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Effective Column Length in Unsymmetrical Frames

Longueur de flambement des cadres asymétriques

Knicklänge in unsymmetrischem Rahmen

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Introduction

In the design of bulding frames, the slenderness ratio of any column is determined by its effective column length rather than its actual unbraced length. The ratio K of the effective column length to the actual unbraced length is of great concern to designers working with unbraced frames. This is because K is always less than 1.0 in braced frames but it is usually greater than 1.0 for unbraced frames subject to lateral sway. The value of K for unbraced frames is usually determined by the alignment chart given in the AISC Manual of Steel Construction [1]. This chart is based on an equation (given in the Guide to Design Criteria for Metal Compression Members [2]) which is the buckling equation for columns in a symmetrical rectangular frame subjected to symmetrical vertical loads at the tops of the columns [3]. As shown in Fig. 1, the frame is assumed to be braced in the direction perpendicular to its plane with moments of inertia of the colums (which resist bending in its plane) $I'_c = I_c$ and subjected to loads $P' = P$. Note that the moments of inertia of the beams are not equal ($I'_b \neq I_b$) and the column bases become fixed if $I'_b = \infty$ hinged if $I'_b = 0$.

Since the AISC alignment chart is based on symmetrical frames symmetrically loaded, the question arises as to what will be the value of K if $P' \neq P$ and $I'_c \neq I_c$. In this paper, the basic buckling equation will be derived and a chart which gives a coefficient for modifying the K values given by the AISC chart will be presented. Since coefficients given by the chart are average values, some errors can be expected in the modified K values obtained. However, the maximum error of the modified K will be at most about 20 per cent and for most pratical cases less than 10 per cent.

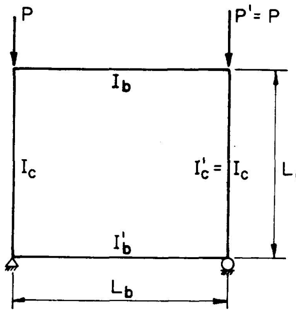

Fig. 1. Symmetrical frame symmetrically loaded.

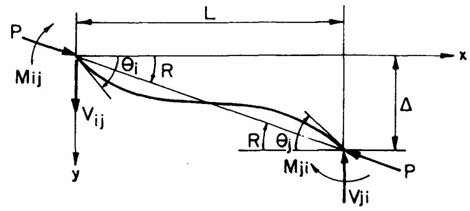

Fig. 2. Notations for the slope deflection equation.

Buckling Equation

The slope deflection method will be used in the derivation of the buckling equation. The slope deflection equation for a member ij subject to an axial load P (Fig. 2) as given in standard textbooks [4, 5] may be written in the following form [6, 7].

$$M_{ij} = \frac{2EI}{L}(2a\,\theta_i + b\,\theta_j - c\,R), \tag{1}$$

in which M_{ij} = moment at end i (M_{ji} at end j), positive clockwise,

E = modulus of elasticity,

I = moment of inertia about an axis perpendicular to the plane of the frame,

L = length of the member,

θ_i, θ_j = slope at ends i and j respectively, positive clockwise,

R = slope of the chord, positive clockwise,

= Δ/L = end deflection/length of member,

$$a = \frac{\phi}{4}\left(\frac{\phi}{1-\phi\cot\phi} + \cot\phi\right), \tag{2a}$$

$$b = \frac{\phi}{2}\left(\frac{\phi}{1-\phi\cot\phi} - \cot\phi\right), \tag{2b}$$

$$c = 2a + b = \frac{\phi^2}{1-\phi\cot\phi} \tag{2c}$$

and

$$\phi = \frac{L}{2}\sqrt{\frac{P}{EI}}. \tag{3}$$

For small values of ϕ

$$a = 1 - \frac{(2\,\phi)^2}{30} - \frac{11\,(2\,\phi)^4}{25000} \cdots, \tag{4a}$$

$$b = 1 + \frac{(2\,\phi)^2}{60} - \frac{13\,(2\,\phi)^4}{25000} \cdots, \tag{4b}$$

for $\phi = 0$
$$a = b = 1. \tag{4c}$$

Letting K be the ratio of the effective column length to the actual unbraced length, then P at buckling becomes the critical load P_{cr} given by

$$P_{cr} = \frac{\pi^2\,E\,I}{(K\,L)^2} \tag{5}$$

and
$$\phi = \frac{L}{2}\sqrt{\frac{P_{cr}}{E\,I}} = \frac{\pi}{2\,K}. \tag{6}$$

Referring to Fig. 3, let

$$P' = \lambda\,P\,(0 \leqq \lambda \leqq 1), \qquad I'_c = \alpha\,I_c\,(0 \leqq \alpha \leqq 1), \tag{7a, b}$$

$$G_A = \frac{I_c/L_c}{I_b/L_b}, \qquad G'_A = \frac{I'_c/L_c}{I_b/L_b} = \alpha\,G_A, \tag{8a, b}$$

$$G_B = \frac{I_c/L_c}{I'_b/L_b}, \qquad G'_B = \frac{I'_c/L_c}{I'_b/L_b} = \alpha\,G_B, \tag{8c, d}$$

in which I_c and I'_c are moments of inertia for the columns, I_b and I'_b are the same for the beams, L_c and L_b are the lengths of the columns and beams respectively as shown in Fig. 3. With the coefficients $a_1, b_1 \ldots$ etc. for the

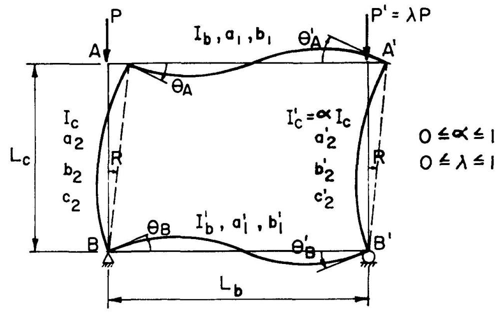

Fig. 3. Unsymmetrical frame unsymmetrically loaded.

members as indicated in Fig. 3, the slope deflection equations for members $A\,A'$ and $A\,B$ are

$$M_{AA'} = \frac{2\,E\,I_b}{L_b}(2\,a_1\,\theta_A + b_1\,\theta_{A'}), \tag{9a}$$

$$M_{AB} = \frac{2\,E\,I_c}{L_c}(2\,a_2\,\theta_A + b_2\,\theta_B - c_2\,R). \tag{9b}$$

For equilibrium at joint A, $\sum M_A = M_{AA'} + M_{AB} = 0$, yields

$$2\,(a_1 + G_A\,a_2)\,\theta_A + G_A\,b_2\,\theta_B + b_1\,\theta_{A'} - G_A\,c_2\,R = 0. \tag{10a}$$

Similarly with $\sum M_B = 0$, $\sum M_{A'} = 0$ and $\sum M_{B'} = 0$, one obtains

$$G_B b_2 \theta_A + 2(a_1' + G_B a_2)\theta_B + b_1' \theta_B' - G_B c_2 R = 0, \tag{10b}$$

$$b_1 \theta_A + 2(a_1 + \alpha G_A a_2')\theta_A' + \alpha G_A b_2' \theta_B' - \alpha G_A c_2' R = 0, \tag{10c}$$

$$b_1' \theta_B + \alpha G_B b_2' \theta_A' + 2(a_1' + \alpha G_B a_2')\theta_B' - \alpha G_B c_2' R = 0. \tag{10d}$$

The shear equilibrium equation

$$M_{AB} + M_{BA} + M_{A'B'} + M_{B'A'} + P L_c R + P' L_c R = 0 \tag{11a}$$

gives $$c_2 \theta_A + c_2 \theta_B + \alpha c_2' \theta_A' + \alpha c_2' \theta_B' - 2[c_2 + \alpha c_2' - (1+\lambda)\phi^2] R = 0, \tag{11b}$$

in which $$\phi^2 = \frac{L_c^2 P}{4 E I_c}. \tag{12}$$

Let $$2[c_2 + \alpha c_2' - (1+\lambda)\phi^2] = d. \tag{13}$$

From Eq. (11) $$R = \frac{c_2}{d}(\theta_A + \theta_B) + \frac{\alpha c_2'}{d}(\theta_A' + \theta_B'). \tag{14}$$

Since there is no axial force in the beams AA' and BB',

$$a_1 = b_1 = a_1' = b_1' = 1. \tag{15}$$

Substituting Eqs. (14) and (15) into Eqs. (10a–d), yields

$$(2 + G_A W_1)\theta_A + G_A W_2 \theta_B + (1 - G_A W_5)\theta_A' - G_A W_5 \theta_B' = 0, \tag{16a}$$

$$G_B W_2 \theta_A + (2 + G_B W_1)\theta_B - G_B W_5 \theta_A' + (1 - G_B W_5)\theta_B' = 0, \tag{16b}$$

$$(1 - G_A W_5)\theta_A - G_A W_5 \theta_B + (2 + G_A W_3)\theta_A' + G_A W_4 \theta_B' = 0, \tag{16c}$$

$$- G_B W_5 \theta_A + (1 - G_B W_5)\theta_B + G_B W_4 \theta_A' + (2 + G_B W_3)\theta_B' = 0, \tag{16d}$$

in which $$W_1 = 2a_2 - \frac{c_2^2}{d}, \qquad W_2 = b_2 - \frac{c_2^2}{d}, \tag{17a, b}$$

$$W_3 = 2\alpha a_2' - \frac{(\alpha c_2')^2}{d}, \qquad W_4 = \alpha b_2' - \frac{(\alpha c_2')^2}{d}, \qquad W_5 = \frac{\alpha c_2 c_2'}{d}. \tag{17c–e}$$

The buckling equation is obtained by setting the determinant of the coefficients of the unknown θ's in Eqs. (16) equal to zero, or

$$\begin{vmatrix} 2 + G_A W_1 & G_A W_2 & 1 - G_A W_5 & -G_A W_5 \\ G_B W_2 & 2 + G_B W_1 & -G_B W_5 & 1 - G_B W_5 \\ 1 - G_A W_5 & -G_A W_5 & 2 + G_A W_3 & G_A W_4 \\ -G_B W_5 & 1 - G_B W_5 & G_B W_4 & 2 + G_B W_3 \end{vmatrix} = 0. \tag{18}$$

Expanding and rearranging Eq. (18), one obtains

$$\begin{aligned} 3V_1 &+ (2U_1 V_1 - U_3 V_2 - U_5 V_3 - U_7 V_4 - U_9 V_5) G_A \\ &+ (U_2 V_1 - U_4 V_2 - U_6 V_3 - U_8 V_4 - U_{10} V_5 + V_6) G_A^2 = 0, \end{aligned} \tag{19}$$

in which

$$\begin{aligned}
U_1 &= W_1 + W_3 + W_5, & U_2 &= W_1 W_3 - W_5^2, \\
U_3 &= W_2 + 2\,W_5, & U_4 &= W_1 W_5 - W_2 W_5, \\
U_5 &= W_4 + 2\,W_5, & U_6 &= W_3 W_5 - W_4 W_5, \\
U_7 &= 2\,W_4 + W_5, & U_8 &= W_1 W_4 - W_5^2, \\
U_9 &= 2\,W_2 + W_5, & U_{10} &= W_2 W_3 - W_5^2, \\
U_{11} &= W_2 W_4 - W_5^2.
\end{aligned} \tag{20a–k}$$

$$\begin{aligned}
V_1 &= 3 + 2\,G_B U_1 + G_B^2 U_2, & V_2 &= G_B U_5 + G_B^2 U_6, \\
V_3 &= G_B U_3 + G_B^2 U_4, & V_4 &= G_B U_7 + G_B^2 U_8, \\
V_5 &= G_B U_9 + G_B^2 U_{10}, & V_6 &= G_B^2 U_{11}^2.
\end{aligned} \tag{21a–f}$$

In Eqs. (20), the W's are obtained from Eqs. (17a–e) with a_2, b_2, etc. obtained from Eqs. (2a–c) and (4a, b), using

$$\phi = \frac{L_c}{2}\sqrt{\frac{P}{E I}} = \frac{\pi}{2K} \tag{22a}$$

for a_2, b_2 and c_2, and

$$\phi = \phi' = \frac{L_c}{2}\sqrt{\frac{P'}{E I_c'}} = \frac{\pi}{2K}\sqrt{\frac{\lambda}{\alpha}} \tag{22b}$$

for a_2', b_2' and c_2'.

Special Cases

The following special cases may be obtained from the general buckling Eq. (18) or (19)

a) If $G_A = 0$, $(I_b = \infty)$, then in Eq. (19), $V_1 = 0$ or

$$3 + 2\,G_B U_1 + G_B^2 U_2 = 0. \tag{23}$$

b) If $G_A = \infty$, $(I_b = 0)$, then in Eq. (19), the coefficient of $G_A^2 = 0$, hence

$$U_2 V_1 - U_4 V_2 - U_6 V_3 - U_8 V_4 - U_{10} V_5 + V_6 = 0 \tag{24a}$$

or

$$\begin{aligned}
&G_B^2 (U_2^2 - 2\,U_4 U_6 - U_8^2 - U_{10}^2 - U_{11}^2) \\
&+ G_B (2\,U_1 U_2 - U_4 U_5 - U_3 U_6 - U_7 U_8 - U_9 U_{10}) + 3\,U_2 = 0.
\end{aligned} \tag{24b}$$

c) If $P = P'$ and $I_c = I_c'$ then $\lambda = 1$, $\alpha = 1$, $W_1 = W_3$ and $W_2 = W_4$. Consider the case $\theta_A = \theta_A'$ and $\theta_B = \theta_B'$.

Let

$$W_1 = W_1', \qquad W_2 = W_2', \qquad W_5 = W_5' \tag{25a–c}$$

for this particular case. Eq. (18) becomes

$$\begin{vmatrix} 3 + G_A (W_1' - W_5') & G_A (W_2' - W_5') \\ G_B (W_2' - W_5') & 3 + G_B (W_1' - W_5') \end{vmatrix} = 0 \tag{26a}$$

or

$$9 + 3\,(G_A + G_B)\,(W_1' - W_5') + G_A G_B [(W_1' - W_5')^2 - (W_2' - W_5')^2] = 0. \tag{26b}$$

Note that $$W_1' - W_5' = 2\left(a_2 - \frac{c_2^2}{d}\right) = \frac{\phi}{2}(\cot\phi - \tan\phi) = \phi\cot 2\phi, \tag{27a}$$

$$W_2' - W_5' = b_2 - 2\frac{c_2^2}{d} = -\frac{\phi}{2}(\cot\phi + \tan\phi), \tag{27b}$$

in which $$d = 4(c_2 - \phi^2) = \frac{4\phi^3\cot\phi}{1-\phi\cot\phi}. \tag{27c}$$

Eq. (26b) becomes

$$9 + 3\phi\cot 2\phi\,(G_a + G_B) - G_A G_B \phi^2 = 0. \tag{28}$$

With $\phi = \frac{\pi}{2K}$, Eq. (28) is identical to the equation given in Guide to Design Criteria for Metal Compression Members [2].

d) If $I_c' = 0$ $(\alpha = 0)$ and $P' \neq 0$ $(\lambda \neq 0)$, then $\phi' = \infty$ and W_3, W_4 and W_5 become indefinite. There will be no solution for Eq. (18). The structure will buckle since there is a load on a column of zero flexural rigidity.

However, if $I_c' = 0$ and $P' = 0$, then $\alpha = 0$ and $W_3 = W_4 = W_5 = 0$.

Let $$W_1 = W_1'', \qquad W_2 = W_2'' \tag{29a, b}$$

for this particular case. Eq. (18) becomes

$$\begin{vmatrix} 2 + G_A W_1'' & G_A W_2'' & 1 & 0 \\ G_B W_2'' & 2 + G_B W_1'' & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \end{vmatrix} = 0 \tag{30a}$$

or $$9 + 6(G_A + G_B) W_1'' + 4 G_A G_B [(W_1'')^2 - (W_2'')^2] = 0 \tag{30b}$$

with $$W_1'' = 2a_2 - \frac{c_2^2}{d} = W_1' - W_5', \tag{31a}$$

$$W_2'' = b_2 - \frac{c_2^2}{d} = W_2' - W_5' \tag{31b}$$

and $$d = 2(c_2 - \phi^2) = \frac{2\phi^3\cot\phi}{1-\phi\cot\phi}. \tag{31c}$$

The case of $I_c' = 0$ and $P' = 0$ is equivalent to the case with the member $A'B'$ omitted and the ends A' of AA' and B' of BB' supported on rollers (see Fig. 4a). It can be seen that Eq. (30b) is the same as Eq. (26b) for the symmetrical frame symmetrically loaded if $2\,G_A$ and $2\,G_B$ are used instead of G_A and G_B. The same result is obtained if $2\,L_b$ is used instead of L_b. This is because in the present case the points of inflection of the beams are at the ends A' and B', whereas in the symmetrical case, the inflection points are located at the midspan of the beams (see Fig. 4b).

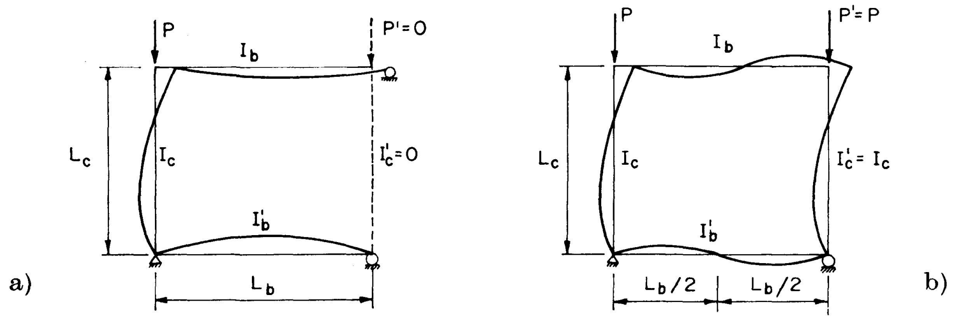

Fig. 4. Comparison of the deflection pattern of the case $P'=0$ and $I'_c=0$ with the symmetrical case.

e) For the case of $P=P'$ and $I=I'_c$ ($\lambda=1$, $\alpha=1$, $W_1=W_3$ and $W_2=W_5$) without the restriction of $\theta_A=\theta_A$ and $\theta_B=\theta_B$, it can be shown that Eq. (18) may be reduced to

$$\begin{aligned}&\{1-(G_A+G_B)(W_1+W_5)+G_A G_B[(W_1+W_5)^2-(W_2-W_5)^2]\}\cdot\\&\cdot\{9+3(G_A+G_B)(W_1-W_5)+G_A G_B[(W_1-W_5)^2-(W_2-W_5)^2]\}=0.\end{aligned}\tag{32}$$

Setting the terms in the second pair of braces equal to zero results an equation which is identical to Eq. (26b) for antisymmetrical buckling. Setting the terms in the first pair of braces equal to zero would result an equation for symmetrical buckling. Since K values for symmetrical buckling is always less than 1 while that of the antisymmetrical buckling always greater than 1 (equal to 1 if $I_b=I'_b=\infty$). The buckling load will be governed by the antisymmetrical case instead of symmetrical case.

Solution of the Buckling Equation and Presentation of Results

Eq. (19) may be solved very simply in the following manner. If the values of α, λ, and G_B are given, values of G_A may be determined for any assumed values of K. Since the equation for G_A is quadratic, negative and complex solutions must be discarded. Also some false or physically meaningless roots must be rejected.

It is noted that Eq. (18) is symmetrical with respect to G_A and G_B. Hence G_A and G_B are interchangeable. Thus Eq. (23) may be regarded as an equation for G_A when $G_B=0$ and Eq. (24) as an equation for G_A when $G_B=\infty$.

Various methods of presenting the results were tried. It was found that for a given pair of α and λ values, the β values defined by the following equation remain approximately constant for various values of G_A and G_B.

$$\beta=\frac{K}{K_0},\tag{33}$$

in which K is the ratio of effective length to actual unbraced length for a given pair of α and λ values corresponding to a pair of specifies G_A and G_B values (referring to the stronger column) and K_0 is the value of K for $\lambda=\alpha=1$ (symmetrical case) with the same values of G_A and G_B.

The ratio β is determined in the following manner. For given values of α, λ and G_B, using Eq. (19), G_A is obtained for some assumed value of K. Using the same G_A and G_B, K_0 is determined from Eq. (28) by Newton's method for solving transcendental equations. Then β is determined from Eq. (33). All computations were carried out on an IBM 360 computer.

For a given pair of α and λ values, an average value of β is determined for various values of G_A and G_B. A set of curves of the average β versus α for various values of λ is plotted in Fig. 5. This figure is to be used in conjunction with Fig. 6 which is reproduced from the AISC Manual [1]. With known values

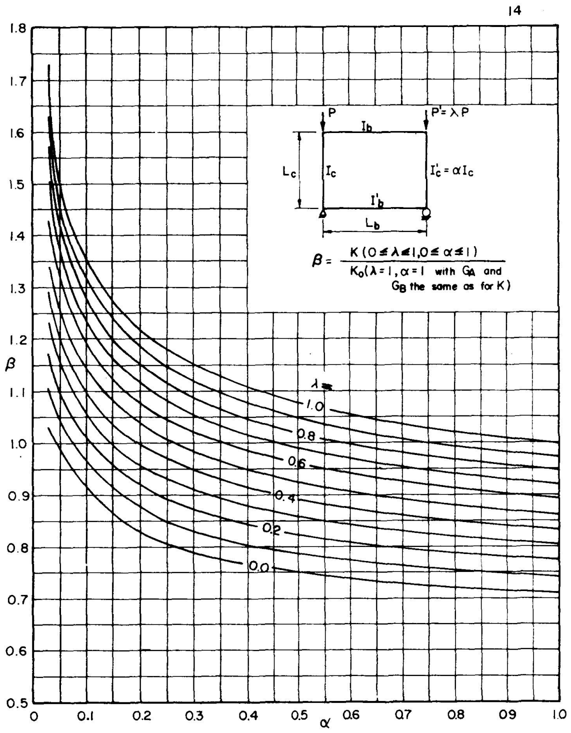

Fig. 5. Curves of average β versus α for various values of λ.

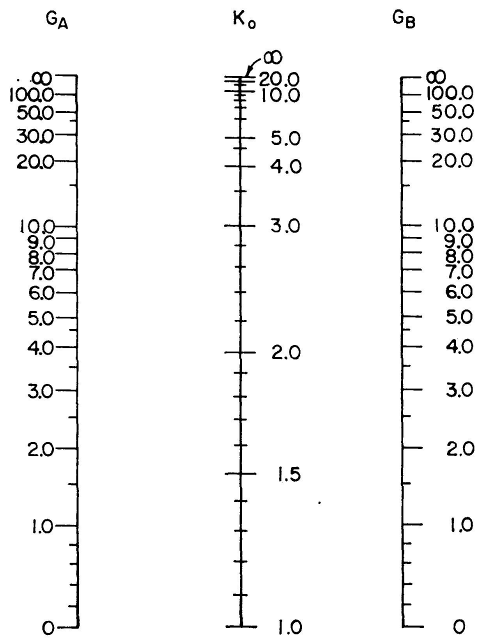

Fig. 6. Chart for the determination of K_0. (After AISC Manual of Steel Construction.)

of λ, α, G_A and G_B, the value of K will be given by βK_0 with β determined from Fig. 5 for a given α and λ and K_0 from Fig. 6 for given values of G_A and G_B.

Since average values of β are used in the plotting, some errors can be expected. It was found that the error increases as α of λ decreases (i.e., as the frame or loading becomes more unsymmetrical). For very small value of α, the error may reach 20 per cent. However, the maximum error is 11.3% for $\alpha=0.3$, 8.9% for $\alpha=0.5$, 4.5% for $\alpha=0.75$ and 1.1% for $\alpha=1.0$. For most practical cases, the error will be less than 10 per cent since α is seldom less than 0.5.

It should be noted that the estimated errors are maximum values. For the ordinary ranges of G_A and G_B, β will be near the average and the error will be much smaller. In case precise K values are needed, Eq. (19) may be solved by the method of linear interpolation (false position) using as first approximate values of $(1 \pm e)K$ in which e is a small fraction representing an estimated error and K is determined from Figs. 5 and 6.

Numerical Examples

Various problems may be solved by using the charts presented herein intelligently. For readers not familiar with the use of the AISC Chart, examples will be shown for both the case of $\beta = 1$ (for $\lambda = 1$ and $\alpha = 1$) and the case of $\beta \neq 1$.

Example 1. A ten story frame is shown in Fig. 7 (a). The moments of inertia of the columns and beams are as shown in the figure. It may be noted that the ratio of G_A and G_B at all joints are equal.

$$(G_A) \text{ at } A = \frac{(I_c/2)/L_c}{(I_b/2)/L_b} = \frac{I_c/L_c}{I_b/L_b},$$

$$(G_B) \text{ at } B = \frac{[(I_c/2)+(I_c/2)]/L_c}{I_b/L_b} = \frac{I_c/L_c}{I_b/L_b},$$

$$(G_A) \text{ at } C = \frac{I_c/L_c}{[(I_b/2)+(I_b/2)]/L_b} = \frac{I_c/L_c}{I_b/L_b},$$

$$(G_B) \text{ at } D = \frac{(I_c+I_c)/L_c}{(I_b+I_b)/L_b} = \frac{I_c/L_c}{I_b/L_b}.$$

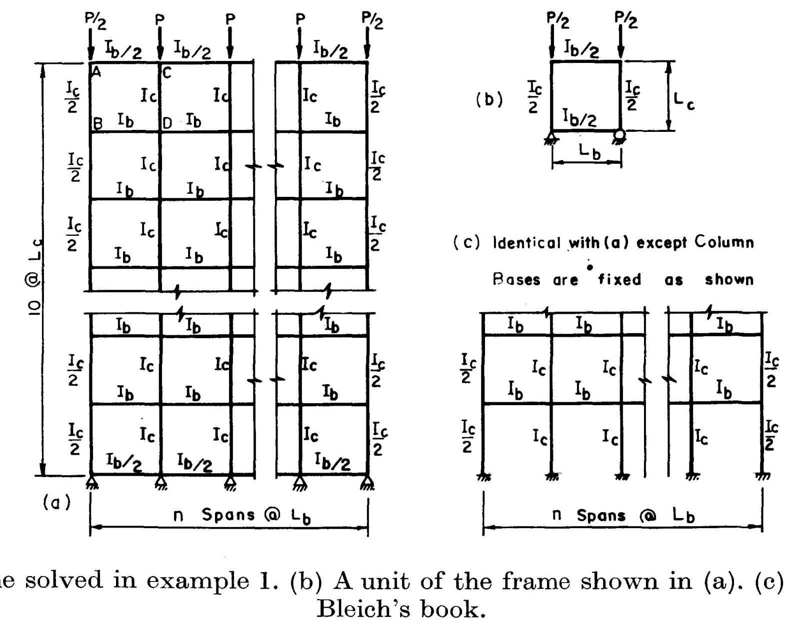

Fig. 7. (a) Frame solved in example 1. (b) A unit of the frame shown in (a). (c) Frame solved in Bleich's book.

The problem may be treated as a superposition of the basic frame as shown in Fig. 7 (b). Noting that $\beta=1$ and $K=K_0$, from Fig. 6, connecting a straight line between the given values on the G_A and G_B scale and finding the intercepts on the K_0 scale, one can easily determine the values as shown in Table 1.

Table 1. K values for the ten story frame shown in Fig. 7

$G_A=G_B=$	1	2	3	4
$K=K_0$	1.31	1.59	1.83	2.05

The values in Table 1 check very well with the curve given in BLEICH's book [4] for the solution of the frame as shown in Fig. 7 (c). The frames shown in Fig. 7 (a) and Fig. 7 (c) differ only in the degree of fixity of the bases of the bottom story columns. However for tall buildings the effect of base fixity of the columns in the bottom story will be negligible.

Example 2. The problem as shown in Fig. 8 (a) was solved by JOHNSON [8].

$$(G_A)_{AD,CF}=\frac{I/L}{2\,I/(\frac{2}{3}\,L)}=(G_A)_{BE}=\frac{2\,I/L}{2\cdot 2\,I/(\frac{2}{3}\,L)}=\frac{1}{3}=0.33\,,$$

$$(G_B)_{AD,CF}=\frac{(I+2\,I)/L}{2\,I/(\frac{2}{3}\,L)}=(G_B)_{BE}=\frac{(2\,I+4\,I)/L}{2\cdot 2\,I/(\frac{2}{3}\,L)}=\frac{3}{3}=1\,,$$

$$(G_A)_{DG,EH,FI}=(G_B)_{AD,CF,BE}=1\,,$$

$$(G_B)_{DG,EH,FI}=0\,.$$

From Fig. 6, $K=K_0=1.21$ for columns AD, CF and BE and $K=K_0=1.15$ for columns DG, EH, FI. The given solution is $P=7.10\,\frac{EI}{L^2}=\frac{\pi^2 EI}{(KL)^2}$ with $K=1.18$ which is exactly equal to the average of 1.21 and 1.15. The frame may be treated as a superposition of two frames shown in Fig. 8 (b). It may be pointed out that the results are good primary because the column stiffness is proportional to the loading.

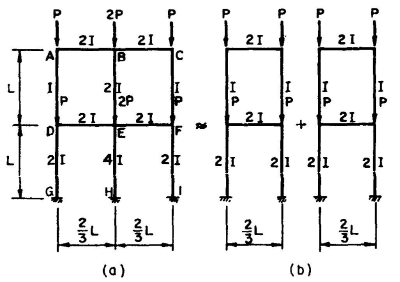

Fig. 8. Frame solved in example 2.

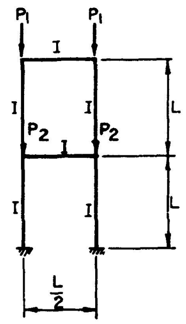

Fig. 9. Frame solved in example 3.

Example 3. The previous example shows that very good results can be obtained by using the AISC Chart for well proportioned rectangular frames.

This example will show the amount of error involved in applying to some other cases. Fig. 9 shows a frame which has been solved in reference 9. The given values of EI/L^2 is 100^T and the solutions are $P_1 = 400^T$ and $(P_1+P_2) = 807^T$ corresponding to $K = K_0 = \pi/\sqrt{4.0} = 1.56$ and $\pi/\sqrt{8.07} = 1.10$ respectively. From Fig. 6, the K value for the top column is 1.24 for $G_A = \frac{1}{2}$ and $G_B = 1$ and that for the bottom column is 1.15 for $G_A = 1$ and $G_B = 0$. Consider the fact that column stiffness is not proportional to loading, the errors involved are reasonable.

Example 4. The four cases shown in Fig. 10 were solved by ZWEIG [10]. A comparison of the results will be of interest. For cases I and III, $G_A = G_B = 0$

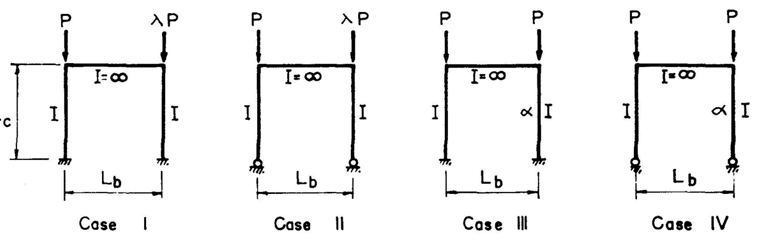

Fig. 10. Frames solved in example 4.

and $K_0 = 1$. For cases II and IV, $G_A = 0$, $G_B = \infty$ and $K_0 = 2$. The values of β for various values of λ and α from Fig. 5 are listed in Tables 2 and 3.

Table 2. β values for cases I and II

$\lambda = \bar{r}^2$	0.16	0.49	0.81	1.0
β	0.76	0.86	0.95	1.0
β (Zweig)	0.765	0.864	0.951	1.0

Table 3. β values for cases III and IV

$\alpha = r^2$	0.16	0.49	0.81	1.0
β	1.27	1.08	1.02	1.0
$\beta' = r\beta$	0.508	0.756	0.918	1.0
β' (Zweig)	0.532	0.812	0.946	1.0
Error	7%	7%	3%	0%

It should be noted that the notations λ and α are respectively corresponding to $\bar{r}^2$ and r^2 given in reference [10] and the β values can easily be derived from the K values given therein. Also for cases III and IV, the critical load in reference [10] is related to the weaker column, thus

$$P = \frac{\alpha \pi^2 E I}{(\beta' K_0 L)^2} = \frac{r^2 \pi^2 E I}{(\beta' K_0 L)^2} = \frac{\pi^2 E I}{(\beta K_0 L)^2}. \quad \text{Hence } \beta' = r\beta.$$

It can be seen that for cases I and II, the β values checks very well and for cases III and IV, the errors are within the maximum values stated previously.

Example 5. This example shown in Fig. 11 (a) was also solved by ZWEIG [10]. It would put a severe test of the versatility of the proposed method. This frame may be treated as a superposition of the frames shown in Fig. 11 (b) and the equivalent frame is as shown in Fig. 11 (c). The conditions of the column bases, however, are dissimilar and can not be replaced by a single

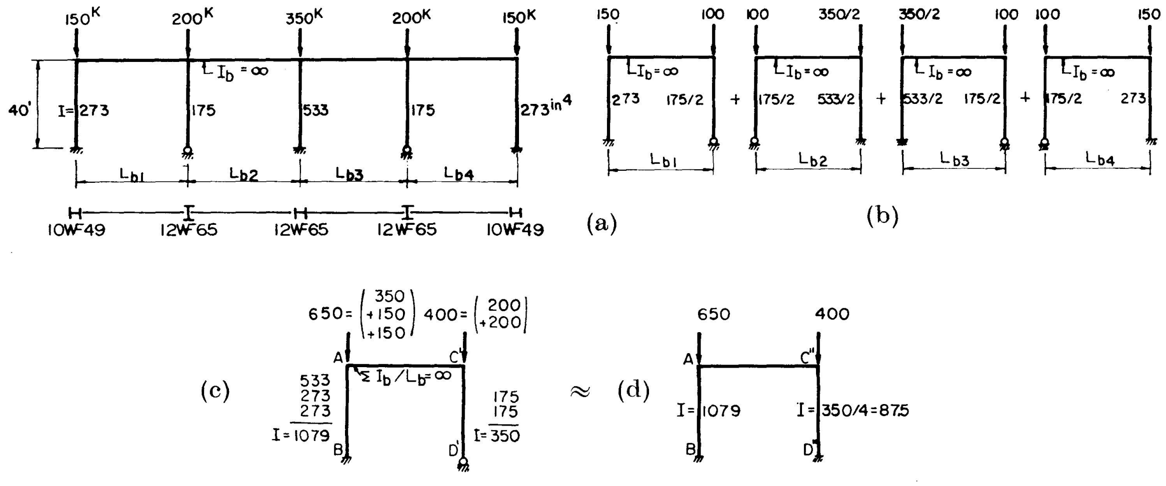

Fig. 11. (a) Frame solved in example 5. (b) Superposition of frames. (c) An equivalent frame. (d) Final equivalent frame.

beam. Nevertheless since column $C'D'$ has $G_A = 0$, $G_B = \infty$ and $K_0 = 2$ (from Fig. 6), it may be replaced by a column of $C''D''$ with $G_A = G_B = 0$ and $K_0 = 1$ if $I_{C''D''} = \frac{I_{C'D'}}{K^2_{C'D'}} = \frac{350}{4} = 87.5$ in^4 as shown in Fig. 11 (d). Noting that $\lambda = 400/650 = 0.615$, $\alpha = 87.5/1079 = 0.081$, and $G_A = G_B = 0$, one obtains $\beta = 1.24$ and $K_0 = 1$ from Figs. 5 and 6 and $K = \beta K_0 = 1.24$. The answer given in reference [10] is $3.14/2.46 = 1.28$. The error is only 3% although the α value is very small in this case.

Discussion of Results and Conclusions

The following conclusions may be reached particularly with reference to Fig. 5.

1. Note that for $\lambda = 1$, on the average, $\beta = 1.08$ for $\alpha = 0.5$ and $\beta = 1.15$ for $\alpha = 0.3$. This means that for equal loads, if the stiffness of the column on one side is only 50 per cent of the stiffness of the column on the other side, the K value is only slightly higher than the K_0 value for the symmetrical case.

2. For equal loads ($\lambda = 1$), K will be significantly higher than K_0 only when α is small (say $\alpha < 0.3$).

3. With the exception of $\lambda = 0$, if the loads and stiffness are in the same proportion ($\lambda = \alpha$), the average β will lie between 0.9 and 1.0.

4. With the exception of $\lambda=0$, β will be greater than 1.0 only when $\lambda>\alpha$.

5. When there is no load on the weaker column ($\lambda=0$), β will have values lying between 0.8 and 0.7 for $0.3 \leqq \alpha \leqq 1.0$ with a smaller β corresponding to a larger α.

6. As bointed out previously, there will be no solution for the case $\alpha=0$ and $\lambda \neq 0$. However, for $\alpha=0$ and $\lambda=0$ roller supports are assumed for the beams, with the weaker column omitted. The K value may be obtained from the AISC alignment chart using $2\,L_b$ instead of L_b in computing the values of G_A and G_B.

7. As pointed out previously, the maximum error in using Fig. 5 would be less than 9 per cent for $\alpha=0.5$. It may reach 20 per cent for small values of α.

8. By intelligently using the concept of considering G_A (similarly G_B) as the ratio of $\sum I_c/L_c$ and $\sum I_b/L_b$ instead of the ratio of I_c/L_c and I_b/L_b, the rectangular frame studied may be considered to represent any story of a multistory, multibay frame as shown in the numerical examples.

Notation

a	see Eq. (2a)
a_1, a_1', a_2, a_2'	values of a for members as shown in Fig. 3
b	see Eq. (2b)
b_1, b_1', b_2, b_2'	values of b for members as shown in Fig. 3
c	see Eq. (2c)
c_1, c_1', c_2, c_2'	values of c for members as shown in Fig. 3
d	see Eq. (13)
e	a small fraction representing an estimated error
E	modulus of elasticity
G_A, G_A', G_B, G_B'	see Eq. (8a–d)
I	moment of inertia about an axis perpendicular to the plane of the frame
I_b, I_b'	moment of inertia of top and bottom beams respectively
I_c, I_c'	moments of inertia of columns as shown in Fig. 3
K	ratio of effective column length to actual unbraced length
K_0	K for the symmetrical case $\lambda=\alpha=1$
L	length of a member
L_b, L_c	lengths of beam and column respectively
M_{ij}	moment at end i of the member ij
P, P'	column axial loads ($P' \leqq P$)
$R=\Delta/L$	chord slope of a member
U_1 to U_{11}	see Eqs. (20a–k)
V_1 to V_6	see Eqs. (21a–f)
W_1 to W_5	see Eqs. (17a–e)

W_1', W_2', W_5'	$W's$ for the symmetrical case $\lambda = \alpha = 1$
W_1'', W_2''	$W''s$ for the case $\lambda = \alpha = 0$
α =	I_c'/I_c $(0 \leqq \alpha \leqq 1)$
β =	K/K_0 using the same G_A and G_B for K and K_0
Δ =	deflection of one end of a member with respect to the other end
θ_i =	slope of a member at end i = rotation at joint i
λ =	P'/P $(0 \leqq \lambda \leqq 1)$
ϕ =	$\frac{L_c}{2}\sqrt{\frac{P}{EI_c}}$
ϕ' =	$\frac{L_c}{2}\sqrt{\frac{P'}{EI_c'}}$

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Summary

In this paper, the basic buckling equation is derived for an unsymmetrically loaded unsymmetrical rectangular frame which may be considered as representing a portion of a multistory multibay frame. The ratio, K, of the effective column length to the actual unbraced length may be considered as equal to value of K_0 obtained by the alignment chart given in the AISC Manual of Steel Construction multiplied by a coefficient β. A chart which gives average values of β is presented. For most practical cases, the maximum error of the

K values obtained from this chart in conjunction with the AISC chart will be less than 10 per cent. Numerical examples are given for illustrating the use of the charts presented.

Résumé

Dans cet article, les auteurs étendent l'equation générale du flambement au cas d'un cadre rectangulaire asymétrique et changé asymétriquement que l'on peut considérer comme une portion d'un cadre à plusieurs étages et à plusieurs ouvertures. Le rapport K (longueur de flambement sur longueur théorique) peut être considéré comme égal à la valeur de K_0, obtenue par les diagrammes du manuel de l'AISC pour la construction métallique, multipliée par un coefficient β. On donne un diagramme indiquant des valeurs moyennes de β. Dans la plupart des cas, l'erreur maximum des valeurs K obtenues à l'aide de ce diagramme et de celui de l'AISC est en dessous de 10%. Pour montrer l'utilisation des diagrammes, on a apporté des exemples numériques.

Zusammenfassung

In diesem Beitrag wird die Grundknickgleichung für einen unsymmetrisch belasteten, unsymmetrisch gebauten und rechteckigen Rahmen hergeleitet, der als Ausschnitt eines vielstöckigen und mit vielen Öffnungen versehenen Rahmens aufgefaßt werden kann. Das Verhältnis K der Knicklänge zur geometrischen Länge kann gleich dem Wert K_0, den man aus dem Schaubild des AISC-Handbuchs für Stahlkonstruktionen herausliest, multipliziert mit einem Beiwert β gesetzt werden. Ein Diagramm für durchschnittliche Werte von β ist abgebildet. Für die meisten praktischen Fälle liegt der maximale Fehler des Wertes K, den man aus dem Schaubild für β und K^0 der AISC erhält, unterhalb zehn Prozent. Numerische Beispiele sind beigefügt worden, um die Anwendung der Schaubilder zu zeigen.

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