Numerical analysis of conoidal shells

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Numerical Analysis of Conoidal Shells

Calcul des voiles conoïdes

Numerische Berechnung von Konoidschalen

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1. Introduction

A "Conoid" is a surface generated by the locus of a variable straight line that moves parallel to a plane, known as the "Director Plane", with one end on a Curve and the other on a Straight Line, termed the "Directrices", Fig. 1. A thin shell assuming the form of this surface, and having the directrices at right angles to the director plane, represents a doubly curved shell of anticlastic type, and has been dealt with in detail by M. SOARE [1]1). The treatment has been further extended by other authors to include the use of a "stress function" for the solution of the basic equations. It should be noted that all

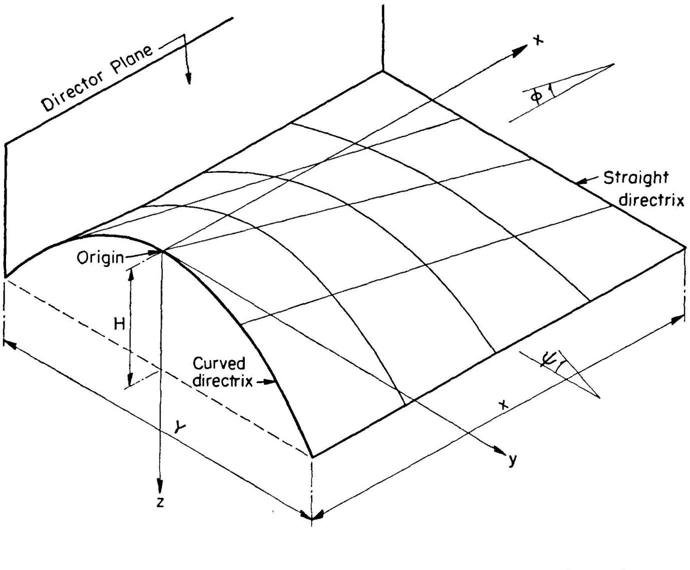

Fig. 1.

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1) The figures in parentheses refer to the numbers in the Bibliography, section 11.

the authors take into consideration two boundary conditions to satisfy the general equation.

The solution of the differential equations of equilibrium of a shell element having a double curvature by direct methods is difficult. It is therefore usual to have recourse to iterative or other methods. The paper presents an alternative method of approach for the direct solution of Conoidal Shells. In addition, the present concept of selecting the number of boundary conditions for such shells is discussed.

2. Equations of Equilibrium of a Shell Element

As has been shown earlier [2], for loading along the z-axis per unit area of projected plane $x-y$, Fig. 2, the following equations satisfy the equilibrium conditions of a shell element when n_x, n_y and t are the components on the $x-y$ plane of the stress resultants, i.e., forces per unit length of shell section.

$$\frac{\partial n_x}{\partial x}+\frac{\partial t}{\partial y}=0, \tag{2.1}$$

$$\frac{\partial n_y}{\partial y}+\frac{\partial t}{\partial x}=0, \tag{2.2}$$

$$n_x\frac{\partial^2 z}{\partial x^2}+n_y\frac{\partial^2 z}{\partial y^2}+2t\frac{\partial^2 z}{\partial x\,\partial y}+q_z=0. \tag{2.3}$$

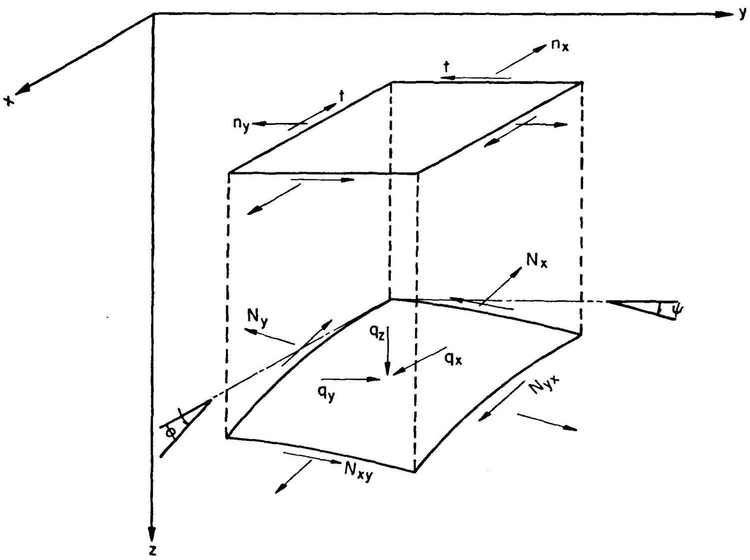

Fig. 2.

For a load $\bar{q}_z$ varying as the weight of the shell, the equivalent load q_z per unit area of projected plane in Eq. (2.3) would be

$$q_z=\bar{q}_z m, \tag{2.4}$$

where
$$m=\frac{\operatorname{Sin}\omega}{\operatorname{Cos}\phi\operatorname{Cos}\psi}=\frac{(1-\operatorname{Sin}^2\phi\operatorname{Sin}^2\psi)^{1/2}}{\operatorname{Cos}\phi\operatorname{Cos}\psi} \tag{2.5}$$

and ω = angle of obliquity between two adjacent sides of the element

$$= \mathrm{Cos}^{-1}(\mathrm{Sin}\,\phi\,\mathrm{Sin}\,\psi). \tag{2.6}$$

The stress resultants and the components are related by the following equations

$$N_x = n_x \gamma; \qquad N_y = n_y/\gamma \quad \text{and} \quad N_{xy} = N_{yx} = t, \tag{2.7}$$

where

$$\gamma = \frac{\mathrm{Cos}\,\psi}{\mathrm{Cos}\,\phi}. \tag{2.8}$$

3. Solution of the Equations

To solve the above partial differential Eqs. (2.1) to (2.3) simultaneously for n_x, n_y and t, it is customary in most cases, to reduce the three equations to a single equation by introducing an auxiliary variable. Using this variable, termed the Stress Function, F, the force terms in Eqs. (2.1) and (2.2) are eliminated and Eq. (2.3) reduces to

$$\frac{\partial^2 F}{\partial y^2}\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 F}{\partial x^2}\frac{\partial^2 z}{\partial y^2} - 2\frac{\partial^2 F}{\partial x\,\partial y}\frac{\partial^2 z}{\partial x\,\partial y} + q_z = 0. \tag{3.1}$$

The conditions at the boundaries of the shell structure must, however, be taken into account in its solution for F. In the present treatment the introduction of any auxiliary function is eliminated and most of the computational work becomes numerical.

4. Geometry of a Parabolic Conoidal Shell

Assuming the curvature of the curved directrix to be a parabola, Fig. 1, with the origin at its crown, the equation for the shell surface is

$$Z = f(x,y) = \frac{C}{2}y^2 + \frac{H}{X}x - \frac{C}{2X}x\,y^2, \tag{4.1}$$

where

$$C = \frac{8H}{Y^2}. \tag{4.2}$$

Thus,

$$\begin{aligned} \frac{\partial z}{\partial x} &= \frac{H}{X}\left(1 - \frac{4}{Y^2}y^2\right) = \tan\phi, \\ \frac{\partial^2 z}{\partial x^2} &= 0, \\ \frac{\partial^2 z}{\partial x\,\partial y} &= -\frac{C}{X}y, \\ \frac{\partial z}{\partial y} &= C\,y\left(1 - \frac{x}{X}\right) = \tan\psi, \\ \frac{\partial^2 z}{\partial y^2} &= C\left(1 - \frac{x}{X}\right). \end{aligned} \tag{4.3}$$

5. Boundary Conditions Influencing the Shell Stresses

In order to ensure the number of boundary or edge conditions required in a particular problem, observation of Eq. (3.1) is necessary. The number would correspond to the number of differentiations of F that have been introduced in the above equation, since the same number of arbitrary integrational constants (or functions) should arise in the expression of F.

In the case of a "conoidal" shell, the General Equation (3.1) takes the form

$$\frac{\partial^2 F}{\partial x^2} f_1(x) + \frac{\partial^2 F}{\partial x\, \partial y} f_2(y) = -q_z. \tag{5.1}$$

Hence the number of integrational constants or functions would be three, as the expression for F has been differentiated three times before introduction into the equation. This number would thus correspond to the number of boundary conditions.

6. Boundary Conditions to be Taken Into Consideration

If the shell is supported on traverses along the two directrices, then because of symmetry about the x-axis, Fig. 1, the three following conditions may be considered

(1) At $y = 0$, $t = 0$.
(2) At $y = Y/2$, $n_y = 0$.
and (3) At $x = 0$, $n_x = 0$.

7. Equations of Equilibrium for Vertical Loading Due to the Actual Weight of the Shell

Eqs. (2.1) and (2.2) remain unchanged whereas Eq. (2.3) becomes

$$n_y (X - x) - 2 t y + \frac{\bar{q}_z X}{C} m = 0. \tag{7.1}$$

8. Direct Solution for the Stresses

Eqs. (2.1), (2.2) and (7.1) have to be solved simultaneously for the unknowns n_x, n_y and t.

Now, differentiating Eq. (7.1) partially with respect to y,

$$(X - x)\frac{\partial n_y}{\partial y} - 2\left(t + y\frac{\partial t}{\partial y}\right) = -\frac{\bar{q}_z X}{C}\frac{\partial m}{\partial y}.$$

From Eq. (2.2),

$$\left[(X-x)\frac{\partial z}{\partial x}+2y\frac{\partial z}{\partial y}\right]\frac{\partial t}{\partial z}+2t=\frac{\bar{q}_z X}{C}\frac{\partial m}{\partial y}. \qquad \text{(A)}$$

Now, from Eq. (4.3),

$$(X-x)\frac{\partial z}{\partial x}=(X-x)\frac{H}{X}\left(1-\frac{4y^2}{Y^2}\right)$$

and

$$2y\frac{\partial z}{\partial y}=2Cy^2\left(1-\frac{x}{X}\right)=\frac{16Hy^2}{XY^2}(X-x).$$

Introducing these values in Eq. (A) above,

$$\frac{\partial t}{\partial z}+\left[\frac{2XY^2}{(X-x)H(Y^2+12y^2)}\right]t=\left[\frac{\bar{q}_z X^2 Y^2}{CH(X-x)(Y^2+12y^2)}\frac{\partial m}{\partial y}\right]$$

or

$$\frac{\partial t}{\partial z}+Pt=R. \qquad \text{(B)}$$

Using an integrating factor

$$u=e^{\int P\,dz},$$

the solution of the equation gives

$$t=\frac{\bar{q}_z X}{2C}\frac{\partial m}{\partial y}+K\,e^{-Pz}. \qquad \text{(C)}$$

Now, from boundary condition 1 (Section 6), $y=0$, $t=0$, $\frac{\partial m}{\partial y}=0$.

$\therefore$ K must be zero.

Hence, the solution of t is given by

$$t=\frac{\bar{q}_z X}{2C}\frac{\partial m}{\partial y} \qquad \text{(8.1)}$$

$$=D\frac{\partial m}{\partial y}. \qquad \text{(8.1a)}$$

Thus,

$$\frac{\partial t}{\partial y}=D\frac{\partial^2 m}{\partial y^2}.$$

From Eq. (2.1),

$$n_x=-D\int\frac{\partial^2 m}{\partial y^2}dx+f(y).$$

From boundary condition-3, $x=0$, $n_x=0$.

$$\therefore\ f(y)=D\left[\frac{\partial^2 m}{\partial y^2}\delta_x\right]_{x=0}=D\,\delta x\,K_x,$$

where

$$K_x=\left[\frac{\partial^2 m}{\partial y^2}\right]_{x=0}. \qquad \text{(8.2)}$$

Hence,

$$n_x=-D\sum_0^x\frac{\partial^2 m}{\partial y^2}\delta x+D\,\delta x\,K_x=-D\,\delta x\left[\sum_0^x\frac{\partial^2 m}{\partial y^2}-K_x\right]. \qquad \text{(8.3)}$$

Similarly, from Eqs. (8.1 a) and (2.2), and boundary condition -2,

$$n_y = -D\,\delta y\left[\sum_0^y \frac{\partial^2 m}{\partial x\,\partial y} - K_y\right], \tag{8.4}$$

where

$$K_y = \sum_0^{Y/2} \frac{\partial^2 m}{\partial x\,\partial y}. \tag{8.5}$$

9. Example

A shell with the following dimensions, Fig. 3, is considered with loading due to its own weight of 25 lb./sqr. ft. along the shell surface including finishes.

$H = 6' - 7\,{}^1/_2''$

$X = 31' - 8\,{}^1/_2''$

$Y = 20' - 0''$

$L = 25' - 0''$

h (shell thickness) $= 1\,{}^1/_4''$

$\bar{q}_z = 25$ lb./sft.

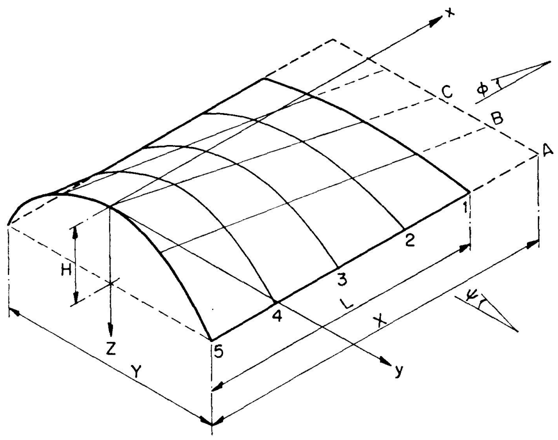

Fig. 3.

Thus, from Eqs. (4.2), (4.3) and (8.1 a),

$$C = 0.1325\ \text{ft.}^{-1}; \qquad D = 3000\ \text{lb.};$$

$$\tan\phi = 0.209\left[1 - \frac{y^2}{100}\right];$$

$$\tan\psi = 0.1325\,y\left[1 - \frac{x}{31.7}\right].$$

Whence the values of $\sin\phi$, $\cos\phi$, $\sin\psi$ and $\cos\psi$ at various nodal points in

Table 1a

ϕ	y	y^2	$\tan\phi = 0.209\,(1\text{-}.01\,y^2)$	Value of ϕ	Sin ϕ	Cos ϕ
ϕ_A	10	100	0	0°	0.000	1.000
ϕ_B	5	25	$0.209\times0.75 = 0.1570$	8° 55′	0.158	0.98792
ϕ_C	0	0	0.209	11° 48′	0.20450	0.97887

Table 1b

ψ	y	x	$\frac{x}{31.7}$	$\tan\psi =$ $0.1325y\left(1-\frac{x}{31\cdot7}\right)$	Value of ψ	Sin ψ	Cos ψ
ψ_{A1}	10	25	0.788	$0.1325\times10\times0.212$ $= 0.2809$	15° 42′	0.27060	0.96269
ψ_{A2}	10	18.75	0.591	$0.1325\times10\times0.409$ $= 0.541925$	28° 27′	0.47639	0.87923
ψ_{A3}	10	12.50	0.394	$0.1325\times10\times0.606$ $= 0.803556$	38° 47′	0.62638	0.77952
ψ_{A4}	10	6.25	0.197	$0.1325\times10\times0.803$ $= 1.063975$	46° 46′	0.72857	0.68498
ψ_{A5}	10	0	0	$= 1.325$	52° 58′	0.79829	0.60229
ψ_{B1}	5	25	0.788	$0.1325\times5\times0.212$ $= 0.14045$	8° 0′	0.13917	0.99027
ψ_{B2}	5	18.75	0.591	$= 0.2709625$	15° 10′	0.26162	0.96517
ψ_{B3}	5	12.50	0.394	$= 0.401778$	21° 54′	0.37299	0.92784
ψ_{B4}	5	6.25	0.197	$= 0.5319875$	28° 1′	0.46973	0.88281
ψ_{B5}	5	0	0	$= 0.6625$	33° 30′	0.55194	0.86163
ψ_{C1}	0	25		0	0	0.000	1.000
ψ_{C2}	0	18.75		0	0	0.000	1.000
ψ_{C3}	0	12.50		0	0	0.000	1.000
ψ_{C4}	0	6.25		0	0	0.000	1.000
ψ_{C5}	0	0		0	0	0.000	1.000

the grid are obtained from Tables 1 (a) and (b). Table 2 then gives values of m, γ and $1/\gamma$.

Values of m at various points are now inserted in the Operational Grid 1, whence $\frac{\partial m}{\partial y}$, $\frac{\partial^2 m}{\partial y^2}$ and $\frac{\partial^2 m}{\partial x \partial y}$ values are also found.

These differentiations are carried out numerically, with the equations in "finite difference" form. Thus, referring to the grid portion in Fig. 4, for panel point 0,

Table 2

Points	$\sin\phi \sin\psi = \cos\omega$	ω	$\sin\omega$
A_1	$\sin\phi_A \ \sin\psi_{A1} = 0.000 \times 0.27060 = 0.000$	90°	1.000
A_2	$\sin\psi_{A2} = 0.000 \times 0.47639 = 0.000$	90°	1.000
A_3	$\sin\psi_{A3} = 0.000 \times 0.62638 = 0.000$	90°	1.000
A_4	$\sin\psi_{A4} = 0.000 \times 0.72857 = 0.000$	90°	1.000
A_5	$\sin\psi_{A5} = 0.000 \times 0.79829 = 0.000$	90°	1.000
B_1	$\sin\phi_B \ \sin\psi_{B1} = 0.158 \times 0.13917 = 0.02199$	88° 44′	0.99976
B_2	$\sin\psi_{B2} = 0.158 \times 0.26162 = 0.041333$	87° 38′	0.99915
B_3	$\sin\psi_{B3} = 0.158 \times 0.37299 = 0.058935$	86° 37′	0.99826
B_4	$\sin\psi_{B4} = 0.158 \times 0.46973 = 0.07422$	85° 44′	0.99721
B_5	$\sin\psi_{B5} = 0.158 \times 0.55194 = 0.087206$	85° 0′	0.99619
C_1	$\sin\phi_C \ \sin\psi_{C1} = 0.2045 \times 0.000 = 0.000$	90°	1.000
C_2	$\sin\psi_{C2} = 0.2045 \times 0.000 = 0.000$	90°	1.000
C_3	$\sin\psi_{C3} = 0.2045 \times 0.000 = 0.000$	90°	1.000
C_4	$\sin\psi_{C4} = 0.2045 \times 0.000 = 0.000$	90°	1.000
C_3	$\sin\psi_{C5} = 0.2045 \times 0.000 = 0.000$	90°	1.000

Points	$\cos\phi \cos\psi$	$m = \dfrac{\sin\omega}{\cos\psi \cos\phi}$	$\gamma = \dfrac{\cos\psi}{\cos\phi}$	$\dfrac{1}{\gamma}$
A_1	$\cos\phi_A \ \cos\psi_{A1} = 1.000 \times 0.96269 = 0.96269$	1.03917	0.96269	1.03875
A_2	$\cos\psi_{A2} = 1.000 \times 0.87923 = 0.87923$	1.13735	0.87923	1.13735
A_3	$\cos\psi_{A3} = 1.000 \times 0.77952 = 0.77952$	1.28284	0.77952	1.28284
A_4	$\cos\psi_{A4} = 1.000 \times 0.68498 = 0.68498$	1.45989	0.68498	1.45999
A_5	$\cos\psi_{A5} = 1.000 \times 0.60229 = 0.60229$	1.66032	0.60229	1.66043
B_1	$\cos\phi_B$ $\cos\psi_{B1} = 0.98792 \times 0.99027 = 0.97838$	1.02185	1.11473	0.99769
B_2	$\cos\psi_{B2} = 0.98792 \times 0.96517 = 0.95356$	1.04781	0.96599	1.02460
B_3	$\cos\psi_{B3} = 0.98792 \times 0.92784 = 0.91666$	1.08901	0.93918	1.06475
B_4	$\cos\psi_{B4} = 0.98792 \times 0.88281 = 0.87116$	1.14469	0.89461	1.11906
B_5	$\cos\psi_{B5} = 0.98792 \times 0.86163 = 0.85124$	1.17028	0.87216	1.14657
C_1	$\cos\phi_C \ \cos\psi_{C1} = 0.97887 \times 1.000 = 0.97887$	1.02158	1.02162	0.97887
C_2	$\cos\psi_{C2} = 0.97887 \times 1.000 = 0.97887$	1.02158	1.02162	0.97887
C_3	$\cos\psi_{C3} = 0.97887 \times 1.000 = 0.97887$	1.02158	1.02162	0.97887
C_4	$\cos\psi_{C4} = 0.97887 \times 1.000 = 0.97887$	1.02158	1.02162	0.97887
C_5	$\cos\psi_{C5} = 0.97887 \times 1.000 = 0.97887$	1.02158	1.02162	0.87887

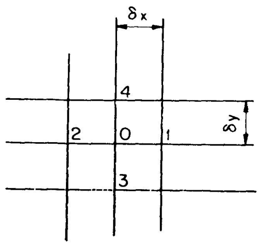

Fig. 4.

$$\begin{aligned} \frac{\partial m}{\partial y} &= \frac{m_3 - m_4}{2\,\delta y} = r, \\ \frac{\partial^2 m}{\partial y^2} &= \frac{\partial r}{\partial y} = \frac{r_3 - r_4}{2\,\delta y}, \\ \frac{\partial^2 m}{\partial x\,\partial y} &= \frac{\partial r}{\partial x} = \frac{r_1 - r_2}{2\,\delta x} \end{aligned} \tag{D}$$

GRID 1

	5		4		3		2		1	
Origin↘	[0]		[0]		[0]		[0]		[0]	
	0.000	1.02158	0.000	1.02158	0.000	1.02158	0.000	1.02158	0.000	1.02158 x → C
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000 ($\delta x = 6.25$ ft.)
	[+191.622]		[+131.493]		[+78.378]		[+34.731]		[+5.277]	
	-0.00320848	1.17028	-0.00301984	1.14469	-0.0025811	1.08901	-0.0019494	1.04781	-0.0015708	1.02185
B	0.009800	0.063874	0.006304	0.043831	0.0038766	0.026126	0.001791	0.011577	0.0003445	0.001759
	[+294]		[+189.12]		[+116.298]		[+53.73]		[+10.335]	
	-0.0055936	1.66032	-0.004738	1.45989	-0.0036104	1.28284	-0.0028256	1.13735	-0.0023144	1.03917
A	0.0068252	0.09800	0.003842	0.06304	0.002528	0.038766	0.001266	0.01791	0.000337	0.003445
	↓ y ($\delta y = 5$ ft.)									

Ref.: Eqs. (8.1a), (8.2) and (8.5).

[t]	
$\frac{\partial^2 m}{\partial x \partial y}$	m
$\frac{\partial^2 m}{\partial y^2}$	$\frac{\partial m}{\partial x}$

Index (from table 2 insert m)

GRID 2

$[K_x]$		5		4		3		2			
	0	-132.031 +0.00880208 ×(-15000)	0	-116.36 +0.00775784 ×(-15000)	0	-92.873 +0.0061915 ×(-15000)	0	-71.625 +0.0047750 ×(-15000)	0	-58.278 +0.0038852 ×(-15000)	
[0.000]	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	→C x
	0	-83.904 +0.0055936 ×(-15000)	-118.2 0.006304 ×(-18750)	-71.07 +0.004738 ×(-15000)	-190.886 0.0101806 ×(-18750)	-54.156 +0.0036104 ×(-15000)	-224.46 0.0119716 ×(-18750)	-42.384 +0.0028256 ×(-15000)	-230.92 0.0123161 ×(-18750)	-34.716 +0.0023144 ×(-15000)	
[0.009800]	0.009800	-0.00320848	0.016104	-0.00301984	0.0199806	-0.0025811	0.0217716	-0.0019494	0.0221161	-0.0015708	B
	0	0	-72.0375 0.0038420 ×(-18750)	0	-119.43 0.006370 ×(-18750)	0	-143.175 0.007636 ×(-18750)	0	-149.49 0.007973 ×(-18750)	0	
[0.0068252]	0.0068252	-0.00880208	0.0106672	-0.00775784	0.0131952	-0.0061915	0.0144612	-0.0047750	0.0147982	-0.0038852	A
		[-0.00880208] ↓ y		[-0.00775784]		[-0.0061915]		[-0.0047750]		[-0.0038852]	← $[k_y]$

Ref.: Eqs. (8.3) and (8.4)

Index $\left((\text{from Grid 1 insert } \sum \frac{\partial^2 m}{\partial x \partial y} \text{ and } \sum \frac{\partial^2 m}{\partial y^2}\right)$

n_x	n_y
$\sum_0^x \frac{\partial^2 m}{\partial y^2}$	$\sum_0^y \frac{\partial^2 m}{\partial x \partial y}$

GRID 3

	5		4		3		2		1	
0	-129.2411	0	-113.8012	0	-90.9105	0	-70.1115	0	-57.0465	C
1.02162	0.97887	1.02162	0.97887	1.02162	0.97887	1.02162	0.97887	1.02162	0.97887	x
0	-96.2018	-105.7429	-79.5315	-179.2763	-57.6626	-216.8261	-43.4266	-257.4134	-34.6358	B
0.87216	1.14657	0.89461	1.11906	0.93918	1.06475	0.96599	1.02460	1.11473	0.99769	
0	0	-49.3442	0	-93.0980	0	-125.8838	0	-143.9125	0	A
0.60229	1.66043	0.68498	1.45999	0.77952	1.28284	0.87923	1.13735	0.96269	1.03875	
	y									

Ref.: Eqs. (2.7) and (2.8)

Index (From table 2 insert γ and $1/\gamma$ and use n_x, n_y from Grid 2)

N_x	N_y
γ	$1/\gamma$

except for the edge panels of the grid, where

$$\begin{aligned} \frac{\partial m}{\partial y} &= \frac{m_3 - m_0}{\delta y} = s, \\ \frac{\partial^2 m}{\partial y^2} &= \frac{\partial s}{\partial y} = \frac{s_3 - s_0}{\partial y}, \\ \frac{\partial^2 m}{\partial x\, \partial y} &= \frac{\partial s}{\partial x} = \frac{s_1 - s_0}{\delta x}. \end{aligned} \tag{E}$$

With the value of $D = 3{,}000\,\text{lb.}$, values of t at various points are thus obtained from Eq. (8.1a) in Grid 1. Operational Grids 2 and 3 thus follow progressively and by similar operations the values of t, n_x and n_y on the $x-y$ plane are finally obtained in "lb./ft. run" units.

The stress resultants are then obtained from Eqs. (2.7) and (2.8) through Grid 3. These are summarised in Table 3.

Table 3

Points	N_x (lb./ft.)	N_y (lb./ft.)	N_{xy} (lb./ft.)
A_1	−143.9125	0	+ 10.335
A_2	−125.8838	0	+ 53.73
A_3	− 93.0980	0	+116.298
A_4	− 49.3442	0	+189.120
A_5	0	0	+294.000
B_1	−257.4134	− 34.6358	+ 5.277
B_2	−216.8261	− 43.4266	+ 34.731
B_3	−179.2763	− 57.6626	+ 78.378
B_4	−105.7429	− 79.5315	+131.493
B_5	0	− 96.2018	+191.622
C_1	0	− 57.0465	0
C_2	0	− 70.1115	0
C_3	0	− 90.9105	0
C_4	0	−113.8012	0
C_5	0	−129.2411	0

10. Remarks

It should be observed that the solutions obtained in section 8 satisfy the boundary conditions, three in number, and thus represent complete solutions corresponding to the General Equation (5.1). Any lesser number of conditions, if considered, would only offer incomplete solutions.

The present problem has been solved using two boundary conditions in accordance with Soare's theory and the results obtained are given in Table 4.

Table 4. Results by Soare's Method

Points	N_x (lb./ft.)	N_y (lb./ft.)	N_{xy} (lb./ft.)
A_1	-322.8	-875.0	$+22.7$
A_2	-294.9	-408.2	$+80.8$
A_3	-231.4	-230.9	$+166.3$
A_4	-114.3	-131.2	$+275.8$
A_5	0	-101.4	$+288.3$
B_1	-450.0	-900.2	$+11.5$
B_2	-381.1	-454.2	$+42.3$
B_3	-343.5	-293.8	$+90.9$
B_4	-204.9	-205.9	$+159.1$
B_5	0	-166.0	$+166.8$
C_1	-550.8	-907.6	0
C_2	-518.1	-470.5	0
C_3	-432.4	-317.4	0
C_4	-268.3	-239.5	0
C_5	0	-192.3	0

The N_y-values in this table are higher, having values along the longitudinal edges. The N_x-values are also higher, while the N_{xy}-values are reasonably close to those obtained by the present method.

Having regard to the fact that the third boundary condition is not taken into consideration, it may be stated that the solution in Soare's paper [1] is only one of several others which satisfy his general equation, and is therefore not "complete".

11. Bibliography

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Summary

The solution of the differential equations of equilibrium for a shell element having a double curvature by direct methods is difficult. It is therefore usual to have recourse to iterative or other methods.

The paper presents an alternative method of approach for the direct solution of Conoidal Shells. In addition, the present concept of selecting the number of boundary conditions for such shells is criticized. The paper includes an example in which the results arising out of these conditions are compared with those obtained by Soare's method.

Résumé

Avec les méthodes directes, il est difficile de résoudre les équations différentielles traduisant l'équilibre d'un élément de voile à double courbure. C'est pour cette raison que l'on recourt généralement à des procédés itératifs ou à d'autres méthodes.

Dans la présente communication, on expose un autre procédé permettant le calcul direct des voiles conoïdes. De plus, on étudie de façon critique la méthode habituelle pour le choix du nombre des conditions au contour de ces voiles. Un exemple est présenté, qui illustre la comparaison des résultats correspondant à ces conditions à ceux donnés par la méthode de Soare.

Zusammenfassung

Eine direkte Lösung der Differentialgleichungen für das Gleichgewicht einer doppeltgekrümmten Schale ist schwierig. Aus diesem Grunde werden üblicherweise iterative Verfahren oder andere Methoden herangezogen.

In dieser Veröffentlichung wird eine andere Methode dargestellt, die eine direkte Lösung bei Konoidschalen erlaubt. Es werden ebenfalls die aktuellen Kriterien zur Wahl der Anzahl der Randbedingungen kritisch durchleuchtet. Die Arbeit enthält ferner ein Beispiel, welches erlaubt, die Ergebnisse, die sich aus diesen Bedingungen ergeben, mit denen, die die Methode Soare liefert, zu vergleichen.