Ultimate Strength Tests of Reinforced Concrete Beams in Combined Torsion, Bending and Shear

Essais de rupture de poutres en béton armé soumises à l'action combinée de la torsion, de la flexion et du cisaillement

Bruchlasttests an Stahlbetonträgern unter kombinierter Beanspruchung aus Drillung, Biegung und Schub

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Introduction

The earliest major effort in the study of the behavior of reinforced concrete members in combined torion, bending and shear was made in Russia. More recently, important work has been done in the United States, Canada, Australia and India. The Russian work resulted in a series of publications by CHINENKOV [1], LIALIN [2], LESSIG [3, 4] and YUDIN [5, 6]. Some of it was experimental and some theoretical, with early emphasis placed on combined torsion and bending moment and combined torsion and shear. YUDIN later combined all three types of loading, but the experiments were conducted on rather small specimens (3.5 by 6.4 inches in cross section). He reported failure modes which were also observed in some of the specimens to be reported on here, namely that the failures were "... typical of tearing the center part of the beam from the end sections". A similar phenomenon may be observed in NYLANDER'S [7] report on tests of frames in which one member, reinforced both longitudinally and transversely, was subjected to torsion.

More recently, PANDIT and WARWARUK [8] tested sixteen specimens in combined torsion, bending and shear. These were larger than the Russian specimens (6 by 12 inches in cross section), but the sequence of loading was different. Whereas the Russians, and apparently all other investigators to date, have increased the torsional, bending and shear loads simultaneously in proportion, PANDIT and WARWARUK subjected their specimens first to some predetermined load level in transverse bending and shear alone, and then twisted them to failure.

Test results from several of their specimens will be checked against the torsional strength predicted by a rational equation to be developed in this paper.

A number of specimens were also tested under combined loading by WALSH, COLLINS, ARCHER and HALL [9], who then developed semi-empirical formulae [10] to predict the strengths of members subjected to the combined loading. The failures observed in these tests were similar to most of those to be described here, i.e. they occurred by rotation of sections of the specimens about hinges which formed near one face after inclined cracks had formed on the other faces. All reinforcement in this series consisted of round, undeformed, mild steel bars. The same type of reinforcement was used in tests conducted by RAMAKRISHNAN and VIJAYARANGAN [11, 12], whose results will also be checked against the theory to be developed here.

Almost all of the above mentioned work contained restrictions on specimen size, method of application or sequence of loading, type of reinforcement, and/or strength of concrete, which affected the results of the tests. Furthermore, reinforcement strains were measured only in some of the Russian tests and by PANDIT and WARWARUK. Other investigations into the effects of combined loading have concerned themselves with prestressed concrete beams and with reinforced concrete beams of cross section other than solid rectangular. Consequently it seemed advisable to extend the earlier work on rectangular reinforced concrete specimens conducted at the University of Kentucky [13, 14]. The results of this had indicated that a member subjected to combined torsion and bending, without shear, would fail by rotation about a hinge in the compression face of the member if the specimen were square in cross section and contained longitudinal reinforcement only or if it were square or rectangular and contained transverse reinforcement, but that it would fail by rotation about a hinge in one of the sides if it were rectangular and only reinforced longitudinally. Since most actual beams may be expected to be subjected to combined torsion, bending and shear, it was decided to study square specimens containing only longitudinal reinforcement and rectangular specimens containing both longitudinal and transverse reinforcement subjected to the triple loading.

Description of the Specimens

Ten beams were loaded in combined torsion, bending and shear. The first six were eight inches by eight inches in cross section and contained essentially only longitudinal reinforcement. The other four were six inches by twelve inches in cross section and contained both longitudinal and transverse reinforcement. An overall view of the test specimens and the loading scheme is given in figure 1. Various ratios of bending moment to torque could be obtained by changing the lengths of the arms. The ratio of shear to bending moment was kept constant by making the total lengths of all specimens equal. The loading and support arms in all specimens were reinforced with three no. 4 bars on the tension side and two no. 4 bars on the compression side. The steel was detailed so that in all arms the two outside tension bars were bent around the beam reinforcement and back into the arm to provide compression reinforcement. The third tension bar was hooked around the bottom beam reinforcement for anchorage. Shear reinforcement in the form of no. 3 or no. 4 closed ties, spaced three inches center to center, was also provided in all arms.

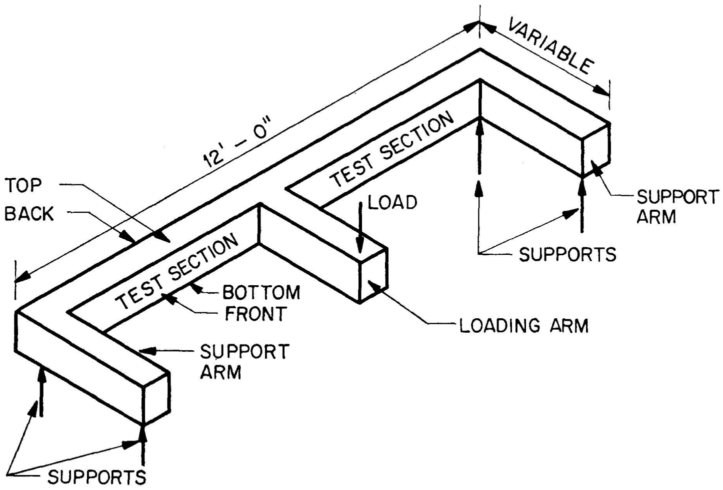

Fig. 1. Specimens and loading scheme.

The longitudinal reinforcement in the test sections of all specimens consisted of three no. 4 bars on the bottom and two no. 4 bars on the top. The test sections of specimens 1 through 6 were reinforced only with the longitudinal steel, except for two no. 3 closed ties placed three inches center to center from each other and from the arm reinforcement at each end of each test section to prevent local failure due to stress concentrations. The test sections of specimens 7 through 10 also contained transverse reinforcement. This consisted of no. 3 closed ties, spaced three inches center to center throughout the test sections except for the first three ties at each end, which were placed at two inches center to center, again in an attempt to prevent local failure. For further details see table I and figure 2.

All reinforcement consisted of intermediate grade deformed bars meeting the requirements of ASTM specifications A-15 and A-305. Yield stresses of the various bars used are given in table I. The concrete for all specimens was commercially obtained transit mix with the following composition: 5 bags cement, 1545 pounds sand, 1890 pounds stone ($\frac{1}{2}$ inch to $\frac{3}{4}$ inch chips), $1\frac{1}{4}$ pounds Pozzolith and, nominally, 37 gallons of water per cubic yard. Since it was considered desirable to have some variation in concrete strength, strict control was not exercised over the mix. The mixing water was added in the truck at the laboratory, but workability of the concrete rather than the precise quantity of water added was used as the controlling factor.

After being cast, the beams were cured at room temperature under wet burlap for seven days and the left to air dry in the laboratory. All specimens

Table I. Beam Properties

Specimen Number	Nominal[1]) M/T ratio	Cross Section (inches)	f_y Longitudinal Reinf. (psi)	Transverse Reinf. (psi)	f_c'[2]) (psi)
1	1	8× 8	43,000	—	6,025
2	2	8× 8	43,000	—	6,025
3	3	8× 8	43,000	—	4,350
4	4	8× 8	43,000	—	4,350
5	5	8× 8	46,250	—	5,100
6	6	8× 8	46,250	—	5,100
7	1	6×12	45,500	50,000	5,460
8	2	6×12	45,500	50,000	5,500
9	4	6×12	45,500	50,000	5,500
10	8	6×12	45,500	50,000	5,460

[1]) The ratios of the actual values varied slightly from this. See table II.
[2]) Average of three cylinders, obtained at time of test of associated specimen.

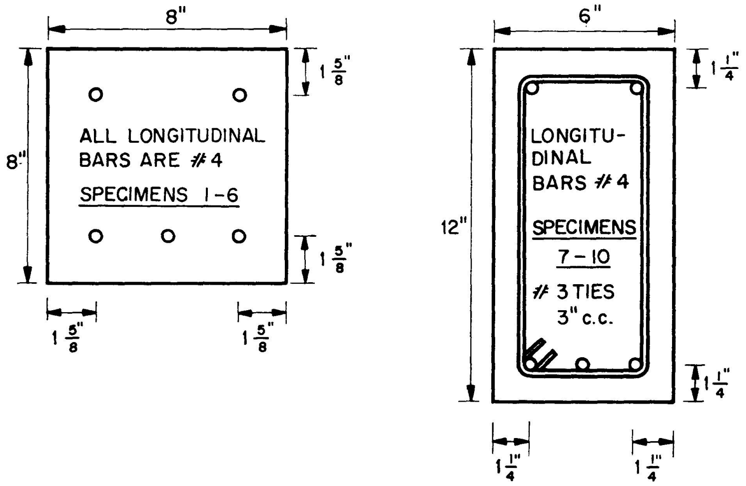

Fig. 2. Specimen cross sections.

were tested between thirty and sixty days after being cast. The accompanying cylinders were tested at the same time as the specimens. Strengths are recorded in table I.

SR-4 type A-7 strain gages were attached to the reinforcement at many points and type AR-1 rosettes were later attached to the concrete at locations where maximum tensile stresses were expected. It was thus possible to monitor the strains in the reinforcement and the concrete during the conduct of the tests and to deduce some information regarding the mode of failure later.

Conduct of Tests and Results

The general test setup is shown in figure 1. The load was applied to the center of each specimen in one to two thousand pound increments, the smaller increments being used near failure. After each increase, the load was held constant for a period of from five to twenty minutes while all strain gage readings and loads were recorded and the crack patterns sketched and photographed. Except for the last one or two increments before failure, recheck of the strain gages showed that little creep occurred during the observation periods.

The behavior of the two sets of specimens was quite similar at relatively low loads, but differed markedly as the loading proceeded. Their final failure modes bore very little relationship to each other. Within each set, however, the response to the loading changed very little from specimen to specimen, though one could easily observe the differences caused by the changes in the moment/torque ratio. For example, in the set of six beams which contained only longitudinal reinforcement, it was possible to see the gradual changes in the crack patterns as the moment/torque ratio progressed from 1 to 6. In beam 1, the initial crack pattern consisted of cracks on all surfaces except the back (see figure 1 for the designations of the surfaces), inclined at between 37 and 45 degrees to the beam axis. This pattern had formed by the time a load equal to 80 percent of the failure load had been reached. Continued loading produced no remarkable deformations until, suddenly, a mechanism formed in one of the test sections, in which two segments of the beam rotated with respect to each other about an S-shaped hinge which formed on the back of the specimen, approximately at mid-height. At the same time, a piece of concrete was spalled off the bottom of the beam. Upon close examination it was found that this piece of concrete was a parallelopiped whose boundaries were: the bottom of the beam, the longitudinal bottom reinforcing bars, and parallel diagonal cracks approximately four inches apart, intersecting the front and back faces of the beam. Specimen 2 behaved very similarly. In this beam the first cracks were noticed when the load had reached approximately 38 percent of the failure load.

Specimen 3 showed considerable influence of bending. The cracks on the bottom made a larger angle with the axis of the beam and the early cracks on the front and back were almost vertical. Specimen 4 appeared to behave as though it were being loaded only in bending, with the classical bending crack pattern, until it suddenly collapsed with the formation of the S-shaped hinge on the back and the spalling of the parallelopiped from the bottom. The major failure crack pattern seemed to partially override the bending cracks and was almost identical with that formed in the previous three specimens. It is shown in figure 3. Specimens 5 and 6 behaved like specimen 4, but some crushing of the top surface and considerable widening of the bottom tension cracks was noticeable before the sudden torsional failure occurred. The strain gage readings indicated that all three bottom reinforcing bars of specimen 6 had yielded before the collapse.

Figure 4 shows a plot of bending moment versus strain for the longitudinal reinforcement in specimen 2. The strains were measured 12 inches from the face of the loading arm but the recorded moments are those at the face of the loading arm. The bending moments at the locations of the gages were approx-

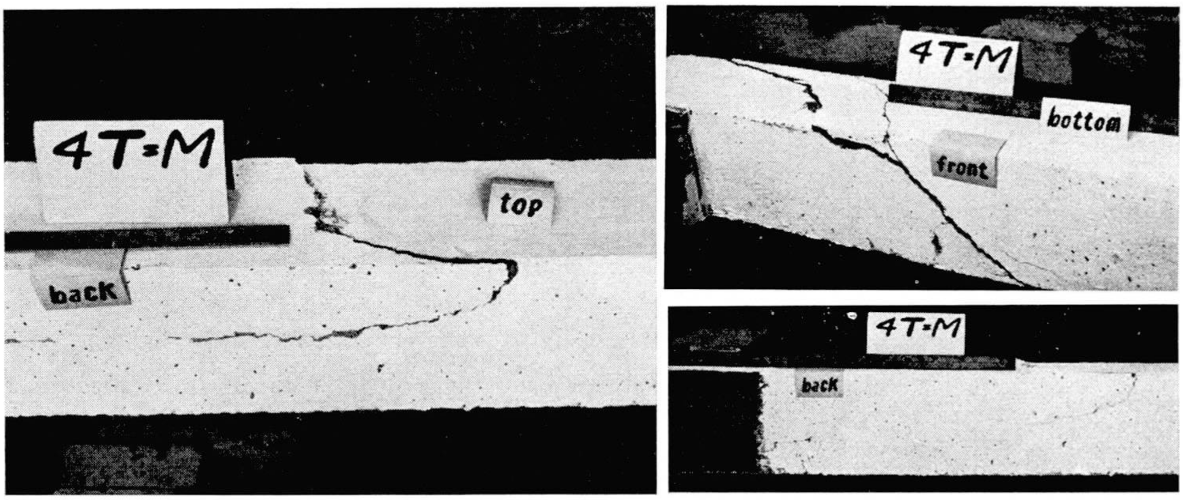

Fig. 3. Failure crack pattern of specimen 4.

imately 80% of those indicated. They are typical of the readings observed in specimens 1 through 6, and clearly indicate that the specimens were bent laterally, convex to the front, to such an extent that the front bars were subjected to an appreciable additional tensile strain. This additional strain caused the front bottom bar to yield long before the ordinary bending moment would have done so. The strain readings on specimens 7 through 10 also indicated more tension in the front than the rear bottom bars, but the differences were much smaller and the top bars were not consistent in this respect. The strain readings for specimen 5 appeared to indicate a reversal of this curvature, but

Table II. Test Results

Specimen Number	Bending Moment[1]) at Failure (in. kips)	Shear at Failure (kips)	Torque at Failure[2]) (inch-kips) about:		Theoretical Ultimate Moment (ACI Code)[4]) (inch-kips)		Theoretical Dowel Torque about Hinge in back (inch-kips) for Crack Spacing	
			Center Line	Hinge in back	Beam	Load Arm	Predicted	Actual
1	75.1	1.18	62.5	67.2	157	171	47.1	58.0
2	113.8	1.78	43.3	50.4	157	171	47.1	43.5
3	105.2	1.60	43.3	49.6	155	168	40.0	43.0
4	153.1	2.40	36.4	46.0	155	168	40.0	36.8
5	179.0	2.80	44.6	55.8	169	182	43.2	38.4
6	194.0	3.03	36.5	48.6	169	182	43.2	38.4
7	77.6	1.17	78.5	82.0	307	322	59.5	79.3
8	231	3.50	120.3	262[3])	307	322	—	—
9	281	4.26	73.2	172[3])	307	322	—	—
10	306	4.64	44.6	108[3])	307	322	—	—

[1]) Taken about the edge of the loading arm rather than about the center of the beam.
[2]) Calculated from the reactions of the test section which failed.
[3]) Calculated as load applied to specimen times length of loading arm to back of beam.
[4]) With ϕ set equal to 1.
— No values are reported here because it was not possible to predict the failure torque for the observed mode of failure.

there is a possibility that the gage wiring was transposed between the front and back bars in this specimen, and it is assumed that this happened.

Table II summarizes the test results. It should be noted that the bending moments at failure were calculated at the edge of the loading arm, i.e. at the ends of the test sections. Since the bending moment varied throughout each test section, it was decided to report only its maximum value. This does not mean, necessarily, that the failure always occurred adjacent to the loading arm. In fact, in specimens 2 and 3 it occurred nearer the supports, and in specimens 1, 4 and 5 the ties next to the loading arm apparently bounded the failure region. In specimen 6 the failed zone intruded on the tied region with some top surface crushing. The mode of failure of specimens 7 through 10 will be discussed later.

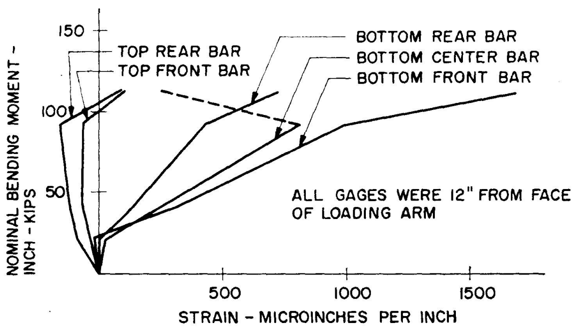

Fig. 4. Strain gage readings for specimen 2.

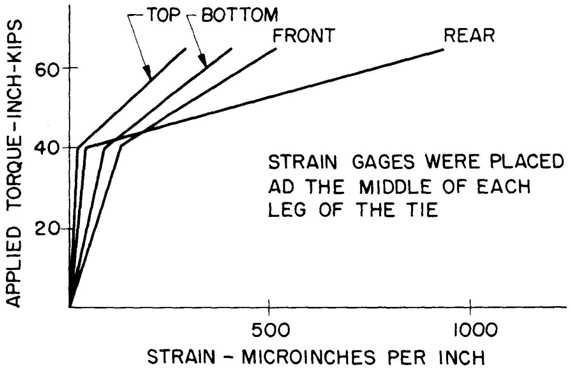

Fig. 5. Strains in one tie of specimen 7.

The shear was essentially constant throughout the test sections of all specimens, as was the torque. Two different torques are given in table II. The first one is the torque in each test section about the central axis of the beam, and the other is the torque in each test section about the hinge which formed in the back. Both were calculated from the support reactions, except in the case of specimens 7 through 10. It will be noticed that the torques about the hinge reported for these specimens were calculated as the product of the applied load and the length of the loading arm, measured to the back of the beam. This was done because this total applied torque is more relevant to the mode of failure observed in specimens 7 through 10. The theoretical ultimate bending strengths of both the beam sections and the loading arms are also given. These were calculated from equation 16–1 of the ACI Building Code with the capacity reduction factor, ϕ, set equal to 1. Equation 16–3 was not applicable since the compression reinforcement could not reach its yield stress. The last section of the table contains values of predicted torsional strength. These will be discussed later.

The behavior of specimen 7, the first of the rectangular, transversely reinforced beams, resembled that of specimens 1 and 2. The first cracks occurred at approximately 50 percent of the failure load. They were inclined at approximately 45 degrees to the longitudinal axis of the beam and were initially observed on all faces except the top. The failure was of essentially the same form as in the earlier specimens, with a hinge forming on the back and a piece of concrete being forced out of the bottom. A small crack was also observed on the top face, at the junction of the loading arm and the main body of the specimen, as though the loading arm were suffering some bending distress at that point. The strain gages on the ties, some of which were attached on all sides, indicated that all the ties were in tension, but that none had reached the tensile yield strain prior to failure. The torque/strain graphs also showed a definite decrease in slope at the cracking load. See figure 5, which gives a rather typical plot of the strain readings.

Specimens 8, 9 and 10 behaved quite differently and their mode of failure is rather difficult to interpret. In all three, combined bending-torsional cracking, similar to that previously described for the square specimens, took place at lower loads. At failure, however, instead of large rotation in one test section, a failure surface formed on both sides of, and close to, the loading arms of specimens 8 and 9, with subsequent torsional rotation about hinges on the back, connecting the two failure surfaces in each member. At the same time the two surfaces were also connected by cracks across the top faces of the loading arms at their junctions with the beams. In fact, the total appearance of the failures was one in which the loading arms failed in tension at the top, simultaneously tearing out a piece of the adjacent beams. See figure 6, which shows specimen 9 after failure. In this case the hinge formed on the back just below the tops of the visible cracks.

These two specimens were also greatly influenced by the bending moments, since the strain readings showed that the tension reinforcement had yielded

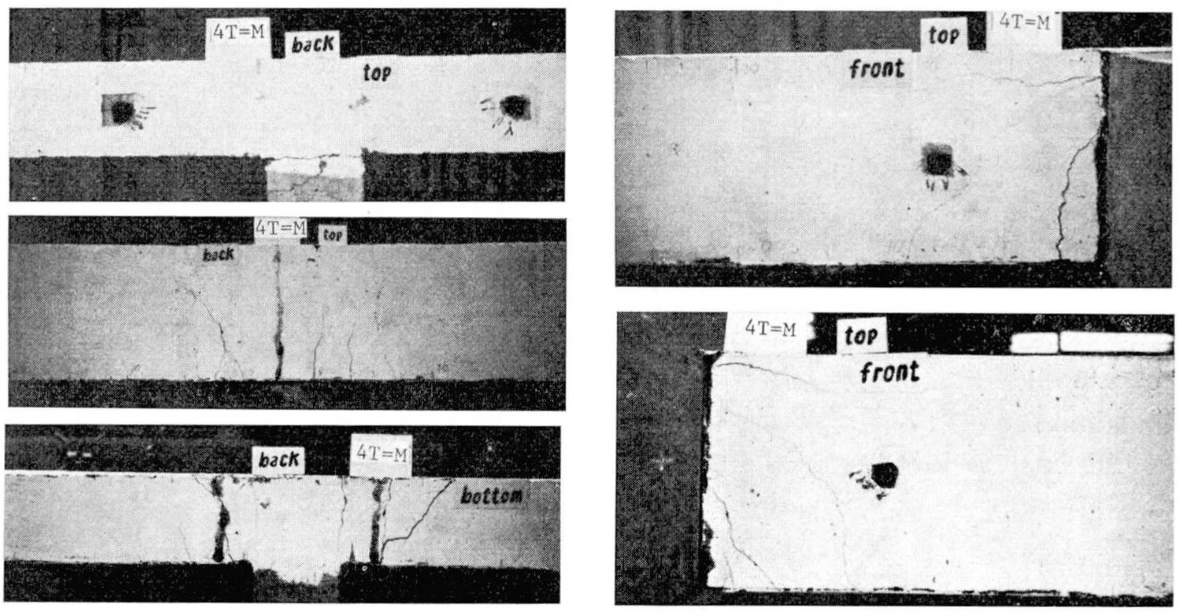

Fig. 6. Failure crack pattern of specimen 9.

prior to failure and, in the case of specimen 9, no torsional cracks appeared on the top surface. Specimen 10 was even more affected by the bending moment. Most cracks prior to failure seemed to be ordinary bending cracks which opened quite wide before the ultimate load was reached. No torsional type cracks appeared on the top surface. There was, however, a very marked crack again across the top of the loading arm and evidence, on the front of the specimen, that the arm had been torn out, taking with it a part of the beam. The tension reinforcement had again passed the yield point before collapse occurred. Strangely enough, the strain gages attached to the ties did not indicate that the ties yielded in any of these members.

Analysis

An analysis of the results observed in these tests requires consideration of three different phenomena. The first is the mode of failure of the square specimens containing only longitudinal reinforcement, the second is the mode of failure of the rectangular specimens containing both longitudinal and transverse reinforcement, and the third is the phenomenon of lateral bending, which may be a part of or may contribute to the other two effects.

Examining these in turn, one finds that the failure of the square specimens can be analysed in the light of a torsional dowel action theory of failure proposed in an earlier paper [13], for members failing by torsional rotation about a hinge in one side. In this, the equilibrium of a parallelopiped of concrete on the bottom of a member, similar to those tested here, was studied. The boundaries of this solid were assumed to be the bottom surface of the specimen, the plane containing the center lines of the longitudinal bottom reinforcement, and two parallel cracks on the bottom surface, spaced a distance, e, apart. The direction of these cracks was, at that time, assumed to be perpendicular to the axis of the member, in order to simplify the derivations and calculations. This agreed fairly well, also, with the test data then available. The forces acting on the top plane of the parallelopiped were then assumed to be the dowel forces from the longitudinal reinforcement, which has to resist most of the torque once the cross section is cracked, and vertical tensile stresses in the concrete which would tend to keep the parallelopiped from being spalled out of the beam. Since no other forces could act on it, the resultants of the vertical components of the dowel forces and the vertical tensile stresses in the concrete had to be in equilibrium. Therefore, analysing the parallelopiped as a biaxially eccentrically loaded tension member, the following equation was obtained for the maximum dowel resisting force the concrete could exert on the critical bar:

$$F_c = \frac{f_t e b}{[1+6(K_2-K_1)]\left(\sin\phi_c + \frac{1}{r_c}\sum_b r_i \sin\phi_i\right)}, \tag{1}$$

where F_c is the maximum bearing force the concrete can exert on a bar designated as the "critical" one; f_t is the modulus of rupture of the concrete; e is the distance between cracks measured parallel to the axis of the beam; b is the width of the beam; K_1 and K_2 are constants of proportionality such that $K_1 e$ is the perpendicular distance from a bending crack to the resultant of the vertical components of the dowel forcds and $K_2 b$ is the perpendicular distance from the side containing the hinge to the same resultant; r is the radial perpendicular distance from the hinge to the center of a longitudinal reinforcing bar, with the subscript c denoting distance to the bar designated as "critical" and the subscript i denoting the distances to the other bars; ϕ with the appropriate subscript is the angle any radius r makes with the vertical side of the beam containing the hinge; and $\sum_b$ indicates summation over the bottom bars only.

Once F_c was known, the total possible dowel resisting torque about the hinge could be expressed as

$$T_r = F_c\left(r_c + \frac{1}{r_c}\sum r_i^2\right), \tag{2}$$

where now the summation extended over all bars except the "critical" one and T_r consequently represented the total resisting torque which could be provided by all bars.

As was mentioned above, the cracks were assumed to be perpendicular to the beam axis in the original derivation. This was not found to be the case in all specimens in this investigation. Consequently, equation 1 was modified to the more general form for the biaxially eccentrically loaded tension member

$$F_c = \frac{f_t}{Q\left(\sin\phi_c + \frac{1}{r_c}\sum_b r_i \sin\phi_i\right)}, \tag{3}$$

where

$$Q = \frac{1}{A} + \frac{e_x}{I_{yy}} X + \frac{e_y}{I_{xx}} Y. \tag{4}$$

A is the cross sectional area of the top surface of the parallopiped; the coordinate system is based on the principal axes of inertia of the parallelogram formed by this surface; e_x and e_y are the coordinates of the resultant of the vertical components of the dowel forces and X and Y are the coordinates of the corner of the parallelogram expected to sustain the maximum tensile stress, all measured within this coordinate system. I_{xx} and I_{yy} are the principal moments of inertia about the indicated axes. e_x and e_y may be found by assuming, as before, that the dowel force exerted on the surrounding concrete by a bar will be proportional to its perpendicular distance from the hinge of rotation and that it will act a distance $K_1 e$ along the bar away from the bending crack. For calculation purposes, a value of $K_1 = \frac{1}{12}$ gave good results previously and was therefore used here again.

As a matter of interest, the value of F_c was calculated for specimens 1 through 6, once with the assumption that the cracks were perpendicular to the beam axis, i.e. using equation 1, and then again with the assumption that they made an angle of 60 degrees with the beam axis, which required use of equation 3. The results of the calculations differed by 3.2 percent, which is quite insignificant, given the physical assumptions on which these equations are based. This then indicates that it is not necessary to be able to predict the orientation of the cracks on the bottom surface accurately in order to apply the equations to a member.

Equations 2 and 3 were used to predict the dowel torional strengths of members 1 through 6 which contained no transverse reinforcement. They were also used to check the dowel strength of member 7 which failed in the same manner; even though transverse reinforcement was present. The results are presented in the last two colums of table II. (Here it should be noted that the torques are taken about the hinge on the back face of the specimen.) The theoretical torsional dowel strength is obviously directly proportional to the crack spacing. The work of BROMS and LUTZ [15, 16] indicated that the average crack spacing to be espected at high reinforcement stresses is approximately twice the effective cover. Based on this, the average crack spacing for members 1 through 6 should have been $3\frac{1}{4}$ inches. In the case of member 7 one would expect the spacing of the transverse reinforcement to be reflected in the crack spacing, which would cause it to be 3 inches. In this member, also, the dowel effect of the ties crossing the cracks had to be included. These figures were used to calculate the torques listed in the next-to-the-last column of table II. In the last column are the dowel torques predicted by the equations when the actual distances between the bending cracks bounding the failure parallelopipeds were used.

The moduli of rupture used in the calculations were taken to be $= 9\sqrt{f_c'}$. Some modulus of rupture specimens had been tested with the control cylinders, but the scatter of the data did not warrant identification of individual control beams with the torsional specimens. Despite the uncertainty of the tensile strength of the concrete and the other assumptions involved, it will be noted that the correlation between the theoretical dowel and the actual failure torques is quite good. The predicted strengths are, in every case, less than those obtained experimentally. This is to be expected, since the theory presented above does not take into account the resistance to rotation which will be provided by the uncracked concrete in the vicinity of the hinge. The dowel torsional strength should thus be a lower bound on the total torsional strength.

The method of analysis was also applied to specimens tested by others under similar loading conditions. The results are shown in table III. The crack spacing used for PANDIT and WARWARUK's specimens was twice the cover, i.e. $3^1/_2$ inches, while that for RAMAKRISHNAN's and VIJAYARANGAN's specimens was $4^1/_2$ inches, which seemed to be approximately the average in the

Table III. Results of Tests by Others

Investigation	Specimen Number	Torque at Failure[1]) (inch-kips) about:		Predicted Dowel[2]) Torque about Hinge in back (inch-kips)
		Center Line	Hinge in back	
PANDIT and WARWARUK [8]	F-1	58	86	73
	F-2	83	102	73
	F-3	89	99	70
	G-1	73	104	70
	G-2	92	111	70
	G-3	103	113	72
	C_{1b}	23	37	23
RAMAKRISHNAN and VIJAYARANGAN [11]	C_3	22	37	23
	C_4	20	33	22
	C_5	23	38	23
	C_{6b}	22	34	22

1) Torque about the hinge was calculated by adding the product of the shear times half the width to the reported center line torque.

2) Using equations 1 and 2, with $f_t = 9\sqrt{f_c'}$ and, where necessary, with f_c' equal to 80% of the cube strength.

figures shown in that paper. BROMS' work would probably not be applicable to the latter specimens, since the reinforcement consisted of plain round bars. There were several factors which would cause inaccuracy. One of these, obviously, is the different sequence of loading used by PANDIT and WARWARUK. Another is the fact that their specimens did contain some transverse reinforcement, though relatively little in the examples chosen. (Specimens *F* contained no. 3 ties at 8 inches center to center and specimens *G* contained no. 2 ties spaced $3^1/_2$ inches center to center.) Nevertheless, one would expect that the ties would have some dowel effect, which was neglected in these calculations and would tend to increase the torsional strength above that predicted. RAMAKRISHNAN and VIJAYARANGAN used very low strength concrete and reported its cube strength. There is, therefore, some question regarding the accuracy of the calculation of the tensile strength.

Another factor which should be taken into account in these comparisons is the stress in the tensile reinforcement at the time of torsional failure. If the applied bending moment is small compared to the moment capacity of the beam, the tensile stresses in the reinforcement will be low. It must then be expected that, for monotonically increasing loading, the crack spacing will be considerably larger than that indicated by BROMS' and LUTZ' work for high reinforcement stresses [17]. This will then, obviously, have the effect of increasing the torque capacity of the specimens. Unfortunately, no valid expressions seem to have yet been devised to relate steel stress and crack spacing under this type of loading condition. However, it seems obvious that in an actual structure one must assume that eventually the crack spacing will reach the minimum predicted by BROMS and LUTZ. If a correction factor for variation of crack spacing with steel stress could be applied to PANDIT'S and WARWARUK'S, and RAMAKRISHNAN'S and VIJAYARANGAN'S test results, the predicted dowel torques of the specimens with lower bending moments would be increased considerably, leading to higher lower bounds and to better correlations in table III.

It was not possible to analyse the mode of failure observed in specimens 8 through 10. As is evident from table II, the loading arms should have been sufficiently strong to prevent bending failure at their intersections with the beam portions of the specimens. Instead, the development of the tension cracks at the intersections clearly indicated that the arm reinforcement was yielding long before final collapse occurred. By dividing the bending moment in the loading arm at failure by the product of the cross sectional area of the arm reinforcement and the yield strength of that reinforcement, it was determined that the centroid of compression for each arm must have been located vertically between the hinge in the back and the usual location of this centroid.

Analyses of this phenomenon were also attempted by taking the moments of the dowel forces of all reinforcement crossing the failure surface about the hinge and also by summing the moments of the axial forces in all the transverse reinforcement crossing the failure surface, but both types of calculations gave predictions of torsional strengths quite different from those actually observed. The type of analysis used for specimens 1 through 7 could not validly be applied to these members since the mode of failure was obviously different. It is apparent, therefore, that the problem of the connections of a member in torsion to its supports must be studied further. In the mean time designers should be very careful and conservative in their detailing of such connections.

The last major phenomenon requiring discussion is the lateral bending which was observed in all specimens. As can be seen from figure 4, there were large differences in the strains observed in the various longitudinal bars. The result was that the front bottom bar yielded at a relatively low transverse bending moment. This behavior can be explained by reference to figure 7, which shows part of a specimen with everything past the failure surface removed. Also shown are the applied loads and reactions, and the horizontal and vertical components of the dowel forces. It is then evident that these

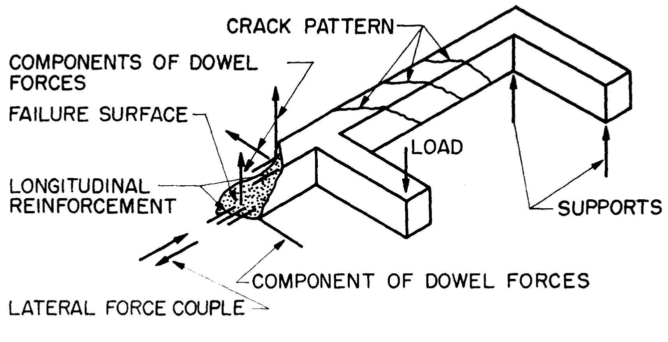

Fig. 7. Development of lateral Moment.

horizontal components will form a couple tending to rotate the free-body about a vertical axis. The resisting couple can only be supplied by the reinforcement, aided perhaps by shearing stresses in the uncracked concrete near the hinge. The magnitude of the applied lateral moment can be found, approximately, by calculating the horizontal components of the dowel forces applied to the concrete by the individual bars both on top and on the bottom, finding the resultant of each set of horinzontal components (note that they will not be equal, which means that some of the horizontal force must be taken in shear by the concrete at the hinge) multiplying each by half the horizontal distance between them and adding the two moments.

Using the same assumptions as those which were used to derive equations 1 through 3, one can find that the horinzontal component of dowel force in the "critical" bar at failure is

$$F_{cH} = F_c \cos\phi_c \tag{5a}$$

and the horizontal component of dowel force in any other bar at failure is

$$F_{iH} = F_i \cos\phi_i = F_c \left(\frac{r_i}{r_c}\right) \cos\phi_i . \tag{5b}$$

If the cracks on the top, bottom and front of each specimen were all inclined at 45 degrees to the beam axis and if the resultant horizontal force acted at the center of each bar group, the horizontal distance between the resultants would be equal to the width of the specimen plus the height, minus the top and bottom cover. Actually, it will be somewhat less than that since some of the cracks will make angles greater than 45 degrees with the axis and since the centroids of the horizontal forces will not be at the centers of the top and bottom surfaces. For calculation purposes it will be convenient to let the length of the lever arm be $K_3\ (b + h)$, where h is the overall height of the member.

It is now possible to write an approximate expression for the lateral bending moment:

$$M_{Lat.} = F_c \left(\cos\phi_c + \frac{1}{r_c} \sum r_i \cos\phi_i\right) \frac{K_3\,(b+h)}{2}, \tag{6}$$

where the summation extends over all bars except the "critical" one. Equation 6 can be rewritten by solving equation 2 for F_c and substituting this into equation 6:

$$M_{Lat.} = \frac{K_3 T_r\,(b+h)\left(\cos\phi_c + \frac{1}{r_c} \sum r_i \cos\phi_i\right)}{2\left(r_c + \frac{1}{r_c} \sum r_i^2\right)} . \tag{7}$$

If desired, this equation can be simplified somewhat before the numerical calculations are carried out. The value of K_3 was taken as 0.7 for specimens 1 through 6, which made $M_{Lat.} \doteq \frac{1}{2} T_r$ for these specimens.

It is then possible to check whether the longitudinal reinforcement is really resisting the lateral moments, by calculating the lateral moment couple set up in the top and bottom reinforcement by the differences in strain between the front and rear bars on the top and on the bottom. This was done for specimens 1 through 6, and the results are presented in table IV. The correspondence between the last two columns of the table is quite striking. Unfortunately it was not possible to make a comparison for specimen 7, since the strain gage on the bottom front bar failed at the beginning of the test. The rest of the specimens exhibited a different mode of failure and no comparison was attempted. As a matter of interest it might be noted that for specimens 7 through 10, $M_{Lat.} \doteq 0.9\, T_r$.

Table IV. Check of Lateral Bending Moments

Specimen Number	Strain difference [1] (micro-inches/inch)		Lateral Bar Moment (inch-kips)	½ Predicted Theoretical Dowel Torque [3] (inch-kips)
	Top bars	Bottom bars		
1	50	600	18.0	23.5
2	30	820	23.5	23.5
3	–200	820	17.1	20.0
4	40	650	19.0	20.0
5	650	200	23.4	21.6
6	–30	130 [2]	9.5 [2]	21.6

[1] Just prior to failure.
[2] All bottom reinforcement had yielded prior to collapse.
[3] For the predicted crack spacing, from table II.
– The minus sign indicates that the front bar was subjected to a smaller tensile or larger compressive strain than the rear bar. This would tend to cause a lateral moment opposite to that of the other bars.

The vertical components of the dowel forces create an upward shear which, in the case of these specimens, was actually larger than the vertical shearing forces. This will simply have the effect of reversing the vertical shearing stresses in the uncracked concrete near the hinge.

In many of the specimens the top surface was crossed by cracks. It was further noticed that in most cases the compression reinforcement was either in tension or contained a very low compression strain at the time of failure, even though the bending moments were considerable. This would lead one to believe that the centroid of compression must have shifted. However, the lever arms of the tension reinforcement, as calculated from observed bending moments and average steel strains at or near failure, were of approximately the magnitude to be expected in pure bending. It is not possible, therefore, to draw any conclusions regarding the effect of torsion on the bending moment capacities of these specimens.

Conclusions

The following conclusions may be drawn from the foregoing:

1. Reinforced concrete beams subjected to combined torsion, bending and shear are likely to fail in torsion by rotation about a hinge on the vertical side on which the shearing stresses due to vertical shear and those due to the torque subtract from each other.

2. When the members contain only longitudinal reinforcement, they are able to resist torsion beyond the cracking load because of the dowel action of the reinforcement. Analysis of this dowel action provides a lower bound on the torsional strength of such members.

3. Interaction between shear and torque is based on the total torsion of the applied loading and the resisting dowel action about the hinge of rotation, for members containing only longitudinal reinforcement. The interaction between bending moment and torsional resistance for such members must be based on the variation in flexural crack spacing with stress in the tension reinforcement.

4. The dowel action will cause lateral bending moments which must be resisted by a rearrangement of the stresses in the reinforcement. The stresses involved are by no means negligible and can cause yielding in some bars at loads much lower than those which would cause yielding in ordinary bending.

5. When the members contain both longitudinal and transverse reinforcement, their connections to supporting or loading members appear to be much weaker than would have been suspected from conventional theory. It is therefore necessary to design such connections for much higher moments than those expected to be applied.

6. No conclusion could be drawn regarding the effect of torsion on the bending moment capacity of a member subjected to combined torsion, bending and shear, though there is some evidence that the lever arm of the tension reinforcement was not appreciably reduced by the torsional effects.

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Summary

Ten reinforced concrete beams were tested in combined torsion, bending and shear. Six of the beams were eight inches by eight inches in cross section and contained only longitudinal reinforcement. The other four were six inches by twelve inches in cross section and contained both longitudinal and transverse reinforcement. The principal variable was the bending moment to torque ratio.

The strains in the members and their modes of failure were examined. Two different modes of failure were observed and a theoretical model for predicting a lower bound on the strengths of the members was developed for one of them. This model was also applied to a limited number of test results reported by others. The correlation was considered to be fairly good. It was discovered that the reinforcement, in resisting torsion, created lateral bending moments in the members. These changed the distribution of stress in the reinforcement, and led to the early yielding of some bars. A theoretical model was also developed to analyse this phenomenon. It agreed well with the test results.

Résumé

Dix poutres en béton armé ont été testées à la torsion, la flexion et le cisaillement combinés. Six de ces poutres avaient une section de 8×8 inches et n'étaient armées qu'en longueur. Les autres quatre avaient une section de 6×12 inches et étaient armées en longueur et en largeur. La principale variable était le rapport de la flexion à la torsion. On a examiné les tensions et le mode de rupture. Deux types de rupture ont été observés et pour l'un des deux, un modèle théorique a été développé, permettant de déterminer la limite inférieure des tensions de rupture. Ce modèle a été contrôlé avec un certain nombre de tests fait par d'autres, et l'on peut dire que la correspondance est assez bonne. On a découvert en outre que l'armature soumise à la torsion, produit des moments de flexion latéraux, ce qui mène à un changement de la répartition des tensions dans l'armature et à l'écoulement prématurée dans certaines barres. Pour analyser ce phénomène, on a développé un deuxième modèle, correspondant très bien avec les resultats des expériences.

Zusammenfassung

Es wurden zehn Stahlbetonträger untersucht, bei gleichzeitiger Torsions-, Biegungs- und Schubbeanspruchung. Darunter hatten sechs einen Querschnitt von 8×8 Zoll und waren nur längsarmiert. Die vier andern hatten einen Querschnitt von 6×12 Zoll und waren sowohl längs- als auch querarmiert. Die wichtigste Unabhängige dabei war das Verhältnis der Biegung zur Torsion. Beobachtet wurden die Spannungen und die Bruchart. Dabei stellte man zwei Bruchtypen fest. Für einen dieser Typen wurde ein Modell entwickelt, das die rechnerische Ermittlung der unteren Grenze der Bruchspannungen erlaubt. Dieses theoretische Modell wurde an verschiedenen fremden Tests geprüft und zeigte eine ziemlich gute Übereinstimmung mit den Meßresultaten. Man stellte fest, daß die torsionsbeanspruchte Stahlbewehrung Biegemomente in Querrichtung hervorruft. Dies änderte die Spannungsverteilung in der Bewehrung und führte bei einigen Bewehrungsstäben zum vorzeitigen Fließen. Um diese Erscheinung zu analysieren, wurde ein zweites Modell entwickelt, das gut mit den Meßwerten übereinstimmt.