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Optimum Design using Linear Programming

L'application de la programmation linéaire au calcul optimal

Optimale Bemessung mittels linearer Programmierung

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1. Introduction

The purpose of this paper is the discussion of direct methods for the optimum design of structures, assuming that the geometrical configuration of the structure and the different loading cases are known. In particular dimensions and resistance values of the individual members shall be determined in such a way that a prescribed safety factor is ensured and the total cost of the structure is a minimum.

Using elastic methods of analysis, the only possible approach so far has been successive trial, due to the very complex interdependence between loads, stiffness properties and elastic stresses of the structure. Such iterativ methods are only in special cases suitable for electronic computation.

However, applying plastic analysis, methods of minimum weight design have been developped (e. g. list of references, reference [4], p. 266). In this paper the structural design problem is presented using linear programming procedures. The use of computers, even in the most simple practical cases, will always be necessary.

The first basic assumption of such methods is the linear dependence of the cost of each individual member or part of the structure and its plastic carrying capacity or "resistance" value. "Resistances" are, for instance, the plastic moments in different cross-sections of a frame structure, or the yield forces in the axially loaded bars of a truss, or, more generally, the maximum "generalized stresses" which can be carried by a fully plastified member or cross-section of the structure. It is then:

$$\text{Cost} = C_0 + \sum_{i=1}^{k} c_i R_i ,$$

where R_1 to R_k are the resistances of the structure, c_1 to c_k the cost coefficients, and C_0 is that part of the structural cost which does not depend upon the resistance distribution and therefore not upon the way in which the structure is designed.

Obviously this part might sometimes be very large compared to the diminution of the structural cost which a sophisticated design can attain. On the other hand, the saving of material, which is the only result that such simple methods can obtain, might not always appear as a decisive optimalization criterium. The problem of the effective economic advantages of such methods arises, a problem which can not really find a definitive answer until many practical cases have been treated and the results of the different design methods critically compared. However, considering that the development of direct optimum design methods has been impossible because of the prohibitive amount of numerical computation it is logical to assume that, with the use of modern computers, such methods will be able, in some cases, to substitute advantageously the actual trial-methods.

The linear program will minimalize the variable part of the cost function. In matrix form it is:

$$\text{Cost} = \sum_{i=1}^{k} c_i R_i = c^t r \rightarrow \text{Minimum} \tag{1}$$

the column-matrices "c" and "r" (each with "k" elements) being defined accordingly.

The basic assumptions of the plastic theory will then have to be considered. In particular, because of the assumption of rigid-plastic behavior of the material, plastic design can not be applied where stability or fatigue criteria govern the design, and also where the material does not show the necessary plastic deformation capacity. If, instead of a rigid-plastic, an elastic-plastic material is assumed, the interesting problem of "deflection stability" arises. As it will be shown below, this criterium will be used to develop a new method of "elastic-plastic" optimum design, the central theme of the present paper.

2. Rigid-Plastic Optimum Design

It will be useful, for a better understanding of the elastic-plastic method, to start by discussing shortly the related method of rigid-plastic optimum design. Both methods will be explained on the example of a simple steel truss structure. Fig. 1 shows its geometrical configuration as well as the three loading cases which the structure has to carry. The dead load is supposed to be known[1]).

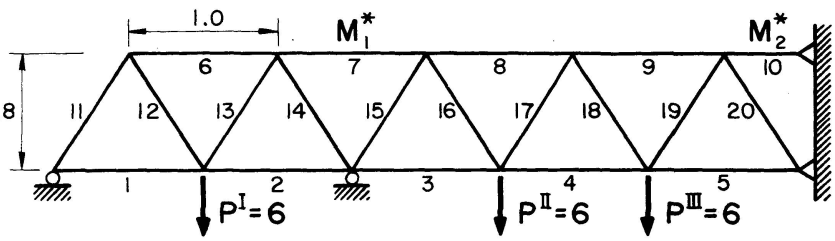
[1]) This assumption is not necessary as soon as linear relationship between dead load and resistances can be established. See reference [1].

The structure has to be designed in such a way that the total steel volume is a minimum. Assuming everywhere a constant yield stress σ_y, the cost coefficients will then be the length l_1 to l_{20} of the bars:

$$\text{Cost} = \sum_{i=1}^{20} c_i R_i = \sigma_y \sum_{i=1}^{20} l_i A_i \rightarrow \text{Minimum}.$$

A_1 to A_{20} being the unknown cross sectional areas of the bars.

The derivation of the linear program is based on the lower-bound theorem of the plastic theory. It would also be possible to derive the same linear program from the upper-bound theorem, thus in a kinematical way, but this would be more complicated (reference [1]). The lower-bound theorem states that "if an equilibrium distribution of stresses can be found which balances the applied loads and is everywhere less than or equal to the yield stress, the



Dead load: $P = 1$ at each nodal point

Equilibrium: $M_i = B_i + \sum_{j=1}^{2} a_{ij} M_j^*$ $(i = 1 \text{ to } 20)$

		Load I		Load II		Load III		A-Matrix		
M_1	=	2.81	V	0.94	V	0.94	+	−0.250	0	$\cdot \begin{bmatrix} M_1^* \\ M_2^* \end{bmatrix}$
M_2		2.81		0.94		0.94		−0.750	0	
M_3		1.56		4.06		2.81		−0.833	−0167.	
M_4		2.25		6.00		6.00		−0.500	−0.500	
M_5		1.56		2.81		4.06		−0.167	−0.833	
M_6		−5.00		−1.25		−1.25		0.500	0	
M_7		0		0		0		1.000	0	
M_8		−2.50		−7.50		−5.00		0.667	0.333	
M_9		−2.50		−5.00		−7.50		0.333	0.667	
M_{10}		0		0		0		0	1.000	
M_{11}		−5.31		−1.77		−1.77		0.472	0	
M_{12}		4.13		0.59		0.59		−0.472	0	
M_{13}		4.13		0.59		0.59		0.472	0	
M_{14}		−5.31		−1.77		−1.77		−0.472	0	
M_{15}		−2.95		−7.67		−5.31		−0.314	0.314	
M_{16}		1.77		6.49		4.13		0.314	−0.314	
M_{17}		−0.59		1.77		−2.95		−0.314	0.314	
M_{18}		−0.59		−2.95		1.77		0.314	−0.314	
M_{19}		1.77		4.13		6.49		−0.314	0.314	
M_{20}		−2.95		−5.31		−7.67		0.314	−0.314	

Fig. 1. Simple truss structure.

structure will not fail. At most it will have reached the ultimate load'' (reference [3], page 6).

The rigid-plastic optimum design methods is now based on the following idea. Because the structure is not yet designed, and the resistance distribution (in the truss of Fig. 1 the distribution of the cross-sectional areas of the bars) therefore not yet known, it is possible to choose any arbitrary equilibrium distribution of stresses and then design the structure in such a way that the plasticity conditions are nowhere violated. The structure designed in such a way, according to the lower-bound theorem, will not fail. For optimalization then, out of all possible equilibrium states, the one which minimizes the cost function has to be chosen.

The problem has significance only for statically indeterminate structures. For, in the case of a statically determinate structure, the equilibrium configuration is uniquely prescribed by the loads and the geometry of the structure. On the other hand the equilibrium configuration in a n-times statically indeterminate system, has ''n'' degrees of freedom corresponding to the ''n'' redundants of the structure acting on a statically determinate base system. The redundants produce ''n'' independent homogeneous[2]) equilibrium configurations, whereas one external loading case, acting on the base system, makes up one inhomogeneous stress state.

If M_1^* to M_n^* are the redundants of the system and M_1 to M_k are the ''generalized stresses'' necessary to design the structure, e. g. the bending moments in ''k'' selected cross-sections of a frame, or the axial forces in the ''k'' bars of a truss, it is possible to write:

$$M_i = B_i + \sum_{j=1}^{n} a_{ij} M_j^* \quad (i = 1 \text{ to } k). \tag{2}$$

The coefficients a_{ij} describe the homogeneous states, a_{ij} being the generalized stress M_i due to the redundant $M_j^* = 1$ acting on the base system. The ''inhomogeneous'' coefficient B_i is the generalized stress M_i caused by the external loads acting on the base system. In matrix form Eq. (2) becomes:

$$m = b + A\, m^* \tag{2}$$

the column-matrices ''m'', ''b'' and ''m^*'', and the matrix ''A'', with ''k'' rows and ''n'' columns, being defined accordingly.

Eq. (2) describes the equilibrium state of the structure completely. The redundants M_1^* to M_n^* are the free parameters of the equilibrium state and will also be free parameters of the plastic design problem. In fact, the redundants of a structure can be chosen arbitrarily without violating any equilibrium condition of the system. Fig. 1 shows the two homogeneous states due to the

[2]) Homogeneous: Selfequilibrating state without external loads. Inhomogeneous: State of equilibrium including external loads.

redundants M_1^* and M_2^* which correspond to the axial forces in the bars 7 and 10, as well as the three inhomogeneous states (with $M_7 = M_{10} = 0$) due to the three loading cases.

To design the structure the necessary plastic resistances have then to be determined. They are non-negative but otherwise also free parameters of the plastic design problem. For the sake of simplicity it will be assumed that each generalized stress corresponds in the structure to a "generalized" resistance, so that there will be "k" resistances to determine. This assumption is not necessary. In practical cases, due to architectural or structural reasons, the same resistance may be required in different parts of the structure, its value being equal to the largest of several generalized stresses. On the other hand, for reinforced concrete frames the resistances i. e. the plastic moments will generally be different for positive and negative moments in the same cross-section.

To arrive at an optimum design, the equilibrium state must be admissible, hence the absolute value of each generalized stress should not be greater than the corresponding resistance:

$$|M_i| \leqq R_i \quad \text{or} \quad -R_i \leqq M_i \leqq R_i \quad (i = 1 \text{ to } k)$$

or

$$\begin{aligned} 0 &\leqq -M_i + R_i \\ 0 &\leqq M_i + R_i \end{aligned} \quad (i = 1 \text{ to } k). \tag{3}$$

Introducing Eq. (2):

$$\begin{aligned} 0 &\leqq -B_i - \sum_{j=1}^{n} a_{ij} M_j^* + R_i \\ 0 &\leqq B_i + \sum_{j=1}^{n} a_{ij} M_j^* + R_i \end{aligned} \quad (i = 1 \text{ to } k). \tag{4}$$

In matrix form, for a certain number "l" of loading cases, the loading case suscript being written above on the left, it is:

$$\begin{aligned} 0 &\leqq -{}^{j}b - A\,{}^{j}m^* + r \\ 0 &\leqq {}^{j}b + A\,{}^{j}m^* + r \end{aligned} \quad (j = 1 \text{ to } l). \tag{5}$$

These restrictions together with the cost function (1)

$$\text{Cost} = c^t r \rightarrow \text{Minimum} \tag{1}$$

constitute the linear program shown in Fig. 2. The U-matrices are unit matrices with "k" diagonal elements.

Generally speaking, the rigid-plastic optimum design problem is a minimum problem where the free parameters are, first of all, the "n" redundants of the system for each of the "l" considered loading cases. They determine the "l" different equilibrium distributions which balance the applied loads, and consequently determine the necessary resistance distribution and the optimum value of the cost function. In the example of Fig. 1 there will be six redundants

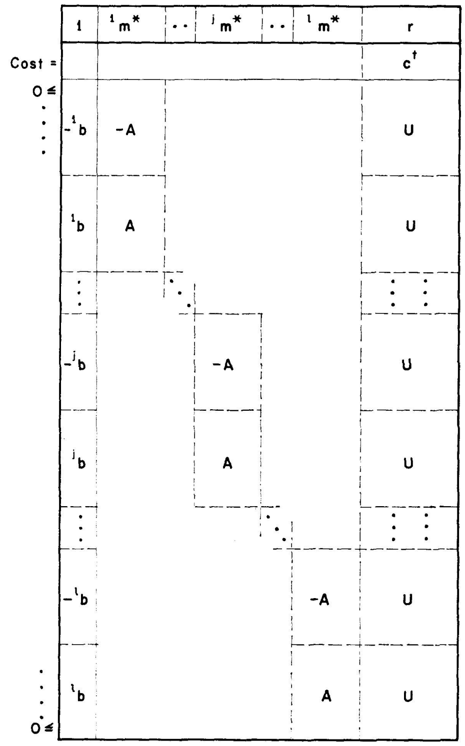


Fig. 2. Table of the linear program for rigid-plastic optimum design.

to determine, two for each of the three loading cases. These redundants will then prescribe the three corresponding optimum axial force distributions in all the bars of the truss and the required cross-sectional areas.

3. Elastic-Plastic Optimum Design

It can be easily shown that, for a structure designed according to the rigid-plastic method, several or even all considered loading cases will represent ultimate loads of the structure in the sense of the simple plastic theory. One of the assumptions of this theory is that the applied loads increase proportionally until the limit load is reached. During this process the ratio between the individual loads remains fixed.

In practice the individual loads act generally in a random manner and hence do not satisfy this condition. This fact leads, for structures made of elastic-plastic material, to the problem of "deflection stability", as unfavorable loading cycles could constantly increase the deflections, so that the structure may finally fail although the ultimate load has never been reached. This mode of failure is termed "incremental collaps". The elastic-plastic optimum design method will avoid the danger of such a mode of failure.

It should be pointed out that this phenomenon of "incremental collaps" is rather an accademic problem. On the other hand the determination of the "stabilizing" or "shake down" load, is in most practical cases, quite complicated. Consequently the danger of incremental collaps has generally been disregarded for practical design purposes. It will be shown, however, that in the design problem, where the loads are known but not the resistance distribution within the structure, it is in many cases most advantageous to design a structure which is able to "shake down".

The "shake down" theorem states that, if a residual homogeneous equilibrium stress state can be superimposed on the elastic stresses of the structure due to external loading, such that the resulting stresses nowhere violate the plasticity conditions, then the structure will be able to "shake down" for any possible loading cycle.

It will now be supposed that it is possible to calculate the elastic "generalized stresses" M_1^{el} to M_k^{el} for each loading case. The maximum and minimum stresses M_1^{max} to M_k^{max} and M_1^{min} to M_k^{min} between which the stress state of the structure would vary if the material would be elastic, are hence known. If a residual homogeneous but otherwise arbitrary stress state is superimposed on these ideal-elastic stresses and if the structure is designed so that the resulting stresses nowhere violate the plasticity conditions, the danger of incremental collaps will be avoided.

Here two points have to be considered. First, a complete elastic analysis of the structure has to be made before the optimum resistance distribution of the structure is known. Second, for all the different loading cases there will be only one unknown homogeneous stress state, which must be determined in such a way that the structural cost will be a minimum. On the contrary, by the rigid-plastic optimum design method, for each loading case a different homogeneous stress state must be determined.

The residual stress state is a homogeneous state. According to Eq. (2) and without the inhomogeneous coefficients B_1 to B_k, it is:

$$M_i^{res} = \sum_{j=1}^{n} a_{ij} M_i^* \quad (i=1 \text{ to } k), \tag{6}$$

where M_1^{res} to M_k^{res} are the generalized stresses that describe the residual state, "n", "a_{ij}" and "M_j^*" being defined in the same way as in Eq. (2).

The plasticity conditions have then to be formulated. Corresponding to Eq. (3) they are:

$$|M_i| = |M_i^{el} + M_i^{res}| \leqq R_i$$

or, for all loading cases together:

$$\begin{aligned} 0 &\leqq -M_i^{max} - M_i^{res} + R_i \\ 0 &\leqq \quad M_i^{min} + M_i^{res} + R_i \end{aligned} \quad (i=1 \text{ to } k). \tag{7}$$

Introducing Eq. (6):

$$\begin{aligned} 0 &\leqq -M_i^{max} - \sum_{j=1}^{n} a_{ij} M_j^* + R_i \\ 0 &\leqq \quad M_i^{min} + \sum_{j=1}^{n} a_{ij} M_j^* + R_i \end{aligned} \quad (i=1 \text{ to } k) \tag{8}$$

in matrix form:

$$
\begin{aligned}
0 &\leqq -m^{max} - A\,m^* + r\,. \\
0 &\leqq m^{min} + A\,m^* + r\,.
\end{aligned}
\tag{8}
$$

These restrictions together with the cost function (1) make up the linear program shown in Fig. 3. The free parameters of the problem are, first of all, the "n" redundants which determine the optimum residual stress state and consequently the optimum resistance distribution of the structure.

	1	m*	r
Cost =			c^t
$0 \leq$	$-m^{max}$	$-A$	U
$0 \leq$	m^{min}	A	U

Fig. 3. Table of the linear program for elastic-plastic optimum design.

A comparison between the linear program tables of Fig. 2 and 3 for rigid-plastic and elastic-plastic optimum design, shows immediately the great advantage of the second method. The dimensions of the matrix of the linear program are very much reduced as soon as there are several loading cases to consider. This is indeed the greatest advantage of the elastic-plastic method, considering that the amount of needed computation time for the solution of a linear program increases approximately with the cube of the matrix dimensions. The solution of the linear program for the truss structure of Fig. 1, with three loading cases requires, for example, 3.5 more storage and a nearly 10 times longer computation time by the rigid-plastic method than by the elastic-plastic one.

Fig. 4 gives the exact values as well as the cost function and the required cross-sectional areas of the bars for optimum rigid-plastic, optimum elastic-plastic and non-optimum elastic design. For the two last cases the elastic axial forces were calulated assuming the same cross section for all bars. As expected the rigid-plastic and the elastic-plastic solution lie near to each other. Other calculated examples support this statement. Thus it can be said that the elastic-plastic solution represents an approximation of the rigid-plastic one if

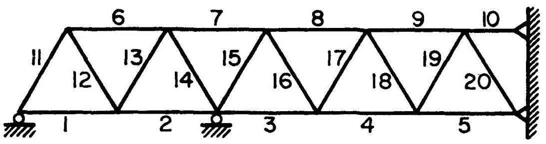
		Design method		
		rigid-plastic	elastic-plastic	ideal-elastic
Cost function		100.0	101.2	111.5
Dimensions of the matrix of the lin. prog.		120×26	40×22	
Computation time for the lin. prog.		30 sec.	4 sec.	
Cross sectional areas of the bars A_1 to A_{20}	1	2.07	2.22	2.30
	2	1.28	1.39	1.28
	3	0.78	0.74	1.05
	4	2.09	2.24	2.54
	5	1.72	1.39	1.10
	6	3.52	3.82	3.98
	7	2.97	3.10	2.78
	8	3.90	3.96	4.26
	9	3.28	3.18	3.48
	10	4.85	5.13	4.85
	11	3.91	4.20	4.35
	12	2.73	3.02	3.17
	13	5.53	5.24	5.09
	14	6.71	6.42	6.27
	15	7.08	7.26	7.25
	16	5.90	6.08	6.07
	17	2.36	2.18	2.19
	18	3.54	3.36	3.37
	19	7.08	7.26	7.27
	20	8.26	8.44	8.45

Fig. 4. Simple truss structure: results.

several loading cases are considered. For, in the case when only one loading configuration is considered, the two solutions are identical.

The second advantage of this methods is, of course, the shake down capacity of the structure. On the other hand this quality has to be payed. The elastic-plastic solution will be therefore, if the same loading cases are considered, always more expensive than the rigid-plastic one. However, this may not be considered as a disadvantage as it may be permissible in an elastic-plastic design, to reduce the load safety factor, the danger of incremental collaps being eliminated by the design method.

The principal disadvantage of the method is that the elastic stiffness properties of the members of the not yet designed structure have to be known to calculate the elastic stress distribution. Two remarks are here indicated. First, for reinforced concrete structures the design problem very often merely consists in finding the most advantageous distribution of the reinforcement, the dimensions and the concrete cross-sections of the structure being already fixed. It is generally assumed for the elastic calculation of the forces in a redundant concrete structure that the reinforcement does not change the stiffness properties of the structural members. Hence there will be no problem in calculating the elastic moment distribution even without any knowledge of the needed reinforcement.

Second, in other cases it will be necessary to make assumptions about these stiffness properties, which might then not correspond to the final design of the structure. However, a difference between the elastic stresses of the assumed and the actual structure endanger only the shake down capacity of the structure but not its rigid-plastic carrying capacity. This one will always be provided unless the plasticity conditions or the equilibrium of the system are not formulated correctly. On the other hand, the optimum elastic-plastic stress distribution which finally governs the design, will generally be only little influenced by the stiffness properties of the structure, less than the elastic stress distribution. It will then be possible, considering also the short computation time, to develop rapidly convergent iteration methods.

It may appear as a disadvantage that, quite contrary to the rigid plastic method, a complete elastic analysis is required. However, it should be considered that for the elastic analysis of structures, the most complicated part will generally be the problem specification, that is the imput phase more than the calculation itself. It will then be advisable to write a program that, with the same imput, both calculates, the elastic analysis as well as the elastic-plastic optimum design (see reference [1]). From the user's point of view it will be of no significance if the elastic analysis has to be made, the necessary computation time being very short in comparison with the time needed for the solution of the linear program.

4. Example

To show the practical possibilities of plastic design methods, the example of a reinforced concrete bridge will shortly be discussed. Fig. 5 shows the top view of a skewed bridge having five girders and several cross beams. The four quadratic columns are rigidly connected with the two main girders.

The shape of the structure is known. Fig. 6 shows its dimensions; Fig. 7 shows the three loading cases for which the bridge has to be designed. All loads and the bridge itself are symmetrical relative to the central point. The objective of the design is to minimize the total volume of the steel reinforcement.

Torsional moments as well as shear froces will be neglected. The design of the bridge will be based on the bending moment distribution. "Resistances" of the structure are therefore the plastic moments in a number of selected cross sections.

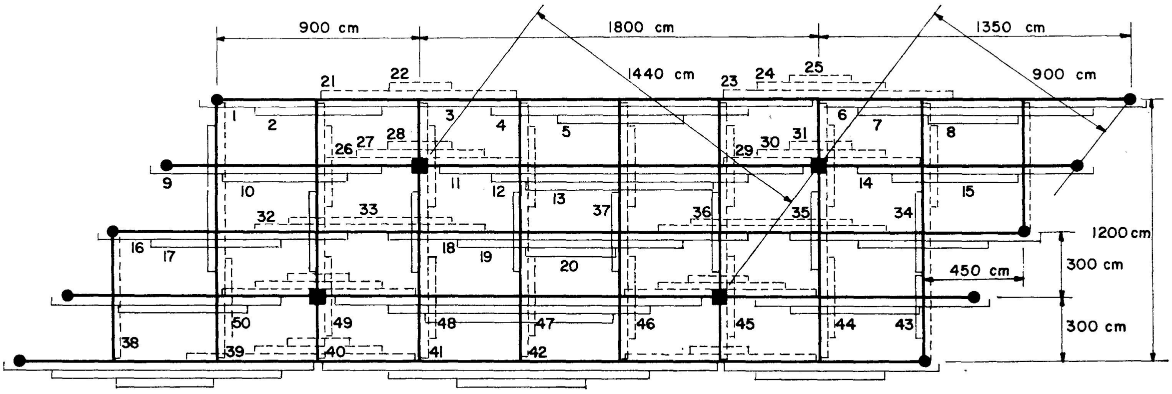


Fig. 5. Top-view and resistance-plan.

● Simple support

■ Column

└───┘ Resistance for positiv bending-moments (Numbers 1. . .20, 34. . .37)

└‐ ‐ ‐┘ Resistance for negativ bending-moments (Numbers 21. . .33, 38. . .50)

Fig. 6.

It can be assumed that the plastic moments R_i vary linearly with the reinforcement:

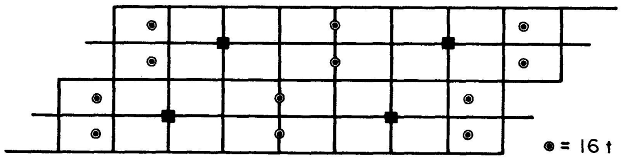
$$R_i = y_i A_i \sigma_y, \tag{9}$$

where A_i is the unknown cross-sectional steel area necessary for developping the plastic moment R_i, σ_y is the steel yield stress and the factor y_i represents the estimated distance between the reinforcement and the resulting compressive force in the concrete. The latter may be taken as independent of the steel percentage in an underreinforced section.

In order to relate the cost function to the total steel volume, the following relationship for the cost coefficient must hold:

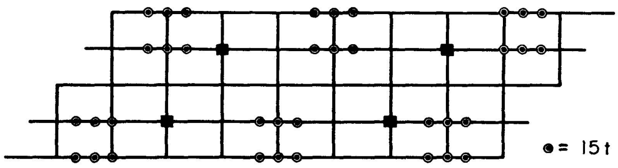
Dead-load: 5.0 t/m on girders

1. Loading-case



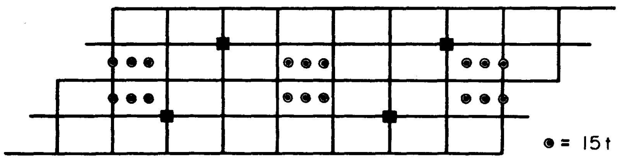
Uniform load: 2.5 t/m on girders + 12 concentrated loads of 16 t

2. Loading-case



36 concentrated loads of 15 t

3. Loading-case



18 concentrated loads of 15 t

Fig. 7. Loading-schedule.

$$c_i = \frac{l_i}{y_i}. \tag{10}$$

"l_i" is the length of the i-th reinforcement, hence the length of the region with the plastic moment R_i. As indicated in Fig. 5 the change of reinforcement occurs at discret points along the length of the girders. Furthermore a minimum percentage of reinforcement has been assumed for all sections, leading to further design restrictions.

Considering Eqs. (9) and (10) the cost function becomes:

$$\text{Cost} = \sum_i c_i R_i = \sum \frac{l_i}{y_i} y_i A_i \sigma_y = \sigma_y \sum_i l_i A_i$$

and therefore will correspond to the total steel volume if the constant factor σ_y is neglected.

It is quite obvious that the cost coefficients can only be determined with a limited degree of accuracy. On the other hand assumptions have to be made about the length and the position of the reinforcements when the bending moment distribution is completely unknown. Fig. 5 shows these assumptions: the numbers there are resistance numbers, the thin lines give the corresponding positions and lengths. It is clear that it will not always be possible to obtain already at the first trial a really useful solution because of the many assumptions which will not correspond to the final design of the structure. Fortunately the optimum plastic stress distribution will generally not be very much influenced by such assumptions and even less influenced will be the value of the cost function, so that a high accuracy will not be necessary in most cases.

The restrictions of the linear program, which are nothing but plasticity checks in different cross sections, are only formulated where they might become decisive for the corresponding plastic moments. By such a judicious selection the matrix dimensions of the linear program can be considerably reduced. On the other hand additional restrictions concerning minimum or maximum plastic moments are formulated in those sections where a minimum reinforcement will in any case be necessary or where a larger plastic moment can not be developped without enlarging the concrete section[3]).

The linear program for rigid-plastic optimum design requires a matrix with 177 rows and 108 columns, and 23′ 34″ computation time. For elastic-plastic design a 112×70 matrix and 5′ 11″ computation time was needed, including the complete elastic analysis of the 19-times statically indeterminated structure for the three loading cases.

The value of the cost function was, for the three design methods:

rigid-plastic	100.0,
elastic-plastic	106.1,
elastic	120.0.

The shake down ability of the bridge requires a cost increase of 6.1%. If no plastic deformations at all are permitted, 20.0% more reinforcement will be necessary. However, these results do not consider that the load safety factor

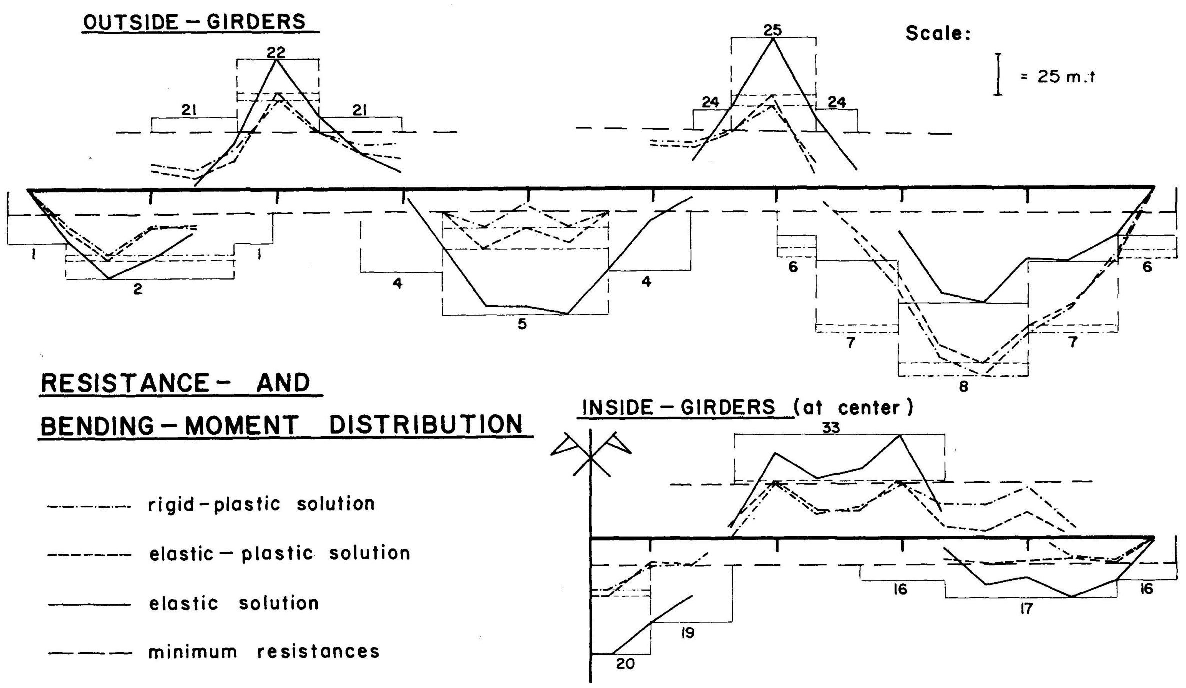


Fig. 8.

[3]) For formulation of design restrictions see references [1] or [2].

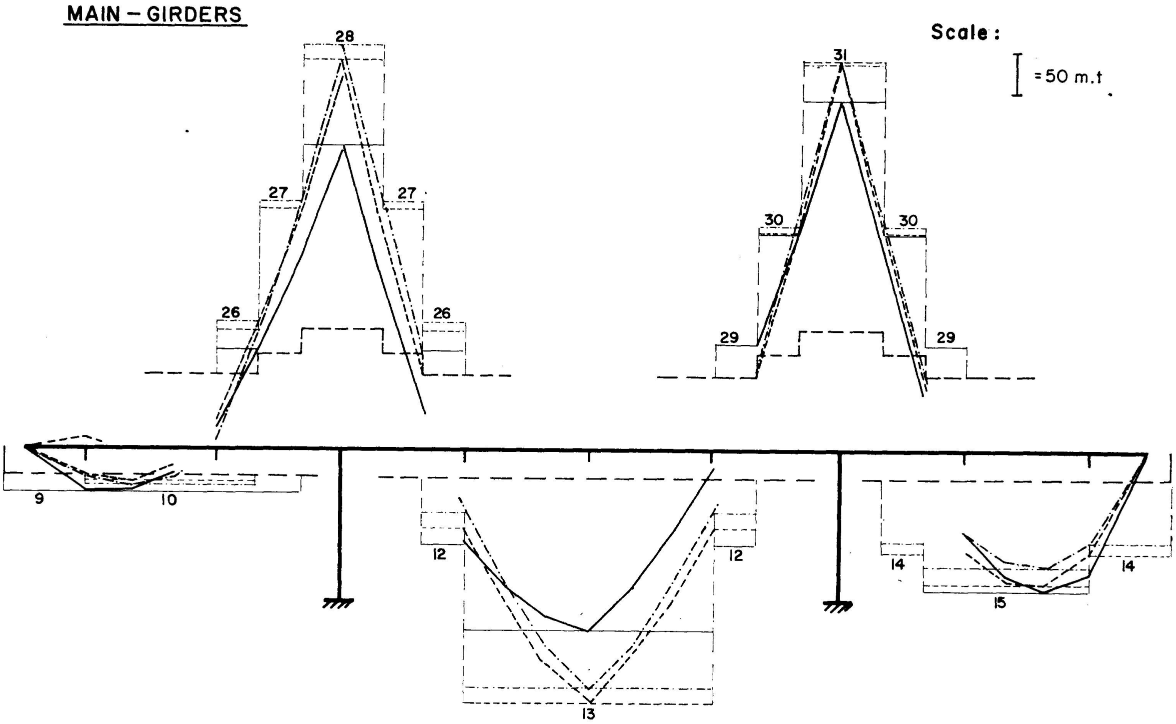


Fig. 9.

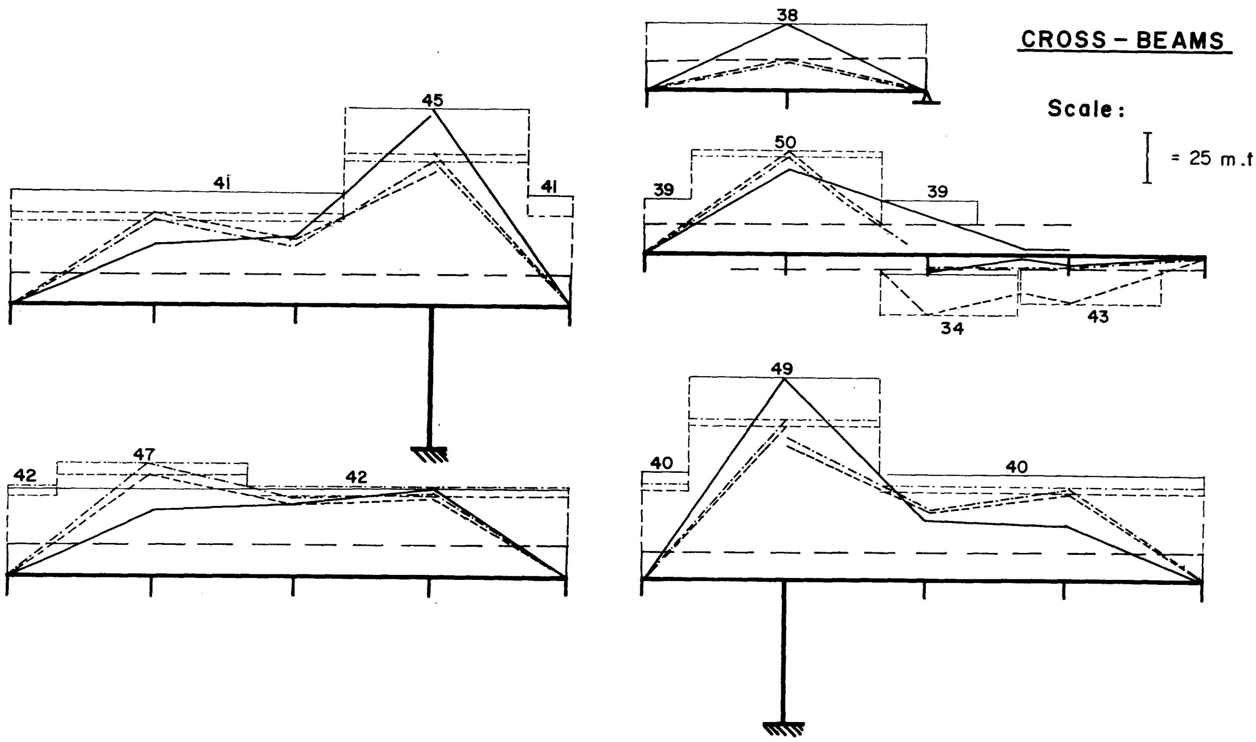


Fig. 10.

will be different for the three methods. This will slightly change the proportions, approaching the elastic-plastic solution to the rigid-plastic one and possibly reducing the advantages of both these solutions in comparison to the elastic one.

Fig. 8, 9 and 10 show the bending moment and the resistance distribution in all members of the bridge for the three design methods. The numbers there are the resistance numbers corresponding to those of Fig. 5. It can be seen that the elastic-plastic and the rigid-plastic solutions lie on the same side relative to the elastic design and generally very near to each other.

Fig. 8 shows also an other interesing result: the optimum plastic bending moments in the central girder are so much reduced that they almost nowhere exceed the minimum resistances. This proves the economic disadvantages of the whole central girder and suggests its elimination in a further development of the bridge design. This is a result which the elastic analysis would not show so clearly.

Outlook

A next step for the development of optimum design methods is certainly an accurate study of their real economic advantages. New procedures of non linear programming might be useful, especially for a better approximation of the cost function. The very important problem of elastic optimum design is still open, its general solution would very much enlarge the application of optimum design methods to a great number of structures for which the assumptions of the plastic theory are not acceptable.

Acknowledgments

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The numerical computations were performed on the CDC 1604-A Computer of the Computation Center at the Swiss Federal Institute of Technology.

List of References

1. E. ANDERHEGGEN: «Optimale Bemessung von Stabtragwerken.» Prom. No. 3710. Swiss Federal Institute of Technology. Zurich, Switzerland, 1965.
2. B. THÜRLIMANN: "Optimum Design of Structures." Lehigh University, Summer Conference 1965.
3. A.S.C.E.: "Commentary on Plastic Design in Steel." 1961.
4. B. G. NEAL: "The Plastic Methods of Structural Analysis." Chapman & Hall Ltd., 2nd. edition, London, 1963.

Summary

The aim of structural design is to develop a structure which exhibits the required safety margins under the prescribed loading configurations and at the same time represents an optimum solution according to appropriately selected criteria.

Applying plastic analysis, methods can be devised to develop directly an optimum design by linear programming procedures. A basic assumption for the applicability of such methods is the linear dependence of the cost of each part or member of the structure and its plastic carrying capacity. Furthermore, all the assumptions connected with plastic analysis are used.

First, an optimum design method is developped for structures under different loading configurations by assuming rigid-plastic behavior of the material. The corresponding linear program, based on the lower bound theorem of the plastic theory, is derived. The discussion of this method will then serve as an introduction to the so called elastic-plastic optimum design method. This is based on the "shake down" theorem and results in the design of a structure which, under the assumption of elastic-plastic behavior of the material, will be able to "stabilize" or to "shake down" for any possible loading cycle. The greatest advantage of this method will be the reduction of the dimensions of the matrix of the linear program, resulting in a considerable reduction of computation time in comparison to the rigid-plastic method. However a complete elastic analysis of the structure will be necessary.

The last part of the paper presents as a practical example the optimum design of a reinforced concrete bridge. The results of an "non-optimum" elastic, optimum elastic-plastic and an optimum rigid-plastic design are compared.

Résumé

L'étude conceptuelle et le dimensionnement ont pour but de créer un ouvrage qui, à la fois, présente la sécurité nécessaire sous les charges prévues et constitue une solution optimale à l'égard de critères déterminés.

Pour appliquer les méthodes de la programmation linéaire, il est fondamentalement nécessaire qu'existe une relation linéaire entre les coûts et la résistance des différents éléments qui composent la construction. En recourant aux méthodes du calcul plastique, on peut ainsi procéder à un dimensionnement optimal direct.

C'est tout d'abord dans le cas du comportement rigide-plastique des matériaux qu'on étudie le dimensionnement optimal d'un ouvrage soumis à différents systèmes de charges. Le programme linéaire correspondant est construit à partir du théorème de la limite inférieure de la théorie plastique. Dans une seconde partie, on exécute le dimensionnement en appliquant les méthodes

du calcul optimal élasto-plastique, calcul fondé sur le principe du «shake-down» de la théorie plastique (stabilisation des déformations). Le réel intérêt de cette méthode tient à ce que la matrice du programme linéaire est plus petite que dans le cas du calcul rigide-plastique et que le temps de calcul se trouve abrégé d'autant. En revanche, il faut en plus effectuer l'analyse élastique de l'ouvrage.

Dans la dernière partie, on indique, à titre d'illustration, l'exemple d'un pont à réseau de poutres en béton armé.

Zusammenfassung

Entwurf und Bemessung haben zum Ziel, ein Bauwerk zu schaffen, welches unter der vorgeschriebenen Belastung die notwendige Sicherheit aufweist und gleichzeitig eine optimale Lösung hinsichtlich festgelegten Kriterien darstellt.

Für die Anwendung linearer Programmierungsmethoden ist grundsätzlich eine lineare Beziehung zwischen Kosten und Widerstand der Elemente eines Tragwerkes notwendig. Unter Anwendung von plastischen Berechnungsverfahren läßt sich auf dieser Basis eine direkte optimale Bemessung vornehmen.

Zuerst wird die optimale Bemessung eines Tragwerkes unter verschiedenen Belastungszuständen für den Fall von starr-plastischem Materialverhalten untersucht. Das entsprechende lineare Programm ist auf dem unteren Grenzwertsatz der Plastizitätstheorie aufgebaut. In einem zweiten Teil wird die elastisch-plastische optimale Bemessung hergeleitet. Sie ist auf dem «Shake down»-Satz der Plastizitätstheorie (Stabilisierung der Verformungen) aufgebaut. Der eigentliche Vorteil dieser Methode liegt in einer Verkleinerung der Matrix des linearen Programms gegenüber der starr-plastischen Lösung und einer entsprechenden Verkürzung der Rechenzeit. Hingegen ist zusätzlich eine elastische Untersuchung des Tragwerkes notwendig.

Im letzten Teil wird zur Illustration das Beispiel einer Trägerrostbrücke in Stahlbeton angeführt.

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