The approximation of stability effects on frames

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The Approximation of Stability Effects on Frames

Une approximation des facteurs influant sur la stabilité de portiques

Näherungen für Stabilitätseinflüsse an Rahmen

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Introduction

The inelastic behavior of multi-story frameworks can be appreciably influenced by stability effects. An approximate method for computing the maximum load carrying capacity of inelastic frames has been proposed by MERCHANT [1]. In this method the maximum capacity as influenced by stability effects, P_m, is related to the rigid-plastic collapse load, P_p, and the elastic critical load, P_c.

$$\frac{1}{P_m} = \frac{1}{P_p} + \frac{1}{P_c}. \tag{1}$$

Horne has shown that the empirical relationship has some theoretical justification for frames bent into a double curvature configuration that is similar to the mode shape for sidesway buckling [2]. Therefore, the prediction is generally valid for frames subjected to appreciable horizontal loads. The available experimental results indicate that the Merchant formula reasonably describes the maximum load carrying capacity of one and two story frames [3].

For the usually encountered framework, the elastic critical load is very much larger than the rigid-plastic collapse load. Consequently, HORNE has noted that the accuracy of Eq. (1) can be maintained without the necessity of having "exact" elastic critical loads [4]. By using approximate buckling loads, the computational work required to use the MERCHANT formula is considerably reduced and the method becomes more practical to employ.

The object of this paper is to compare the results obtained from the Merchant prediction with the collapse loads obtained by the more accurate method of computerized elastic-plastic analysis. The elastic and collape load behavior of eight multi-story frames, with and without stability effects, has already been described by this author [5]. The previously reported research is used to compute crude estimates of the elastic critical loads. In addition, the exact critical loads are computed and used to demonstrate that the Merchant prediction remains virtually unaltered by the use of approximate buckling loads.

Computation of Critical Loads

The determination of "exact" elastic critical loads requires considerable computational effort. Critical loads have been obtained by several methods: moment distribution, stiffness matrix techniques and eigenvalue computations. The latter two methods are suitable for the computerized investigations of large frameworks, but require the performance of repeated analyses. STEVENS and SCHMIDT have presented additional techniques based on the amplification of artificially introduced components of the buckled shape [6]. Their methods utilize the Southwell plot in order to obtain initial estimates of the critical load. Improvements of the estimated critical load are obtained by iterative techniques. The process can be terminated prematurely if approximate critical loads are all that is needed.

A few researchers have reported methods for computing approximate critical loads to be specifically used in the Merchant formula. HORNE has used rigid-plastic-rigid analysis to derive approximate elastic critical loads [4]. STEVENS has used energy techniques for the same purpose [7]. The approach used by Stevens results in the definition of the maximum allowable horizontal structure sway as a function of the elastic critical load. In either case, the approximation requires the performance of some sort of analysis, although the work is considerably reduced from that required for an exact computation of critical load.

Approximate Critical Loads by Elastic Analyses

The results of elastic analyses can be used to approximate critical loads by the following procedure.

Assume that the deformations obtained from the usual first order, linear-elastic frame analysis are denoted by the vector $\bar{y}$ (see Fig. 1). Let the vector be described in terms of all of the i mode shapes of the frame, $\bar{y}_i$, associated with all of the i critical loads, P_i.

$$\bar{y} = a_1\bar{y}_1 + a_2\bar{y}_2 + a_3\bar{y}_3 + \ldots a_i\bar{y}_i, \tag{2}$$

where, a_i represents the participation of the ith mode in the description of the deflected shape. Furthermore, assume that the deformations given by a second order analysis (stability effects included) are $\bar{y}_*$. Then the relationship between

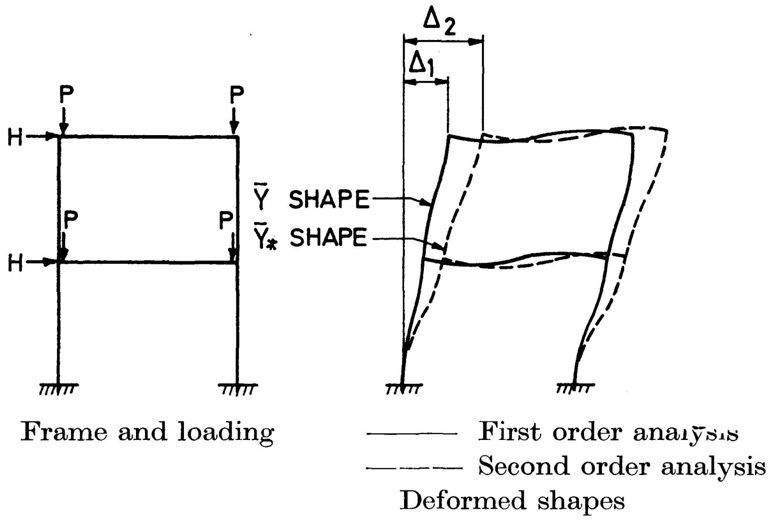


Fig. 1. Elastic frame deformations.

the amplified $\bar{y}_*$ deformations and the first order deformations is given by

$$\bar{y}_* = \frac{a_1 \bar{y}_1}{1 - \frac{P}{P_1}} + \frac{a_2 \bar{y}_2}{1 - \frac{P}{P_2}} + \frac{a_3 \bar{y}_3}{1 - \frac{P}{P_3}} + \cdots \frac{a_i \bar{y}_i}{1 - \frac{P}{P_i}}, \tag{3}$$

where, P represents the vertical load set imposed on the structure.

If the loads are such that the first order deformations an *entirely* those of the first mode, then the ratio of the second and first order deflections would be a constant value throughout the frame. Denoting the ratio of $\bar{y}_*$ to $a_1 \bar{y}_1$ by α, and using λ_c to represent the proportional multiplier of the loads P needed to cause the first critical load set, P_1,

$$\alpha = \frac{1}{1 - \frac{1}{\lambda_c}}. \tag{4a}$$

Solving for λ_c,

$$\lambda_c = \frac{\alpha}{\alpha - 1}. \tag{4b}$$

Although Eqs. (4) are true only for a very special case, the behavior of a frame subjected to appreciable horizontal loads is such that the structure primarily deforms into the double curvature configuration similar to that of the most critical sidesway buckling mode. Since only approximate critical loads are needed for use in the Merchant prediction, Formula (4b) can be utilized. Furthermore, α can be approximated by the ratio of second order to first order horizontal sway at the top of the frame. With reference to Fig. 1, this ratio is given by Δ_2/Δ_1. Therefore, the approximate critical load can be directly obtained from first and second order elastic frame analyses. It is to be noted that this method is essentially the starting point for the more accurate iterative computation of critical loads presented by STEVENS and SCHMIDT [6]. They have also suggested that the ratio α be taken as the average of the deformation

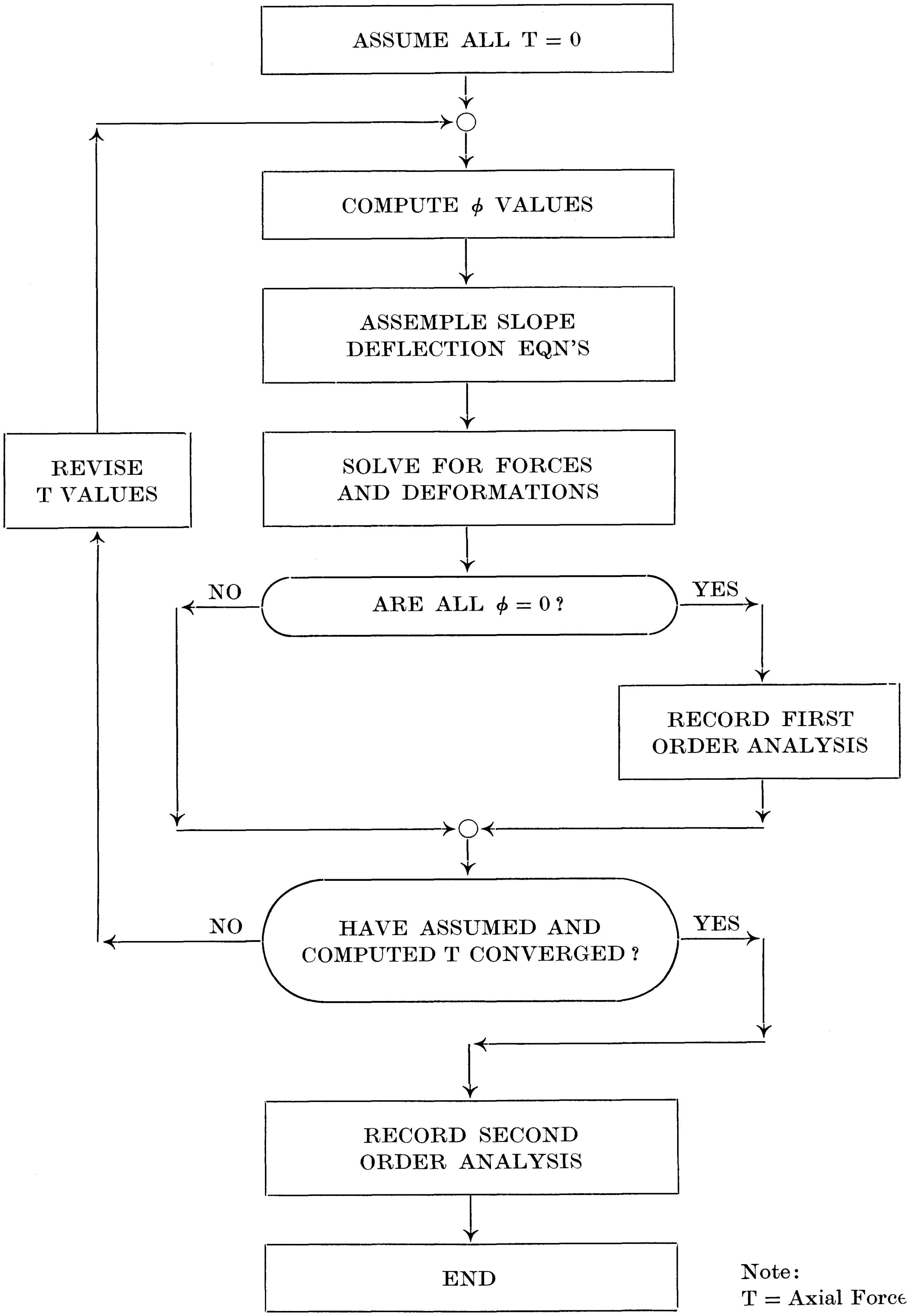


Fig. 2. Flow chart; unified elastic analysis.

ratios at all of the stories of the frame. For the particular application considered here, the refinement will be shown to be unnecessary.

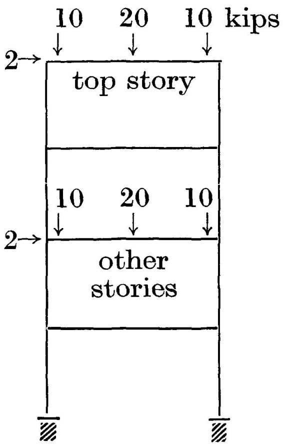
Second order elastic analysis has been programmed by HARRISON [8] and has been used by several other researchers as an integral part of elastic-plastic computer programs. For the computation of approximate critical loads, it is advantageous to unify first and second order analysis in a single program. One convenient method can be based on the slope deflection equations with stability functions that have been presented by BLEICH [9]. In this method, the slope deflection equations have coefficients dependent on the axial loads acting on the members. For a member subjected to axial load, T, and having stiffness and length of EI and L respectively, the stability factor, Φ, determines the slope deflection coefficients. The factor Φ is given by,

$$\Phi = L\left[\sqrt{\frac{T}{EI}}\right]. \tag{5}$$

When Φ is set equal to zero, the coefficients for the usual linear analyses are obtained. Thus, it is possible to obtain both first and second order analyses by formulating a second order analysis by an iterative procedure starting from the assumption that all Φ values are zero. The first computation describes the linear-elastic behavior of the frame. As the values of Φ are continually refined to account for more precise axial loads, the iteration quickly converges to yield second order deformations. The process is described by the flow chart shown in Fig. 2.

Frames and Loadings

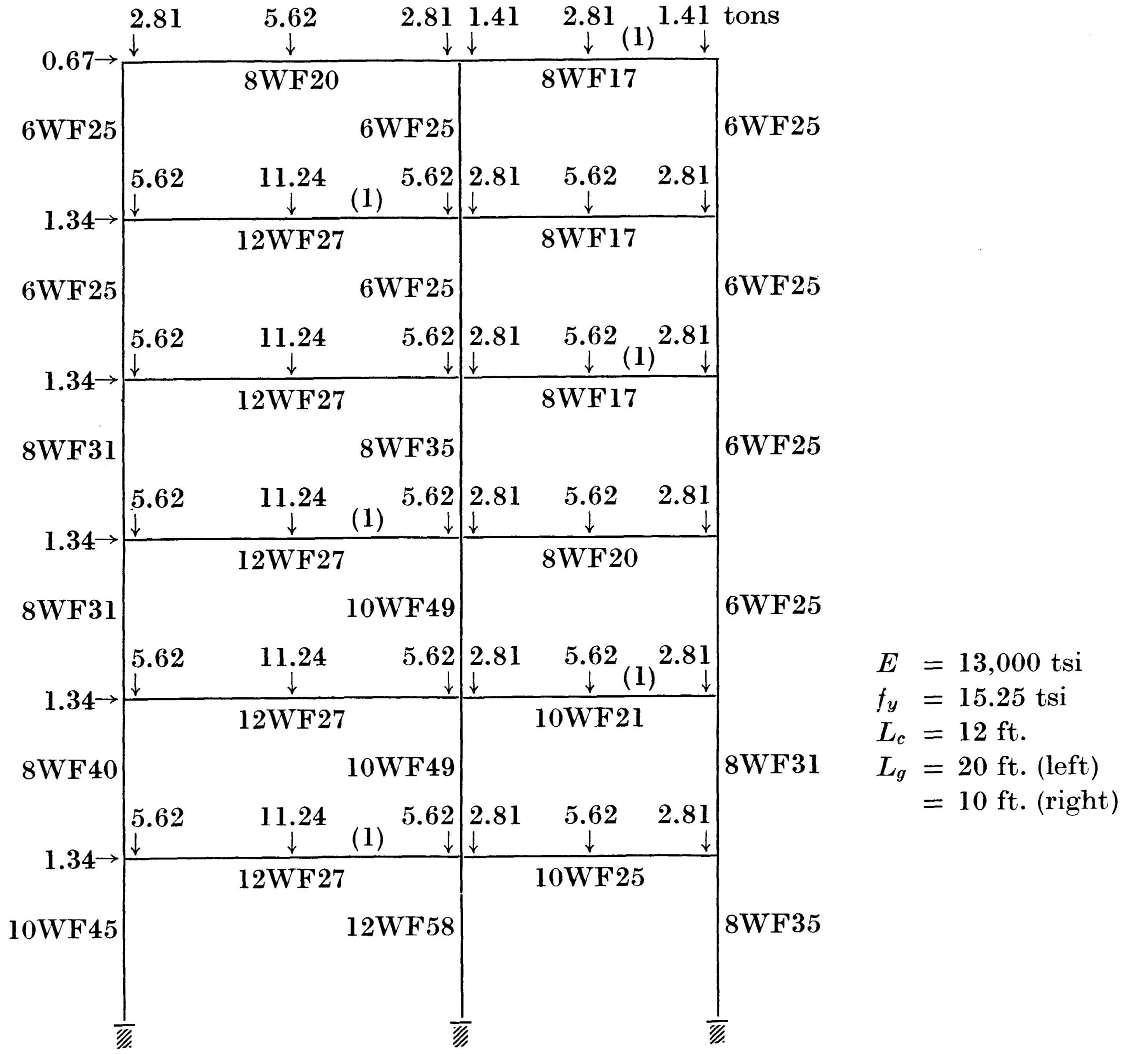
The dimensions, properties and loadings for the eight frames that have been analyzed are shown in Figs. 3 through 6. Modulus of elasticity, yield stress, column length and girder length (E, f_y, L_c and L_g respectively) are shown for each frame. The wide flange sections that have been used are also



All members are fictitious

$I = 144$ in.^{4} $\quad f_y = 36$ ksi
$M_p = 160$ ft.k. $\quad L_c = 25$ ft.
$A = 11.75$ in.^{2} $\quad L_g = 20$ ft.
$E = 30{,}000$ ksi

Fig. 3. Frame 4—1.



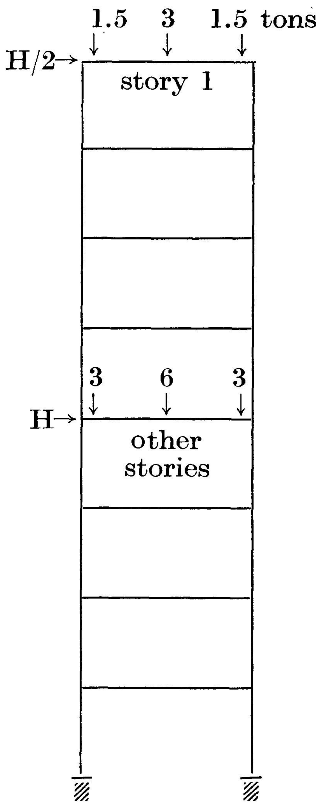
Note: Frame 6—1 shown. Frame 6—2 is identical except for half vertical loads in girder spans marked [1].

Fig. 4. Six story frames.

indicated on the figures. Where fictitious or approximate members have been used, the fully plastic moment, M_p, the moment of inertia, I, and the area, A, are also indicated. Frame designation is given by a numeral indicating the number of stories, followed by an identification number after a dash. Thus $8-2$ indicates the second of the eight story frames to be considered.

The working loads are indicated on the figures. Horizontal forces are applied at each story and a three point concentrated load system has been applied on each girder. The three point system (1/4 of the total load at each girder end and 1/2 at the center of the girder) is used to simulate uniform loads. In several cases, the loadings are given in ton units (1 ton $=$ 2.24 kips) for frames obtained from the British literature.

Frame $4-1$ has been chosen to illustrate a frame particularly susceptible to stability effects. The six-story, two-bay frames are identical with respect to members, configuration and horizontal loadings. Frame $6-1$ is subjected



Columns

Story	Frame 8—1	Frame 8—2	Frames 8—3 and 8—4
1	6 WF 15.5	6 WF 15.5	6 WF 15.5
2	8 WF 31	8 WF 24	8 WF 24
3	do	do	8 WF 28
4	8 WF 35	8 WF 28	8 WF 40
5	10 WF 49	10 WF 39	10 WF 45
6	12 WF 79	12 WF 58	10 WF 60
7	14 WF 87	14 WF 68	10 WF 66
8	14 WF 95	14 WF 74	14 WF 68

Girders

Story	Frame 8—1	Frame 8—2	Frames 8—3 and 8—4
1	8 WF 17	8 WF 20	8 WF 17
2	8 WF 20	10 WF 21	10 WF 21
3	10 WF 21	10 WF 25	10 WF 29
4	12 WF 27	12 WF 31	12 WF 31
5	14 WF 30	14 WF 34	14 WF 34
6	14 WF 34	14 WF 38	14 WF 43[1])
7	do	14 WF 43	14 WF 53
8	14 WF 38	14 WF 48	14 WF 61

E = 13,000 tsi
f_y = 15.25 tsi
$L_c = L_g$ = 10 ft.
H = 2.2 t (Frames 8—1, 2 and 3)
1.1 t (Frame 8—4)

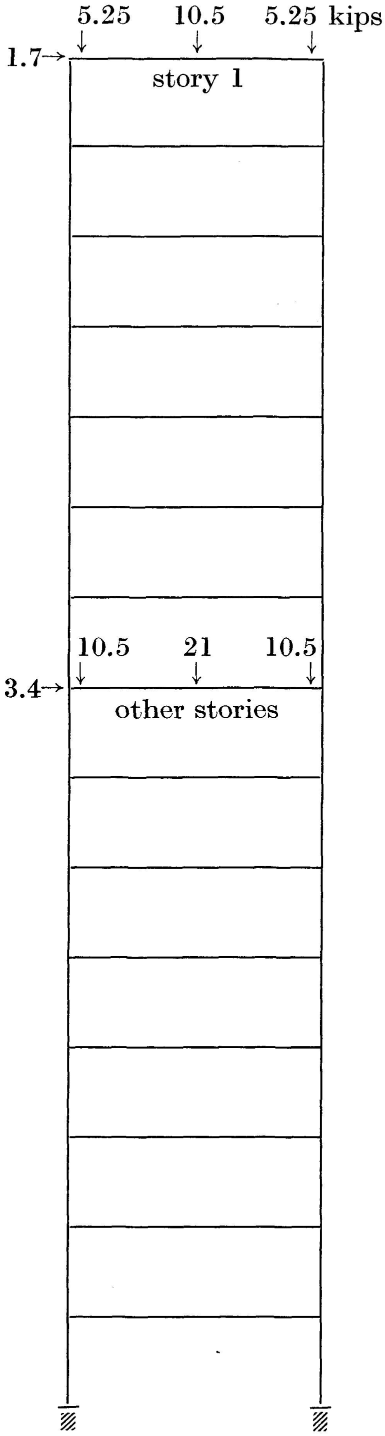
[1]) Fictitious M_p used (99.7 ft.tons).

Fig. 5. Eight story frames.

to full vertical loads, wheras the companion, 6 – 2, has vertical loads applied in a "checkerboard pattern" of full and half loading. All of the eight story frames are identical in dimensions and vertical loading. Frames 8 – 1, 8 – 2, and 8 – 3 also have identical horizontal loadings, their only difference being in the proportions of the members. Frame 8 – 4 is identical to 8 – 3, except that only half of the horizontal forces are applied. Frame 15 – 1 is the most slender of the structures considered, having a height-to-width ratio of 10.5 to 1. For a more complete description of the source of the frames, the reader is referred to Reference [5].

Results of the Analyses

The results of all the exact and approximate calculations are summarized in Table 1. Previously performed elastic-plastic analyses have been used to furnish values of the amplification factors at working loads and the first and second order collapse load factors (α, λ_p and λ_2 respectively). The elastic-



Story	Columns	Girders
1	8 WF 17	8 WF 20
2	8 WF 28	10 WF 25
3	10 WF 39	12 WF 27
4	do	12 WF 36
5	12 WF 40	do
6	12 WF 50	12 WF 45
7	12 WF 58	14 WF 53
8	14 WF 61	14 WF 61
9	14 WF 74	do
10	14 WF 84	14 WF 74
11	14 WF 111	14 WF 78
12	do	14 WF 84
13	do	16 WF 88
14	14 WF 127	16 WF 96
15	14 WF 136	18 WF 96

$E = 30{,}000$ ksi
$f_y = 36$ ksi
$L_c = 14$ ft.
$L_g = 20$ ft.

Fig. 6. Frame 15—1.

plastic analyses include the effects of bending deformations only, since axial deformations had negligible effects on the maximum frame capacity [5]. In all cases, monotonically increasing proportional loads were applied up to the maximum frame capacity. The plastic moment capacity of each member was continually for existing axial load in accordance with the bi-linear formula of the A.I.S.C. [10].

Table 1. Critical loads and collapse loads

Frame	From Elastic-Plastic Analysis, Reference 5			Elastic Critical Loads, λ_c			Merchant Loads, λ_m, Formula 1 [5]			Exact $\frac{\lambda_m}{\lambda_2}$
	α [1]	λ_p [2]	λ_2 [3]	Exact	Approx. [4]	Approx. ÷ Exact	Exact	Approx.	Approx. ÷ Exact	
4—1	1.2961	2.067	1.286	4.00	4.38	1.10	1.36	1.40	1.03	1.06
6—1	1.1021	1.571	1.367	9.89	10.8	1.09	1.36	1.37	1.01	0.99
6—2	1.0750	1.719	1.524	13.2	14.3	1.08	1.52	1.53	1.01	1.00
8—1	1.0267	1.649	1.414	33.5	38.5	1.15	1.57	1.58	1.01	1.10
8—2	1.0273	1.511	1.424	30.6	37.6	1.23	1.44	1.45	1.01	1.01
8—3	1.0244	1.814	1.646	37.1	42.0	1.13	1.73	1.74	1.01	1.05
8—4	1.0243	2.905	2.836	37.1	42.2	1.14	2.69	2.76	1.03	0.95
15—1	1.0779	1.632	1.403	12.2	13.8	1.13	1.44	1.46	1.01	1.03

[1]) $\alpha = \Delta_2/\Delta_1$ = Amplification of top sway at working loads.
[2]) λ_p = Load factor for rigid-plastic collapse.
[3]) λ_2 = Load factor at collapse, second-order elastic-plastic analysis.
[4]) Approximate critical loads computed from Formula 4b.
[5]) Exact and approximate λm values are based on exact and approximate critical loads respectively.

$$\begin{aligned} M_{pc} &= M_p && (P/P_y \leqq 0.15), \\ M_{pc} &= 1.18\, M_p\,[1 - P/P_y] && (P/P_y > 0.15). \end{aligned} \tag{6}$$

Here, the reduced and fully plastic moments are given by M_{pc} and M_p respectively. The existing axial load in the member is P, and the fully yielded capacity of the section is denoted by P_y. It is to be noted that information for calculating α was automatically obtained by the performance of first and second order elastic-plastic analysis.

The critical loads, λ_c, were computed by two methods. Exact values were calculated by formulating the slope deflection equations with stability functions and equating the determinant of the resulting stiffness matrix to zero [9]. For this purpose, each in-span girder load was replaced by two equal loads acting over the adjacent columns. Trial values of load factor were chosen and the determinant of the stiffness matrix was calculated. By means of interpolation, accurate values of the critical loads were obtained. Approximate critical loads were obtained by substituting the previously determined values of α into Eq. (4b). Exact and approximate Merchant loads were obtained by using the values of λ_p in conjunction with the exact and approximate critical loads.

The approximately calculated critical loads were anywhere from 8 to 23 percent higher than the true critical loads. Thus, the approximations are considered to be crude. Improved estimates of λ_c could have been obtained by averaging the values of α obtained at each story level of the frame. In the case of Frame 4 – 1, the values of α from the top to the bottom story were 1.2961, 1.3161, 1.3398 and 1.3375 respectively. By using any one of the stories to describe α, the greatest variation in λ_c would have been from 3.94 to 4.38. The value of λ_c would have been 4.10 if averaging were employed. However, the crude value of λ_c is seen to be entirely adequate when employed in the already empirical Merchant formula. The approximate Merchant load is at worst 3% higher than the exact Merchant load.

For the frames in question, the Merchant formula gave reasonable estimates of the maximum frame capacity. At worst, the empirical prediction was 5% low or 10% high. The predictions for Frames $6-1$, $6-2$ and $8-2$ were almost identical to the values obtained by second order elastic-plastic analysis. However, the virtue of the Merchant formula is seen to be its ability to describe the correct trends of frame behavior. Frame $4-1$ lost 38 percent of its potential load carrying capacity due to stability effects — Formula 1 predicted a 37 percent loss. All of the other frames were relatively insensitive to stability effects, losing at most 14 percent of their rigid-plastic capacity. Here, the Merchant prediction tended to considerably underestimate stability effects. Yet, when Eq. (1) denoted insignificant frame stability effects, the effects were indeed insignificant. In no case did the Merchant prediction describe an incorrect trend of frame behavior. It is to be noted that the Merchant formula has been used here in conjunction with reduced plastic moments. The validity of using reduced plastic moments is justified solely by the correctness of the empirical results.

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The remainder of the research has been performed in the Department of Civil Engineering of the University of Kentucky.

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Summary

A method for computing crude estimates of elastic critical loads has been utilized in the empirical predictions of maximum frame capacity. The crude estimates have been shown to be more than adequate when used solely for the Merchant prediction. Furthermore, the empirical Merchant prediction described the correct general trends of carrying capacity that were obtained by second order elastic-plastic analysis.

Résumé

Pour obtenir une prévision empirique des capacités de charges maximales de huit portiques plans, à plusieurs étages, rectangulaires et non entretoisés, on s'est servi d'une méthode permettant une estimation grossière des charges élastiques critiques. Or pour la «prévision Merchant», cette estimation grossière se montre plus que suffisante. Un des avantages de la méthode proposée est qu'elle permet de contrôler le comportement élastique des portiques.

Les capacités maximales des portiques, trouvées avec la «prévision Merchant» ont été comparées aux résultats obtenus par un calcul élasto-plastique du second ordre. Dans tous les cas, la relation empirique donne l'allure générale correcte de la capacité de charge. Ainsi, on a tout intérêt à se servir de la formule de Merchant, avec des charges critiques approximées et des charges de rupture rigide-plastique estimées, pour détecter d'éventuels portiques particulièrement sensibles à des instabilités.

Zusammenfassung

Ein Verfahren zur groben, schätzenden Berechnung elastischer, kritischer Lasten ist für die empirische Voraussage maximaler Tragfähigkeit von acht mehrstöckigen, unversteiften, rechteckigen und ebenen Stockwerkrahmen angewandt worden. Die grobe Schätzung ist durch die Merchant-Voraussage (Merchant prediction) mehr als genügend. Ein Vorteil der vorgeschlagenen Methode besteht darin, daß das elastische Rahmenverhalten für Prüfungen verwendbar ist. Die maximale Tragfähigkeit der Merchant-Voraussage ist mit den Ergebnissen der elasto-plastischen Analyse zweiter Ordnung verglichen worden. In allen Fällen beschreibt die empirische Beziehung die genaue Richtung der Tragfähigkeit. Demgemäß möge der Gebrauch der genäherten, kritischen Lasten und die geschätzten Traglasten in den Merchant-Formeln für Rahmen verwendet werden, die möglicherweise empfindlich auf große Stabilitätseinflüsse sind.