Analysis of curved box girder bridges by finite strip method

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Analyse de ponts courbes en caisson avec la méthode Finite-strip

Analyse von gebogenen Kastenträger-Brücken mittels der Finite-Strip-Methode

M. S. CHEUNG
Ph. D., Postdoctoral Fellow, Dept. of Civil Engineering University of Calgary, Calgary, Alberta, Canada

Y. K. CHEUNG
Ph. D., MICE, M. I. Struct. E. Professor of Civil Engineering, University of Calgary

Introduction

The finite strip method has been applied successfully to the analysis of right box girder bridges [1], and to curved bridge decks [2]. In this paper, the method is extended to deal with curved box-girder bridges of constant width and supported along radial lines. In the past, a number of papers have been published in which the writers treated the curved box girders as curved beams of prismatic thin-walled closed cross-section (for example [3], [4]). Such an assumption is obviously unsuitable for many of the wider box girder sections commonly used in bridge design practice, in which a fair amount of transverse bending occurs in the flanges. The longitudinal stress in the top and bottom flanges of such wider sections are also far from being uniform.

The finite strip method used in the present analysis incorporates all the bending and membrane actions, and since the displacement functions employed in formulating the strip stiffness matrices are compatible functions, it is expected that the results should converge to the correct solution with increasingly finer mesh divisions.

The standard finite element method using a shell element can be used to analyse multi-spanned curved box girder bridges with internal diaphragms and skewed supports, and is therefore extremely powerful and versatile. However, in a paper by SISODIYA et al. [5], it was pointed out that more research is needed to develop elements which permit the use smaller number of equations and to reduce computing time and programming effort, before the method can be widely accepted by bridge designers. In the same paper, it was also pointed out that the finite strip method provides a solution which can be conveniently used in practical design for simply-supported constant width curved box girder bridges, because it requires relatively short computer time and smaller computer storage.

A further advantage of the finite strip method which should be mentioned here is that the amount of data input is reduced drastically as a result of the strip idealization.

Basic Assumptions and General Philosophy

The basic assumptions used in the present analysis are as follows:

1. The webs and flanges are made up of isotropic or cylindrical orthotropic materials.
2. The box girder is bounded by concentric circular arcs and by two radial planes at the supports.
3. The girder is supported at each radial edge by a diaphragm which is infinitely stiff in its own plane but infinitely flexible out of plane.

In the finite strip method, the bridge is divided into a number of strips supported at their radial ends. A displacement function of the form $f(r)\Phi(\theta)$ or $f(z)\Phi(\theta)$ is chosen for the strip, in which $f(r)$ or $f(z)$ is a polynomial with undetermined displacement parameters for the sides i and j (see Figs. 1b and 1c) and $\Phi(\theta)$ is a Fourier series which satisfies automatically the support conditions. The external loads are also resolved into the same Fourier series for the corresponding displacement components. It can be easily proved that the series will in fact uncouple, so that a term by term analysis can be conducted and then all the results are summed together. In the subsequent derivations it will be assumed that we are dealing with one term (say the mth) of the series.

The adoption of a continuous function from one radial support to the other eliminates the need for subdividing the bridge in the θ-direction, as in the standard finite element method and consequently the analysis of a curved box girder bridge is very little different from that of a plane frame.

Development of Stiffness Matrices

The general formulation of the stiffness matrix in the finite strip method can be found in references [1] and [2] and shall not be repeated here. In the text which follows, only the displacement functions and some of the important matrices will be given.

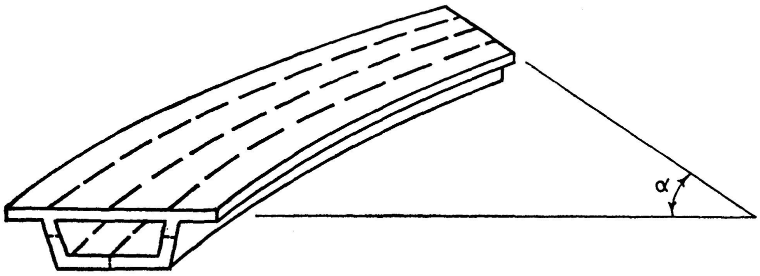


Fig. 1a. A Curved Box Bridge and Its Idealization into Strips.

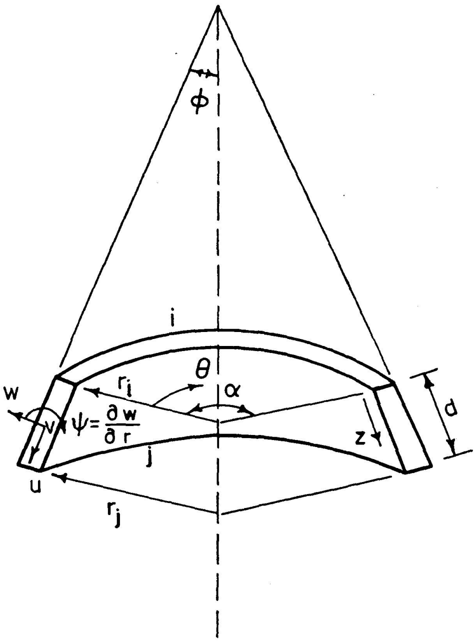


Fig. 1b. A Conical Web Strip.

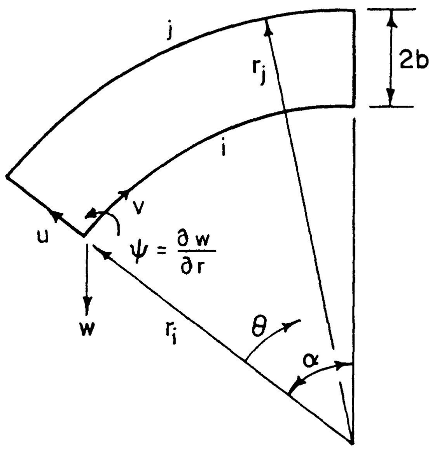


Fig. 1c. A Flange Strip.

A. Top and Bottom Flanges

The top and bottom flanges of the box girder are flat plates which are curved in plan, and therefore the membrane and bending actions are uncoupled and can be treated separately. In the following paragraphs we shall discuss firstly the in-plane and then the bending aspects of the strip, and finally combine the two together.

1. In-plane Stiffness Matrix

Displacement functions:

$$u_m = \left[\left(1-\frac{R}{2}\right)u_{im}+\frac{R}{2}u_{jm}\right]\sin\frac{m\pi\theta}{\alpha},$$
$$v_m = \left[\left(1-\frac{R}{2}\right)v_{im}+\frac{R}{2}v_{jm}\right]\cos\frac{m\pi\theta}{\alpha}, \tag{1}$$

in which $R=\frac{r-r_i}{b}$, and $b=\frac{r_j-r_i}{2}$.

Strain displacement relationship:

$$\left\{\begin{matrix}\epsilon_r\\ \\ \epsilon_\theta\\ \\ \gamma_{r\theta}\end{matrix}\right\}_m = \left\{\begin{matrix}\dfrac{\partial u}{\partial r}\\ \\ \dfrac{1}{r}\dfrac{\partial v}{\partial\theta}+\dfrac{u}{r}\\ \\ \dfrac{1}{r}\dfrac{\partial u}{\partial\theta}+\dfrac{\partial v}{\partial r}-\dfrac{v}{r}\end{matrix}\right\}_m = [P_m]\{\delta_{pm}\}, \tag{2}$$

where $\{\delta_{pm}\}$ is equal to $\{u_{im}, v_{im}, u_{jm}, v_{jm}\}^T$.

Property matrix:

$$[D_p] = \left\{\begin{matrix}K_r & K_1 & 0\\ K_1 & K_\theta & 0\\ 0 & 0 & K_{r\theta}\end{matrix}\right\}, \tag{3}$$

in which $K_r = \dfrac{E_r t}{1-\nu_r\nu_\theta}$, $K_1 = \nu_\theta K_r$, $K_\theta = \dfrac{E_\theta t}{1-\nu_r\nu_\theta}$ and $K_{r\theta} = G_{r\theta} t$.

The strain matrix $[P_m]$ and stiffness matrix $[S_{pm}]$ are given in Appendix Ia and Ib respectively. Note that the integration with respect to *dr* is to be performed numerically.

2. *Bending Stiffness Matrix*

Displacement function:

$$w_m = \left[\left(1-\frac{3}{4}R^2+\frac{1}{4}R^3\right)w_{im}+b\left(R-R^2+\frac{R^3}{4}\right)\psi_{im} + \left(\frac{3}{4}R^2-\frac{1}{4}R^3\right)w_{jm}+b\left(\frac{R^3}{4}-\frac{R^2}{2}\right)\psi_{jm}\right]\sin\frac{m\pi\theta}{\alpha}. \tag{4}$$

Strain displacement relationship:

$$\left\{\begin{matrix}-\chi_r\\ \\ -\chi_\theta\\ \\ \chi_{r\theta}\end{matrix}\right\}_m = \left\{\begin{matrix}-\dfrac{\partial^2 w}{\partial r^2}\\ \\ -\dfrac{1}{r}\left(\dfrac{1}{r}\dfrac{\partial^2 w}{\partial\theta^2}+\dfrac{\partial w}{\partial r}\right)\\ \\ -\dfrac{2}{r}\left(\dfrac{\partial^2 w}{\partial r\,\partial\theta}-\dfrac{1}{r}\dfrac{\partial w}{\partial\theta}\right)\end{matrix}\right\}_m = [B_m]\{\delta_{bm}\}, \tag{5}$$

where $\{\delta_{bm}\}$ is equal to $\{w_{im}, \psi_{im}, w_{jm}, \psi_{jm}\}^T$.

Property matrix:

$$[D_b] = \left\{\begin{matrix}D_r & D_1 & 0\\ D_1 & D_\theta & 0\\ 0 & 0 & D_{r\theta}\end{matrix}\right\}, \tag{6}$$

in which $D_r = \frac{E_r t^3}{12(1-\nu_r \nu_\theta)}$, $D_\theta = \frac{E_\theta t^3}{12(1-\nu_r \nu_\theta)}$,

$$D_{r\theta} = \frac{G_{r\theta} t^3}{12} \quad \text{and} \quad D_1 = \nu_\theta D_r = \nu_r D_\theta .$$

The strain matrix $[B_m]$ and stiffness matrix $[S_{bm}]$ are listed in Appendix IIa and IIb respectively. There the integration with respect to dr is also to be performed numerically.

3. *Combined Stiffness Matrix*

For ease of operation in the overall analysis the two stiffness matrices are combined into a shell stiffness matrix in the following manner:

$$[S_m] = \left[\begin{array}{cc|cc} [S_{pm}]_{ii} & 0 & [S_{pm}]_{ij} & 0 \\ 0 & [S_{bm}]_{ii} & 0 & [S_{bm}]_{ij} \\ \hline [S_{pm}]_{ji} & 0 & [S_{pm}]_{jj} & 0 \\ 0 & [S_{bm}]_{ji} & 0 & [S_{bm}]_{jj} \end{array}\right], \tag{7}$$

in which the subscripts i and j refer to the two sides of a strip.

B. Curved Interior and Exterior Webs of Box Girder

Each web is in general a part of a conical frustum (Fig. 1b), but becomes a cylindrical panel when it is in a vertical position. For such a curved surface the membrane and bending actions cannot be uncoupled, and a 8×8 stiffness matrix has to be formed directly.

Displacement functions:

$$\begin{aligned} u_m &= \left[\left(1-\frac{z}{d}\right) u_{im} + \left(\frac{z}{d}\right) u_{jm}\right] \sin\frac{m\pi\theta}{\alpha}, \\ v_m &= \left[\left(1-\frac{z}{d}\right) v_{im} + \left(\frac{z}{d}\right) v_{jm}\right] \cos\frac{m\pi\theta}{\alpha}, \\ w_m &= \left[\left(1-\frac{3z^2}{d^2}+\frac{2z^3}{d^3}\right) w_{im} + \left(z-\frac{2z^2}{d}+\frac{z^3}{d^2}\right)\psi_{im}\right. \\ &\quad \left. + \left(\frac{3z^2}{d^2}-\frac{2z^3}{d^3}\right) w_{jm} + \left(\frac{z^3}{d^2}-\frac{z^2}{d}\right)\psi_{jm}\right] \sin\frac{m\pi\theta}{\alpha}. \end{aligned} \tag{8}$$

Strain displacement relationship:

$$\begin{Bmatrix} \epsilon_z \\ \epsilon_\theta \\ \gamma_{z\theta} \\ \chi_z \\ \chi_\theta \\ \chi_{z\theta} \end{Bmatrix}_m = \begin{Bmatrix} \dfrac{\partial u}{\partial z} \\ \dfrac{1}{r}\dfrac{\partial v}{\partial \theta} + \dfrac{w\cos\phi + u\sin\phi}{r} \\ \dfrac{1}{r}\dfrac{\partial u}{\partial \theta} + \dfrac{\partial v}{\partial z} - \dfrac{v\sin\phi}{r} \\ -\dfrac{\partial^2 w}{\partial z^2} \\ -\dfrac{1}{r^2}\dfrac{\partial^2 w}{\partial \theta^2} + \dfrac{\cos\phi}{r^2}\dfrac{\partial v}{\partial \theta} - \dfrac{\sin\phi}{r}\dfrac{\partial w}{\partial z} \\ 2\left(-\dfrac{1}{r}\dfrac{\partial^2 w}{\partial z \partial \theta} + \dfrac{\sin\phi}{r^2}\dfrac{\partial w}{\partial \theta} + \dfrac{\cos\phi}{r}\dfrac{\partial v}{\partial z} - \dfrac{\sin\phi\cos\phi}{r^2} v\right) \end{Bmatrix}_m = [T_m]\{\delta_m\}, \tag{9}$$

where $\{\delta_m\}$ is equal to $\{u_{im}, v_{im}, w_{im}, \psi_{im}, u_{jm}, v_{jm}, w_{jm}, \psi_{jm}\}^T$.

Property matrix:

$$[D_t] = \begin{bmatrix} K_z & K_2 & 0 & 0 & 0 & 0 \\ K_2 & K_\theta & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{z\theta} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_z & D_2 & 0 \\ 0 & 0 & 0 & D_2 & D_\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{z\theta} \end{bmatrix}, \tag{10}$$

in which $K_z = \dfrac{E_z t}{1-\nu_z\nu_\theta}$, $K_2 = \nu_\theta K_z$, $K_{z\theta} = G_{z\theta} t$, $K_\theta = \dfrac{E_\theta t}{1-\nu_z\nu_\theta}$,

$D_z = \dfrac{E_z t^3}{12(1-\nu_z\nu_\theta)}$, $D_2 = \nu_\theta D_z$, $D_{z\theta} = \dfrac{G_{z\theta} t^3}{12}$. $D_\theta = \dfrac{E_\theta t^3}{12(1-\nu_z\nu_\theta)}$,

The strain matrix $[T_m]$ and stiffness $[S_m]$ are listed in Appendix IIIa and IIIb respectively. Note that the terms T_{ij} used in Appendix IIIb refer to the corresponding coefficients of the matrix $[T_m]$.

Transformation and Assembly

The procedure for transforming the stiffness matrix of a strip into a common set of coordinates and subsequently the assembly of the transformed matrix into the overall matrix of the structure are identical to the methods used for a plane frame, and therefore will not be discussed here. Readers who are interested should refer to any textbook on matrix analysis of structures or to reference [1].

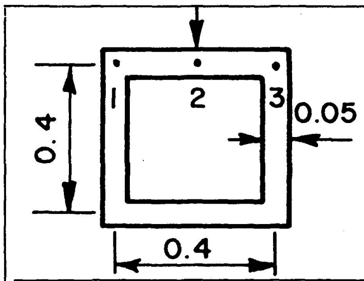
Numerical Examples

The first example involves the analysis of a nearly straight box girder, using the curved strip program, and then comparing the results with those due to a straight box girder program [1], which has been well tested.

The length of the straight box girder is taken as unity, while the curved box girder has a radius of 1000 units and a subtended angle of 0.001 radian. A unit load is applied on the top slab at the centre of the girder.

The comparison of two sets of results for the central section is given in Table 1. Since nearly identical results have been produced by the two different programs, it can be concluded that the curved strip program is correct, and that it can be used successfully for analysing straight box girder bridges by using very large radius in conjunction with a very small subtended angle.

Table 1. Comparison of Results Obtained by Straight Strip Program and Curved Strip Program



	1		2		3	
	Straight Strip	Curved Strip	Straight Strip	Curved Strip	Straight Strip	Curved Strip
w	27.511	27.561	168.421	165.561	27.511	27.573
σ_x	–11.051	–10.910	–5.917	–5.780	–11.051	–10.904
σ_y	–24.118	–24.158	–9.451	–9.493	–24.118	–24.128
M_x	–0.105	–0.103	0.214	0.209	–0.105	–0.103
M_y	–0.031	–0.030	0.161	0.157	–0.031	–0.030

The second example involves the analysis of a multi-celled, curved box girder bridge (Fig. 2a) under unit central point load at three different radial positions, i.e., outer web, middle web and inner web. The numbering of nodal lines and strips can be found in Fig. 2b, in which a very narrow band matrix will result from such a scheme. A total of 20 non-zero terms of the series has been used for the analysis. The distribution of membrane forces and bending moments for the central section of the bridge are given in Fig. 3a to Fig. 3c. As a matter of interest, a straight bridge with the same cross-section and span length equal to the circumferential span of the curved bridge at the middle web is analysed for a central point load, and the results are given in brackets inside the same figure.

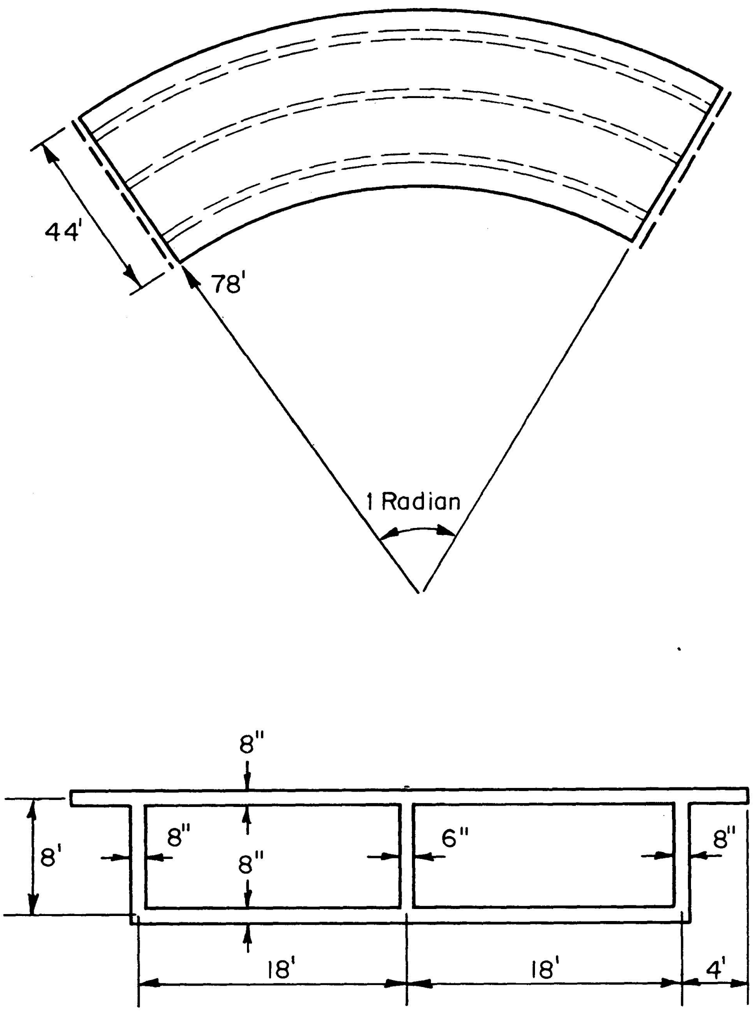


Fig. 2a. Plan and Section of the Curved Box Girder Bridge in Example 2 ($E = 1$, $v = 0.16$).

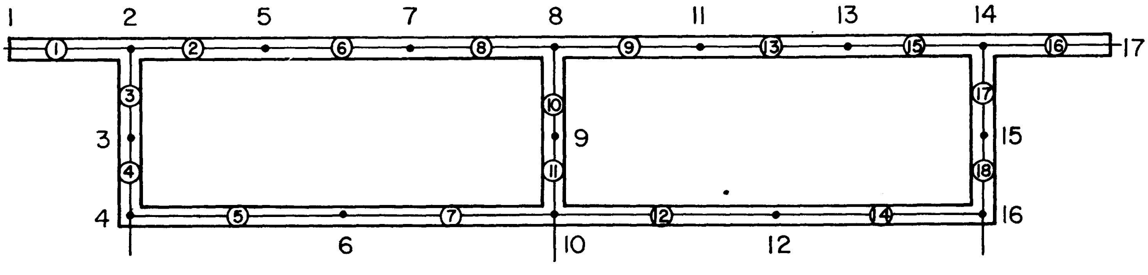


Fig. 2b. Numbering of Nodal Lines and Strips for Bridge in Example 2.

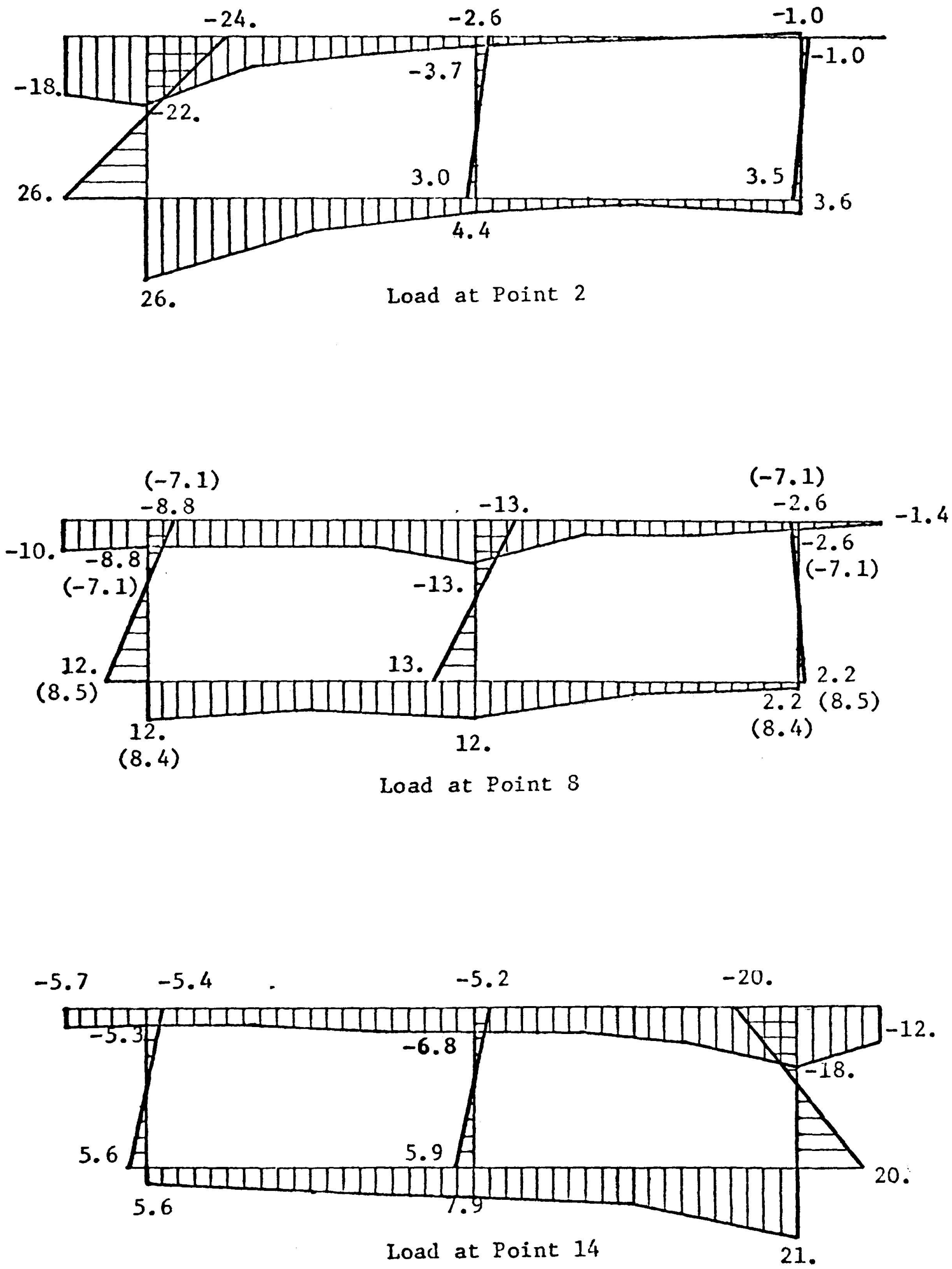


Fig. 3a. $N_\theta(10^{-2})$ Distribution in the Mid-Section of Bridge Due to Load at Three Different Positions.

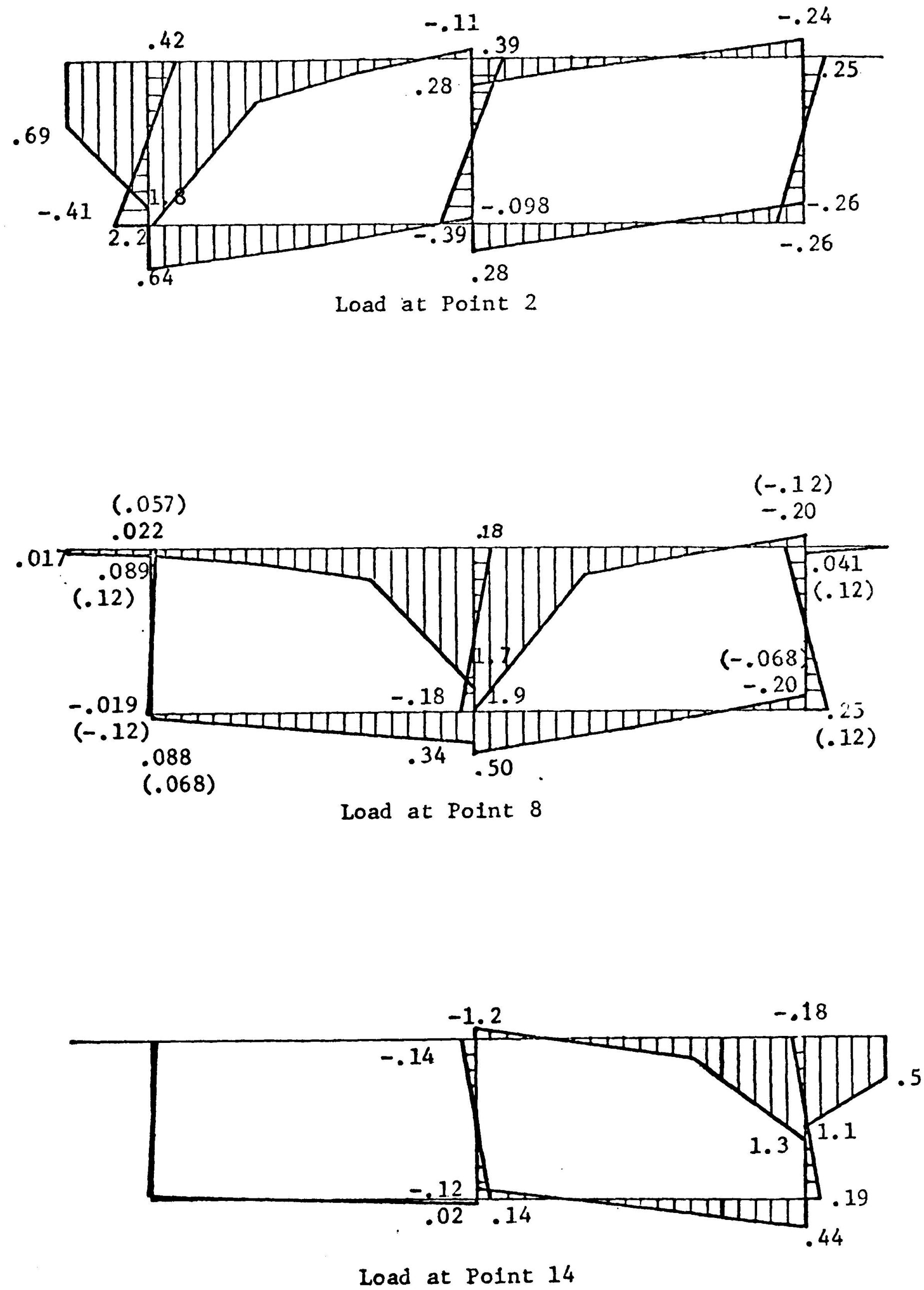


Fig. 3b. $M_\theta(10^{-2})$ Distribution in the Mid-Section of Bridge Due to Load at Three Different Positions.

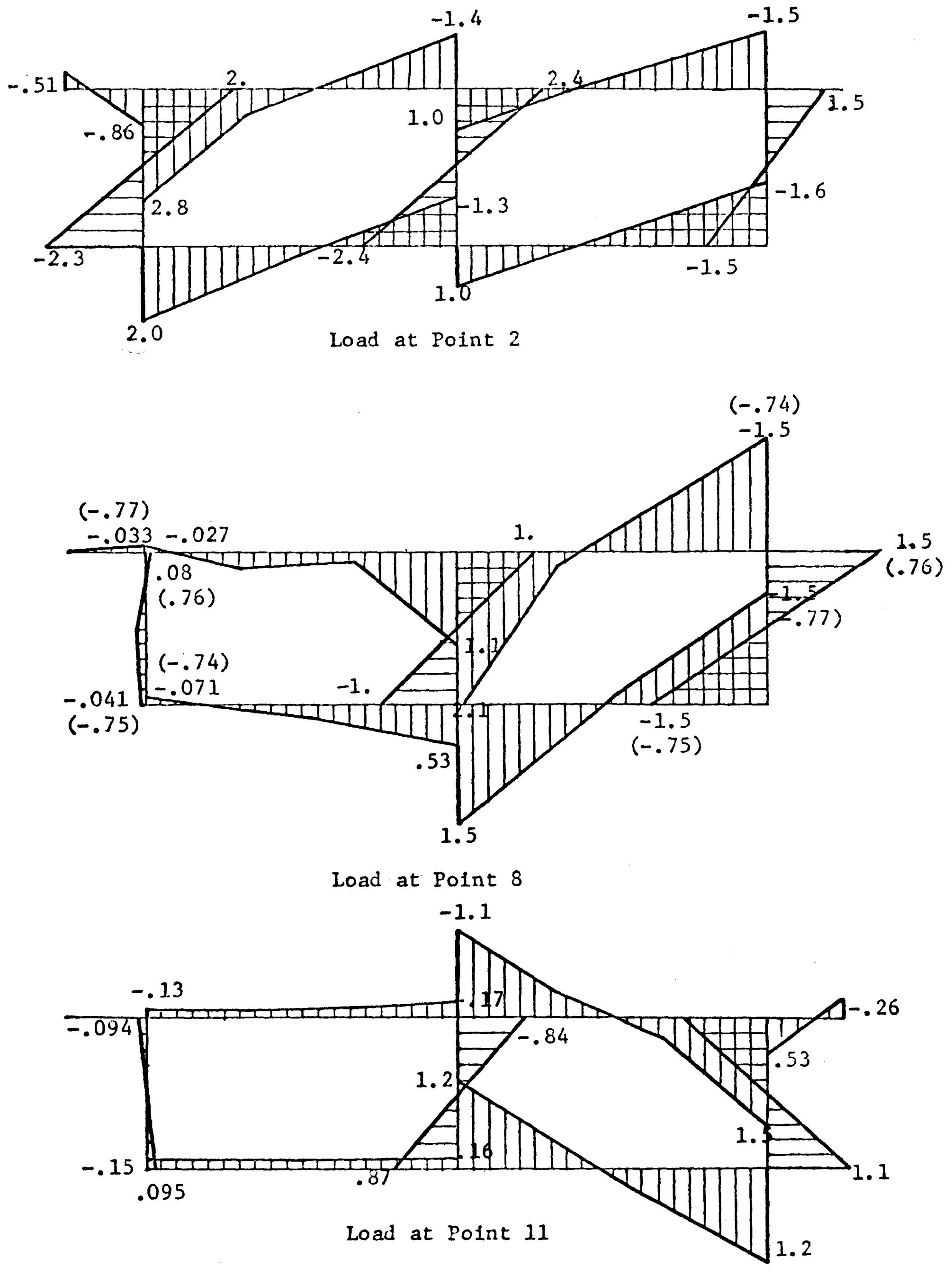


Fig. 3c. $M_r\,(10^{-2})$ Distribution in the Mid-Section of Bridge Due to Load at Three Different Positions.

Conclusions

It has been demonstrated that the finite strip method can be used successfully for the analysis of curved as well as straight box girder bridges. The method is simple but versatile, and requires minimal computer storage and execution time.

Nomenclature

$2b$	width of the flange strip.
d	width of the conical strip.
E_r, E_θ, E_z	orthotropic material properties.
$\nu_r, \nu_\theta, G_{r\theta}, G_{z\theta}$	
γ, θ	polar coordinates for the curved bridge.
t	thickness of the strip.
u, v, w, ψ	displacement parameters of a strip.
α	subtended angle of curved bridge.
ϕ	angle of inclination between conical segment and vertical axis.

Appendix I a

Strain Matrix of Flange Strip for In-Plane Actions

$$[P_m] = \begin{bmatrix} -\frac{1}{2b}S & 0 & \frac{1}{2b}S & 0 \\ \frac{1}{r}\left(1-\frac{R}{2}\right)S & \frac{-1}{r}\left(1-\frac{R}{2}\right)k_m S & \frac{1}{r}\left(\frac{R}{2}\right)S & \frac{-1}{r}\left(\frac{R}{2}\right)k_m S \\ \frac{1}{r}\left(1-\frac{R}{2}\right)k_m C & -\frac{1}{2b}C-\frac{1}{r}\left(1-\frac{R}{2}\right)C & \frac{1}{r}\left(\frac{R}{2}\right)k_m C & \frac{1}{2b}C-\frac{1}{r}\left(\frac{R}{2}\right)C \end{bmatrix}$$

$$\left(k_m = \frac{m\pi}{\alpha}, \quad S = \sin k_m\theta, \quad C = \cos k_m\theta\right).$$

Appendix Ib

Stiffness Matrix of Flange Strip for In-Plane Actions

$[\overline{S}_{pm}] =$

$K_r P_{11}^2 + 2K_1 P_{11} P_{21} + K_\theta P_{21}^2 + K_{r\theta} P_{31}^2$			
$K_r P_{11} P_{12} + K_1 P_{12} P_{21} + K_1 P_{11} P_{22} + K_\theta P_{21} P_{22} + K_{r\theta} P_{31} P_{32}$	$K_r P_{12}^2 + 2K_1 P_{12} P_{22} + K_\theta P_{22}^2 + K_{r\theta} P_{32}^2$	Symmetrical (P_{ij} refers to coefficients in strain matrix)	
$K_r P_{11} P_{13} + K_1 P_{21} P_{13} + K_1 P_{11} P_{23} + K_\theta P_{21} P_{23} + K_{r\theta} P_{31} P_{33}$	$K_r P_{12} P_{13} + K_1 P_{22} P_{13} + K_1 P_{12} P_{23} + K_\theta P_{22} P_{23} + K_{r\theta} P_{32} P_{33}$	$K_r P_{13}^2 + 2K_1 P_{23} P_{13} + K_\theta P_{23}^2 + K_{r\theta} P_{33}^2$	
$K_r P_{11} P_{14} + K_1 P_{21} P_{14} + K_1 P_{11} P_{24} + K_\theta P_{21} P_{24} + K_{r\theta} P_{31} P_{34}$	$K_r P_{12} P_{14} + K_1 P_{22} P_{14} + K_1 P_{12} P_{24} + K_\theta P_{22} P_{24} + K_{r\theta} P_{32} P_{34}$	$K_r P_{13} P_{14} + K_1 P_{23} P_{14} + K_1 P_{13} P_{24} + K_\theta P_{23} P_{24} + K_{r\theta} P_{33} P_{34}$	$K_r P_{14}^2 + 2K_1 P_{24} P_{14} + K_\theta P_{24}^2 + K_{r\theta} P_{34}^2$

$$[S_{pm}] = \frac{\alpha}{2} \int_{r_i}^{r_j} [\overline{S}_{pm}] \, r \, dr.$$

Appendix IIa. *Strain Matrix of Flange Strip in Bending*

$$[B_m] = \begin{bmatrix} \left(-\frac{3R}{2b^2}+\frac{3}{2b^2}\right)S & \left(\frac{2}{b}-\frac{3R}{2b}\right)S & \left(\frac{3R}{2b^2}-\frac{3}{2b^2}\right)S & \left(\frac{1}{b}-\frac{3R}{2b}\right)S \\ \frac{1}{r^2}\left(1-\frac{3}{4}R^2+\frac{1}{4}R^3\right)k_m^2 S + \frac{1}{r}\left(\frac{3R}{2b}-\frac{3R^2}{4b}\right)S & \frac{b}{r^2}\left(R-R^2+\frac{R^3}{4}\right)k_m^2 S + \frac{1}{r}\left(2R-1-\frac{3R^2}{4}\right)S & \frac{1}{r^2}\left(\frac{3}{4}R^2-\frac{1}{4}R^3\right)k_m^2 S + \frac{1}{r}\left(\frac{3R^2}{4b}-\frac{3R}{2b}\right)S & \frac{b}{r^2}\left(\frac{R^3}{4}-\frac{R^2}{2}\right)k_m^2 S + \frac{1}{r}\left(R-\frac{3R^2}{4}\right)S \\ \frac{2}{r}\left(\frac{3R}{2b}-\frac{3R^2}{4b}\right)k_m C + \frac{2}{r^2}\left(1-\frac{3}{4}R^2+\frac{1}{4}R^3\right)k_m C & \frac{2}{r}\left(-1+2R-\frac{3R^2}{4}\right)k_m C + \frac{2b}{r^2}\left(R-R^2+\frac{R^3}{4}\right)k_m C & \frac{2}{r}\left(\frac{-3R}{2b}+\frac{3R^2}{4b}\right)k_m C + \frac{2}{r^2}\left(\frac{3}{4}R^2-\frac{1}{4}R^3\right)k_m C & \frac{2}{r}\left(R-\frac{3R^2}{4}\right)k_m C + \frac{2b}{r^2}\left(\frac{R^3}{4}-\frac{R^2}{2}\right)k_m C \end{bmatrix}$$

$$\left(k_m = \frac{m\pi}{\alpha}, \quad S = \sin k_m\theta, \quad C = \cos k_m\theta\right).$$

Appendix IIb. *Stiffness Matrix of Flange Strip in Bending*

$$[\bar{S}_{bm}] = \begin{bmatrix} D_r B_{11}^2 + 2D_1 B_{11} B_{21} + D_\theta B_{21}^2 + D_{r\theta} B_{31}^2 & & & \\ D_r B_{11} B_{12} + D_1 B_{12} B_{21} + D_1 B_{11} B_{22} + D_\theta B_{21} B_{22} + D_{r\theta} B_{31} B_{32} & D_r B_{12}^2 + 2D_1 B_{12} B_{22} + D_\theta B_{22}^2 + D_{r\theta} B_{32}^2 & \multicolumn{2}{c}{\text{Symmetrical } (B_{ij} \text{ refers to coefficients in strain matrix})} \\ D_r B_{11} B_{13} + D_1 B_{21} B_{13} + D_r B_{11} B_{23} + D_\theta B_{21} B_{23} + D_{r\theta} B_{31} B_{33} & D_r B_{12} B_{13} + D_1 B_{22} B_{13} + D_1 B_{12} B_{23} + D_\theta B_{22} B_{23} + D_{r\theta} B_{32} B_{33} & D_r B_{13}^2 + 2D_1 B_{23} B_{13} + D_\theta B_{23}^2 + D_{r\theta} B_{33}^2 & \\ D_r B_{11} B_{14} + D_1 B_{21} B_{14} + D_1 B_{11} B_{24} + D_\theta B_{21} B_{24} + D_{r\theta} B_{31} B_{34} & D_r B_{12} B_{14} + D_1 B_{22} B_{14} + D_1 B_{12} B_{24} + D_\theta B_{22} B_{24} + D_{r\theta} B_{32} B_{34} & D_r B_{13} B_{14} + D_1 B_{23} B_{14} + D_1 B_{13} B_{24} + D_\theta B_{23} B_{24} + D_{r\theta} B_{33} B_{34} & D_r B_{14}^2 + 2D_1 B_{24} B_{14} + D_\theta B_{24}^2 + D_{r\theta} B_{34}^2 \end{bmatrix}$$

$$[S_{bm}] = \frac{\alpha}{2}\int_{r_i}^{r_j} [\bar{S}_{bm}]\, r\, dr.$$

Appendix IIIa. *Strain Matrix of Conical Web Strip*

$[T_m] =$

$-\frac{1}{d}S$	0	0	0	$\frac{1}{d}S$	0	0	0
$\frac{1}{r}\left(1-\frac{z}{d}\right)S\,S_\phi$	$-\frac{1}{r}\left(1-\frac{z}{d}\right)k_m S$	$\frac{1}{r}\left(1-\frac{3z^2}{d^2}+\frac{2z^3}{d^3}\right)S\,C_\phi$	$\frac{1}{r}\left(z-\frac{2z^2}{d}+\frac{z^3}{d^2}\right)S\,C_\phi$	$\frac{1}{r}\left(\frac{z}{d}\right)S\,S_\phi$	$-\left(\frac{z}{r\,d}\right)k_m S$	$\frac{1}{r}\left(\frac{3z^2}{d^2}-\frac{2z^3}{d^3}\right)S\,C_\phi$	$\frac{1}{r}\left(\frac{z^3}{d^2}-\frac{z^2}{d}\right)S\,C_\phi$
$\frac{1}{r}\left(1-\frac{z}{d}\right)k_m C$	$-\frac{1}{d}C-\frac{1}{r}\left(1-\frac{z}{d}\right)C\,S_\phi$	0	0	$\frac{1}{r}\left(\frac{z}{d}\right)k_m C$	$\frac{1}{d}C-\frac{1}{r}\left(\frac{z}{d}\right)C\,S_\phi$	0	0
0	0	$\left(\frac{6}{d^2}-\frac{12z}{d^3}\right)S$	$\left(\frac{4}{d}-\frac{6z}{d^2}\right)S$	0	0	$\left(\frac{-6}{d^2}+\frac{12z}{d^3}\right)S$	$\left(\frac{-6z}{d^2}+\frac{2}{d}\right)S$
0	$-\frac{1}{r^2}\left(1-\frac{z}{d}\right)k_m S\,C_\phi$	$\frac{1}{r^2}\left(1-\frac{3z^2}{d^2}+\frac{2z^3}{d^3}\right)k_m^2 S-\frac{1}{r}\left(-\frac{6z}{d^2}+\frac{6z^2}{d^3}\right)S\,S_\phi$	$\frac{1}{r^2}\left(z-\frac{2z^2}{d}+\frac{z^3}{d^2}\right)k_m^2 S-\frac{1}{r}\left(1-\frac{4z}{d}+\frac{3z^2}{d^2}\right)S\,S_\phi$	0	$-\frac{1}{r^2}\left(\frac{z}{d}\right)k_m S\,C_\phi$	$\frac{1}{r^2}\left(\frac{3z^2}{d^2}-\frac{2z^3}{d^3}\right)k_m^2 S-\frac{1}{r}\left(\frac{6z}{d^2}-\frac{6z^2}{d^3}\right)S\,S_\phi$	$\frac{1}{r^2}\left(\frac{z^3}{d^2}-\frac{z^2}{d}\right)k_m^2 S-\frac{1}{r}\left(\frac{3z^2}{d^2}-\frac{2z}{d}\right)S\,S_\phi$
0	$-\frac{2}{r}\left(\frac{1}{d}\right)C\,C_\phi-\frac{2}{r^2}\left(1-\frac{z}{d}\right)C\,S_\phi\,C_\phi$	$\frac{2}{r}\left(\frac{6z}{d^2}-\frac{6z^2}{d^3}\right)k_m C+\frac{2}{r^2}\left(1-\frac{3z^2}{d^2}+\frac{2z^3}{d^3}\right)k_m C\,S_\phi$	$\frac{2}{r}\left(-1+\frac{4z}{d}-\frac{3z^2}{d^2}\right)k_m C+\frac{2}{r^2}\left(z-\frac{2z^2}{d}+\frac{z^3}{d^2}\right)k_m C\,S_\phi$	0	$\frac{2}{r}\left(\frac{1}{d}\right)C\,C_\phi-\frac{2}{r^2}\left(\frac{z}{d}\right)C\,S_\phi\,C_\phi$	$\frac{2}{r}\left(\frac{6z^2}{d^3}-\frac{6z}{d^2}\right)k_m C+\frac{2}{r^2}\left(\frac{3z^2}{d^2}-\frac{2z^3}{d^3}\right)k_m C\,S_\phi$	$\frac{2}{r}\left(\frac{2z}{d}-\frac{3z^2}{d^2}\right)k_m C+\frac{2}{r^2}\left(\frac{z^3}{d^2}-\frac{z^2}{d}\right)k_m C\,S_\phi$

$$\left(k_m=\frac{m\,\pi}{\alpha},\quad S=\sin k_m\theta,\quad C=\cos k_m\theta\right),\quad S_\phi=\sin\phi,\quad C_\phi=\cos\phi.$$

Appendix IIIb

Stiffness Matrix of Conical Web Strip

$$
[\bar{S}_m]_{ii} = \left[\begin{array}{l|l|l|l}
\begin{array}{l} K_z T_{11}^2 + 2 K_2 T_{21} T_{11} \\ + K_\theta T_{21}^2 + K_{z\theta} T_{31}^2 \\ + D_z T_{41}^2 + 2 D_2 T_{51} T_{41} \\ + D_\theta T_{51}^2 + D_{z\theta} T_{61}^2 \end{array} &
\begin{array}{l} K_z T_{12} T_{11} + K_2 T_{22} T_{11} \\ + K_2 T_{12} T_{21} + K_\theta T_{22} T_{21} \\ + K_{z\theta} T_{32} T_{31} + D_z T_{42} T_{41} \\ + D_2 T_{52} T_{41} + D_2 T_{42} T_{51} \\ + D_\theta T_{52} T_{51} + D_{z\theta} T_{62} T_{61} \end{array} &
\begin{array}{l} K_z T_{13} T_{11} + K_2 T_{23} T_{11} \\ + K_2 T_{13} T_{21} + K_\theta T_{23} T_{21} \\ + K_{z\theta} T_{33} T_{31} + D_z T_{43} T_{41} \\ + D_2 T_{53} T_{41} + D_2 T_{43} T_{51} \\ + D_\theta T_{53} T_{51} + D_{z\theta} T_{63} T_{61} \end{array} &
\begin{array}{l} K_z T_{14} T_{11} + K_2 T_{24} T_{11} \\ + K_2 T_{14} T_{21} + K_\theta T_{24} T_{21} \\ + K_{z\theta} T_{34} T_{31} + D_z T_{44} T_{41} \\ + D_2 T_{54} T_{41} + D_2 T_{44} T_{51} \\ + D_\theta T_{54} T_{51} + D_{z\theta} T_{64} T_{61} \end{array} \\ \hline
\begin{array}{l} K_z T_{11} T_{12} + K_2 T_{21} T_{12} \\ + K_2 T_{11} T_{22} + K_\theta T_{21} T_{22} \\ + K_{z\theta} T_{31} T_{32} + D_z T_{41} T_{42} \\ + D_2 T_{51} T_{42} + D_2 T_{41} T_{52} \\ + D_\theta T_{51} T_{52} + D_{z\theta} T_{61} T_{62} \end{array} &
\begin{array}{l} K_z T_{12}^2 + 2 K_2 T_{22} T_{12} \\ + K_\theta T_{22}^2 + K_{z\theta} T_{32}^2 \\ + D_z T_{42}^2 + 2 D_2 T_{52} T_{42} \\ + D_\theta T_{52}^2 + D_{z\theta} T_{62}^2 \end{array} &
\begin{array}{l} K_z T_{13} T_{12} + K_2 T_{23} T_{12} \\ + K_2 T_{13} T_{22} + K_\theta T_{23} T_{22} \\ + K_{z\theta} T_{33} T_{32} + D_z T_{43} T_{42} \\ + D_2 T_{53} T_{42} + D_2 T_{43} T_{52} \\ + D_\theta T_{53} T_{52} + D_{z\theta} T_{63} T_{62} \end{array} &
\begin{array}{l} K_z T_{14} T_{12} + K_2 T_{24} T_{12} \\ + K_2 T_{14} T_{22} + K_\theta T_{24} T_{22} \\ + K_{z\theta} T_{34} T_{32} + D_z T_{44} T_{42} \\ + D_2 T_{54} T_{42} + D_2 T_{44} T_{52} \\ + D_\theta T_{54} T_{52} + D_{z\theta} T_{64} T_{62} \end{array} \\ \hline
\begin{array}{l} K_z T_{11} T_{13} + K_2 T_{21} T_{13} \\ + K_2 T_{11} T_{23} + K_\theta T_{21} T_{23} \\ + K_{z\theta} T_{31} T_{33} + D_z T_{41} T_{43} \\ + D_2 T_{51} T_{43} + D_2 T_{41} T_{53} \\ + D_\theta T_{51} T_{53} + D_{z\theta} T_{61} T_{63} \end{array} &
\begin{array}{l} K_z T_{12} T_{13} + K_2 T_{22} T_{13} \\ + K_2 T_{12} T_{23} + K_\theta T_{22} T_{23} \\ + K_{z\theta} T_{32} T_{33} + D_z T_{42} T_{43} \\ + D_2 T_{52} T_{43} + D_2 T_{42} T_{53} \\ + D_\theta T_{52} T_{53} + D_{z\theta} T_{62} T_{63} \end{array} &
\begin{array}{l} K_z T_{13}^2 + 2 K_2 T_{23} T_{13} \\ + K_\theta T_{23}^2 + K_{z\theta} T_{33}^2 \\ + D_z T_{43}^2 + 2 D_2 T_{53} T_{43} \\ + D_\theta T_{53}^2 + D_{z\theta} T_{63}^2 \end{array} &
\begin{array}{l} K_z T_{14} T_{13} + K_2 T_{24} T_{13} \\ + K_2 T_{14} T_{23} + K_\theta T_{24} T_{23} \\ + K_{z\theta} T_{34} T_{33} + D_z T_{44} T_{43} \\ + D_2 T_{54} T_{43} + D_2 T_{44} T_{53} \\ + D_\theta T_{54} T_{53} + D_{z\theta} T_{64} T_{63} \end{array} \\ \hline
\begin{array}{l} K_z T_{11} T_{14} + K_2 T_{21} T_{14} \\ + K_2 T_{11} T_{24} + K_\theta T_{21} T_{24} \\ + K_{z\theta} T_{31} T_{34} + D_z T_{41} T_{44} \\ + D_2 T_{51} T_{44} + D_2 T_{41} T_{54} \\ + D_\theta T_{51} T_{54} + D_{z\theta} T_{61} T_{64} \end{array} &
\begin{array}{l} K_z T_{12} T_{14} + K_2 T_{22} T_{14} \\ + K_2 T_{12} T_{24} + K_\theta T_{22} T_{24} \\ + K_{z\theta} T_{32} T_{34} + D_z T_{42} T_{44} \\ + D_2 T_{52} T_{44} + D_2 T_{42} T_{54} \\ + D_\theta T_{52} T_{54} + D_{z\theta} T_{62} T_{64} \end{array} &
\begin{array}{l} K_z T_{13} T_{14} + K_2 T_{23} T_{14} \\ + K_2 T_{13} T_{24} + K_\theta T_{23} T_{24} \\ + K_{z\theta} T_{33} T_{34} + D_z T_{43} T_{44} \\ + D_2 T_{53} T_{44} + D_2 T_{43} T_{54} \\ + D_\theta T_{53} T_{54} + D_{z\theta} T_{63} T_{64} \end{array} &
\begin{array}{l} K_z T_{14}^2 + 2 K_2 T_{24} T_{14} \\ + K_\theta T_{24}^2 + K_{z\theta} T_{34}^2 \\ + D_z T_{44}^2 + 2 D_2 T_{54} T_{44} \\ + D_\theta T_{54}^2 + D_{z\theta} T_{64}^2 \end{array}
\end{array}\right]
$$

$$
[\bar{S}_m]_{jj} = \begin{bmatrix}
\begin{array}{l} K_z T_{15}^2 + 2K_2 T_{25} T_{15} \\ + K_\theta T_{25}^2 + K_{z\theta} T_{35}^2 \\ + D_z T_{45}^2 + 2D_2 T_{55} T_{45} \\ + D_\theta T_{55}^2 + D_{z\theta} T_{65}^2 \end{array} &
\begin{array}{l} K_z T_{16} T_{15} + K_2 T_{26} T_{15} \\ + K_2 T_{16} T_{25} + K_\theta T_{26} T_{25} \\ + K_{z\theta} T_{36} T_{35} + D_z T_{46} T_{45} \\ + D_2 T_{56} T_{45} + D_2 T_{46} T_{55} \\ + D_\theta T_{56} T_{55} + D_{z\theta} T_{66} T_{65} \end{array} &
\begin{array}{l} K_z T_{17} T_{15} + K_2 T_{27} T_{15} \\ + K_2 T_{17} T_{25} + K_\theta T_{27} T_{25} \\ + K_{z\theta} T_{37} T_{35} + D_z T_{47} T_{45} \\ + D_2 T_{57} T_{45} + D_2 T_{47} T_{55} \\ + D_\theta T_{57} T_{55} + D_{z\theta} T_{67} T_{65} \end{array} &
\begin{array}{l} K_z T_{18} T_{15} + K_2 T_{28} T_{15} \\ + K_2 T_{18} T_{25} + K_\theta T_{28} T_{25} \\ + K_{z\theta} T_{38} T_{35} + D_z T_{48} T_{45} \\ + D_2 T_{58} T_{45} + D_2 T_{48} T_{55} \\ + D_\theta T_{58} T_{55} + D_{z\theta} T_{68} T_{65} \end{array} \\
\begin{array}{l} K_z T_{15} T_{16} + K_2 T_{25} T_{16} \\ + K_2 T_{15} T_{26} + K_\theta T_{25} T_{26} \\ + K_{z\theta} T_{35} T_{36} + D_z T_{45} T_{46} \\ + D_2 T_{55} T_{46} + D_2 T_{45} T_{56} \\ + D_\theta T_{55} T_{56} + D_{z\theta} T_{65} T_{66} \end{array} &
\begin{array}{l} K_z T_{16}^2 + 2K_2 T_{26} T_{16} \\ + K_\theta T_{26}^2 + K_{z\theta} T_{36}^2 \\ + D_z T_{46}^2 + 2D_2 T_{56} T_{46} \\ + D_\theta T_{56}^2 + D_{z\theta} T_{66}^2 \end{array} &
\begin{array}{l} K_z T_{17} T_{16} + K_2 T_{27} T_{16} \\ + K_2 T_{17} T_{26} + K_\theta T_{27} T_{26} \\ + K_{z\theta} T_{37} T_{36} + D_z T_{47} T_{46} \\ + D_2 T_{57} T_{46} + D_2 T_{47} T_{56} \\ + D_\theta T_{57} T_{56} + D_{z\theta} T_{67} T_{66} \end{array} &
\begin{array}{l} K_z T_{18} T_{16} + K_2 T_{28} T_{16} \\ + K_2 T_{18} T_{26} + K_\theta T_{28} T_{26} \\ + K_{z\theta} T_{38} T_{36} + D_z T_{48} T_{46} \\ + D_2 T_{58} T_{46} + D_2 T_{48} T_{56} \\ + D_\theta T_{58} T_{56} + D_{z\theta} T_{68} T_{66} \end{array} \\
\begin{array}{l} K_z T_{15} T_{17} + K_2 T_{25} T_{17} \\ + K_2 T_{15} T_{27} + K_\theta T_{25} T_{27} \\ + K_{z\theta} T_{35} T_{37} + D_z T_{45} T_{47} \\ + D_2 T_{55} T_{47} + D_2 T_{45} T_{57} \\ + D_\theta T_{55} T_{57} + D_{z\theta} T_{65} T_{67} \end{array} &
\begin{array}{l} K_z T_{16} T_{17} + K_2 T_{26} T_{17} \\ + K_2 T_{16} T_{27} + K_\theta T_{26} T_{27} \\ + K_{z\theta} T_{36} T_{37} + D_z T_{46} T_{47} \\ + D_2 T_{56} T_{47} + D_2 T_{46} T_{57} \\ + D_\theta T_{56} T_{57} + D_{z\theta} T_{66} T_{67} \end{array} &
\begin{array}{l} K_z T_{17}^2 + 2K_2 T_{27} T_{17} \\ + K_\theta T_{27}^2 + K_{z\theta} T_{37}^2 \\ + D_z T_{47}^2 + 2D_2 T_{57} T_{47} \\ + D_\theta T_{57}^2 + D_{z\theta} T_{67}^2 \end{array} &
\begin{array}{l} K_z T_{18} T_{17} + K_2 T_{28} T_{17} \\ + K_2 T_{18} T_{27} + K_\theta T_{28} T_{27} \\ + K_{z\theta} T_{38} T_{37} + D_z T_{48} T_{47} \\ + D_2 T_{58} T_{47} + D_2 T_{48} T_{57} \\ + D_\theta T_{58} T_{57} + D_{z\theta} T_{68} T_{67} \end{array} \\
\begin{array}{l} K_z T_{15} T_{18} + K_2 T_{25} T_{18} \\ + K_2 T_{15} T_{28} + K_\theta T_{25} T_{28} \\ + K_{z\theta} T_{35} T_{38} + D_z T_{45} T_{48} \\ + D_2 T_{55} T_{48} + D_2 T_{45} T_{58} \\ + D_\theta T_{55} T_{58} + D_{z\theta} T_{65} T_{68} \end{array} &
\begin{array}{l} K_z T_{16} T_{18} + K_2 T_{26} T_{18} \\ + K_2 T_{16} T_{28} + K_\theta T_{26} T_{28} \\ + K_{z\theta} T_{36} T_{38} + D_z T_{46} T_{48} \\ + D_2 T_{56} T_{48} + D_2 T_{46} T_{58} \\ + D_\theta T_{56} T_{58} + D_{z\theta} T_{66} T_{68} \end{array} &
\begin{array}{l} K_z T_{17} T_{18} + K_2 T_{27} T_{18} \\ + K_2 T_{17} T_{28} + K_\theta T_{27} T_{28} \\ + K_{z\theta} T_{37} T_{38} + D_z T_{47} T_{48} \\ + D_2 T_{57} T_{48} + D_2 T_{47} T_{58} \\ + D_\theta T_{57} T_{58} + D_{z\theta} T_{67} T_{68} \end{array} &
\begin{array}{l} K_z T_{18}^2 + 2K_2 T_{28} T_{18} \\ + K_\theta T_{28}^2 + K_{z\theta} T_{38}^2 \\ + D_z T_{48}^2 + 2D_2 T_{58} T_{48} \\ + D_\theta T_{58}^2 + D_{z\theta} T_{68}^2 \end{array}
\end{bmatrix}
$$

$$[\bar{S}_m]_{ij} = \begin{bmatrix}
\begin{array}{l} K_z T_{15} T_{11} + K_2 T_{25} T_{11} \\ + K_2 T_{15} T_{21} + K_\theta T_{25} T_{21} \\ + K_{z\theta} T_{35} T_{31} + D_z T_{45} T_{41} \\ + D_2 T_{55} T_{41} + D_2 T_{45} T_{51} \\ + D_\theta T_{55} T_{51} + D_{z\theta} T_{65} T_{61} \end{array} &
\begin{array}{l} K_z T_{16} T_{11} + K_2 T_{26} T_{11} \\ + K_2 T_{16} T_{21} + K_\theta T_{26} T_{21} \\ + K_{z\theta} T_{36} T_{31} + D_z T_{46} T_{41} \\ + D_2 T_{56} T_{41} + D_2 T_{46} T_{51} \\ + D_\theta T_{56} T_{51} + D_{z\theta} T_{66} T_{61} \end{array} &
\begin{array}{l} K_z T_{17} T_{11} + K_2 T_{27} T_{11} \\ + K_2 T_{17} T_{21} + K_\theta T_{27} T_{21} \\ + K_{z\theta} T_{37} T_{31} + D_z T_{47} T_{41} \\ + D_2 T_{57} T_{41} + D_2 T_{47} T_{51} \\ + D_\theta T_{57} T_{51} + D_{z\theta} T_{67} T_{61} \end{array} &
\begin{array}{l} K_z T_{18} T_{11} + K_2 T_{28} T_{11} \\ + K_2 T_{18} T_{21} + K_\theta T_{28} T_{21} \\ + K_{z\theta} T_{38} T_{31} + D_z T_{48} T_{41} \\ + D_2 T_{58} T_{41} + D_2 T_{48} T_{51} \\ + D_\theta T_{58} T_{51} + D_{z\theta} T_{68} T_{61} \end{array} \\
\begin{array}{l} K_z T_{15} T_{12} + K_2 T_{25} T_{12} \\ + K_2 T_{15} T_{22} + K_\theta T_{25} T_{22} \\ + K_{z\theta} T_{35} T_{32} + D_z T_{45} T_{42} \\ + D_2 T_{55} T_{42} + D_2 T_{45} T_{52} \\ + D_\theta T_{55} T_{52} + D_{z\theta} T_{65} T_{62} \end{array} &
\begin{array}{l} K_z T_{16} T_{12} + K_2 T_{26} T_{12} \\ + K_2 T_{16} T_{22} + K_\theta T_{26} T_{22} \\ + K_{z\theta} T_{36} T_{32} + D_z T_{46} T_{42} \\ + D_2 T_{56} T_{42} + D_2 T_{46} T_{52} \\ + D_\theta T_{56} T_{52} + D_{z\theta} T_{66} T_{62} \end{array} &
\begin{array}{l} K_z T_{17} T_{12} + K_2 T_{27} T_{12} \\ + K_2 T_{17} T_{22} + K_\theta T_{27} T_{22} \\ + K_{z\theta} T_{37} T_{32} + D_z T_{47} T_{42} \\ + D_2 T_{57} T_{42} + D_2 T_{47} T_{52} \\ + D_\theta T_{57} T_{52} + D_{z\theta} T_{67} T_{62} \end{array} &
\begin{array}{l} K_z T_{18} T_{12} + K_2 T_{28} T_{12} \\ + K_2 T_{18} T_{22} + K_\theta T_{28} T_{22} \\ + K_{z\theta} T_{38} T_{32} + D_z T_{48} T_{42} \\ + D_2 T_{58} T_{42} + D_2 T_{48} T_{52} \\ + D_\theta T_{58} T_{52} + D_{z\theta} T_{68} T_{62} \end{array} \\
\begin{array}{l} K_z T_{15} T_{13} + K_2 T_{25} T_{13} \\ + K_2 T_{15} T_{23} + K_\theta T_{25} T_{23} \\ + K_{z\theta} T_{35} T_{33} + D_z T_{45} T_{43} \\ + D_2 T_{55} T_{43} + D_2 T_{45} T_{53} \\ + D_\theta T_{55} T_{53} + D_{z\theta} T_{65} T_{63} \end{array} &
\begin{array}{l} K_z T_{16} T_{13} + K_2 T_{26} T_{13} \\ + K_2 T_{16} T_{23} + K_\theta T_{26} T_{23} \\ + K_{z\theta} T_{36} T_{33} + D_z T_{46} T_{43} \\ + D_2 T_{56} T_{43} + D_2 T_{46} T_{53} \\ + D_\theta T_{56} T_{53} + D_{z\theta} T_{66} T_{63} \end{array} &
\begin{array}{l} K_z T_{17} T_{13} + K_2 T_{27} T_{13} \\ + K_2 T_{17} T_{23} + K_\theta T_{27} T_{23} \\ + K_{z\theta} T_{37} T_{33} + D_z T_{47} T_{43} \\ + D_2 T_{57} T_{43} + D_2 T_{47} T_{53} \\ + D_\theta T_{57} T_{53} + D_{z\theta} T_{67} T_{63} \end{array} &
\begin{array}{l} K_z T_{18} T_{13} + K_2 T_{28} T_{13} \\ + K_2 T_{18} T_{23} + K_\theta T_{28} T_{23} \\ + K_{z\theta} T_{38} T_{33} + D_z T_{48} T_{43} \\ + D_2 T_{58} T_{43} + D_2 T_{48} T_{53} \\ + D_\theta T_{58} T_{53} + D_{z\theta} T_{68} T_{63} \end{array} \\
\begin{array}{l} K_z T_{15} T_{14} + K_2 T_{25} T_{14} \\ + K_2 T_{15} T_{24} + K_\theta T_{25} T_{24} \\ + K_{z\theta} T_{35} T_{34} + D_z T_{45} T_{44} \\ + D_2 T_{55} T_{44} + D_2 T_{45} T_{54} \\ + D_\theta T_{55} T_{54} + D_{z\theta} T_{65} T_{64} \end{array} &
\begin{array}{l} K_z T_{16} T_{14} + K_2 T_{26} T_{14} \\ + K_2 T_{16} T_{24} + K_\theta T_{26} T_{24} \\ + K_{z\theta} T_{36} T_{34} + D_z T_{46} T_{44} \\ + D_2 T_{56} T_{44} + D_2 T_{46} T_{54} \\ + D_\theta T_{56} T_{54} + D_{z\theta} T_{66} T_{64} \end{array} &
\begin{array}{l} K_z T_{17} T_{14} + K_2 T_{27} T_{14} \\ + K_2 T_{17} T_{24} + K_\theta T_{27} T_{24} \\ + K_{z\theta} T_{37} T_{34} + D_z T_{47} T_{44} \\ + D_2 T_{57} T_{44} + D_2 T_{47} T_{54} \\ + D_\theta T_{57} T_{54} + D_{z\theta} T_{67} T_{64} \end{array} &
\begin{array}{l} K_z T_{18} T_{14} + K_2 T_{28} T_{14} \\ + K_2 T_{18} T_{24} + K_\theta T_{28} T_{24} \\ + K_{z\theta} T_{38} T_{34} + D_z T_{48} T_{44} \\ + D_2 T_{58} T_{44} + D_2 T_{48} T_{54} \\ + D_\theta T_{58} T_{54} + D_{z\theta} T_{68} T_{64} \end{array}
\end{bmatrix}$$

$$[\bar{S}_m]_{ji} = [\bar{S}_m]_{ij}, \quad [S] = \frac{\alpha}{2} \int_0^d [\bar{S}]\, r\, dz.$$

References

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Summary

The analysis of curved box girder bridges by the Finit Strip Method is presented in this paper. Furthermore it has been demonstrated that the computer programme can also be used for analysing straight box girder bridges by making the radius very large and the subtended angle very small. The displacement functions for the strips are given in the text while the strain and stiffness matrices are given in the Appendices.

Résumé

Dans ce travail on traite l'analyse de ponts courbes à l'aide de la méthode Finite-strip. On a en outre démontré que le programme de la machine électronique peut aussi être utilisé pour l'analyse des ponts droits en caisson, en choisissant un rayon, si possible très grand, et l'angle compris très petit. Dans le texte on donne aussi les commentaires sur les fonctions de déplacement des couvre-joints, tandisque les matrices des tensions et de rigidité des couvre-joints sont traitées à l'appendice.

Zusammenfassung

In der vorstehenden Arbeit wird die Analyse gebogener Brücken mittels der Finite-Strip-Methode behandelt. Ausserdem wurde nachgewiesen, dass das Computerprogramm auch zur Analyse von geraden Brücken mit Kastenträgern benützt werden kann, indem man den Radius möglichst gross und den gegenüberliegenden Winkel möglichst klein wählt. Im Text werden die Verschiebungsfunktionen für die Laschen erläutert, während die Matrizen für Spannungen und Steifigkeit der Laschen im Anhang behandelt werden.

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