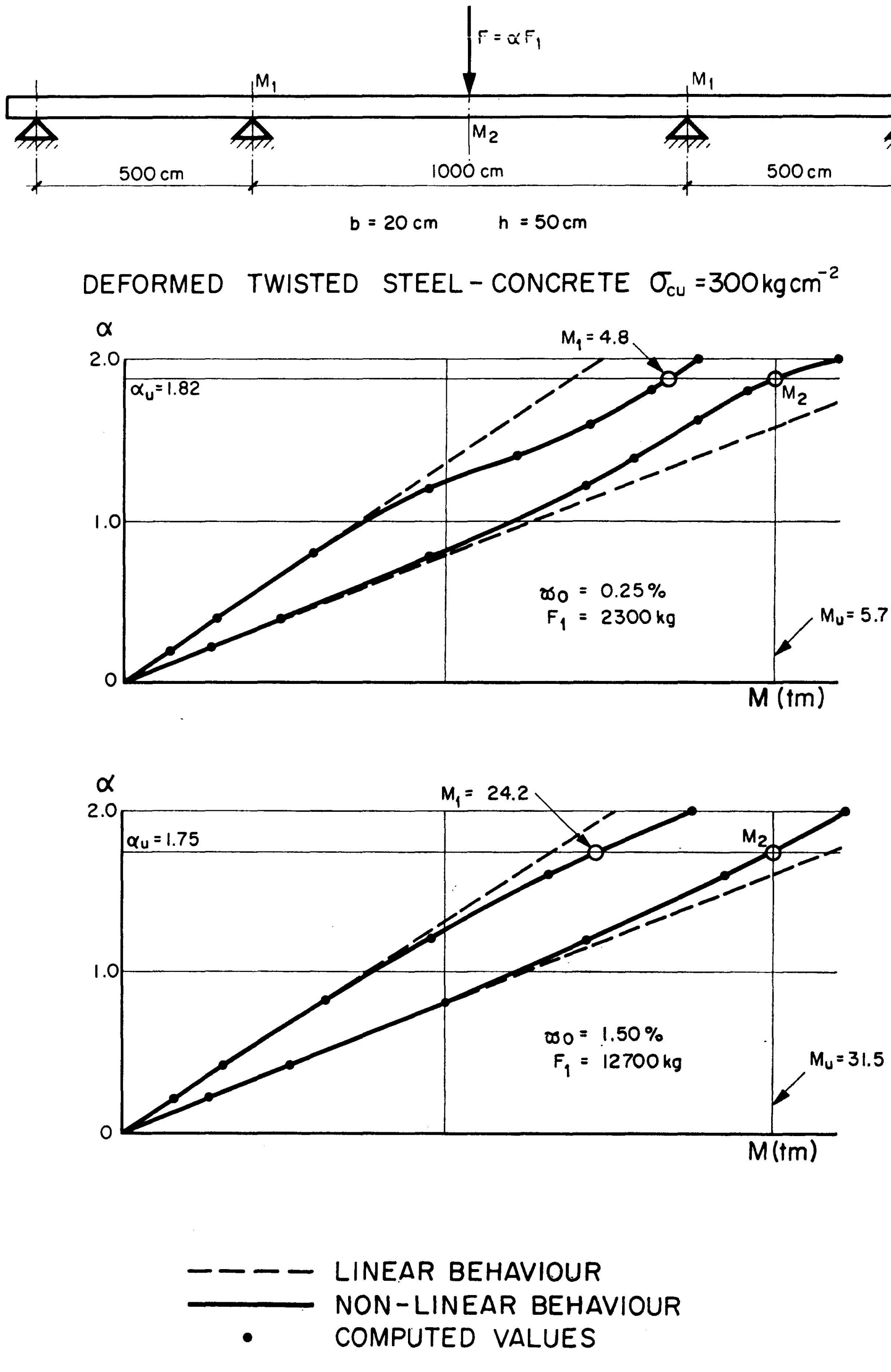

F = αF1
M1
M1
M2
500 cm
1000 cm
500 cm
b = 20 cm h = 50 cm
DEFORMED TWISTED STEEL - CONCRETE σcu = 300 kg cm-2
α
2.0
1.0
0
αu = 1.82
M1 = 4.8
M2
ϖo = 0.25 %
F1 = 2300 kg
Mu = 5.7
M (tm)
α
2.0
1.0
0
αu = 1.75
M1 = 24.2
M2
ϖo = 1.50 %
F1 = 12700 kg
Mu = 31.5
M (tm)
LINEAR BEHAVIOUR
NON-LINEAR BEHAVIOUR
COMPUTED VALUES

Fig. 13.

7. Instances of Application

The use of the program is exemplified for two simple structural types.

The first example concerns two parallel stanchions of different stiffness acted upon by a horizontal force, Fig. 12. The floor system is assumed to be much stiffer than the stanchions.

Stanchion 2 is four times stiffer than stanchion 1 and, as both are considered to have the same percentage of reinforcement (deformed twisted steel), the ultimate strength of number 2 is double that of number 1.

When the ultimate moment of stanchion 2 is reached, the bending moment in stanchion 1 is 90 and 68% of its ultimate value, respectively for the percentages of reinforcement of 0.25 and 1.50%.

The second example concerns a three-span continuous symmetric beam, Fig. 13, under a concentrated force applied at the middle of the central span. The ratio between the central and the lateral spans is 2.

Concrete rectangular beams reinforced with deformed twisted steel with percentages of reinforcement of 0.25 and 1.50% are considered.

For linear behaviour, the moment at the support, M_1, is 60% of the middle span moment, M_2.

Fig. 13 shows how the moments change as the load increases. When the ultimate moment at the middle of the central span is attained, the bending moments at the supports are 84 and 77% of the ultimate moment, respectively for percentages of reinforcement of 0.25 and 1.50%.

Some comparisons of analytical and experimental results already performed showed a good agreement between both kinds of values.

In the near future it is intended systematically to compare experimental results available with results yielded by the present method and to use it to improve the understanding of the non-linear behaviour of structures.

8. Conclusion

The possibilities deriving from the use of the established program are deemed important for investigating the behaviour of reinforced and prestressed plane structures made up of linear elements, The program may also be useful in the study of structures made of other materials.

The basic assumptions of the method are very general and, since the moment-curvature diagrams are introduced in the data, they can very easily be changed. The geometry of the structure being given by the displacement transformation matrix, any type of structure can be considered without difficulty.

The accuracy of the results is very good and can be checked by changing the load increments and or the number of iterative cycles.

Acknowledgements

The authors wish to thank Dr. Palma Fernandes and Miss Lina de Oliveira for their help in programming the sub-routines for the computation of the moment-curvature diagrams and of the stiffness matrices, respectively.

Symbols *)

ϵ_c concrete strain.

ϵ_{cu} ultimate concrete strain.

ϵ_s tensile steel strain.

ϵ_{su} ultimate tensile steel strain.

ϵ_s' compression steel strain.

σ_c concrete stress.

σ_{cu} ultimate concrete stress (cylinder test strength).

σ_s tensile steel stress.

σ_{su} ultimate tensile steel stress.

σ_s' compression steel stress.

ρ radius of curvature.

$\theta = h/\rho = \epsilon_c + \epsilon_s$ reduced curvature.

$m = \dfrac{M}{b\,h^2\,\sigma_{cu}}$ reduced bending moment.

$n = \dfrac{N}{b\,h\,\sigma_{cu}}$ reduced normal force.

(k) stiffness of the bars.

(K) stiffness of the structure.

(a) displacement transformation matrix.

$\{F\} = \{F_0\} + \alpha\,\{F_1\}$ forces applied at the nodes.

$\{p\} = \{p_0\} + \alpha\,\{p_1\}$ uniform distributed loads applied on the bars.

$\{r\}$ displacements (translations and rotations).

$\{M\}$ moments at the ends of the bars.

$\{V\}$ shear forces at the ends of the bars.

α load parameter.

β number of a bar.

γ relaxation factor.

n cycle number.

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*) The symbols that refer to the geometry of a reinforced concrete section are indicated in fig. 3.

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Summary

A method for the analysis of the non-linear behaviour of reinforced concrete structures with linear elements is presented.

The elasto-plastic properties of the linear elements are determined from stress-strain diagrams selected for concrete and steel. The latter are corrected to take into account the contribution of the concrete enveloping the tensile reinforcement.

For defining the behaviour of structures formed by linear elements, a matric analysis by the stiffness method is used. Successive load increments are considered and the resulting displacements and internal forces are computed considering stiffness matrices, the elements of which vary in function of the existing forces.

Examples of the behaviour of some simple structures are presented.

Résumé

Les auteurs présentent une méthode permettant d'étudier le comportement non linéaire d'ossatures en béton armé composées d'éléments rectilignes.

Les propriétés élasto-plastiques des éléments sont déduites des diagrammes contraintes-dilatations admis pour le béton et l'acier. Ces derniers sont corrigés pour tenir compte de l'effet du béton enrobant les armatures tendues.

Les auteurs définissent le comportement des ossatures composées d'éléments rectilignes par des matrices de rigidité. En considérant les accroissements successifs de la charge, on calcule les déplacements correspondants et les efforts internes à partir des matrices de rigidité dont les éléments varient en fonction des forces existantes.

Des exemples du comportement de quelques ossatures simples sont présentés.

Zusammenfassung

Die vorliegende Arbeit befaßt sich mit einer Methode zur Untersuchung des nichtlinearen Verhaltens von Stahlbetontragwerken, welche aus linearen Elementen zusammengesetzt sind.

Die elastoplastischen Eigenschaften der linearen Elemente werden durch die für Beton und Stahl ermittelten Spannungs-Dehnungsdiagramme bestimmt. Die letzteren sind so korrigiert, daß der Einfluß des die Zugarmierung umhüllenden Betons darin enthalten ist.

Das Verhalten von Tragwerken aus linearen Elementen wird mit Hilfe einer Matrizenrechnung nach der Steifigkeitsmethode beschrieben. Bei schrittweiser Laststeigerung werden jeweils die Verformungen und die inneren Kräfte ermittelt. Diese Berechnung erfolgt unter Berücksichtigung der Steifigkeitsmatrizen, deren Elemente in Funktion der vorhandenen Kräfte variieren.

Am Schluß werden Beispiele des Verhaltens einiger einfacher Tragwerke dargestellt.