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Solutions of Two Dimensional Problems of Elasticity Without Use of the Stress Function

Solutions de problèmes d'élasticité bidimensionnels sans emploi de la fonction de tension

Lösungen zweidimensionaler Elastizitätsprobleme ohne Benützung der Spannungsfunktion

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1. Introduction and Method of Solution

The usual method of solving two-dimensional problems of elasticity is to introduce a stress function such that the equations of equilibrium are satisfied. The compatibility equations, which are expressed in terms of the stress function, are then solved for the stress function. This procedure, however, gives no indication of how to satisfy any boundary conditions. It is the purpose of this paper to show that it is often advantageous to first specify one of the stresses from observation of a loaded surface and then use the governing equations to solve the problem if only in view of Saint-Venant's Principle.

Consider any loaded surface, that of a thin plate for example, shown in Fig. 1. Use coordinates such that the loaded surface coincides with a line parallel to one of the coordinate axis. Then the stress perpendicular to the surface may be of the form

$$\sigma_\eta = \sum_{i=0}^{n} Y_i(\eta)\, \xi^i .$$

For a closed form solution to exist the equations of equilibrium and compatibility must then be able to be integrated to determine the other stresses in terms of $Y_i(\eta)$ and its derivatives and any functions of integration which may arise. These functions are then integrated and the resulting constants are adjusted to satisfy the boundary conditions often only in view of Saint-Venant's

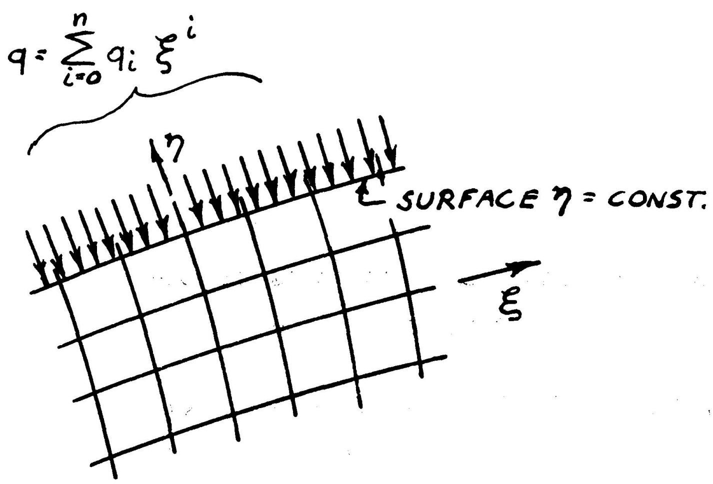

Fig. 1. A loaded surface of a thin plate.

Principle. This procedure will now be illustrated for several two dimensional problems.

2. Application of the Method

Let the loaded surface ($\eta = y = \text{const.}$) in Fig. 1 be a horizontal line and hence solve plane stress beam problems using cartesian coordinates x and y. The governing Eqs. [1] (zero body forces) in addition to boundary conditions are

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0, \qquad \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0, \qquad \nabla^2 (\sigma_x + \sigma_y) = 0. \tag{1}$$

Eliminating τ_{xy} from these equations shows that both σ_x and σ_y are biharmonic functions. Because of the type of load at the surface y = const., the stress σ_y must be of the form

$$\sigma_y = \sum_{i=0}^{n} Y_i(y) x^i, \quad i = 0, 1, 2, \ldots n, \tag{2}$$

where $Y_i(y)$ is a function of y to be determined. The two equations of equilibrium yield the other stresses as

$$\sigma_x = \sum_{i=0}^{n} \frac{Y_i'' x^{i+2}}{(i+1)(i+2)} + x\alpha'_{(y)} + \beta_{(y)}, \qquad \tau_{xy} = -\sum_{i=0}^{n} \frac{Y_i' x^{i+1}}{(i+1)} - \alpha_{(y)}, \tag{3}$$

where α and β are arbitrary functions of integration. Substituting in the equations of compatibility yields the reoccurrence relations

$$Y_i'''' + 2(i+1)(i+2) Y_{i+2}'' + (i+1)(i+2)(i+3)(i+4) Y_{i+4} = 0 \tag{4}$$

and

$$\beta'' + 2(Y_0'' + Y_2) = 0 \qquad a''' + 2Y_1'' + 6Y_3 = 0. \tag{5}$$

The forms of Eqs. (4) were to be expected since σ_y is biharmonic. Note that the solutions for odd and even polynomial loads are uncoupled.

Consider first the examples $Y_i = 0$ therefore $\sigma_y = 0$ and

$$\sigma_x = (2c_3 y + c_4)x + c_1 y + c_2, \qquad \tau_{xy} = -(c_3 y^2 + c_4 y + c_5),$$

where $c_1 - c_5$ are constants of integration. The function β itself solves the case of pure bending ($\beta = c_1 y$) and pure tension or compression of a beam ($\beta = c_2$). The function $\alpha = c_3 y^2 + c_5$ solves the problem of a cantilever beam loaded at its free end (see article 21 of Ref. [1]).

Consider next the solutions when Y_i is not zero. Depending on the location of the origin of coordinates the case $i=0$ solves the problems of uniformly loaded beam of article 22 of Ref. [1] and the uniformly loaded cantilever beam of article 413 of Ref. [2]. The problem of a cantilever beam subjected to a linearly varying load (see art. 23 of Ref. [1]) is solved by letting $i=1$. To solve problems of a beam subjected to a parabolic load it is necessary that σ_y be of the form

$$\sigma_y = Y_0 + Y_2 x^2, \qquad (i = 0 \text{ and } 2).$$

Note that, in general, if the load is of the form $q = q_i \xi^i$ the stress σ_η must be of the form

$$\sigma_\eta = \sum_n Y_n(\eta)\,\xi^n, \qquad (n = i, i-2, i-4).$$

As noted before, it will be found that the large number of solutions represented by (4) and (5) will be subject to Saint-Venant's Principle.

The problem of an anisotropic beam is no more difficult than that of the isotropic beam just treated. The generalized Hook's Law, when the xy plane is the plane of elastic symmetry (using notation of Ref. [3]), is

$$\epsilon_x = a_{11}\sigma_x + a_{12}\sigma_y + a_{16}\tau_{xy}, \qquad \epsilon_y = a_{12}\sigma_x + a_{22}\sigma_y + a_{26}\tau_{xy},$$
$$\gamma_{xy} = a_{16}\sigma_x + a_{26}\sigma_y + a_{66}\tau_{xy}$$

and the third of Eqs. (1) becomes

$$a_{11}\frac{\partial^2 \sigma_x}{\partial y^2} + (a_{66} + 2a_{12})\frac{\partial^2 \sigma_x}{\partial x^2} + a_{22}\frac{\partial^2 \sigma_y}{\partial x^2} + 2a_{16}\frac{\partial^2 \tau_{xy}}{\partial y^2} + 2a_{26}\frac{\partial^2 \tau_{xy}}{\partial x^2} = 0. \qquad (6)$$

The reoccurring relations (4) and (5) for this case are

$$\frac{a_{11} Y_i''''}{(i+1)(i+2)} - \frac{2a_{16} Y_{i+1}'''}{(i+2)} + (a_{66} + 2a_{12}) Y_{i+2}''$$
$$- 2a_{26}(i+3) Y_{i+3}' + a_{22}(i+3)(i+4) Y_{i+4} = 0, \qquad (7)$$
$$a_{11}\beta_y'' - 2a_{16}\alpha_{(y)}'' + (a_{66} + 2a_{12}) Y_0'' - 2a_{16} Y_1' + 2a_{22} Y_2 = 0,$$
$$a_{11}\alpha_{(y)}''' - 2a_{16} Y_0''' + (a_{66} + 2a_{12}) Y_1'' - 4a_{26} Y_2' + 6a_{22} Y_3 = 0.$$

Eqs. (7), unlike those of (4) and (5) (for even and odd loads) are completely coupled, however, they uncouple for an orthotropic beam.

More closed form solutions will be subject to Saint-Venant's Principle. For example, since α and β are coupled the solution for an anisotropic cantilever problem of art. 21 of Ref. [1] will not be exact but will only be valid in view of Saint-Venant's Principle [3]. Eq. (7) will yield the solutions given in articles 14–16 of Ref. [3] as well as many others.

The boundary value problems solved in Ref. [4] may also be solved by first specifying the stress in the direction of the load. The analysis of deep beams treated in Ref. [5] may be solved by first specifying σ_y (see Fig. 7 of Ref. [5]) in the form

$$\sigma_y = \sum_{n=0}^{\infty} Y_n(y)\cos\alpha_n x + \sum_{m=0}^{\infty} X_m(x)\cos\beta_m y .$$

Let the loaded surface ($\eta = \theta = \text{const.}$) in Fig. 1 be a straight line inclined to the horizontal and hence solve the wedge problems of articles 38, 39, and 45 of Ref. [1] using polar coordinates r and θ. The solutions are

$$\sigma_\theta = \sum_{i=0}^{n} Y_i(\theta)\, r^i, \qquad \tau_{r\theta} = -\sum_{i=0}^{n} \frac{Y_i'\, r^i}{(i+2)} - \frac{M_{(\theta)}}{r^2},$$

$$\sigma_r = \sum_{i=0}^{n} \left[\frac{Y_i''}{(i+2)} + Y_i\right] \frac{r^i}{(i+1)} - \frac{M_\theta'}{r^2} + \frac{N_\theta}{r},$$

where
$$\begin{aligned} Y_0 &= A_0\theta + B_0 + A_2\sin 2\theta + B_2\cos 2\theta,\\ Y_i &= A_i\sin i\theta + B_i\cos i\theta + A_{i+2}\sin(i+2)\theta + B_{i+2}\cos(i+2)\theta, \qquad (8)\\ N_\theta &= c_1\sin\theta + c_2\cos\theta, \qquad M_\theta = c_3\sin 2\theta + c_4\cos 2\theta + c_5 \end{aligned}$$

and the A's, B's and C's are constants of integration. Such solutions are discussed in art. 45 of Ref. [1]. Note that in this case the solutions for all the polynomial loads are uncoupled. Let $Y_i = 0$ and hence note that $N(\theta)$ solves the problems of a force acting on the end of a wedge described in art. 38 of Ref. [1]. The function $M(\theta)$ solves the problem of a wedge subjected to a concentrated bending moment discussed in art. 39 of Ref. [1]. $Y_i \neq 0$ yields the solutions of problems of a wedge loaded along the faces (art. 45 of Ref. [1]). When the material is cylindrically orthotropic or anisotropic, the wedge problems analogous to (8) may also be solved. Some of these solutions may be in closed form[1]) others must be solved numerically.

The half plane and the wedge problems treated in Ref. [5] as well as the wedge problems just solved (but having rectangular orthotropy) may also be treated in the proposed manner by first transforming Eq. (6) ($a_{16} = a_{26} = 0$) into polar coordinates. The integrations (eq. with variable coef.) in the solutions of such problems may prevent closed form solutions.

More problems of plane stress may be solved by assuming the loaded surface of Fig. 1 to be different curves such as a circle for example using polar and bipolar coordinates [7].

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[1]) Those of Ref. [6] and the all the solutions for orthotropic materials for example.

The problem of a circular disk or a long cylinder with or without concentric holes subjected to any loading at the outer surface or the hole may also be treated in the proposed manner. As a simpler example, consider a disk subjected to any radial load which may be expanded in a Fourier Series as

$$q(\theta) = q_0 + \sum_{n=1}^{\infty} q(n) \cos n\theta . \tag{9}$$

σ_r and σ_θ must be of the form

$$\sigma_r = q_0 + \sum_{n=1}^{\infty} f(r) \cos n\theta \quad \text{and} \quad \sigma_\theta = q_0 + \sum_{n=1}^{\infty} g(r) \cos n\theta$$

and the equations of equilibrium yield

$$\tau_{r\theta} = \sum_{n=1}^{\infty} \frac{n}{r^2} \int (r g)\, dr \sin n\theta .$$

The compatibility equations yield

$$\begin{aligned} f &= A_1 r^n + A_2 r^{-n} + A_3 r^{n-2} + A_4 r^{-n-2}, \\ g &= -\left[\frac{n+2}{n-2}\right] A_1 r^m - \left[\frac{n+2}{n-2}\right] A_2 r^{-n} - A_3 r^{n-2} - A_4 r^{-n-2}, \end{aligned} \tag{10}$$

where $A_1 - A_4$ are constants of integration which may now be determined from the boundary condition (9) and the three remaining conditions on the surface and at the inside surface of the hole. The general case of any radial and tangential loading may be solved by beginning with a more general assumption than Eq. (9). Problems treated in this section are solved using complex variables in chapter 8 of Ref. [8]. The solution (10) may be modified to include orthotropic material and hence solve problems of the type indicated in Ref. [9].

Consider next a body of revolution subjected to an axially symmetrical load distribution such as the tangential load $q_{(t)}$ shown in Fig. 2.

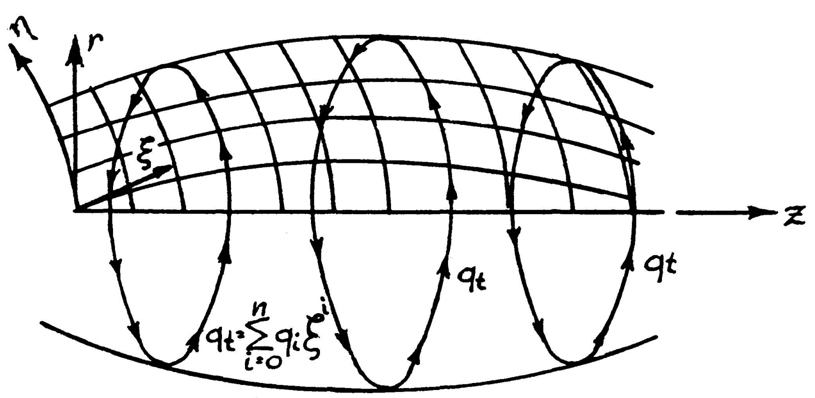

Fig. 2. A body of revolution subjected to a tangential load.

Let the body of revolution in Fig. 2 be a cylinder ($\eta = r$, $\xi = z$) and hence solve problems of a non-homogeneous ($G = G(r)$) cylinder of constant cross section subjected to a tangential load $q_{(t)} = \sum_{i=0}^{n} q_i z^i$ as well as couples applied at the ends. The governing equations are

$$\frac{\partial}{\partial z}\left[\frac{\tau_{\phi r}}{G r}\right] = \frac{\partial}{\partial r}\left[\frac{\tau_{\phi z}}{G r}\right], \qquad \frac{\partial}{\partial r}(r^2 \tau_{r\phi}) + \frac{\partial}{\partial z}(r^2 \tau_{\phi z}) = 0.$$

The stress equations must be of the form

$$\tau_{r\phi} = \sum_{i=0}^{n} R_i(r) z^i, \qquad \tau_{\phi z} = -\sum_{i=0}^{n} \frac{(r^2 R_i)'}{r^2} \frac{z^{i+1}}{(i+1)} - \frac{F(r)}{r^2},$$

where $R_i(r)$ is to be determined, $F(r)$ is an arbitrary function of integration and

$$\frac{(i+1)(i+2) R_{i+2}}{G r} + \left[\frac{(r^2 R_i)'}{G r^3}\right]' = 0, \qquad \left[\frac{F}{r^3 G}\right]' + \frac{R_1}{G r} = 0.$$

When G is constant it can be canceled out of the equations. Note that the solutions for odd and even polynomial loads are again uncoupled. When R_i is zero and G is constant the elementary torsion formula, for a circular bar twisted by end couples, results. The problem in the appendix of Ref. [10] may now be generalized. $R_i \neq 0$ yields solutions of problems of torsion of certain nonhomogeneous (as well as homogeneous) cylinders subjected to tangential surface loads as well as end couples. The solution may be easily extended to bars of cylindrically orthotropic material.

Consider next the body of revolution of Fig. 2 to be a frustrum of a cone ($\eta = \theta$, $\xi = \rho$) and hence solve problems of conical bars subjected to tangential load $q_t = \sum_{i=0}^{n} q_i \rho^i$ as well as twisting couples applied at the ends. The governing equations in spherical coordinates may be derived in the form

$$\rho \frac{\partial \tau_{\theta\phi}}{\partial \rho} = \sin\theta \frac{\partial}{\partial \theta}\left[\frac{\tau_{\rho\phi}}{\sin\theta}\right], \qquad \frac{\partial \tau_{\theta\phi}}{\partial \theta} + 2\cot\theta\, \tau_{\theta\phi} = -\frac{1}{\rho^2}\frac{\partial}{\partial \rho}(\rho^3 \tau_{\rho\phi}). \tag{11}$$

For this type of loading the stresses must be of the form

$$\tau_{\theta\phi} = \sum_{i=1}^{n} F_i(\theta) \rho^i, \qquad \tau_{\rho\phi} = \sum_{i=1}^{n}\left[\sin\theta \int \frac{F_i(\theta)}{\sin\theta} d\theta\right] i \rho^i + I_{(\rho)} \sin\theta,$$

where

$$F_i(\theta) = \frac{dH_i(\theta)}{d\theta}\sin\theta, \qquad \alpha = \cos\theta, \qquad H_i(\alpha) = A_n \frac{dP_n(\alpha)}{d\alpha} + B_n \frac{dQ_n(\alpha)}{d\alpha},$$
$$n = \frac{-1 \pm (2i+3)}{2} \quad \text{and} \quad I_{(\rho)} = \frac{C_0}{\rho^3}, \quad C_0 = \text{const.} \tag{12}$$

$P_n(\alpha)$ and $Q_n(\alpha)$ are the associated Legendre Functions[2]). The solutions for all polynomial loads are uncoupled. $F_i(\theta) \neq 0\,(C_0 = 0)$ yields solutions of problems of torsion of conical bars subjected to tangential polynomial surface loads as well as end couples.

The function $I_{(\rho)}(F_i(\theta) = 0)$ corresponds to Föpple's problem [12] of a conical bar subjected to end couples[3]).

The Föpple problem[4]) may also be solved when the bar material is orthotropic such that

$$\tau_{\phi z} = G_{\phi z}\gamma_{\phi z}, \qquad \tau_{r\phi} = G_{r\phi}\gamma_{r\phi}, \qquad \alpha = \frac{G_{r\phi}}{G_{\phi z}}.$$

Transforming the governing compatibility equation and the stresses into spherical coordinates yields

$$\tau_{r\phi} = \tau_{\rho\phi}\sin\theta\,, \qquad \tau_{z\phi} = \tau_{\rho\phi}\cos\theta \quad \text{for } \tau_{\theta\phi} = 0,$$

$$(1-\alpha)\,\rho^2\cos\theta\frac{\partial}{\partial\rho}\left(\frac{\tau_{\rho\phi}}{\rho}\right) - \sin\theta\frac{\partial\tau_{\rho\phi}}{\partial\theta} - \alpha\cos\theta\frac{\partial}{\partial\theta}(\tau_{\rho\phi}\cot\theta) = 0. \qquad (13)$$

The equilibrium Eq. (11) gives $\tau_{\rho\phi} = K_{(\theta)}/\rho^3$ where $K_{(\theta)}$ must be evaluated from (13). The solution is

$$\tau_{\rho\phi} = \frac{C\sin\theta}{\rho^3[1-(1-\alpha)\cos^2\theta]^{5/2}}$$

and hence
$$\tau_{r\phi} = \frac{C\,r^2}{(r^2+\alpha\,z^2)^{5/2}}, \qquad \tau_{z\phi} = \frac{C\,r\,z}{(r^2+\alpha\,z^2)^{5/2}}.$$

The constant C is evaluated in terms of the applied torques T as

$$C = -\frac{3\,T}{2\,\pi\left[\frac{2}{(\alpha)^{1/2}} - \frac{2}{(\alpha+\tan^2\beta)^{1/2}} - \frac{\tan^2\beta}{(\alpha+\tan^2\beta)^{3/2}}\right]}.$$

This problem is one of several throughout this manuscript which is solved by first equating one stress to zero and then solving for the remaining stresses. These solutions correspond to problems where the continuum is subjected to concentrated loads.

Let again the body of revolution of Fig. 2 be a uniform cylinder subjected to loads $q_{(z)} = \sum_{i=0}^{n} q_i z^i$ instead of the tangential loads $q_{(t)}$ shown. Starting with the assumption

$$\sigma_z = \sum_{i=0}^{n} Z_i(z)\,r^i$$

[2]) A table of these functions is given on page 133 of Ref. [11].

[3]) See also Ref. [1] p. 345 and Ref. [8] p. 104.

[4]) As well as those corresponding to solution (12).

the problem of the symmetrically loaded circular plate discussed in art. 133 of Ref. [1] may be generalized to include additional ($i > 0$) polynomial loading conditions. This problem will not be pursued further but consider the special case when $Z_i(z) = 0$. The governing equations, in cylindrical coordinates yield

$$\tau_{rz} = \frac{G_{(z)}}{r}, \qquad G_z'''' = 0, \qquad Z_1''(z) = 0, \qquad Z_2'' + \frac{\nu K}{(1+\nu)} + (2-\nu)\,G' = 0,$$

$$\sigma_r = [K-(1+\nu)\,G']\,\tfrac{1}{2}(\ln r - \tfrac{1}{2}) + \tfrac{1}{2}(Z_1 - G') + Z_2/r^2,$$

$$\sigma_r = [K-(1+\nu)\,G']\,\tfrac{1}{2}(\ln r + \tfrac{1}{2}) + \tfrac{1}{2}(Z_1 + G') - Z_2/r^2.$$

These equations solve approximately the problem of a solid circular plate, subjected to a concentrated load P at the center and supported by shear loads at the circular edges, as follows:

$$\text{Let} \quad G_{(z)} = c_3\left[1-\left(\frac{z}{c}\right)^2\right], \quad c_3 = \frac{3P}{8\pi c}, \quad K = 0 \quad \text{and} \quad \tau_{rz} = \frac{3P}{8\pi c r}\left[1-\left(\frac{z}{c}\right)^2\right],$$

so that τ_{rz} is zero on the upper and the lower surfaces of the plate (Fig. 202 Ref. [1]). The equations for Z_1 and Z_2 may be integrated to give

$$Z_1 = c_4 z \quad \text{and} \quad Z_2 = -\frac{2(1-\nu)\,c_3 z^3}{3c^2}.$$

For thin plates Z_2 may be neglected since Z^3/r^2 is small in comparison to z. The constant c_4 may be evaluated from the boundary condition $\sigma_r = 0$ at $r = a$ and hence

$$\sigma_r = \frac{3(1+\nu)\,Pz}{8\pi c^3}\ln\frac{r}{a}, \qquad \sigma_\theta = \frac{3Pz}{8\pi c^3}\left[(1+\nu)\ln\frac{r}{a} - (1-\nu)\right].$$

These equations agree with Eqs. (90)—(91) of Ref. [13] obtained by using the theory for bending of thin plates.

Other problems of two dimensional elasticity for isotropic or certain anisotropic material may be solved[5]) using the procedure enunciated. The examples treated are ample proof that it is not always best to solve two dimensional problems of elasticity by first introducing a stress function.

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Summary

Solutions of two-dimensional problems of elasticity are developed without first expressing the governing equations in terms of a stress function. This method uses coordinates such that a loaded surface coincides with a curve parallel to one of the coordinate axis, which leads to an expression for one of the stresses. The equations of elasticity may then often be solved for the remaining stresses if only in view of Saint-Venant's Principle. Closed form solutions may be obtained more readily by this method rather than by the stress function method. This method applies to homogeneous, non-homogeneous, isotropic and anisotropic materials whenever integration of the equations permits a solution.

Résumé

On résoud des problèmes d'élasticité bidimensionnels sans d'abord employer d'équations avec fonctions de tension. Cette méthode emploie des coordonnées où la surface de poids appliqué coincide avec une courbe parallèle à l'un des axes coordonnés, ce qui mène à la formule d'une des tensions. Souvent on peut ainsi résoudre l'équation d'élasticité pour les autres tensions en se servant du principe de Saint-Venant. Cette méthode permet plus facilement d'arriver à des solutions exactes que celle employant une fonction de tension. La méthode est valable pour des matériaux homogènes, non-homogènes, isotropes et anisotropes dans tous les cas où il existe une solution pour l'équation différentielle.

Zusammenfassung

Es werden Lösungen von zweidimensionalen Elastizitätsproblemen entwickelt, wobei man nicht erst die zugehörigen Gleichungen in Form einer Spannungsfunktion auszudrücken braucht. Bei der Lösung werden Koordinaten in der Weise benützt, dass eine Belastungsfläche mit einer Kurve zusammenfällt, welche parallel zu einer der Koordinatenachsen verläuft; dies führt dann zu einem Ausdruck für eine der Spannungen. Unter Benutzung des Saint-Venant-Prinzips können dann auch die Gleichungen für die anderen Spannungen gelöst werden. Mit dieser Methode kann man eher zu geschlossenen Lösungen gelangen als mit der Spannungsfunktions-Methode.