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Prismatic Shells Continuous over Transverse Diaphragms

Voiles prismatiques continus sur diaphragmes transversaux

Durchlaufende Faltwerke mit Querscheiben

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Introduction

The increasing interest in the analysis of prismatic shell structures, also known as folded or hipped plate structures, in the last few years has led to the development of several techniques of analysis. GAAFAR [1]¹), YITZHAKI [2], SCORDELIS [3], PARME and SBAROUNIS [4] and MEEK [5], among others, suggested approximate methods for analyzing simply supported prismatic shell structures. Reviews of the various approximate methods for analyzing such structures can be found in the report of the ASCE Task Committee on folded plate construction [6], in which an extensive list of references is given, and in the work of POWELL [7]. An exact method for analyzing the same problem was presented by GOLDBERG and LEVE [8]. More recently the analysis of continuous prismatic shell structures has received some attention. YITZHAKI and REISS [9], SHARMA and GOYAL [10], and BEAUFAIT [11] discussed approximate methods for analyzing continuous prismatic shell structures. Exact solutions for such problems have been suggested by LEE, PULMANO and LIN [12], PULMANO and LEE [13], and GOLDBERG, GUTZWILLER and LEE [14].

This paper presents a method for analyzing multiple bay, multiple span prismatic shell structures, continuous over intermediate transverse diaphragms and simply supported at the two end diaphragms as shown in Fig. 1.

The proposed method is based on the following assumptions: The material is homogeneous and isotropic with equal moduli of elasticity in tension and

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¹) Numbers in brackets refer to items in the list of References.

compression; it is not strained beyond its elastic limit; plane sections of each individual plate remain plane after bending; the displacements are small for each plate; the supporting diaphragms are infinitely rigid in their planes but flexible normal to their planes; and the influence of the membrane forces on the bending of the plate is neglected. These assumptions result in tow fourth-order differential equations, which govern the bending of the plate under the action of the normal load component and the membrane action of the plate under the in-plane load components.

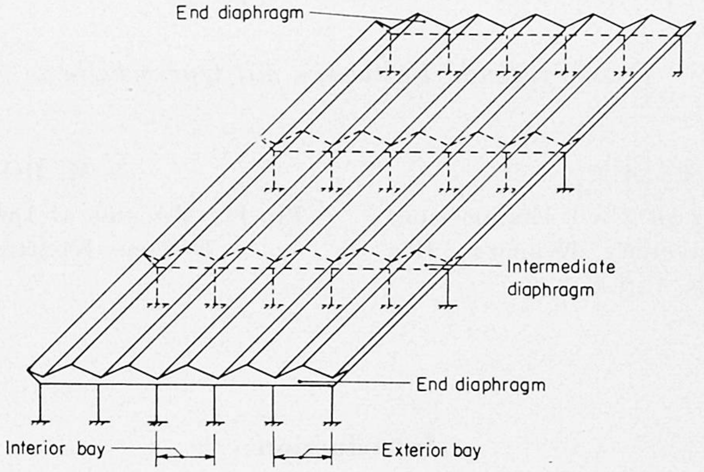


Fig. 1. Prismatic shells with intermediate diaphragms.

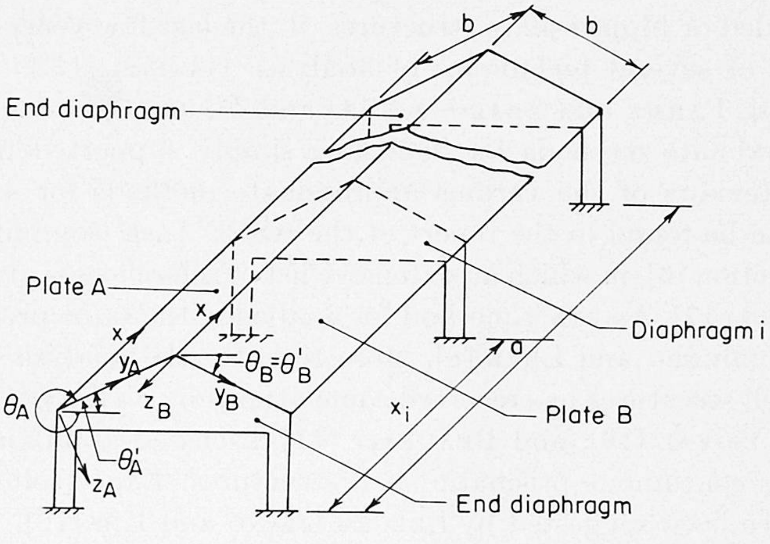
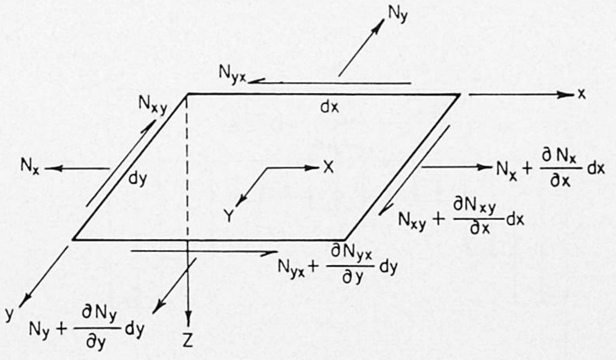
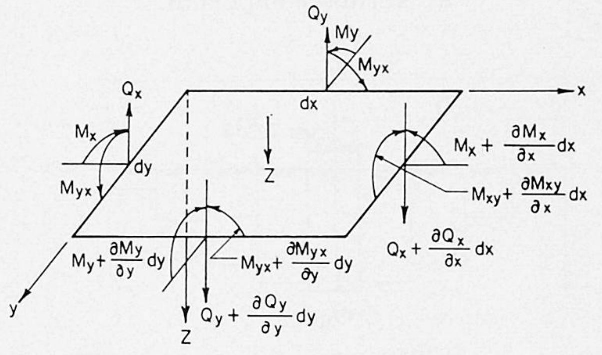


Fig. 2. Orientation of coordinate axes.

The adoption of a clear sign convention is essential especially when the problem is programmed for computer analysis. The three orthogonal coordinate axes, x, y and z are taken, respectively, along the longitudinal, transverse and normal directions, as shown in Fig. 2, with the corner as the origin. The angle θ, which defines the slope of each plate, is measured clockwise from a horizonal plane passing through the origin to the positive y axis. θ' is the absolute value of the acute angle between the horizontal plane and the plane of the plate. The positive directions of the load components X, Y and Z, and the stress resultants are shown in Figs. 3a and 3b. The displacement components, u, v and w are positive in the positive directions of the coordinate axes x, y and z, respectively.



a) Membrane stress resultants.



b) Bending stress resultants.

Fig. 3. Positive directions of stress resultants and load components.

The method of analysis consists of the following steps:

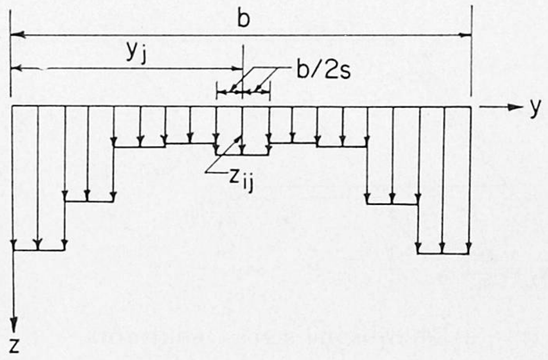
a) The reactions at an intermediate diaphragm are resolved into components along the normal and transverse directions, respectively. The exact distributions of the diaphragm reactions are not known a priori and will be approximated by uniform step functions for the normal and transverse components, shown in Figs. 4a and 4b respectively, which are then expaned into double Fourier series satisfying the boundary conditions at the two transverse edges.

b) The normal and transverse displacements at the center of each step reaction resulting from the applied loads, in the absence of the intermediate diaphragms, are determined.

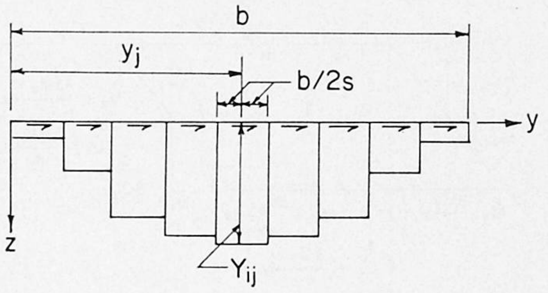
c) The flexibility influence coefficients at these locations are calculated by applying unit step reactions along the normal and transverse directions, respectively.

d) Using the results of steps *b* and *c*, the correct value of the step reactions along the normal and transverse directions are determined from the compatability conditions that the normal and transverse displacements at the center of the step reactions vanish.

e) With the step reactions at the intermediate diaphragms along the normal and transverse directions known, superposition of steps b, c give the total stress resultants and displacements of the structure due to the applied loads.



a) Normal component.



b) Transverse component.

Fig. 4. Approximation by step functions of reactions of intermediate diaphragm at $x=x_i$.

Bending Action

The differential equation governing the bending of the plates [15] subjected to normal load component Z is given by

$$\nabla^4 w = Z/D, \tag{1}$$

in which $D = E h^3/[12(1-\mu^2)]$, h denotes plate thickness, E the modulus of elasticity, and μ Poisson's ratio. The general solution of Eq. (1) is

$$w = w_p + w_c, \tag{2}$$

in which w_p is a particular integral and w_c the complementary solution. A particular integral satisfying Eq. (1) and the boundary conditions at the two transverse edges can be obtained by expanding w_p and Z in double series of the form

$$w_p = \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} w_{mn} \sin \alpha x \cos \beta y , \tag{3}$$

$$Z(x, y) = \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} Z_{mn} \sin \alpha x \cos \beta y . \tag{4}$$

where $\alpha = m \pi / a$, $\beta = n \pi / b$, and W_{mn} and Z_{mn} are Fourier coefficients. Substituting Eqs. (3), (4) into Eq. (1) gives

$$W_{mn} = \frac{Z_{mn}}{D(\alpha^2 + \beta^2)^2}, \tag{5}$$

in which $$Z_{mn} = \frac{4}{a b} \int_0^a \int_0^b Z(x, y) \sin \alpha x \cos \beta y \, dx \, dy \quad \text{for } n \neq 0 , \tag{6}$$

$$Z_{m0} = \frac{2}{a} \int_0^a Z(x, y) \sin \alpha x \, dx \quad \text{for } n = 0 . \tag{7}$$

In view of Eq. (3) it can be seen that the complementary solution w_c, satisfying the homogeneous part of Eq. (1) and the boundary conditions at $x = 0$ and $x = a$, can be taken in the form

$$w_c = \sum_{m=1}^{\infty} \Psi_m \sin \alpha x , \tag{8}$$

where Ψ_m is a function of y only. Substituting Eq. (8) into Eq. (1) and setting $Z = 0$, it can be shown that Ψ_m takes the form

$$\Psi_m = K \{ B_{1m} e^{\alpha y} + B_{2m} e^{-\alpha y} + B_{3m} \alpha y e^{\alpha y} + B_{4m} \alpha y e^{-\alpha y} \} , \tag{9}$$

where K is a load factor introduced to nondimentionalize the constants of integration B_{1m} to B_{4m} and will be defined for particular loading later.

The complete solution is then obtained by substituting Eqs. (3), (8), (9) into Eq. (2), yielding

$$\begin{aligned} w = \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} W_{mn} \sin \alpha x \cos \beta y + K \sum_{m=1}^{\infty} \{ & B_{1m} e^{\alpha y} + B_{2m} e^{-\alpha y} \\ & + B_{3m} \alpha y e^{\alpha y} + B_{4m} \alpha y e^{-\alpha y} \}^2 \sin \alpha x . \end{aligned} \tag{10}$$

The corrsponding bending stress resultants [15], shown in Fig. 3b, are given by

$$\begin{aligned} M_x = {} & D \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} (\alpha^2 + \mu \beta^2) W_{mn} \sin \alpha x \cos \beta y + D K \sum_{m=1}^{\infty} \alpha^2 \{ B_{1m} (1 - \mu) e^{\alpha y} \\ & + B_{2m} (1 - \mu) e^{-\alpha y} + B_{3m} [\alpha y (1 - \mu) - 2 \mu] e^{\alpha y} \\ & + B_{4m} [\alpha y (1 - \mu) + 2 \mu] e^{-\alpha y} \} \sin \alpha x , \end{aligned} \tag{11}$$

$$\begin{aligned} M_y = {} & D \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} (\beta^2 + \mu \alpha^2) W_{mn} \sin \alpha x \cos \beta y - D K \sum_{m=1}^{\infty} \alpha^2 \{ B_{1m} (1 - \mu) e^{\alpha y} \\ & + B_{2m} (1 - \mu) e^{-\alpha y} + B_{3m} [\alpha y (1 - \mu) + 2] e^{\alpha y} \\ & + B_{4m} [\alpha y (1 - \mu) - 2] e^{-\alpha y} \} \sin \alpha x , \end{aligned} \tag{12}$$

$$\begin{aligned} M_{xy} = M_{yx} = {} & D(1-\mu) \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \alpha \beta W_{mn} \cos \alpha x \sin \beta y \\ & - D(1-\mu) K \sum_{m=1}^{\infty} \alpha^2 \{B_{1m} e^{\alpha y} - B_{2m} e^{-\alpha y} \\ & + B_{3m} (\alpha y + 1) e^{\alpha y} + B_{4m} (1 - \alpha y) e^{-\alpha y}\} \cos \alpha x , \end{aligned} \tag{13}$$

$$\begin{aligned} Q_x & = D \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \alpha (\alpha^2 + \beta^2) W_{mn} \cos \alpha x \cos \beta y \\ & - 2 D K \sum_{m=1}^{\infty} \alpha^3 \{B_{3m} e^{\alpha y} - B_{4m} e^{-\alpha y}\} \cos \alpha x , \end{aligned} \tag{14}$$

$$\begin{aligned} Q_y & = - D \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \beta (\alpha^2 + \beta^2) W_{mn} \sin \alpha x \sin \beta y \\ & - 2 D K \sum_{m=1}^{\infty} \alpha^3 \{B_{3m} e^{\alpha y} + B_{4m} e^{-\alpha y}\} \sin \alpha x , \end{aligned} \tag{15}$$

$$\begin{aligned} R_x & = D \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \alpha \{\alpha^2 + \beta^2 (2 - \mu)\} W_{mn} \cos \alpha x \cos \beta y \\ & - D K \sum_{m=1}^{\infty} \alpha^3 \{B_{1m} (1-\mu) e^{\alpha y} + B_{2m} (1-\mu) e^{-\alpha y} \\ & + B_{3m} [\alpha y (1-\mu) + 2(2-\mu)] e^{\alpha y} + B_{4m} [\alpha y (1-\mu) - 2(2-\mu)] e^{-\alpha y}\} \cos \alpha x , \end{aligned} \tag{16}$$

$$\begin{aligned} R_y & = - D \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \beta [\beta^2 + \alpha^2 (2 - \mu)] W_{mn} \sin \alpha x \sin \beta y \\ & + D K \sum_{m=1}^{\infty} \alpha^3 \{B_{1m} (1-\mu) e^{\alpha y} - B_{2m} (1-\mu) e^{-\alpha y} \\ & + B_{3m} [\alpha y (1-\mu) - (1+\mu)] e^{\alpha y} + B_{4m} [\alpha y (\mu - 1) - (1+\mu)] e^{-\alpha y}\} \sin \alpha x , \end{aligned} \tag{17}$$

in which M_x, M_y and M_{xy} are the longitudinal, transverse and torsional moments per unit length respectively, Q_x and Q_y the longitudinal and transverse shearing force per unit length respectively, and R_x and R_y the longitudinal and transverse edge reactions per unit length respectively.

Membrane Action

For the common case where the longitudinal in-plane load component X is zero, the differential equation governing the plane stress problem of the plates [16] subjected to the transverse load component Y is given by

$$\nabla^4 \phi = \frac{\partial^2}{\partial x^2} (\int Y \, dy) - \mu \frac{\partial Y}{\partial y}, \tag{18}$$

where ϕ is an Airy stress function related to the stress resultants by

$$N_x = \frac{\partial^2 \phi}{\partial y^2}, \tag{19}$$

$$N_y = \frac{\partial^2 \phi}{\partial x^2} - \int Y \, dy, \tag{20}$$

$$N_{xy} = -\frac{\partial^2 \phi}{\partial x \, \partial y}, \tag{21}$$

in which N_x, N_y and N_{xy} are the longitudinal normal force, the transverse normal force, and the membrane shearing force, per unit length, respectively.

The general solution of Eq. (18) takes the form

$$\phi = \phi_p + \phi_c, \tag{22}$$

in which ϕ_p is a particular integral of Eq. (18) and ϕ_c the complementary solution.

A particular integral satisfying Eq. (18) and the boundary conditions at the two transverse edges $x = 0$ and $x = a$, can be obtained by expanding ϕ_p and Y in double series of the form

$$\phi_p = \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \phi_{mn} \sin \alpha x \cos \beta y, \tag{23}$$

$$Y(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} Y_{mn} \sin \alpha x \sin \beta y, \tag{24}$$

in which

$$Y_{mn} = \frac{4}{a b} \int_0^a \int_0^b Y(x, y) \sin \alpha x \sin \beta y \, dx \, dy. \tag{25}$$

Integrating Eq. (24) with respect to y yields

$$\int Y \, dy = -\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} Y_{mn} \frac{1}{\beta} \sin \alpha x \cos \beta y + F(x), \tag{26}$$

in which $F(x)$ is a constant of integration and can be expanded in a single series of the form

$$F(x) = \sum_{m=1}^{\infty} F_m \sin \alpha x, \tag{27}$$

substituting Eq. (27) into Eq. (26) leads to

$$\int Y \, dy = -\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} Y_{mn} \frac{1}{\beta} \sin \alpha x \cos \beta y + \sum_{m=1}^{\infty} F_m \sin \alpha x, \tag{28}$$

substituting Eqs. (23), (24), (28) into Eq. (18) and solving for ϕ_{mn} yield

$$\phi_{m0} = -\frac{1}{\alpha^2} F_m \qquad \text{for } n = 0, \tag{29}$$

$$\phi_{mn} = \frac{\alpha^2 - \mu \beta^2}{\beta (\alpha^2 + \beta^2)^2} Y_{mn} \qquad \text{for } n \neq 0. \tag{30}$$

Referring to Eqs. (19) to (21), it is seen that F_m given by Eq. (29) has no effect on the in-plane stresses and is given only for completeness of the solution.

In view of Eqs. (19), (23), the complementary solution ϕ_c satisfying the boundary conditions at the transverse edges $x=0$ and $x=a$, can be taken in the form

$$\phi_c = \sum_{m=1}^{\infty} \xi_m \sin\alpha x\,, \tag{31}$$

in which ξ_m is a function of y only. Substituting Eq. (31) into Eq. (18) and setting $Y=0$, it can be shown that the solution of the resulting equation takes the form

$$\xi_m = L\{A_{1m}\,e^{\alpha y} + A_{2m}\,e^{-\alpha y} + A_{3m}\,\alpha\,y\,e^{\alpha y} + A_{4m}\,\alpha\,y\,e^{-\alpha y}\}\,, \tag{32}$$

in which L is another load factor which nondimentionalizes the costants of integration A_{1m} to A_{4m}.

The complete solution of Eq. (18) is obtained by substituting Eqs. (23), (31), (32) into Eq. (22), giving

$$\begin{aligned}\phi = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\phi_{mn}\sin\alpha x\cos\beta y + L\sum_{m=1}^{\infty}\{A_{1m}\,e^{\alpha y} + A_{2m}\,e^{-\alpha y}\\ + A_{3m}\,\alpha\,y\,e^{\alpha y} + A_{4m}\,\alpha\,y\,e^{-\alpha y}\}\sin\alpha x\,.\end{aligned} \tag{33}$$

Substituting Eqs. (24), (33) into Eqs. (19) to (21) gives the membrane stress resultants

$$\begin{aligned}N_x = &-\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\phi_{mn}\,\beta^2\sin\alpha x\cos\beta y + L\sum_{m=1}^{\infty}\alpha^2\{A_{1m}\,e^{\alpha y}\\ &+ A_{2m}\,e^{-\alpha y} + A_{3m}(2+\alpha y)\,e^{\alpha y} + A_{4m}(-2+\alpha y)\,e^{-\alpha y}\}\sin\alpha x\,,\end{aligned} \tag{34}$$

$$\begin{aligned}N_y = &-\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\phi_{mn}\,\alpha^2\sin\alpha x\cos\beta y - L\sum_{m=1}^{\infty}\alpha^2\{A_{1m}\,e^{\alpha y}\\ &+ A_{2m}\,e^{-\alpha y} + A_{3m}\,\alpha\,y\,e^{\alpha y} + A_{4m}\,\alpha\,y\,e^{-\alpha y}\}\sin\alpha x\,,\end{aligned} \tag{35}$$

$$\begin{aligned}N_{xy} = &\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\phi_{mn}\,\alpha\beta\cos\alpha x\sin\beta y - L\sum_{m=1}^{\infty}\alpha^2\{A_{1m}\,e^{\alpha y}\\ &- A_{2m}\,e^{-\alpha y} + A_{3m}(1+\alpha y)\,e^{\alpha y} + A_{4m}(1-\alpha y)\,e^{-\alpha y}\}\cos\alpha x\,.\end{aligned} \tag{36}$$

Membrane Displacements

The relationship between the membrane stress resultants and the longitudinal and transverse displacements of the middle plate surface are [16]

$$N_x = \frac{E\,h}{(1-\mu^2)}\left(\frac{\partial u}{\partial x} + \mu\frac{\partial v}{\partial y}\right), \tag{37}$$

$$N_y = \frac{E\,h}{(1-\mu^2)}\left(\frac{\partial v}{\partial y} + \mu\frac{\partial u}{\partial x}\right), \tag{38}$$

$$N_{xy} = \frac{E\,h}{2\,(1+\mu)}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right). \tag{39}$$

Substituting Eqs. (34), (35) into Eqs. (37), (38) and solving for u and v yield

$$\begin{aligned} u = {} & \frac{1}{E\,h}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left\{\phi_{mn}\frac{\beta^2}{\alpha} - \mu\,\phi_{mn}\,\alpha + \mu\,Y_{mn}\frac{1}{\alpha\,\beta}\right\}\cos\alpha\,x\cos\beta\,y \\ & - \frac{L}{E\,h}\sum_{m=1}^{\infty}\alpha\,\{A_{1m}(1+\mu)\,e^{\alpha y} + A_{2m}(1+\mu)\,e^{-\alpha y} + A_{3m}(2+\alpha\,y+\mu\,\alpha\,y)\,e^{\alpha y} \\ & + A_{4m}(-2+\alpha\,y+\mu\,\alpha\,y)\,e^{-\alpha y}\}\cos\alpha\,x + \frac{1}{E\,h}S\,(y)\,, \end{aligned} \tag{40}$$

$$\begin{aligned} v = {} & \frac{1}{E\,h}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left\{\mu\,\phi_{mn}\,\beta - \phi_{mn}\frac{\alpha^2}{\beta} + Y_{mn}\frac{1}{\beta^2}\right\}\sin\alpha\,x\sin\beta\,y \\ & - \frac{L}{E\,h}\sum_{m=1}^{\infty}\alpha\,\{A_{1m}(1+\mu)\,e^{\alpha y} - A_{2m}(1+\mu)\,e^{-\alpha y} - A_{3m}[(1-\mu) - \alpha\,y\,(1+\mu)]\,e^{\alpha y} \\ & - A_{4m}[(1-\mu) + \alpha\,y\,(1+\mu)]\,e^{-\alpha y}\}\sin\alpha\,x + \frac{1}{E\,h}R\,(x)\,, \end{aligned} \tag{41}$$

where $\frac{1}{E\,h}S\,(y)$ and $\frac{1}{E\,h}R\,(x)$ are constants of integration. These can be expanded into single series of the form

$$S\,(y) = \frac{S_0}{2} + \sum_{n=1}^{\infty}S_n\cos\beta\,y\,, \tag{42}$$

$$R\,(x) = \sum_{m=1}^{\infty}R_m\sin\alpha\,x\,, \tag{43}$$

in which

$$S_n = \frac{2}{b}\int_0^b S\,(y)\cos\beta\,y\,dy \qquad (n = 0, 1, 2 \ldots)\,, \tag{44}$$

$$R_m = \frac{2}{a}\int_0^a R\,(x)\sin\alpha\,x\,dx \qquad (m = 1, 2, 3 \ldots)\,. \tag{45}$$

Substituting Eqs. (36), (40), (41) into Eq. (39), multiplying both sides of the resulting equation by $\cos\beta\,y$ and integrating with respect to x from zero to a give

$$R_m = \sum_{n=1}^{\infty}\phi_{mn}\frac{(\alpha^2+\beta^2)^2}{\alpha^2\beta}\sin\beta\,y - \sum_{n=1}^{\infty}\frac{Y_{mn}(\alpha^2-\mu\,\beta^2)}{\alpha^2\beta^2}\sin\beta\,y\,. \tag{46}$$

Repeating the same proceduer, but multiplying both sides by $\sin\beta\,y$, integrating with respect to y from zero to b and giving regard to Eq. (46) leads to $S_n = 0$ for all values of n.

It can be readily verified that the solutions, Eqs. (10), (40), (41), satisfy the boundary conditions at the two simply supported transverse edges at $x = 0$ and $x = a$, i. e., $N_x = v = w = M_x = 0$.

Applied Loads

For a uniformly distributed load in the longitudinal and transverse directions, the Fourier coefficients Z_{mn} and Y_{mn} defined by Eqs. (6), (7), (25) are given respectively by

$$Z_{mn} = 0 \qquad \text{for even } m \text{ or } n \neq 0\,, \tag{47}$$

$$Z_{m0} = \frac{4Z}{m\pi} \qquad \text{for odd } m, n = 0\,, \tag{48}$$

$$Y_{mn} = 0 \qquad \text{for even } m, n\,, \tag{49}$$

$$Y_{mn} = \frac{16\,Y}{m\,n\,\pi^2} \quad \text{for odd } m, n\,. \tag{50}$$

For uniformly distributed dead load q per unit area of the middle surface, the load components are defined by

$$Y = q \sin\theta\,, \tag{51}$$

$$Z = q \cos\theta\,. \tag{52}$$

The corresponding dimensional factor K and L in Eqs. (9), (32) are

$$K = a^4 q \cos\theta / D\,, \tag{53}$$

$$L = a^3 q \sin\theta\,. \tag{54}$$

The components of a uniformly distributed live load p per unit area of horizontal projection are

$$Y = p \sin\theta \cos\theta\,, \tag{55}$$

$$Z = p \cos^2\theta\,. \tag{56}$$

The corresponding K and L in Eqs. (9), (32) are, in this case

$$K = a^4 p \cos^2\theta / D\,, \tag{57}$$

$$L = a^3 p \sin\theta \cos\theta\,. \tag{58}$$

Reactions of Intermediate Diaphragms

The line reactions of an intermediate transverse diaphragm, diaphragm i at $x = x_i$, are resolved into normal and transverse components along the z and y axes respectively for each individual plate. An approximation for the distribution of the normal component $Z(x_i, y)$ can be obtained by dividing the length b of the diaphragm into s equal divisions and assuming that the intensity of the line reaction in each division is uniform. This approximation is represented by the step function shown in Fig. 4a. The intensity of the step reaction, the center of which is located at $x = x_i$ and $y = y_i$, is defined by Z_{ij}. Similarly the distribution of the transverse component $Y(x_i, y)$ of the reactions of diaphragm i will be approximated by another step function as shown in

Fig. 4b. The intensity of the step reaction, the center of which is located at $x = x_i$ and $y = y_i$, is defined by Y_{ij}.

In view of Eqs. (4), (24), the step reactions Z_{ij} and Y_{ij} can be represented by the double Fourier series

$$Z_{ij} = \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} Z_{ijmn} \sin \alpha x \cos \beta y\,, \tag{59}$$

$$Y_{ij} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} Y_{ijmn} \sin \alpha x \sin \beta y\,, \tag{60}$$

in which

$$Z_{ijmn} = \frac{4}{a\,b} \lim_{\eta \to 0} \int\limits_{x_i-\eta}^{x_i+\eta} \int\limits_{y_j-c}^{y_j+c} \frac{Z_{ij}}{2\,\eta} \sin \alpha x \cos \beta y \, dx \, dy = \frac{8\,Z_{ij}}{a\,b\,\beta} \sin \beta c \sin \alpha x_i \cos \beta y_j \qquad \text{for } n \neq 0\,, \tag{61}$$

$$Z_{ijm0} = \frac{2}{a\,b} \lim_{\eta \to 0} \int\limits_{x_i-\eta}^{x_i+\eta} \int\limits_{y_j-c}^{y_j+c} \frac{Z_{ij}}{2\,\eta} \sin \alpha x \, dx \, dy = \frac{4\,c\,Z_{ij}}{a\,b} \sin \alpha x_i \qquad \text{for } n = 0\,, \tag{62}$$

$$Y_{ijmn} = \frac{4}{a\,b} \lim_{\eta \to 0} \int\limits_{x_i-\eta}^{x_i+\eta} \int\limits_{y_j-c}^{y_j+c} \frac{Y_{ij}}{2\,\eta} \sin \alpha x \sin \beta y \, dx \, dy = \frac{8\,Y_{ij}}{a\,b\,\beta} \sin \beta c \sin \alpha x_i \sin \beta y_j \tag{63}$$

and $c = b/(2\,s)$.

For the normal step reaction Z_{ij}, K and L in Eqs. (9), (32) are given by

$$K = a^3\,Z_{ij} \cos \theta / D\,, \tag{64}$$

$$L = a^2\,Z_{ij} \sin \theta\,. \tag{65}$$

For the transverse step reaction Y_{ij}, K and L take the form

$$K = a^3\,Y_{ij} \cos \theta / D\,, \tag{66}$$

$$L = a^2\,Y_{ij} \sin \theta\,. \tag{67}$$

The value of Z_{ij} and Y_{ij} are obtained by applying the compatability conditions that the normal and transverse displacement components at the center of each step reaction are zeros. These conditions are expressed by

$$\sum_{i=1}^{r} \sum_{j=1}^{s} w_{klij}\,Z_{ij} + w_k^0 = 0\,, \tag{68}$$

$$\sum_{i=1}^{r} \sum_{j=1}^{s} v_{klij}\,Y_{ij} + v_{kl}^0 = 0\,, \tag{69}$$

in which r denotes the number of intermediate transverse diaphragms, s the number of step reactions in each diaphragm defined previously, w_{kl}^0 and v_{kl}^0 the normal and transverse displacement components at point x_k, y_l respectively due to the applied loads alone, and w_{klij} and v_{klij} the normal and transverse displacement components at point x_k, y_l due to $Z_{ij} = 1$ and $Y_{ij} = 1$ respectively. These values of w and v are computed by means of Eqs. (10), (41) respectively.

Solution for Typical Interior Bay

As an illustrative example, consider the two-plate symmetrical bay in a continuous prismatic shell roof such as the one shown in Fig. 1. Assuming that the number of bays is large, for the purpose of analyzing a typical interior bay this number may be assumed to be infinite for design purposes. Hence each longitudinal edge lies in a vertical plane of symmetry and only one plate need be considered. The boundary conditions on the two longitudinal edges are

$$N_{xy} = 0\,, \tag{70}$$

$$\frac{\partial w}{\partial y} = 0\,, \tag{71}$$

$$v \cos\theta - w \sin\theta = 0\,, \tag{72}$$

$$N_y \sin\theta + R_y \cos\theta = 0\,. \tag{73}$$

Consider plate B of an interior bay shown in Fig. 2, which is subjected to uniformly distributed dead and/or live loads, and normal and transverse step reactions at the intermediate transverse diaphragms. Substituting the Fourier coefficients Z_{mn} and Y_{mn} from Eqs. (48), (50); the load factors K and L from Eqs. (53), (45) for dead load, and from Eqs. (57), (58) for live load; the Fourier coefficients Z_{ijmn} and Y_{ijmn} from Eqs. (61) to (63) for the step reactions; the corresponding load factors K and L from Eqs. (64) to (67); the displacement components w and v from Eqs. (10), (41); and the stress resultants R_y, N_y, N_{xy} from Eqs. (17), (35), (36) into Eqs. (70) to (73) lead to, for $y=0$,

$$A_{1m} - A_{2m} + A_{3m} + A_{4m} = 0\,, \tag{74}$$

$$B_{1m} - B_{2m} + B_{3m} + B_{4m} = 0\,, \tag{75}$$

$$\begin{aligned} A_{1m} + A_{2m} + m\pi\{&-B_{1m}(1-\mu) + B_{2m}(1-\mu) + B_{3m}(1+\mu) + B_{4m}(1+\mu)\}\cot^2\theta \\ &+ \lambda \frac{16\,(b/a)}{m^3\pi^5} \sum_{n=1,3,5} \frac{1}{n^2}\left\{1 - \frac{m^4 - (\mu\, m^2 n^2 a^2/b^2)}{[m^2 + (n^2 a^2/b^2)]^2}\right\} \\ &+ \epsilon \frac{8 \sin\alpha x_i}{m^2 \pi^4 (a/b) \sin\theta} \sum_{n=1,2,3}^{\infty} \frac{1}{n^2}\left\{1 - \frac{m^2 - (\mu\, n^2 a^2/b^2)}{[m + (n^2 a^2/m\, b^2)]^2}\right\} \sin\beta c \sin\beta y_j\,, \end{aligned} \tag{76}$$

$$\begin{aligned} A_{1m}(1+\mu) &- A_{2m}(1+\mu) - A_{3m}(1-\mu) - A_{4m}(1-\mu) \\ &+ \left(\frac{a}{h}\right)^2 \frac{12\,(1-\mu)}{m\pi}(B_{1m} + B_{2m}) = -\lambda \left(\frac{a}{h}\right)^2 \frac{48\,(1-\mu^2)}{(m\pi)^6} \\ &- \sigma \left(\frac{a}{h}\right)^2 \frac{48\,(1-\mu^2)\sin\alpha x_i}{m\pi^5 \cos\theta}\left\{\frac{(c/b)}{m^4} + \frac{2}{\pi}\sum_{n=1,2,3}^{\infty} \frac{\sin\beta c \cos\beta y_j}{n\,[m^2 + (n^2 a^2/b^2)]^2}\right\} \end{aligned} \tag{77}$$

and, for $y = b$,

$$A_{1m} e^{\alpha b} - A_{2m} e^{-\alpha b} + A_{3m}(1 + \alpha b)\, e^{\alpha b} + A_{4m}(1 - \alpha b)\, e^{-\alpha b} = 0\,, \tag{78}$$

$$B_{1m} e^{\alpha b} - B_{2m} e^{-\alpha b} + B_{3m}(1 + \alpha b)\, e^{\alpha b} + B_{4m}(1 - \alpha b)\, e^{-\alpha b} = 0\,, \tag{79}$$

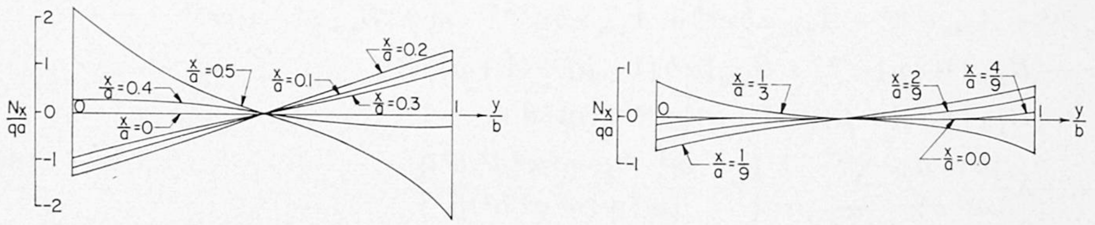
$$\begin{aligned}
&A_{1m}\, e^{\alpha b} + A_{2m}\, e^{-\alpha b} + A_{3m}\, \alpha b\, e^{\alpha b} + A_{4m}\, \alpha b\, e^{-\alpha b} - m\pi \{B_{1m} (1-\mu)\, e^{\alpha b} \\
&\quad - B_{2m} (1-\mu)\, e^{-\alpha b} + B_{3m} [\alpha b (1-\mu) - (1+\mu)]\, e^{\alpha b} \\
&\quad - B_{4m} [\alpha b (1-\mu) + (1+\mu)]\, e^{-\alpha b}\} \cot^2\theta = \\
&\quad -\lambda \frac{16\,(b/a)}{m^3 \pi^5} \sum_{n=1,3,5} \frac{1}{n^2} \left\{1 - \frac{m^4 - (\mu m^2 n^2 a^2/b^2)}{[m^2 + (n^2 a^2/b^2)]^2}\right\} \qquad (80) \\
&\quad + \epsilon \frac{8 \sin \alpha x_i}{m^2 \pi^4 (a/b) \sin\theta} \sum_{n=1,2,3}^{\infty} \frac{(-1)^n}{n^2} \left\{1 - \frac{m^2 - (\mu n^2 a^2/b^2)}{[m + (n^2 a^2/m b^2)]^2}\right\} \sin\beta c \sin\beta y_j ,
\end{aligned}$$

$$\begin{aligned}
&A_{1m} (1+\mu)\, e^{\alpha b} - A_{2m} (1+\mu)\, e^{-\alpha b} - A_{3m} \{(1-\mu) - \alpha b (1+\mu)\}\, e^{\alpha b} \\
&\quad - A_{4m} \{(1-\mu) + \alpha b (1+\mu)\}\, e^{-\alpha b} + \left(\frac{a}{h}\right)^2 \frac{12\,(1-\mu^2)}{m\pi} \{B_{1m}\, e^{\alpha b} \\
&\quad + B_{2m}\, e^{-\alpha b} + B_{3m}\, \alpha b\, e^{\alpha b} + B_{4m}\, \alpha b\, e^{-\alpha b}\} = \qquad (81) \\
&\quad -\lambda \left(\frac{a}{h}\right)^2 \frac{48\,(1-\mu^2)}{(m\pi)^6} - \sigma \left(\frac{a}{h}\right)^2 \frac{48\,(1-\mu^2) \sin\alpha x_i}{m \pi^5 \cos\theta} \left\{\frac{(c/b)}{m^4}\right. \\
&\quad \left. + \frac{2}{\pi} \sum_{n=1,2,3}^{\infty} (-1)^n \frac{\sin\beta c \cos\beta y_j}{n\,[m^2 + (n^2 a^2/b^2)]^2}\right\}.
\end{aligned}$$

In Eqs. (76), (77), (80), (81), the values of λ, ϵ and σ are set as follows. When the structure is subjected to uniformly distributed dead and/or live loads in the absence of the intermediate diaphragms, $\lambda = 1$ and $\sigma = \epsilon = 0$. For the case of the normal step reaction Z_{ij}, acting alone, $\sigma = 1$ and $\lambda = \epsilon = 0$. Similarly when the transverse step reaction Y_{ij} acts alone, $\epsilon = 1$ and $\lambda = \sigma = 0$. The eight constants of integration A_{1m} to A_{4m} and B_{1m} to B_{4m} for each harmonic are readily computed by means of Eqs. (74) to (81), and the intensities of the step reactions Z_{ij} and Y_{ij} by means of Eqs. (68), (69).

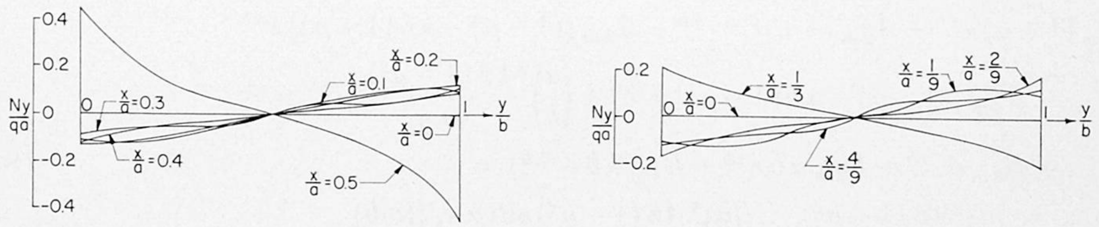
The analysis of the interior bay of a continuous prismatic shell structure subjected to uniform dead load of intensity q, is programmed for a CDC 3400 computer following the steps outlined above. The shell parameters are taken as $a/h = 200$, $b/a = 0.15$, $\mu = 0.2$ and $\theta = \pi/6$. The stress resultants and displacements for one intermediate diaphragm located at midspan and two intermadiate diaphragms at the third points, are calculated to the number of terms where the last term is less than $^1/_4\%$ of the partial sum of the series.

The accuracy of the method is dependent upon the number of step reactions s, which is taken as 5, 7 and 9. For $s = 9$, the maximum normal and transverse displacement components at the intermediate diaphragms are less than 1% of the maximum normal and transverse displacement components in the structure respectively. For design purposes, this degree of accuracy seems to be sufficient and $s = 9$ will be adopted in the illustrative examples. Better accuracy can of course be obtained by using larger value of s. The stress resultants, the normal component of the displacements and the step reactions at the intermediate diaphragms are shown in Figs. 5 to 10. It is seen, as expected, that the magnitude of N_x, N_{xy}, and w decreases with the increase of the number of interme-



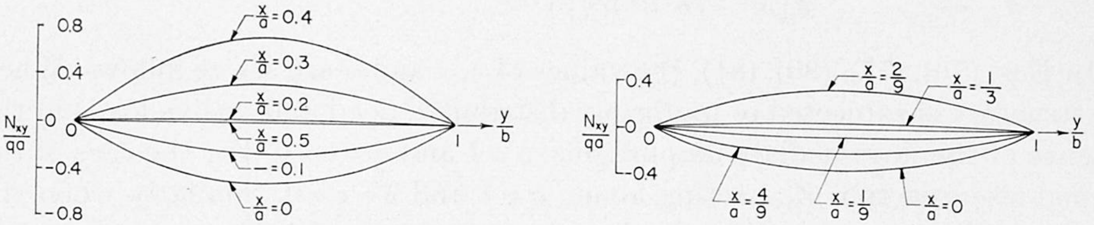
a) One intermediate diaphragm. b) Two intermediate diaphragms.

Fig. 5. Longitudinal normal force, N_x.



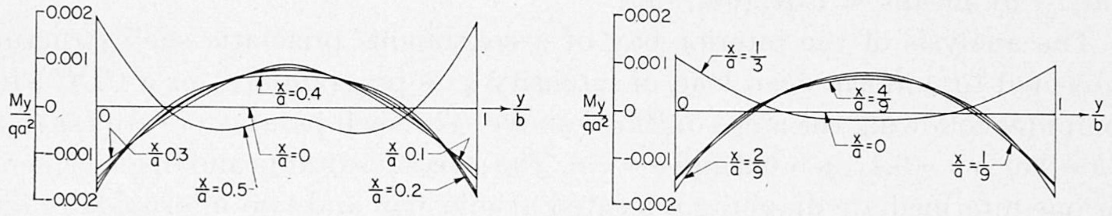
a) One intermediate diaphragm. b) Two intermediate diaphragms.

Fig. 6. Transverse normal force, N_y.



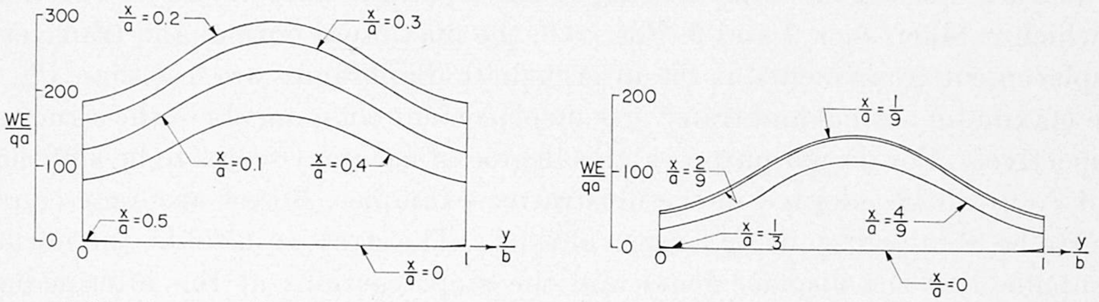
a) One intermediate diaphragm. b) Two intermediate diaphragms.

Fig. 7. Shear force, N_{xy}.



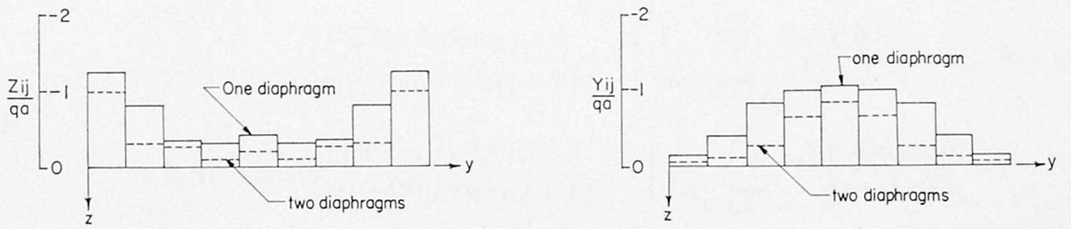
a) One intermediate diaphragm. b) Two intermediate diaphragms.

Fig. 8. Transverse bending moment, M_y.



a) One intermediate diaphragm. b) Two intermediate diaphragms.

Fig. 9. Normal displacement component, w.



a) Normal step reactions, Z_{ij}. b) Transverse step reactions, Y_{ij}.

Fig. 10. Normal and transverse components of step reactions at intermediate diaphragms.

diate diaphragms, as shown in Figs. 5, 7, 9 respectively, whereas the magnitude of N_y and M_y remains practically the same except at the intermediate diaphragms as shown in Figs. 6, 8. It is expected that the errors in N_y and M_y at the intermediate diaphragms are larger than elsewhere due to the fact that the normal and transverse displacement components at the intermediate diaphragms vanish only at the center of each step reaction as prescribed by Eqs. (68), (69). The small error in the normal displacement component at the diaphragms can be seen in Fig. 9.

Discussions and Conclusions

To compare the results of the proposed method with those obtained by GOLDBERG, GUTZWILLER and LEE [14] using finite difference technique, a two-plate, single bay prismatic shell structure, similar to the one shown in Fig. 2, simply supported on one transverse edge and fixed on the other, with both longitudinal edge free and subjected to uniformly distributed live load of intensity p, is analyzed by the proposed method. This structure may be treated as one half of a continuous prismatic shell structure, twice as long in longitudinal span, simply supported on the two transverse edges and continuous over a center diaphragm. The center longitudinal edge lies in a plane of symmetry and the two outer longitudinal edges are free. Hence only one plate need be analyzed.

The boundary conditions on the free longitudinal edge are

$$N_{xy} = 0, \tag{82}$$
$$N_y = 0, \tag{83}$$
$$M_y = 0, \tag{84}$$
$$R_y = 0. \tag{85}$$

Taking $y=0$ as the free longitudinal edge, as in plate A of Fig. 2, appropriate substitution for the stress resultants in Eqs. (82) to (85) as before leads to

$$A_{1m} - A_{2m} + A_{3m} + A_{4m} = 0, \tag{86}$$
$$B_{1m}(1-\mu) - B_{2m}(1-\mu) - B_{3m}(1+\mu) - B_{4m}(1+\mu) = 0, \tag{87}$$

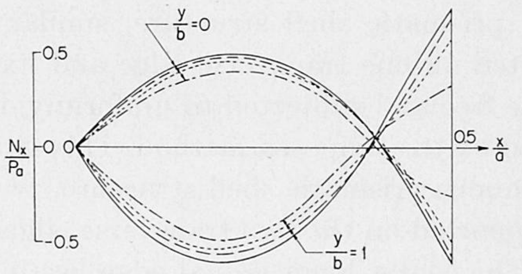
$$
\begin{aligned}
A_{1m}+A_{2m} &= \lambda\frac{16\,(b/a)}{m^3\,\pi^5}\sum_{n=1,3.5}^{\infty}\frac{1}{n^2}\left\{1-\frac{1-(\mu\,n^2\,a^2/m^2\,b^2)}{[1+(n^2\,a^2/m^2\,b^2)]^2}\right\}\\
&\quad+\epsilon\frac{8\,(b/a)\sin\alpha\,x_i}{m^2\,\pi^4\sin\theta}\sum_{n=1,2,3}^{\infty}\frac{1}{n^2}\left\{1-\frac{1-(\mu\,n^2\,a^2/m^2\,b^2)}{[1+(n^2\,a^2/m^2\,b^2)]^2}\right\}\sin\beta\,c\sin\beta\,y_j,
\end{aligned}
\tag{88}
$$

$$
\begin{aligned}
B_{1m}(1-\mu)+B_{2m}(1-\mu)+2\,B_{3m}-2\,B_{4m} &= \lambda\frac{4\,\mu}{(m\,\pi)^5}\\
&\quad+\sigma\frac{4\sin\alpha\,x_i}{(m\,\pi)^4\cos\theta}\left\{\mu\,(c/b)+\frac{2}{\pi}\sum_{n=1,2,3}^{\infty}\frac{\mu+(n^2\,a^2/m^2\,b^2)}{n\,[1+(n^2\,a^2/m^2\,b^2)]^2}\sin\beta\,c\cos\beta\,y_j\right\}.
\end{aligned}
\tag{89}
$$

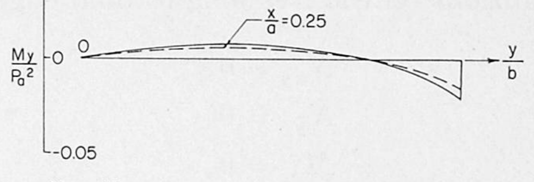
The boundary conditions on the central longitudinal edge, which lies in a plane of symmetry, are given by Eqs. (78) to (81).

The solution is obtained in identical manner as before. A comparison of the stress resultants N_x and M_y is shown in Fig. 11. It is seen that the proposed solution is in good agreement with the extrapolated solution given by GOLDBERG, GUTZWILLER and LEE [14]. The agreement is closer in N_x than M_y.

The proposed method of analysis is quite general and can be used for the analysis of any bay of a multiple span, multiple bay prismatic shell structures simply supported on the two transverse edges and continuous over intermediate transverse diaphragms. An extension of the method to include flexible intermediate transverse diaphragms presents no fundamental difficulties. In



a) Longitudinal distribution of longitudinal normal force, N_x.



b) Transverse distribution of transverse bending moment, M_y.

Fig. 11. Comparison of solutions by GOLDBERG, GUTZWILLER and LEE and proposed method.

—·—·—·	coarse grid	GOLDBERG, GUTZWILLER and LEE [14]
– – – – – –	fine grid	GOLDBERG, GUTZWILLER and LEE [14]
—— ——	extrapolation	GOLDBERG, GUTZWILLER and LEE [14]
————	proposed method	

this case, Eqs. (68), (69) should be modified to take into consideration the deflections of the intermediate diaphragms due to the step reactions.

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Summary

The theory of elasticity is applied to the analysis of multiple bay, multiple span prismatic shell structures, continuous over intermediate transverse diaphragms and simply supported at the two end diaphragms. The exact distribution of intermediate diaphragm reactions is approximated by uniform step functions expanded into double Fourier series. A comparison of tbe results with those obtained by a finite difference technique is also presented.

Résumé

La théorie de l'élasticité est appliquée au calcul des formes prismatiques à baies multiples et à travées multiples, continues sur des diaphragmes intermédiaires et simplement appuyées aux deux diaphragmes extrêmes. On obtient une approximation de la distribution réelle des réactions aux diaphragmes intermédiaires en développant des fonctions en escalier uniformes en double série de Fourier. On compare également les résultats ainsi obtenus avec ceux donnés par l'application d'une méthode aux différences finies.

Zusammenfassung

Die Elastizitätstheorie wird auf die Analyse mehrfeldriger, durchlaufender Faltwerke, mit Querscheiben an den Enden und bei den Zwischenunterstützungen, angewandt. Die genaue Verteilung der Reaktionen auf den Zwischenquerscheiben wird angenähert ausgedrückt durch Stufenfunktionen, die in doppelte Fourierreihen entwickelt werden. Ein Vergleich mit den durch ein Differenzenverfahren ermittelten Ergebnissen ist ebenfalls dargestellt.