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Nonlinear Inelastic Dynamics of Multi-Story Buildings

Dynamique non-linéaire inélastique de bâtiments à multiples étages

Nichtlineare, unelastische Dynamik von Hochhäusern

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Introduction

It is of interest to explore further and elaborate on the General Report on "Dynamic Effects of Wind and Earthquake" by D. Sfintesco in the *Preliminary Publication* of the Eighth Congress of IABSE on the effects of nonlinear and inelastic deformation of multi-story buildings due to strong-motion earthquake [1].

In the general report, plastic deformation is correctly stated to act as a damping mechanism under high intensity earthquakes. In addition, damage is incurred whenever yielding takes place and the damage is accumulative. The damage may be assessed by integrating the area under the hysteresis loops of the moment-curvature diagram for each member since the method of analysis discussed herein accounts for the columns in flexure as continuous members, including the nonlinear and inelastic effects, instead of lumping the inelastic strain at discrete points as plastic hinges or bilinear hinges.

In this discussion a method of shear building analysis by direct stiffness presented by Saul, Fleming and Lee [2] for a bilinear hysteretic strain hardening moment-curvature material law is solved numerically by the fourth order Runge-Kutta formulation for a 10-story building subjected to an interval of the ground acceleration record of the north-south component of the 1940 El Centro, California earthquake to obtain very accurate response records of lateral deflection, column end moments, and other parameters of interest. In a shear building, the floor systems act as rigid diaphrams and the columns, which are rigidly attached to the floor systems, resist all lateral forces and thereby undergo relative displacement between floors. In the analyses, the column segments are massless and continuous; the mass of each story is concentrated at the floor levels.

The nonlinear set of coupled differential equations of motion are numerically integrated over variable steps of time which, in this situation, are governed by either the pulse duration of the seismogram or a smaller interval used to pinpoint phase transitions in the bilinear hysteretic moment-curvature relationship. The response curves obtained, which are the histories of deflection, moment, etc., contain all the effects inherent in the assumptions including nonlinearities, inelastic deformations, and vibration in higher modes. It is notable that the response over the short time increment of integration is not assumed to be linear, but, in fact, follows the material law for shear-deflection which, integrated from the assumed bilinear moment-curvature relationship, is linear when elastic but otherwise nonlinear.

From the response data obtained, illustrative response curves for lateral deflection and column end moments are presented as well as moment-curvature diagrams and envelopes of maximum deflection and moment. Deflected shapes at particular times are also shown. Three values of the strain hardening coefficient were used in the analyses which define an elastic structure, an elasto-plastic structure, and a moderately strain hardened structure. Although damping was neglected in the problem solutions, it is available in the formulation and can be easily included. Since damping restricts vibration, it may be instructive to include it.

A frame building with flexible girders may be sized so that yielding must primarily take place in the girders. However, structures in which the floor beams are composite with reinforced concrete floors or are otherwise more rigid with respect to the columns, will force the inelastic deformation into the columns. Inelastic strains of increasing magnitude, accompanied with large deflections, may indicate that the structure is collapsing since stiffness in the inelastic material regime is a function of relative lateral deflection.

All equations herein are based on an n-story building with n-degrees of freedom in the plane. Thus, summation over repeated indices are understood to be from 1 to n and free indices identify a particular element.

Theoretical Background of the Inelastic Analysis

The complete derivation of the analysis is somewhat lengthy and given elsewhere [2] so only high points of the theory are repeated here. The material law and the resistance function of the structure are reviewed as items of principal interest.

The material represented by a bilinear strain-hardening hysteretic moment-curvature relationship can be given as

$$M_i = N_i + K_{ij}(\Phi_j - \Psi_j). \tag{1}$$

where M_i is the bending moment, Φ_j is the curvature, N_i and Ψ_j are the moment and curvature values at a material phase change and are constant throughout a phase, and the stiffness is

$$K_{ij} = \begin{cases} EI & \text{if } i=j \text{ and the phase is elastic,} \\ \alpha EI & \text{if } i=j \text{ and the phase is plastic,} \\ 0 & \text{if } i \neq j, \end{cases} \tag{2}$$

in which α is the coefficient of strain hardening. When $\alpha = 1$ the material remains elastic and as $\alpha \to 0$ an elastic-perfectly plastic material is represented. In addition, variable values of moment and curvature within a material phase, zero at each phase change, are defined as

$$L_i = M_i - N_i, \tag{3}$$

$$\Omega_i = \Phi_i - \Psi_i \tag{4}$$

so that

$$L_i = K_{ij}\Omega_j. \tag{5}$$

Thus, the material law is defined.

The equations of motion of an n-story multiple bay shear building with framed columns of an elastic-linear strain hardening inelastic material subjected to seismic motion are given as

$$m_{ij}\ddot{u}_j + C_{ij}\dot{u}_j + A_{ij}(u)u_j + B_i(u) = W_i G f(t), \tag{6}$$

where m_{ii} is the mass of the i-th story, the C_{ij} are viscous damping coefficients, W_i is the weight of story i, G is the magnitude factor of the earthquake (maximum fraction of acceleration due to gravity), $f(t)$ represents the unitized time variation of the earthquake; u_j, $\dot{u}_j$, and $\ddot{u}_j$ are the relative displacement, velocity, and acceleration, respectively, of the j-th floor, with respect to the base, the $A_{ij}(u)$ are stiffness influence coefficients, and $B_i(u)$ is a residual vector proportional to the accumulated inelastic strain in the columns. Furthermore, the resistance function of the structure, R_i for the i-th floor, is variable and may be represented as a function of relative displacement by

$$R_i(u) = A_{ij}(u)u_j + B_i(u), \tag{7}$$

where

$$A_{ij}(u) = S_{ik}K_{kr}\Gamma_{rj}(u), \tag{8}$$

$$B_i(u) = S_{ik}(N_k - K_{kr}\Gamma_{rj}(u)T_{jp}\Lambda_p), \tag{9}$$

in which T_{jp} and S_{ik} are constant for a particular structure, their being defined by geometry, Λ_p accounts for residual strain at phase changes, N_k is the bending moment at a phase change, and $\Gamma_{rj}(u)$ is a nonlinear function of $\Omega_i(u)$ the phase variable curvature which is a function of relative displacement. When $\alpha \to 0$ $\Gamma_{rj}(u)$ is singular so a very small value of α, say $\alpha = 0.0001$, is used in calculations for the elastic-perfectly plastic case.

A detailed derivation of the analysis which includes complete definitions of all the tensors given herein is given in reference [2].

The Method of Computation

Numerical Integration

The equations of motion, Eq. (6), without damping, were integrated numerically using the Runge-Kutta fourth-order method [3]. The equations of motion are written

$$\ddot{u}_i = F(t, u_i, \dot{u}_i) = \frac{1}{m_{(ii)}} [G W_i f(t) - A_{ij}(u) u_j - B_i(u)] \tag{10}$$

and the solution at time $t+\tau$, where $\tau = \Delta t$, is

$$u_i(t_1+\tau) = u_i(t_1) + \tau \dot{u}_i(t_1) + \tfrac{1}{6}\tau(a_i + b_i + c_i), \tag{11A}$$

$$\dot{u}_i(t_1+\tau) = \dot{u}_i(t_1) + \tfrac{1}{6}(a_i + 2b_i + 2c_i + d_i), \tag{11B}$$

in which

$$a_i = \tau F[t_1, u_i(t_1), \dot{u}_i(t_1)], \tag{12A}$$

$$b_i = \tau F[t_1 + \tfrac{1}{2}\tau, u_i(t_1) + \tfrac{1}{2}\tau \dot{u}_i(t_1), \dot{u}_i(t_1) + \tfrac{1}{2}a_i], \tag{12B}$$

$$c_i = \tau F[t_1 + \tfrac{1}{2}\tau, u_i(t_1) + \tfrac{1}{2}\tau \dot{u}_i(t_1) + \tfrac{1}{4}\tau a_i \dot{u}_i(t_1) + \tfrac{1}{2}b_i], \tag{12C}$$

$$d_i = \tau F[t_1 + \tau, u_i(t_1) + \tau \dot{u}_i(t_1) + \tfrac{1}{2}\tau b_i, \dot{u}_i(t_1) + c_i]. \tag{12D}$$

Note that the $A_{ij}(u)$ and $B_i(u)$ are functions of displacement and are recalculated for each of the values in Eqs. (12) using the extrapolation values of the displacement function in the formulation.

The duration of the time increment $\Delta t = \tau$ should be as large as possible to conserve computation time and retain a valid solution. However, it may have to be decreased to pinpoint the coordinates of a material phase transition or the characteristics of the forcing function. Since the duration of Δt may be changed with each integration and the time between pulse changes of an earthquake accelerogram is considerably smaller than the period of a tall building, a larger time increment well within the duration of pulse change coordinates of the seismogram and a smaller time increment of 1/20 to 1/4 of the larger to locate the material phase transitions should be chosen for the integration.

Computer Program

A program was written to perform the computation with a digital computer. Once the story heights, weights, and column stiffnesses and yield moments are processed to obtain the physical parameters describing the structure, seismogram cards are read which give the coordinates of acceleration and time delineating peak values. As each seismogram card is read integration proceeds, linearly interpolating seismogram values, at the larger time increment until a material phase transition has been passed. At that time, the last set of calculations are rejected and integration proceeds from the former time with the smaller time increment until the material phase transition is established. Control then returns to the larger time increment until a material phase transition is again detected, at which time the process is repeated.

Each cycle of integration required the reestablishment of the variable matrices, a number of matrix operations, and a check on each column for possible material phase transitions. Lateral floor deflections; column moments, shears, curvature, and angle change moments; residual values of column moments, curvature, and angle change moments; as well as the stiffness influence coefficients and the residual force vector are obtained and printed with each integration cycle.

The procedure is essentially a direct stiffness method and, therefore, matrix operations of multiplication or addition only are required.

The Design Building

For purposes of this study, a 10-story shear building was designed for which the parameters of a column segment are given in Table 1. The funda-

Table 1. 10-story column segment of design building

Story	Height (ft.)	Weight (kips)	I (in.^{4})
1	15	58.5	1166
2	12	58.5	1166
3	12	58.5	1166
4	12	56.5	1166
5	12	56.5	583
6	12	56.5	583
7	12	49.5	583
8	12	49.5	389
9	12	49.5	389
10 (Roof)	12	44.0	389

mental period of the elastic structure is 1.5 seconds using $E = 30{,}000$ ksi for the material. The first column of 4 stories length extends through the first story although this obviously decreases its stiffness. This built in defect not only sensed response effects more rapidly than the rest of the structure as expected but also appears to provide a probable collapse mechanism, especially in the elasto-plastic case.

The initial yield moment for a column is reduced because of the presence of axial force and is given, arbitarily, by

$$N_i' = \epsilon I_i, \tag{13}$$

where ϵ is a factor defined by

$$\epsilon = 1.076 \frac{F_y}{d}. \tag{14}$$

F_y is the yield stress of the material, and d is the depth of the section. Eq. (14) is derived as follows: From AISC formula 21 [4]:

$$\frac{N_i'}{M_{pi}} = 1.18\left(1 - \frac{P}{P_y}\right), \tag{15}$$

where $P_y = F_y A$, P is the axial force, and

$$M_{pi} \cong 1.14 \frac{2 F_y I}{d}. \tag{16}$$

Since maximum $P = 0.6 P_y$ (Section 2.3 of [4]) substitution of Eq. (16) into Eq. (15) yields Eq. (14). Thus, for the A 36 steel 12 inch columns used in the problem $\epsilon = 0.269$ ft.-kips/in.[4]. In addition, the yield moment of column 1 was further reduced by a ratio of the story heights of adjacent columns cubed to account for its lesser stiffness. This factor is empirical and may prove to be excessive or unnecessary, however, results of analyses without it were not significantly different.

The building is tapered in stiffness and mass. It is representative of a commonplace type of rectangular design although the analysis can handle buildings which decrease in the number of bays with height.

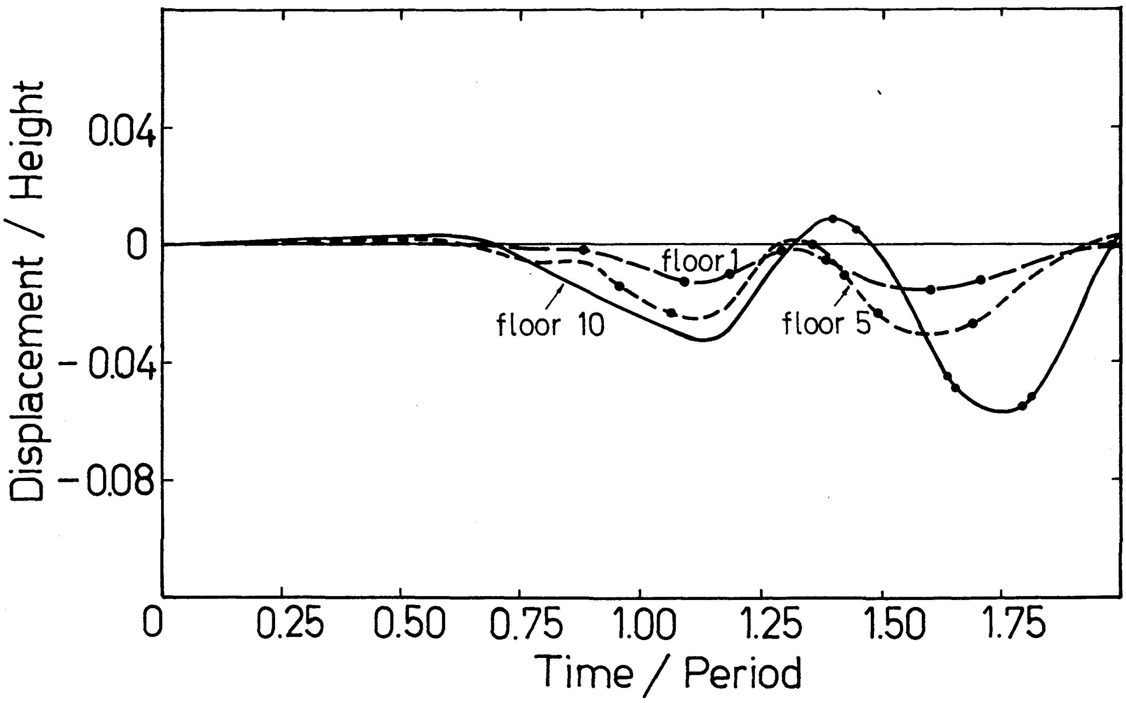

Fig. 1. Deflection response curves for floors 1, 5 an 10 ($\alpha = 0.10$).

Response Calculations

The north-south component of the May 18, 1940 El Centro, California ground acceleration, which is well known and somewhat of a standard in earthquake engineering studies, was used as the dynamic excitation in this study. It is representative of a moderately strong earthquake. The maximum pulse has an acceleration of about 32% of gravity. The most intensive portion of the earthquake was in the second through fifth seconds of its duration although intensive ground movement continued for a half minute. To conserve computer time the problem calculations were generally for 3 seconds of earthquake duration although several solutions were for more extensive periods of time. The

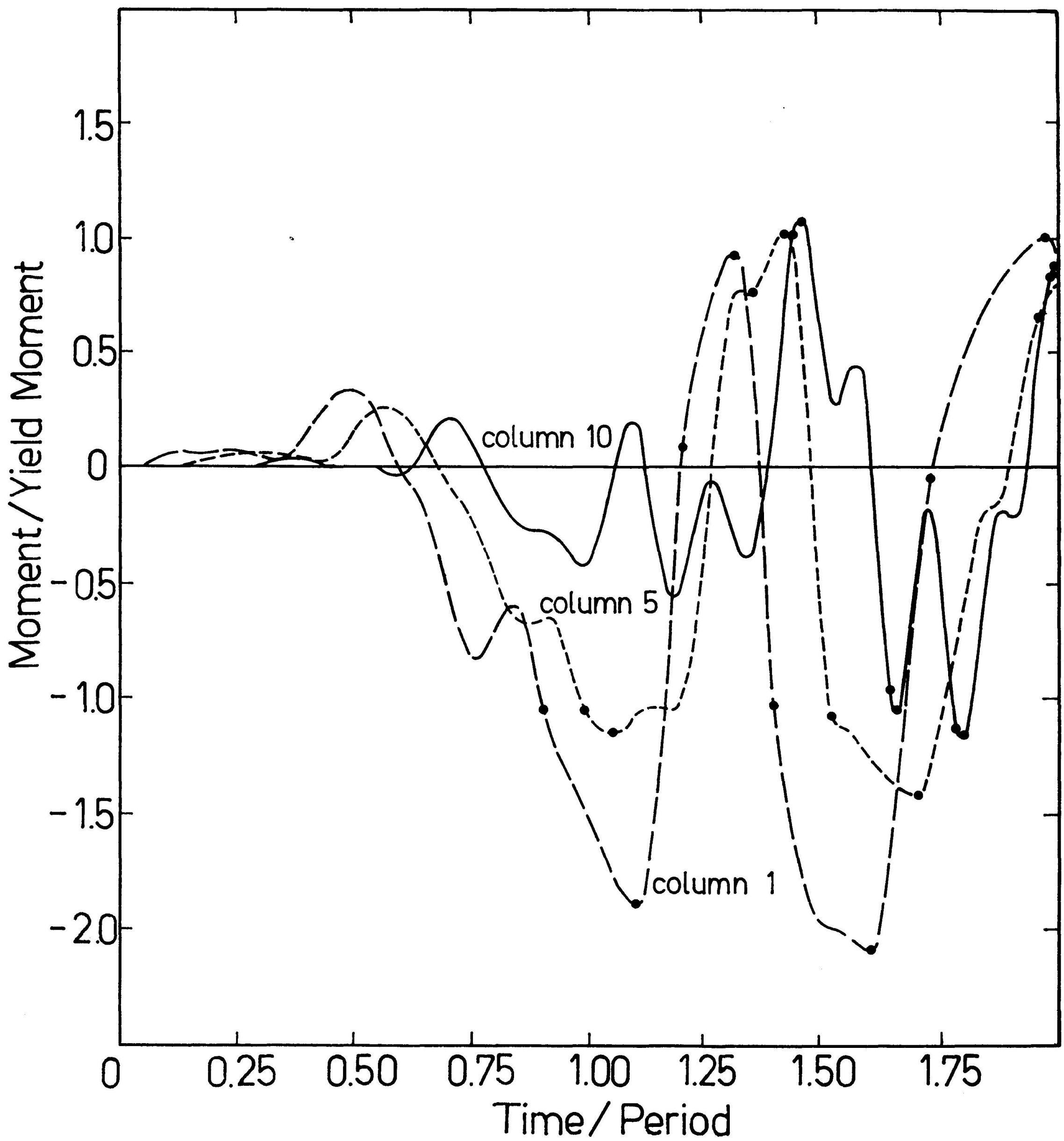

Fig. 2. End moment response Curves for columns 1, 5 and 10 ($\alpha = 0.10$).

results obtained for this particular excitation are only indicative of earthquake response in general since each has its own characteristics.

The design building, with strain-hardening coefficients of $\alpha = 0.00001$ (elasto-plastic), $\alpha = 0.10$ (moderate strain-hardening) and $\alpha = 1.0$ (elastic) was subjected to 3 seconds of excitation. Stress resultants of moment, deflection, curvature, influence coefficients, the resistance forces including residuals, and the excitation were recorded at each time interval of integration, as well as other parameters. From the data representative curves were plotted.

Results of the Analysis

Histories of the deflections and column end moments of floors 1, 5 and 10 with a strain hardening coefficient of $\alpha = 0.10$ are shown in Figs. 1 and 2. The response of floor 10 for the elasto-plastic case, $\alpha = 0.00001$, a strainhardened case in which $\alpha = 0.10$, and the elastic case in which $\alpha = 1.00$ are shown in Figs. 3 and 4. Material regime changes are marked on the curves indicating when excursions take place into the plastic phase. Although excessive deflections were noted for the elasto-plastic case, which may indicate incipient collapse, the bending moment is, of course, contained. However, in the presence of strain

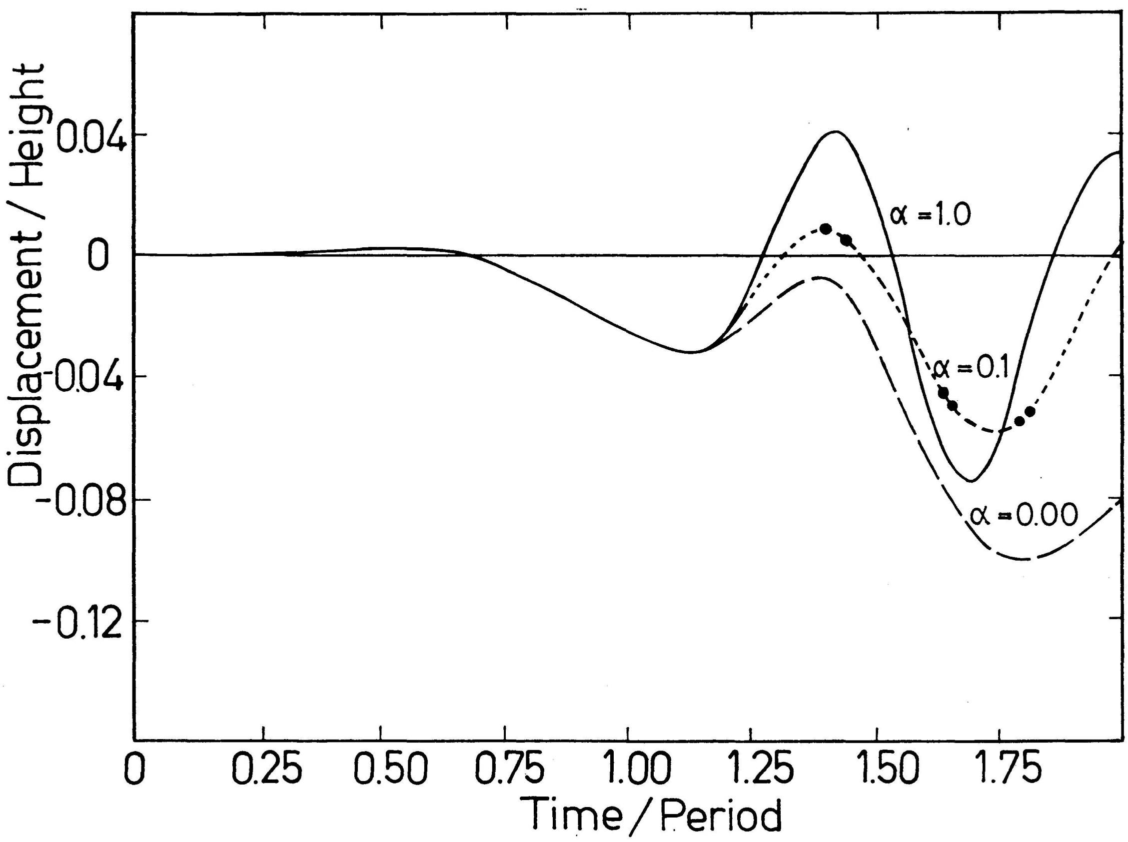

Fig. 3. Deflection response curves for floor 10 for $\alpha = 0.0$, $\alpha = 0.1$ and $\alpha = 1.0$.

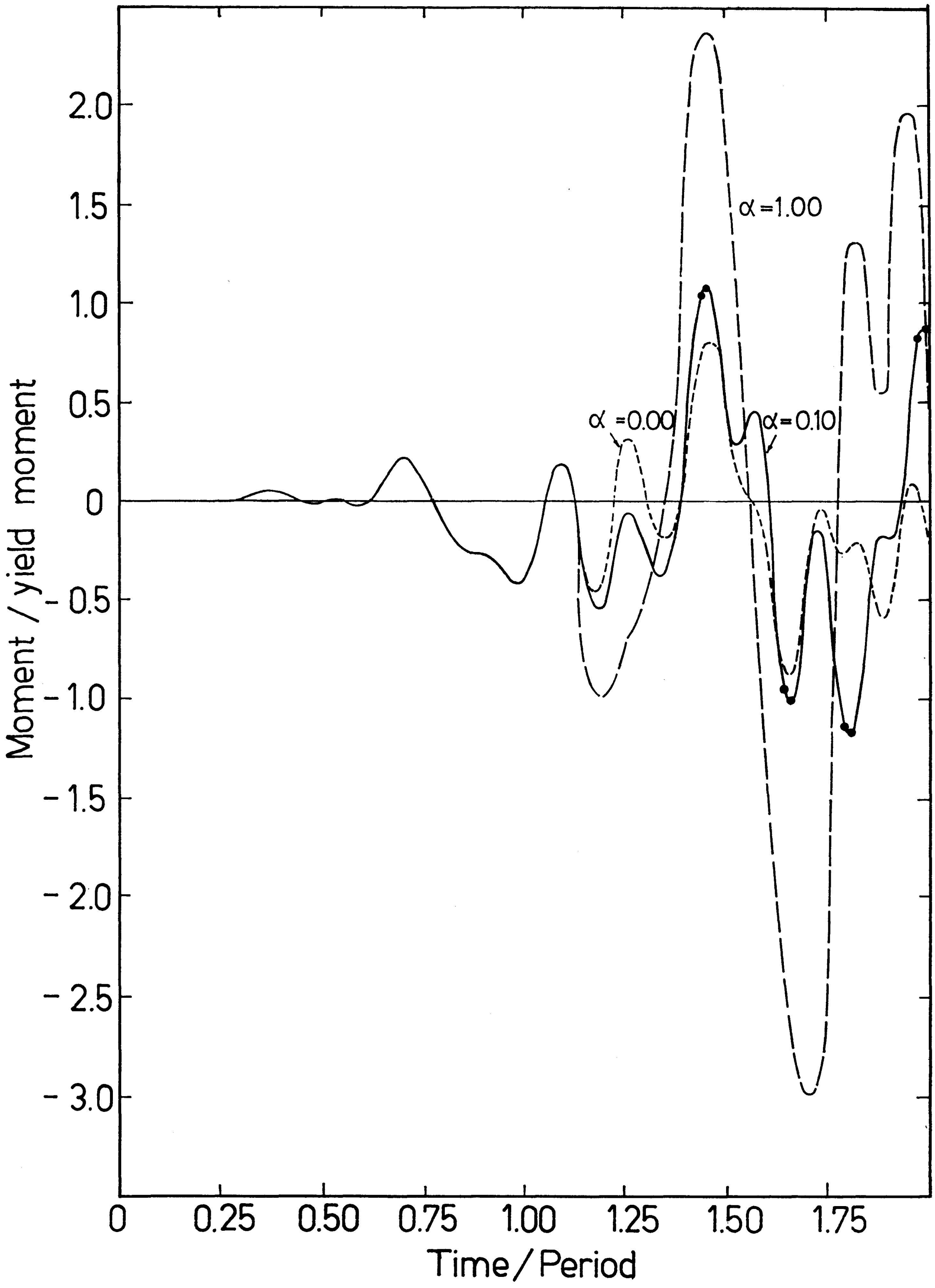

Fig. 4. End moment response curve of the 10th story column for $\alpha=0.00$, $\alpha=0.10$ and $\alpha=1.00$.

hardening, although deflections are considerably decreased with respect to the elasto-plastic case, the magnitude of bending moment in the first story column indicates that it is undergoing excessive deformation although the higher columns are not.

The moment-curvature excursions for columns 1 and 10 with a strain hardening coefficient of $\alpha = 0.10$ are shown in Fig. 5. The amount of plastic deformation can be easily seen from this type of diagram. The limbs of the curves are either positive velocity or negative velocity and their directions are marked. The area under this diagram is a measure of energy and so constitutes a record of cumulative damage. After the ground acceleration has died out

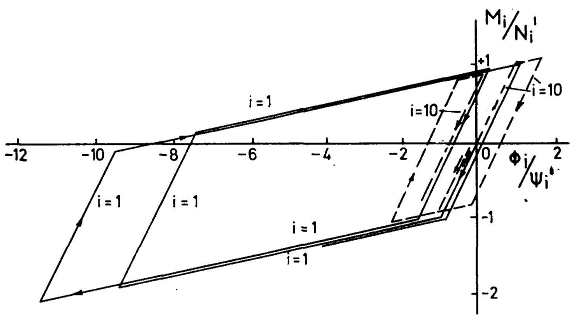

Fig. 5. End moment-curvature history of columns 1 and 10 for $\alpha = 0.1$,

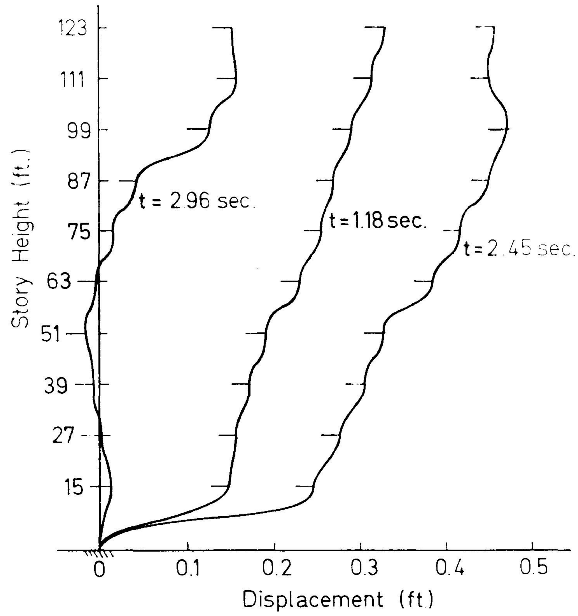

Fig. 6. Deflected position of 10-story column at selected times ($\alpha = 0.10$).

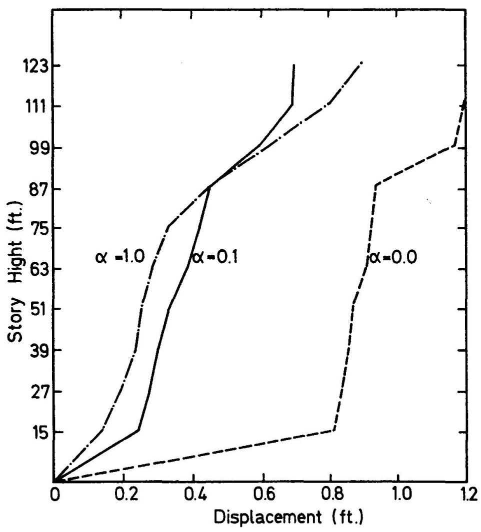

Fig. 7. Envelope of maximum story deflections for $\alpha = 0.00$, $\alpha = 0.10$ and $\alpha = 1.00$ with 3 second duration.

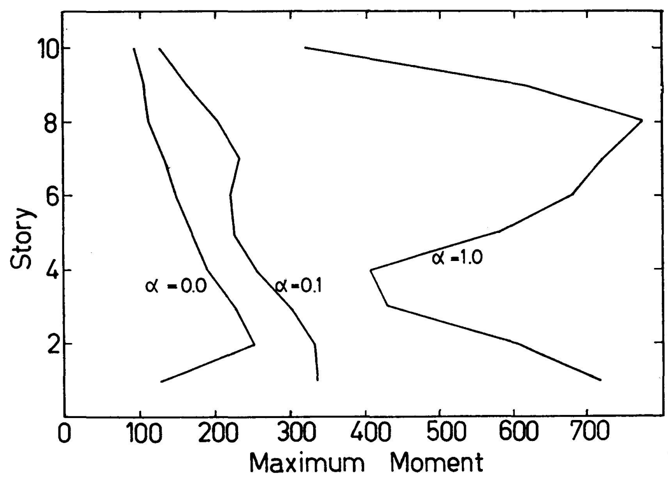

Fig. 8. Envelope of maximum column moments of each story for $\alpha = 0.0$, $\alpha = 0.1$ and $\alpha = 1.0$ within a 3 second duration. (Moments in ft.-kips.)

the structure, if it has not collapsed, will vibrate elastically about a new axis where the offset in curvature is the permanent set.

The deflected position of the 10-story column segment at three randomly selected times is of illustrative interest in that it depicts a physical view of the vibration. Of particular interest is the behavior of the first story column due to its greater relative flexibility to the columns immediately above. The reduced stiffness and yield moment of column 1 is immediately evident in Figs. 5 and 6.

Of major interest to the analyst are the maximum values of deflection and stress resultants at each story which occured during the earthquake. Maximum values can be taken from the response curves and plotted as an envelope as is shown in Fig. 7 for maximum deflections and in Fig. 8 for maximum bending moments for the three levels of strain hardening used in the solution. Maximum shears are directly proportional to the end moments and are given by

$$V_i = \frac{2}{h_{(i)}} M_i . \tag{17}$$

The moment in a column is related to the difference in lateral deflection of its adjoining floors. Thus, the change in deflection between stories in Fig. 7 relates to the moment in Fig. 8, but only in a general sense since the relationship is not linear.

Conclusions

It is shown that a structure idealized as a shear building can be very precisely, within the framework of prior assumptions, analyzed for its response under seismic loading by numerically integrating the nonlinear equations of motion. The columns are treated as continuous members and so inelastic strains also have continuity. No conclusions can be drawn with respect to the behavior of shear buildings in general from the limited analyses in this paper. However, the behavior of the structure analyzed is studied in depth. It is immediately apparent that the lesser stiffness of column 1 and its reduced yield moment has a pronounced adverse effect on the structure. It is felt that a more accurate method of yield moment reduction due to axial force should be used.

Acknowledgment

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Summary

A direct stiffness method of analysis of multi-story shear buildings of a bilinear hysteretic material is used to find their response to ground motion. The nonlinear coupled equations of motion of a building are precisely integrated with the Runge-Kutta Method of numerical integration to obtain their deformation and stress resultant history due to ground acceleration records of the 1940 El Centro earthquake. A 10-story structure is analyzed at three levels of strain hardening in the post-elastic material regime. It is shown how damage may be assessed.

Résumé

Une méthode de rigidité directe permet l'analyse des réactions aux mouvements du sol d'une construction à plusieurs étages, assemblée avec des matériaux à histérésie bilinéaire. Les équations de mouvement d'une construction, non-linéaires, couplées, sont résolues avec précision par l'intégration numérique de la méthode Runge-Kutta, ce qui donne les déformations et les contraintes dues aux accélérations du sol lors du séisme d'El Centro en 1940. Un bâtiment à 10 étages est analysé dans le domaine post-élastique du matériau, à trois échelons de durcissement. Il est démontré comment des dégâts peuvent être évités.

Zusammenfassung

Um die Wirkung der Bodenbewegung (infolge Erdbebens) auf ein Hochhaus aus Material, welches bilineare Hysteresis aufweist, beschreiben zu können, wird ein direktes Drehwinkelverfahren angewandt. Die nichtlinearen, gekoppelten Bewegungsgleichungen sind genau nach Runge-Kutta integriert worden, um den Verlauf der Verformungen und Spannungen infolge der Beschleunigungen des El-Centro-Erdbebens zu erhalten, welches 1940 stattfand. Ein zehnstöckiges Tragwerk ist für drei Stufen innerhalb des Verfestigungsbereiches untersucht worden. Es folgt daraus, wie Schaden abgewendet werden kann.