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A Tubular Steel Dome

Une coupole triangulée en tubes métalliques

Eine Gitterkuppel aus Stahlröhren

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General Description

The dome is spherical in form, however, it is made up of plane curves superimposed in such a way as to give a spherical shape. Fig. 1 is an overall view of the building. Fig. 2 is a reproduction of the structural model. This last model was made for purposes of studying geometry, stresses and erection. One hundred and ninety-two plane curves acting as meridians are laid over one another. The meridians are all identical plane curves approximately 90 feet long. The hoops are of varying length and are offset to the inside by a distance of 12 inches. See Figs. 3 and 10. Since the 192 meridians all converge at the top, a lantern is provided to eliminate congestion. The entire dome is supported on a ring girder box shaped in cross-section which in turn rests on eight supports.

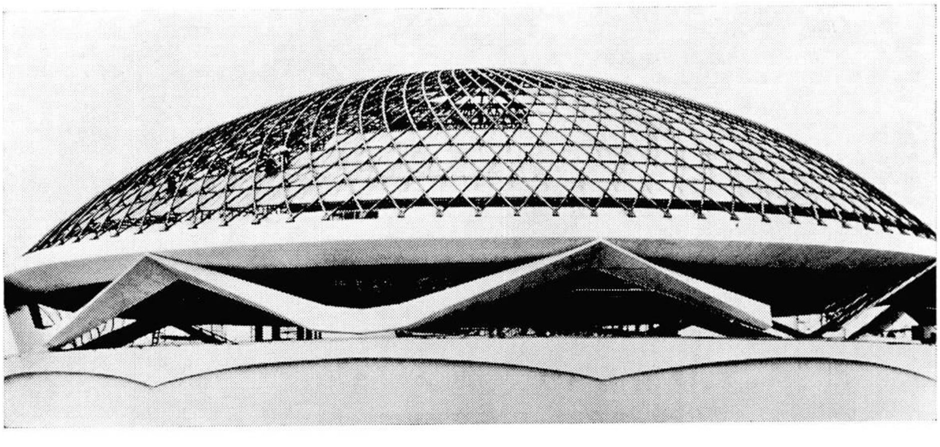

Fig. 1.

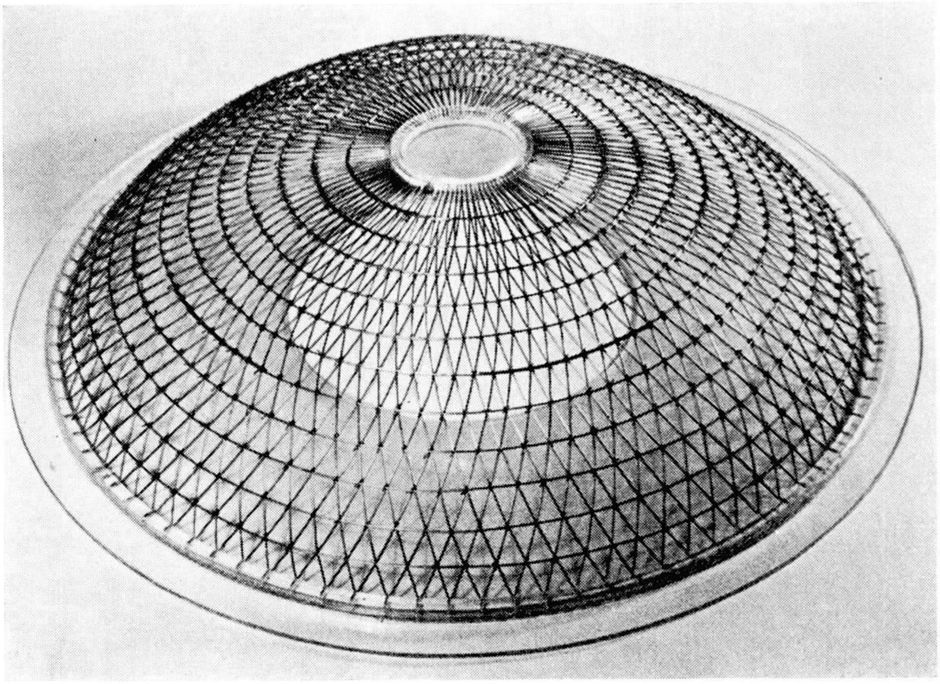

Fig. 2. Structural model.

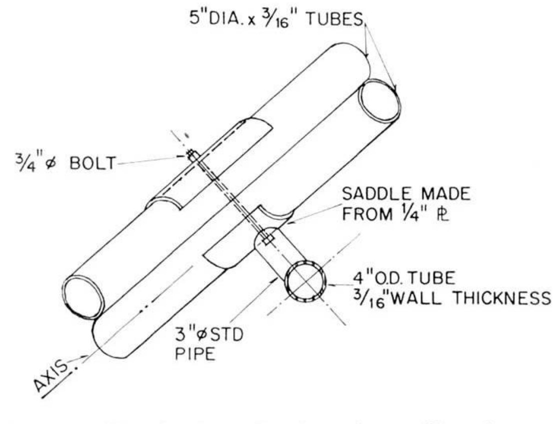

Fig. 3. Typical node showing offset hoop.

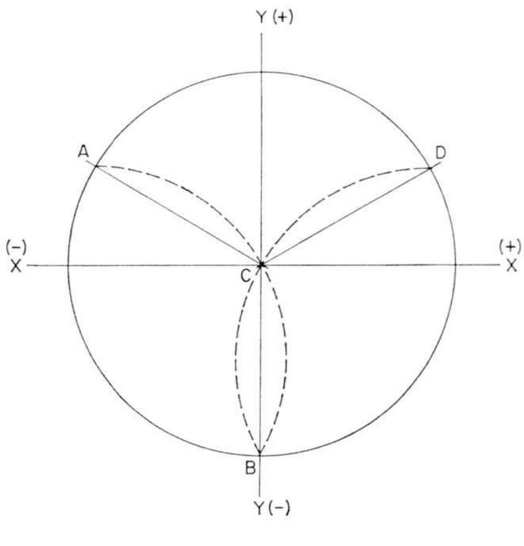

Fig. 4. Plan of dome.

Geometry

All 192 meridians would pass through the pole of the dome if they were extended. One-half of the number of meridians, i. e., 96, lie on top of the other half. Also, the top meridians sweep in one direction and the bottom in another. It was necessary to calculate the intersection points of the meridians and hoops for all three x, y, and z positions in Cartesian Coordinates. The points given are A, B, C, and D on Fig. 4. From knowing these points and from knowing that the equation for a plane through A, B, and C is

$$a_1 x + b_1 y + c_1 z = 100, \tag{1}$$

and through B, C, and D is

$$a_2 x + b_2 y + c_2 z = 100, \tag{2}$$

we get two equations with three unknowns. The equation for a sphere is

$$x^2 + y^2 + (z+k)^2 = a^2. \tag{3}$$

Thus, we have three equations with three unknowns. All intersections of meridians with hoops (henceforth called nodes) may now be calculated.

Stress Calculations

The factors to be considered in stress calculations are as follows:

1. Buckling of the entire dome as a shell.
2. Buckling of a single tube similar to a column between nodal points.
3. Effect of half-span or wind loads.
4. Effect on the dome of deflection of the ring girder.
5. Temperature stresses.
6. Development of "pick-up" in stress at nodal points.
7. Effect of hoops being offset from meridians.
8. Bending of meridians at ring.

The above items will now be discussed.

Buckling

The buckling of a solid dome is given by [1]

$$q_{cR} = \frac{2\,E\,h^2}{a^2 \sqrt{3\,(1-\nu^2)}}. \tag{4}$$

Buckling is dependent on the moment of inertia per unit width of the cross section and the area per unit width. For a solid dome are $I = h^3/12$ and $A = h$ per unit width. Therefore, Eq. (4) can be converted to

$$q_{cR} = \frac{4\,E\sqrt{I\,A}}{a^2}. \tag{5}$$

It is now necessary to compute the moment of inertia and area per unit width for the lattice dome and substitute in (5) above. To do this the I and A of the meridian and hoop tubes were calculated as individual tubes and added together with no allowance for interaction due to being connected together at the nodes. However, allowance was made for the fact that the meridian tubes do not actually fall on a true meridian line due to the "sweep" of the meridian tubes. The I and A were computed in two directions at right angles to each other and then arithmetically averaged. The two directions chosen were along the hoop direction and along a true meridian direction. By this procedure a factor of safety of 10 against overall buckling was obtained.

This may seem at first sight to be excessive. However, spherical shells buckle at loads far below that indicated by classic theory. One of the reasons for this reduction in capacity is initial roughness or eccentricity. One of the effects of the offset hoops is to give an eccentricity which is equivalent to an initial roughness. Having determined the overall buckling load on the shell, it is now necessary to determine buckling of an individual tube between nodes. Here a smaller factor of safety can be applied. Since the members are circular arcs and not chords there is an eccentricity due to the deviation from a straight line. This deviation was computed to be 0.83 inches. Each member is assumed to be pin-connected at both ends since there is no bending restraint at the nodes. The combined bending and direct stress on a member gave a factor of safety of about 5.

Partial Loadings on Dome

The wind stresses were determined by assuming pressure on the windward side and suction on the leeward. Therefore, one-half the code requirement of 25 lbs. per square foot was applied as both a suction and a pressure. The equations for this load are [2]

$$p_\pi = -p \operatorname{Sin}\phi \operatorname{Cos}\varphi, \tag{6}$$

and the equations for the internal stresses are

$$N_\phi = \frac{-pa(2+\operatorname{Cos}\phi)(1-\operatorname{Cos}\phi)\operatorname{Cos}\phi\operatorname{Cos}\varphi}{3(1+\operatorname{Cos}\phi)\operatorname{Sin}\phi} = \frac{-pa}{3}c_1\operatorname{Cos}\varphi, \tag{7}$$

$$N_{\phi\varphi} = \frac{-pa(2+\operatorname{Cos}\phi)(1-\operatorname{Cos}\phi)\operatorname{Sin}\varphi}{3(1+\operatorname{Cos}\phi)\operatorname{Sin}\phi} = \frac{-pa}{3}c_2\operatorname{Sin}\varphi, \tag{8}$$

$$N_\varphi = \frac{-pa(3+4\operatorname{Cos}\phi+2\operatorname{Cos}^2\phi)(1-\operatorname{Cos}\phi)\operatorname{Cos}\varphi}{3(1+\operatorname{Cos}\phi)\operatorname{Sin}\phi} = \frac{-pa}{3}c_3\operatorname{Cos}\varphi. \tag{9}$$

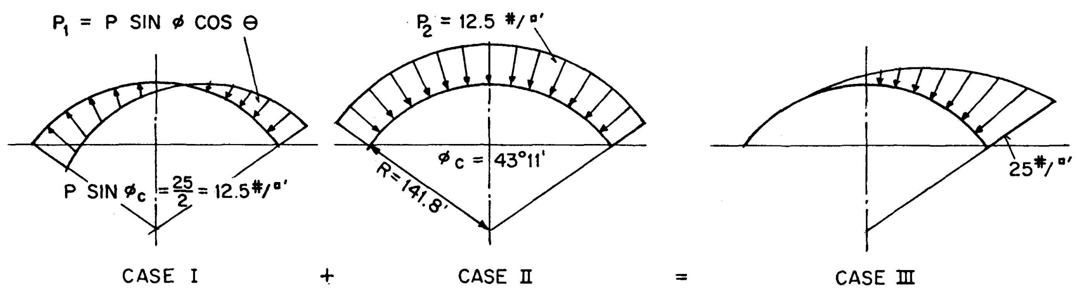

Fig. 5.

For half-span loading the principal of symmetry and anti-symmetry was used. See Fig. 5. Case I and II added together give Case III which shows a live load of 25 lbs. per square foot on one side. There is some error introduced in using a radial load instead of a vertical load and also because the load diminishes to zero at the top instead of remaining constant. However, these errors were partially compensated for by making the half-span loading conservative, i. e., 25 lbs. per square foot. By comparing the wind analysis with the half-span analysis it was found that one condition governed in one part of the shell and the other in another part.

Temperature Stress

Two conditions create stresses due to temperature. The first is the rise and fall of the air temperature during the day and from summer to winter. This causes the temperature of the entire exposed meridians to change. The second type of temperature change investigated is that which heats one side of the dome while the other is unheated., i. e., a differential temperature change. For

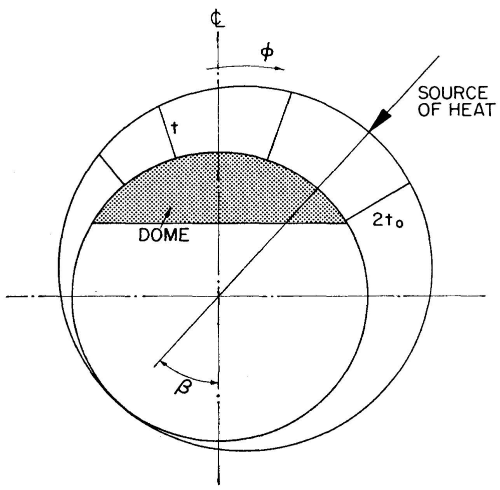

Fig. 6.

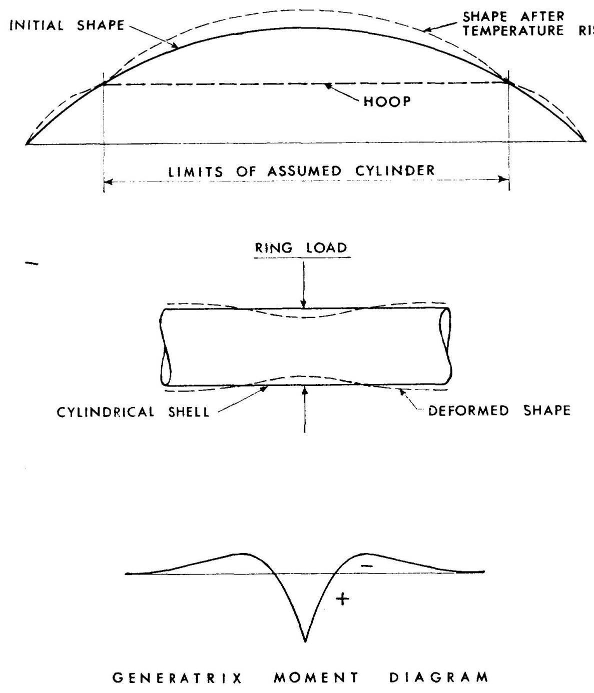

Fig. 7. Effect of temperature rise.

differential temperature change see Fig. 6. The equation for this type of temperature change is given by [3]

$$t = t_0 (1 + \mathrm{Cos}\,\phi\,\mathrm{Cos}\,\beta + \mathrm{Sin}\,\phi\,\mathrm{Sin}\,\beta\,\mathrm{Cos}\,\varphi). \tag{10}$$

Here the effect is aggravated by the fact that the hoops are on the inside of the dome and the meridians are on the outside. The effect of expanding or contracting the hoops while holding the meridians constant is similar to the effect caused in a cylindrical shell by a stiff unyielding ring and a relatively flexible shell. See Fig. 7. The longitudinal or generatrix bending moment in such a shell is given by [4]

$$\delta = -\frac{12(1-\nu^2)Q}{2\beta^3 E h^3} \quad \text{and} \quad M_x = \frac{-E h^3 d^2 \omega}{12(1-\nu^2)dx^2}. \tag{11}$$

See Fig. 7 for the distribution of this moment.

Effect on Dome of Support Deflection

Thus far the dome analysis has been based upon an unyielding support. Actually, the dome rests upon a continuous curved ring girder 200π or 628 feet long supported on eight points. This ring girder also supports the floor of the building, hence, it is subject to deflections from either floor or roof live load. As the girder deflects, stresses are induced in the dome. The girder can only deflect vertically as the torsional deflection is restrained by moment connections from the radial floor beams. The deflection of a continuous circular ring girder on isolated supports is given by [5]

$$\delta = \frac{R^3}{N^4}\left\{F\left(\frac{a_1}{2EI} + \frac{b_1}{2GK}\right) + P\left(\frac{a_1}{4EI} + \frac{b_1}{4KG}\right)\right\}. \tag{12}$$

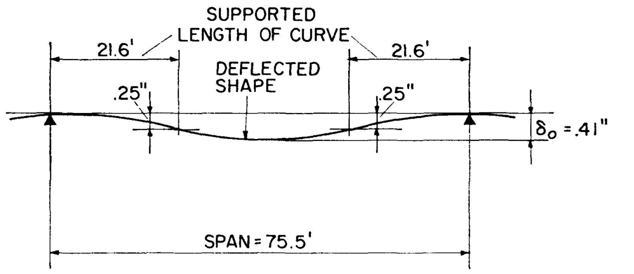

Fig. 8. Developed deflection curve of part of ring girder.

Fig. 8 shows the developed deflection curve of one span of the girder, wherein the deflection curve is assumed to be

$$\omega_x = \frac{\omega_0}{2}\left(1 - \text{Cos}\frac{2\pi x}{L}\right). \tag{13}$$

Since the maximum deflection was found to be only 0.41″ and the entire dome is quite flexible, it was not believed necessary to equate the deflections of the girder and the dome exactly. This could have been done by means of a Fourier Series. Instead it was arbitrarily assumed that 0.25″ was acceptable and anything larger than this must be accounted for. Let $\omega_x = 0.25''$ and solving for X in (13) we get 21.6′ at each end of each span as the amount of supported length upon which the whole dome must rest when live load is a maximum. Accordingly, the members adjacent to the ring girder in the 21.6′

length were checked to carry the entire load. The effect of this concentration of load is rapidly dissipated in the upper members of the dome.

Development of "Pick-Up" in Stress

Each node behaves in a manner similar to a panel point in a truss. The loads in the meridians are different on each side of the joint, hence, the joint must develop this increment of load. The simplest manner of doing this would be to wrap the meridians with a malleable steel bolt which would grip the tubes by friction. However, the clamping force required to develop this increment of load calculated to crush the wall of the tube, particularly since the meridians are exposed and hence, the wet coefficient of friction of steel on steel would have to be used. Eventually it was decided to use a bolt for the pick-up in load although this requires careful locating of the bolt holes prior to assembly.

Effect of Offset Hoops

The principal effect of offsetting the hoops is to put bending in the meridians. Since the offset is radial, the effective lever arm varies from zero at the pole of the dome to a maximum at the base. These effective offset lengths multiplied by their lever arms thus give moments at the nodes. These moments were distributed by conventional moment distribution along the meridians.

Bending of Meridians at Ring

As with most spherical shells there are incompatible deflections at the edge of the dome due to the hoops being under compression while the ring girder is under tension. This effect was minimized by providing a loose connection between the meridiand ans the ring girder. This has somewhat the same effect as placing a roller under the shell. See Figs. 9 and 10.

Practical Considerations

The dome was relatively expensive insofar as cost per pound of steel was concerned, however since the total poundage was comparatively light (less than 9 lbs. per square foot) the overall cost was not prohibitive. The principal factor contributing to the high unit cost was the uncertainty factor of erection. It was not known that the bolt holes through the nodes would line up properly until the dome was actually assembled. Also the speed of erection was unknown as there was no precedent for erecting this type of dome with United States

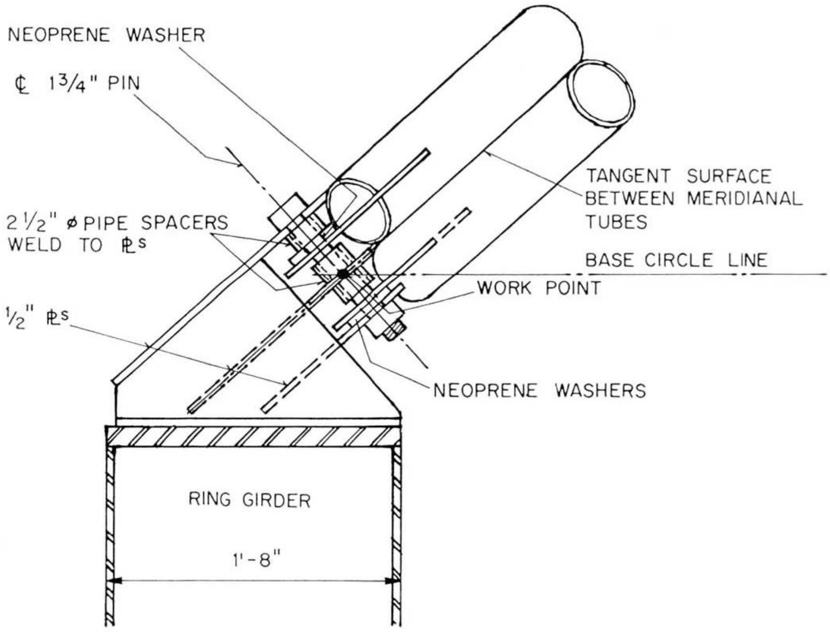

Fig. 9. Connection of meridians to ring girder.

Fig. 10.

equipment and methods. In actuality the dome was erected in less time than anticipated and with no delays of any consequence.

The covering of the roof is a lighweight steel deck (see Fig. 1). Insulation is placed over this deck and a flexible silicone covering on top of the insulation. Furring channels and plaster on the underside complete the roof construction.

It is important that this plaster surface be kept relatively free from cracks and that no leakage occur as the plaster surface is used somewhat as a planetarium dome to receive light displays.

Conclusions

It is practical to build a tubular steel dome with offset hoops and exposed meridians. Such a dome is not inordinately expensive. Furthermore movement may be kept within limits such that a simple economical roof may be used.

Notation

h = thickness of a solid shell.
$A = h$ = area of shell per unit width.
$I = \frac{h^3}{12}$ = moment of inertia of shell per unit width.
q = load per unit area on dome.
q_{cR} = load per unit area causing buckling.
a = radius of curvature of dome.
σ = unit stress on dome due to radial load.
ν = Poisson's Ratio.
p = wind load per unit area.
φ = position angle in horizontal plane.
ϕ = position angle in vertical plane.
N_ϕ = meridian force.
N_φ = hoop force.
$N_{\phi\varphi}$ = shear force.
δ = deflection.
R = radius of base circle of dome = 100 ft.
N = number of supports of ring girder = 8.
F = total concentrated loads on ring girder in pounds.
p = total uniform load on ring girder in pounds.
Q = radial load on a cylindrical shell
a_1 = constant in ring girder equation.
b_1 = constant in ring girder equation.

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Summary

A tubular steel dome is selected as the roof structure for the General Electric Building at the New York World's Fair for 1964/65. The span of the dome is 200 ft. in diameter. The selection is based on architectural appearance and ease of dismantling. Similar tubular steel domes have been built in Europe, however, in the example herein presented, the hoops are offset from the meridians giving built-in eccentricities. Buckling is of prime concern although temperature effects are also of importance because part of the structure is exposed and part inside the roof covering.

Résumé

La couverture du pavillon de la General Electric à l'Exposition Internationale de New York, en 1964—1965, a été réalisée sous la forme d'une coupole triangulée en tubes métalliques, le diamètre de l'aire couverte valant 61 m. Cette solution a été adoptée pour des raisons architecturales et à cause du démontage facile. On a réalisé en Europe des coupoles semblables mais, dans le cas présent, les ceintures horizontales sont décalées par rapport aux tubes méridiens, ce qui provoque des excentricités. Le problème le plus important est celui du voilement; l'influence de la température est aussi d'une certaine importance, parce qu'une partie de l'ossature est à l'air libre tandis que l'autre est à l'intérieur de la couverture.

Zusammenfassung

Für die Dachkonstruktion des General Electric Building an der New Yorker Weltausstellung wurde eine Gitterkuppel aus Stahlröhren gewählt, mit einem Durchmesser von 200 Fuß (60,96 m). Die Wahl fiel auf diese Konstruktion auf Grund des architektonischen Aussehens und der einfachen Abbaumöglichkeit. Ähnliche Schalen wurden bereits in Europa errichtet, allerdings sind in der hier beschriebenen Schale die Horizontalringe von den Meridianen abgesetzt angeordnet, wodurch baulichbedingte Exzentrizitäten auftreten. Das Durchschlagsproblem ist von Hauptinteresse, obwohl auch die Temperatureinflüsse wichtig sind, weil ein Teil des Tragwerkes im Freien bleibt und ein Teil durch die Dachhaut geschützt wird.