Three dimensional lateral load analysis of multistory structures

Three Dimensional Lateral Load Analysis of Multistorey Structures

Analyse tridimensionnelle de charge latérale aux structures à plusieurs étages

Dreidimensionale seitliche Lastanalyse von mehrstöckigen Bauten

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Introduction

Multistory buildings consisting of more than one type of stiffening elements, namely frames, simple or coupled prismatic or nonprismatic shear walls, etc., must generally be analyzed by considering the interaction between the various elements. In case of structures with symmetric lay-out in plane and symmetric lateral load, the statical scheme may be reduced to a two-dimensional one, while in the asymmetric case a three-dimensional scheme must be considered.

The lateral load analysis of multistory structures consists of two types of methods: 1. The discrete method whereby a highly redundant statically indeterminate structure is obtained and for which a large capacity high speed digital computer is essential. 2. The continuous method whereby the discrete structure is replaced by a continuous one, by substituting horizontal connecting beams by a uniform continuous lamella system; the solution is approximate but can be obtained manually or by the aid of a small electronic digital computer.

The discrete method for three-dimensional analysis includes the one presented by WEAVER [5] which treats structures with rectangular patterned lay-out and prismatic beams and columns, as space frames with horizontal rigid lamellas at floor levels. WINOKUR [6] treats structures consisting of plane frames, prismatic or nonprismatic shear walls with the principal axes not parallel to system axes, by a three-dimensional approach. A continuous method for three-dimensional analysis was given by the author in a previous paper [3]. It treats structures consisting of simple or coupled, prismatic or nonprismatic shear walls and frames with asymmetric lay-out in plane, normal strains in frame columns and shear walls being neglected.

The object of this paper is to provide a more complete three-dimensional analysis including the influence of normal strains in frame columns and shear walls. As mentioned by CLOUGH [2] and WEAVER [5] the influence of normal strains is considerable in this case and is not to be neglected. The unknown of the problem are the continuous functions of the shear forces in the lamella system. These functions are derived from a system of second degree nonhomogeneous differential equations with constant coefficients. The homogeneous part of the solution is related to an eigenvalue problem of the degree equal to the number of unknown shear force functions. Knowing the eigenvalues and eigenvectors of the homogeneous part, the problem may be reduced by separation of variables to second order nonhomogeneous linear differential equations, each one having as unknown a basic function. The number of independent equations equals the number of unknown shear force functions. With the basic functions known all interior forces and displacements of the individual stiffening elements may be established.

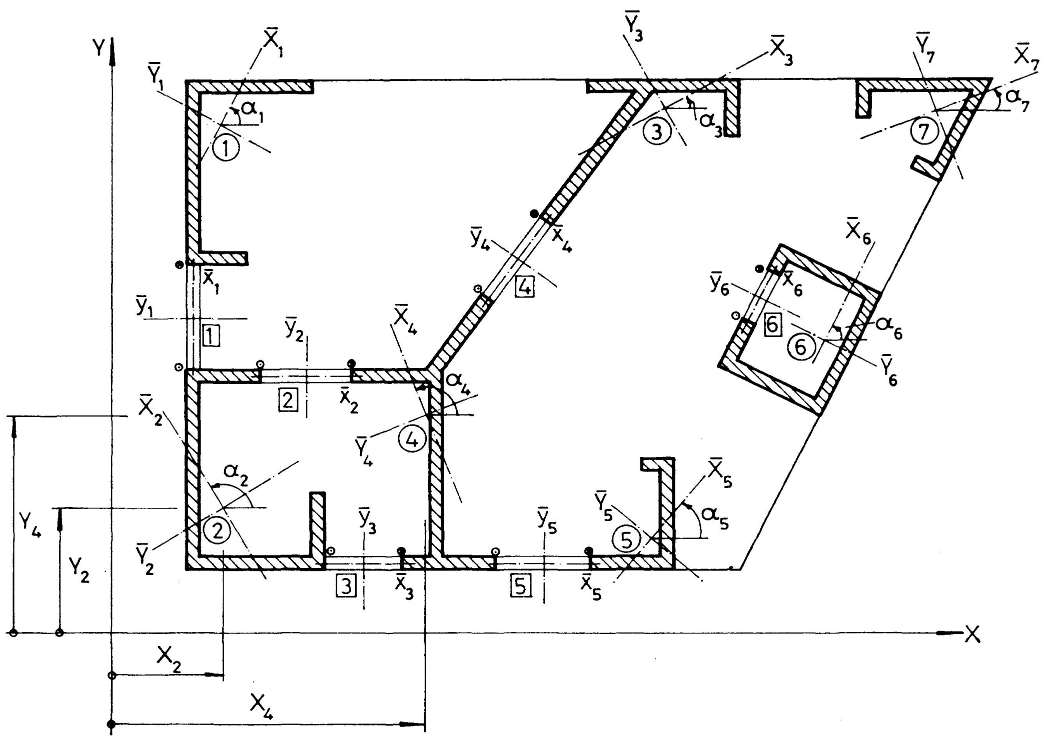


Fig. 1. Floor plane of asymmetric structure.

Fig. 1 shows a horizontal section of an asymmetric structure, where the circled numbers refer to the shear walls and those enclosed in squares refer to connecting beams transformed into a continuous lamella system. The principal axes of the shear wall cross sections are denoted with $\overline{X}_i$ and $\overline{Y}_i$ which are element axes, and overall system axes being denoted with X, Y, Z.

Assumptions and Limitations

1. Floors are undeformable in their planes and have no stiffness normal to these planes.
2. The discrete structure is substituted by a continuous one in which connecting beams are replaced by a uniform distributed lamella system.

Geometric and Elastic Properties of Shear Walls

The relation between internal actions and displacements, including warping effect and neglecting Saint-Venant torsion, in element axes expressed in matrix form, is

$$\bar{\boldsymbol{Q}}_i = -\bar{\boldsymbol{K}}_i \bar{\boldsymbol{D}}_i(z)''', \tag{1}$$

in which

$$\bar{\boldsymbol{Q}}_i = \begin{Bmatrix} \bar{Q}_{xi} \\ \bar{Q}_{yi} \\ \bar{Q}_{\theta i} \end{Bmatrix}. \tag{2}$$

Terms $\bar{Q}_{xi}$ and $\bar{Q}_{yi}$ represent the respective shear forces in the $\bar{X}_i$ and $\bar{Y}_i$ directions and $\bar{Q}_{\theta i}$ the torque in shear wall i.

The stiffness matrix of shear wall i is

$$\bar{\boldsymbol{K}}_i = \begin{bmatrix} E\bar{J}_{yi} & 0 & 0 \\ 0 & E\bar{J}_{xi} & 0 \\ 0 & 0 & EJ_{wi} \end{bmatrix}, \tag{3}$$

in which $\bar{J}_{xi}$ and $\bar{J}_{yi}$ = the respective moments of inertia about the $\bar{X}_i$ and $\bar{Y}_i$ axes; and J_{wi} = the sectorial moment of inertia of shear wall i referred to shear center.

The displacement vector of shear wall i in local axes passing through shear center is:

$$\bar{\boldsymbol{D}}_i(z) = \begin{Bmatrix} \bar{u}_i(z) \\ \bar{v}_i(z) \\ \bar{\theta}_i(z) \end{Bmatrix}, \tag{4}$$

in which $\bar{u}_i(z)$, $\bar{v}_i(z)$ = the respective displacement functions in $\bar{X}_i$ and $\bar{Y}_i$ directions; and $\bar{\theta}_i(z)$ = a function of rotation about $\bar{Z}_i$ axis.

Equilibrium equations of the whole structure refer to the (X, Y, Z) system axes. Transformation of the geometric and physical properties of the shear walls into this system is possible by combined rotation and translation (5).

The rotation matrix is

$$\boldsymbol{R}_i = \begin{bmatrix} \cos\alpha_i & \sin\alpha_i & 0 \\ -\sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5}$$

and she translation is expressed by

$$T_i = \begin{bmatrix} 1 & 0 & -Y_i \\ 0 & 1 & X_i \\ 0 & 0 & 1 \end{bmatrix}, \tag{6}$$

in which X_i and Y_i = global coordinates of shear center of element i. Transformation of the displacement vector of element i is given by

$$\bar{D}_i(z) = R_i\, T_i\, D(z), \tag{7}$$

in which

$$D(z) = \begin{Bmatrix} u(z) \\ v(z) \\ \theta(z) \end{Bmatrix}. \tag{8}$$

Terms $u(z)$, $v(z)$ = the respective displacement functions in the X and Y directions of the system origin; and $\theta(z)$ = a function of rotation about the Z-axis.

Stiffness matrices K_i for the system axes are obtained by the congruent transformation (5).

$$K_i = T_i^T R_i^T \bar{K}_i R_i T_i, \tag{9}$$

in which R_i^T and T_i^T = the transpose of R_i and T_i respectively.

Transformation of the action vector into the system axes is given by (5).

$$Q_i = T_i^T R_i^T \bar{Q}_i. \tag{10}$$

Substituting Eqs. (1), (7) and (9) into Eq. (10) results

$$Q_i = -K_i D(z)'''. \tag{11}$$

Geometric and Elastic Properties of Connecting Beams

In evaluating the influence of connecting beams on the shear wall it is assumed that the stiffness of the latter is high compared with that of the former, in which case the walls may be regarded as rigid indeformable bodies. In this case the mid point of the connecting beam is a contraflexure point with zero bending moment. Cutting the connecting beam at this point, only

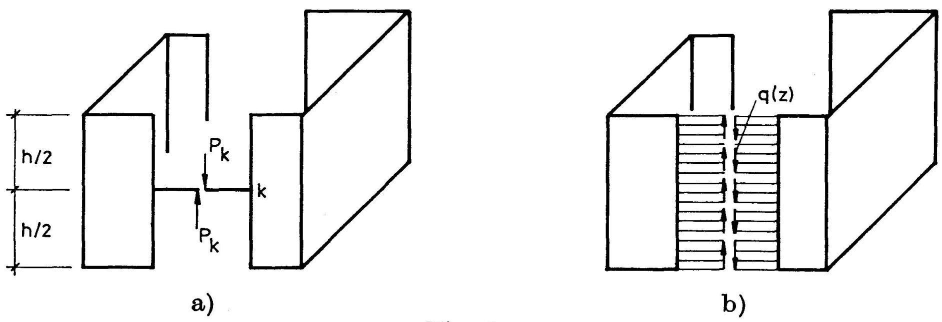


Fig. 2.

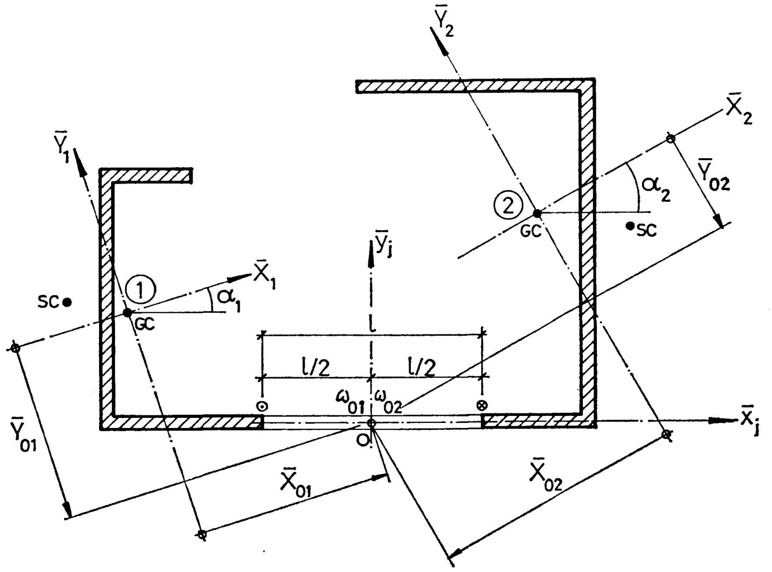
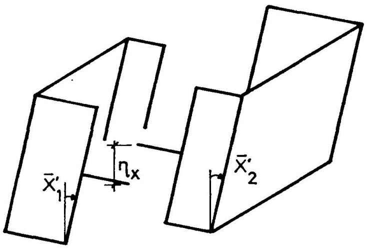
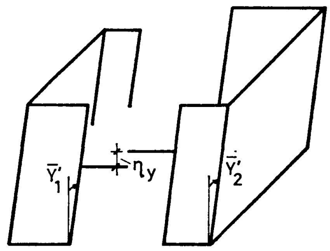


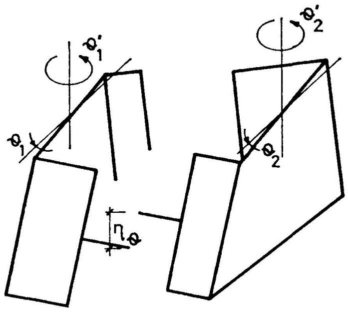
Fig. 3. Cross section of coupled nonprismatic shear wall.



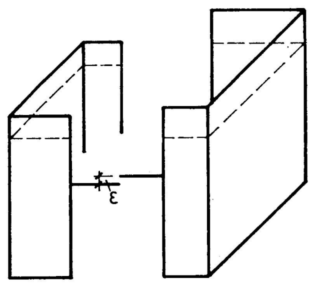
a) Due to displacement in $\bar{X}$ direction only.



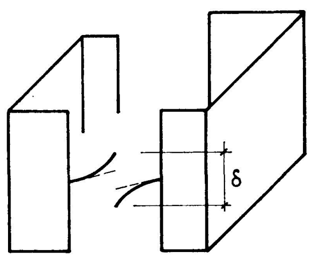
b) Due to displacement in $\bar{Y}$ direction only.



c) Due to warping only.



d) Due to displacement in $\bar{Z}$ direction only.



e) Due to shear and bending deformation of the connecting beams.

Fig. 4. Gaps in cut ends due to displacements in $\bar{X}$, $\bar{Y}$ and $\bar{Z}$ directions and rotation.

a shear force at the two sides of the cut, as shown in Fig. 2a, will suffice to provide continuity at this point. This shear force is transformed in a equivalent distributed shear force (by dividing it by story height), as shown in Fig. 2b.

Fig. 3 shows a cross section of a coupled shear wall with the cut connecting beam at the mid point. Due to lateral deflection and torsion of the shear walls, the cut ends will incur a gap η (see Fig. 4a, b, c), which written in matrix form is

$$\eta_j = \eta_{jx} + \eta_{jy} + \eta_{j\theta} = \boldsymbol{e}_l^T \overline{\boldsymbol{D}}_l(z)' - \overline{\boldsymbol{e}}_r^T \overline{\boldsymbol{D}}_r(z)', \tag{12}$$

in which

$$\boldsymbol{e}_l = \begin{Bmatrix} \overline{X}_{l0} \\ \overline{Y}_{l0} \\ w_{l0} \end{Bmatrix}, \tag{13}$$

$$\boldsymbol{e}_r = \begin{Bmatrix} \overline{X}_{r0} \\ \overline{Y}_{r0} \\ w_{r0} \end{Bmatrix}. \tag{14}$$

In Eqs. (13) and (14) $\overline{X}_{l0}$, $\overline{Y}_{l0}$ and $\overline{X}_{r0}$, $\overline{Y}_{r0}$ = coordinates of mid point 0, in left element centroid local axes respective right; and w_{l0}, w_{r0} = sectorial coordinates of point 0 respectively to left and right element shear center.

Substituting Eq. (7) into Eq. (12) results

$$\eta_j(z) = (\boldsymbol{e}_l^T \boldsymbol{T}_l \boldsymbol{R}_l - \boldsymbol{e}_r^T \boldsymbol{T}_r \boldsymbol{R}_r) \boldsymbol{D}(z)', \tag{15}$$

denoting

$$\boldsymbol{e}_j = \boldsymbol{e}_l^T \boldsymbol{T}_l \boldsymbol{R}_l - \boldsymbol{e}_r^T \boldsymbol{T}_r \boldsymbol{R}_r. \tag{16}$$

Eq. (15) gets

$$\eta_j(z) = \boldsymbol{e}_j \boldsymbol{D}(z)'. \tag{17}$$

The gap at cut end due to axial displacements of the shear walls may be written as

$$\delta_j(z) = -\boldsymbol{U}_j^T \boldsymbol{F} \boldsymbol{U} \int_0^z dz \int_z^H \boldsymbol{q}(z)\, dz - \boldsymbol{v}_j^T \boldsymbol{F} \int_0^z dz \int_z^H \boldsymbol{p}\, dz, \tag{18}$$

in which

$$\boldsymbol{F} = \begin{bmatrix} \frac{1}{EA_1} & 0 & \dots & 0 \\ 0 & \frac{1}{EA_2} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \frac{1}{EA_n} \end{bmatrix}, \tag{19}$$

$$\boldsymbol{q}(z) = \begin{Bmatrix} q_1(z) \\ q_2(z) \\ \vdots \\ q_m(z) \end{Bmatrix}, \tag{20}$$

$$\boldsymbol{p} = \begin{Bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{Bmatrix}. \tag{21}$$

It is assumed that the shear force function $q_j(z)$ in lamella system is positiv when producing tension in the left shear wall (denoted with ⊙) and compression in the right one (denoted with ⊗). In Eq. (21) $\boldsymbol{p}$ = the exterior vertical load vector acting along G. C. lines of the shear walls. In Eq. (18) $\boldsymbol{U}_j, \boldsymbol{V}_j$ = Boolean vectors related to positiv action of respectively $q_j(z)$ and p_i. If tension is produced by $q_j(z)$ in the shear walls connected by lamella system j, a plus one appears in vector U_j, and a minus one if compression, and a zero for all other coupled shear walls. If the exterior vertical loads acting in the left or right shear wall connected by lamella system j are in the positive direction of $q_j(z)$ (upward in the left and downward in the right), a plus one appears in vector V_j and for the negative direction a minus one, and a zero for all other shear walls. For the example building given in Fig. 1 the matrix $\boldsymbol{U}$ containing all vectors $\boldsymbol{U}_j$ is

$$\boldsymbol{U} = (\boldsymbol{U}_1, \boldsymbol{U}_2, \ldots \boldsymbol{U}_6) = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{22}$$

Due to shear and bending of the connecting beam the gap which results, is

$$\varphi_j = f_j q_j(z), \tag{23}$$

in which

$$f_j = \left(\frac{L_j^3}{12 E J_j} + \frac{L_j}{G \bar{A}_j}\right) h. \tag{24}$$

In Eq. (24) $\bar{A}_j$ = the effective area of the connecting beam cross section; and J_j its moment of inertia.

Compatibility Equation

The compatibility condition at the cut end of the connecting beams at row j may be expressed as follows:

$$-\eta_j + \delta_j + \varphi_j = 0. \tag{25}$$

Substituting Eqs. (17), (18) and (23) into Eq. (25) and differentiating twice, results

$$-\boldsymbol{e}_j \boldsymbol{D}(z)''' + f_j q_j(z)'' - \boldsymbol{U}_j^T \boldsymbol{F} \boldsymbol{U} \boldsymbol{q}(z) - \boldsymbol{V}_j^T \boldsymbol{F} \boldsymbol{p} = 0. \tag{26}$$

Denoting

$$\boldsymbol{f} = \begin{bmatrix} f_1 & 0 & \ldots & 0 \\ 0 & f_2 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & f_m \end{bmatrix}, \tag{27}$$

$$\boldsymbol{e} = \begin{Bmatrix} e_1 \\ e_2 \\ \vdots \\ e_m \end{Bmatrix}. \tag{28}$$

The system of differential equations expressing compatibility at cut ends of all lamella systems may be expressed in matrix form as:

$$-\boldsymbol{e}\,\boldsymbol{D}(z)''' + \boldsymbol{f}\,\boldsymbol{q}(z)'' - \boldsymbol{U}^T \boldsymbol{F}\,\boldsymbol{U}\,\boldsymbol{q}(z) - \boldsymbol{V}^T \boldsymbol{F}\,\boldsymbol{p} = 0. \tag{29}$$

For the example building given in Fig. 1, assuming that all vertical loads act downward, the matrix V containing all vectors V_j is

$$\boldsymbol{V} = (\boldsymbol{V}_1, \boldsymbol{V}_2, \ldots \boldsymbol{V}_6) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{30}$$

Equilibrium Equation

Denoting
$$\boldsymbol{e}_j^T = \boldsymbol{R}_l^T \boldsymbol{T}_l^T \boldsymbol{e}_l - \boldsymbol{R}_r^T \boldsymbol{T}_r^T \boldsymbol{e}_r \tag{31}$$
and
$$\boldsymbol{e}^T = (\boldsymbol{e}_1^T, \boldsymbol{e}_2^T, \ldots \boldsymbol{e}_m^T). \tag{32}$$

The basic equation of bending for the whole structure in system coordinates may be written in the form

$$-\boldsymbol{K}\,\boldsymbol{D}(z)''' = \boldsymbol{Q}_0 + \boldsymbol{e}^T \boldsymbol{q}(z), \tag{33}$$

in which
$$\boldsymbol{K} = \sum_{i=1}^{n} \boldsymbol{K}_i, \tag{34}$$

$$\boldsymbol{Q}_0 = \begin{Bmatrix} Q_{x0} \\ Q_{y0} \\ Q_{\theta 0} \end{Bmatrix}. \tag{35}$$

Terms Q_{x0} and Q_{y0} represent the respective shear forces in X and Y directions and $Q_{\theta 0}$ the torque through system axis Z due to exterior loading.

Isolating $\boldsymbol{D}(z)$ from Eq. (33) and substituting in Eq. (29) results

$$+\boldsymbol{e}\,\boldsymbol{K}^{-1} \boldsymbol{Q}_0 + \boldsymbol{e}\,\boldsymbol{K}^{-1} \boldsymbol{e}^T \boldsymbol{q}(z) - \boldsymbol{U}^T \boldsymbol{F}\,\boldsymbol{U}\,\boldsymbol{q}(z) - \boldsymbol{V}^T \boldsymbol{F}\,\boldsymbol{p} + \boldsymbol{f}\,\boldsymbol{q}(z)'' = \boldsymbol{0}. \tag{36}$$

Denoting
$$\boldsymbol{a} = \boldsymbol{U}^T \boldsymbol{F}\,\boldsymbol{U}, \tag{37}$$
$$\boldsymbol{b} = -\boldsymbol{e}\,\boldsymbol{K}^{-1} \boldsymbol{e}^T, \tag{38}$$
$$\boldsymbol{c} = -\boldsymbol{e}\,\boldsymbol{K}^{-1} \boldsymbol{Q}_0 + \boldsymbol{V}^T \boldsymbol{F}\,\boldsymbol{p}. \tag{39}$$

Eq. (36) may be written in the form

$$\boldsymbol{f}\,\boldsymbol{q}(z)'' - (\boldsymbol{a} + \boldsymbol{b})\,\boldsymbol{q}(z) = \boldsymbol{c}. \tag{40}$$

Solution of the System of Differential Equations

The solution of Eq. (40) is involved with an eigenvalue problem of the matrix

$$\boldsymbol{S} = \boldsymbol{f}^{-1}(\boldsymbol{a}+\boldsymbol{b}). \tag{41}$$

The eigenvalues r of the matrix $\boldsymbol{S}$ are the roots of the m degree characteristic equation.

$$\det(\boldsymbol{f}^{-1})\det(r\boldsymbol{f}-\boldsymbol{a}-\boldsymbol{b}) = 0. \tag{42}$$

Matrices $\boldsymbol{f}$, $\boldsymbol{a}$ and $\boldsymbol{b}$ are real and symmetric and $\boldsymbol{f}$ is positive definite, thus the eigenvalues of matrix $\boldsymbol{S}$ are real.

Denoting with

$$r_i = \lambda_i^2 \tag{43}$$

the eigenvector $\boldsymbol{A}_i$ defined by an arbitrary constant for each eigenvalue r_i, is determined by solving the homogeneous system of algebraic equations

$$\lambda^2\boldsymbol{f}-(\boldsymbol{a}+\boldsymbol{b}) = \boldsymbol{0}. \tag{44}$$

It is to be mentioned that between two distinct eigenvectors $\boldsymbol{A}_i$ and $\boldsymbol{A}_j$ orthogonality relation is satisfied, i.e.

$$\boldsymbol{A}_i^T(\boldsymbol{a}+\boldsymbol{b})\boldsymbol{A}_j = \boldsymbol{A}_i^T\boldsymbol{f}\boldsymbol{A}_j = 0, \qquad \text{for } i \neq j \tag{45}$$

and for $i=j$

$$\lambda^2\boldsymbol{A}_i^T\boldsymbol{f}\boldsymbol{A}_i = \boldsymbol{A}_i^T(\boldsymbol{a}+\boldsymbol{b})\boldsymbol{A}_i. \tag{46}$$

The vector $\boldsymbol{q}(z)$ may be expressed with aid of the eigenvectors in the following form

$$\boldsymbol{q}(z) = \sum_{i=1}^{m}\boldsymbol{A}_i\psi_i(z) \tag{47}$$

or in matrix form

$$\boldsymbol{q}(z) = \boldsymbol{A}\boldsymbol{\psi}(z), \tag{48}$$

in which

$$\boldsymbol{\psi}(z) = \begin{Bmatrix}\psi_1(z)\\ \psi_2(z)\\ \vdots\\ \psi_m(z)\end{Bmatrix}, \tag{49}$$

$$\boldsymbol{A} = (\boldsymbol{A}_1, \boldsymbol{A}_2, \ldots \boldsymbol{A}_m). \tag{50}$$

Substituting Eq. (47) into Eq. (40) results

$$\boldsymbol{f}\sum_i\boldsymbol{A}_i\psi_i(z)''-(\boldsymbol{a}+\boldsymbol{b})\sum_i\boldsymbol{A}_i\psi_i(z) = \boldsymbol{c}. \tag{51}$$

Premultiplying Eq. (51) with $\boldsymbol{A}_i^T$ and considering Eq. (45) results

$$\boldsymbol{A}_i^T\boldsymbol{f}\boldsymbol{A}_i\psi_i(z)''-\boldsymbol{A}_i^T(\boldsymbol{a}+\boldsymbol{b})\boldsymbol{A}_i\psi_i(z) = \boldsymbol{A}_i^T\boldsymbol{c}. \tag{52}$$

Substituting Eq. (46) into Eq. (52) results

$$\psi_i(z)'' - \lambda_i^2 \psi_i(z) = \boldsymbol{\varrho}_i \boldsymbol{c}, \tag{53}$$

in which

$$\boldsymbol{\varrho}_i = \frac{\boldsymbol{A}_i^T}{\boldsymbol{A}_i^T \boldsymbol{f} \boldsymbol{A}_i}. \tag{54}$$

The general solution of Eq. (53) for λ_i real, is [4]:

$$\psi_i(z) = \psi_i(0)\operatorname{ch}\lambda_i z + \frac{1}{\lambda_i}\psi_i(0)'\operatorname{sh}\lambda_i z + \frac{1}{\lambda_i}\boldsymbol{\varrho}_i \int_0^z \boldsymbol{c}\operatorname{sh}\lambda_i(z-\xi)\,d\xi \tag{55a}$$

and for λ_i imaginary:

$$\psi_i(z) = \psi_i(0)\cos\lambda_i z + \frac{1}{\lambda_i}\psi_i(0)'\sin\lambda_i z + \frac{1}{\lambda_i}\boldsymbol{\varrho}_i \int_0^z \boldsymbol{c}\sin\lambda_i(z-\xi)\,d\xi, \tag{55b}$$

in which ξ is a dummy variable.

The solution which satisfies the boundary conditions

$$\psi_i(0) = 0, \tag{56}$$

$$\psi_i(H)' = 0, \tag{57}$$

for λ_i real, is given by

$$\psi_i(z) = \frac{-1}{\lambda_i \operatorname{ch}\lambda_i H}\boldsymbol{\varrho}_i \operatorname{sh}\lambda_i z \int_0^H \boldsymbol{c}\operatorname{ch}\lambda_i(H-\xi)\,d\xi + \frac{1}{\lambda_i}\boldsymbol{\varrho}_i \int_0^z \boldsymbol{c}\operatorname{sh}\lambda_i(z-\xi)\,d\xi \tag{58a}$$

and for λ_i imaginary by

$$\psi_i(z) = \frac{-1}{\lambda_i \cos\lambda_i H}\boldsymbol{\varrho}_i \sin\lambda_i z \int_0^H \boldsymbol{c}\cos\lambda_i(H+\xi)\,d\xi + \frac{1}{\lambda_i}\boldsymbol{\varrho}_i \int_0^z \boldsymbol{c}\sin\lambda_i(z-\xi)\,d\xi. \tag{58b}$$

Knowing the functions $\psi_i(z)$ the shear forces in the continuous lamella system "j" are determined with Eq. (48) and in the connecting beam at story k having ordinate Z_k.

$$P_{kj} = \int_{Z_k-h/2}^{Z_k+h/2} q_j(z)\,dz. \tag{59}$$

The displacement vector may be calculated by three successive integrations of Eq. (33). The three vectors of the integration constants are determined from the boundary conditions

$$\boldsymbol{D}(0) = \boldsymbol{0}, \tag{60}$$

$$\boldsymbol{D}(0)' = \boldsymbol{0}, \tag{61}$$

$$\boldsymbol{D}(H)'' = \boldsymbol{0}. \tag{62}$$

The shear forces and torque in shear wall i, are

$$\overline{\boldsymbol{Q}}_i(z) = \begin{Bmatrix} \overline{Q}_{xi}(z) \\ \overline{Q}_{yi}(z) \\ \overline{Q}_{\theta i}(z) \end{Bmatrix} = -\overline{\boldsymbol{K}}_i \overline{\boldsymbol{D}}_i(z)''' + \sum_l \boldsymbol{e}_l \boldsymbol{q}_l(z) - \sum_r \boldsymbol{e}_r \boldsymbol{q}_r(z). \tag{63}$$

In Eq. (63) the sum on l refers to connecting beams joining the shear wall from left and r that from right.

The bending moments and bimoments in shear wall i are given by

$$\overline{\boldsymbol{M}}_i(z) = \begin{Bmatrix} \overline{M}_{xi}(z) \\ \overline{M}_{yi}(z) \\ \overline{B}_i(z) \end{Bmatrix} = -\overline{\boldsymbol{K}}_i \overline{\boldsymbol{D}}_i(z)''. \tag{64}$$

The normal forces in the shear walls i are given by

$$N_i(z) = \sum_j \int_z^H q_j(z)\,dz + \int_z^H p_i\,dz. \tag{65}$$

In Eq. (65) the sum on j refers to lamella systems connected to shear wall i.

The above presented method may be generalized for structures having step wise variations in shear wall stiffness and/or stiffness or span of connecting beams.

The general solution given by Eq. (53) depends on two constants $\psi_i(0)$ and $\psi_i(0)'$.

Let us assume a structure with a sudden change in the geometric characteristics of shear walls and/or connecting beam at level $Z=d$. For analysis purposes the structure may be seen as divided in two zones each one with constant geometric characteristics. The lower zone from level $Z=0$ to $Z=d$ will be denoted with I and the one from $Z=d$ to $Z=H$ with II. For each zone the solution for $\boldsymbol{q}(z)$ may be written in the form

$$\boldsymbol{q}_I(z) = \boldsymbol{A}_I \boldsymbol{\psi}_I(z), \tag{66}$$

$$\boldsymbol{q}_{II}(z) = \boldsymbol{A}_{II} \boldsymbol{\psi}_{II}(z). \tag{67}$$

Each of these solutions depends on the corresponding arbitrary constants $\psi(0)$ and $\psi(0)'$. They are determined from the following boundary conditions:

At level $Z=0$ with zero shear forces in the lamella system

$$\boldsymbol{q}_I(0) = \boldsymbol{0}. \tag{68}$$

At level $Z=d$ with equal shear forces in the lamella systems of zone I and II

$$\boldsymbol{q}_I(d) = \boldsymbol{q}_{II}(d). \tag{69}$$

At level $Z=d$ with equal variations in shear forces in the lamella systems of zone I and II

$$\boldsymbol{q}_I(d)' = \boldsymbol{q}_{II}(d)'. \tag{70}$$

At level $Z=H$ with variations in shear forces in the lamella system equal to zero

$$\boldsymbol{q}_{II}(H) = \boldsymbol{0}. \tag{71}$$

These boundary conditions will result in a system of $4m$ linear algebraic equations. Since the condition $\boldsymbol{q}_I(0)=\boldsymbol{0}$ is reduced to $\psi_I(0)=0$ the system may be reduced to $3m$ equations. Generally for a structure with g changes in characteristics a system of $(2g+1)m$ equations will be obtained.

Conclusions

On the basis of the present method a general program may be prepared for lateral load analysis of multistory structures with arbitrary lay out of stiffening elements and sudden changes in characteristics including the effect of exterior vertical loads. The influence of normal strains in shear walls and columns are included.

List of Symbols and Notations

Upper bar	magnitudes in particular system of axes
i	index for shear wall
j	index for connecting beam or lamella system
A	cross section area
$\boldsymbol{A}_i$	eigenvector belonging to eigenvalue λ_i
B	bimoment
$\boldsymbol{D}(z)$	displacement vector
E	modulus of elasticity
$\boldsymbol{F}$	flexibility matrix to normal strains
G	shear modulus
J_{xi}, J_{yi}, J_{wi}	moment of inertia in X respective Y direction and sectorial moment of inertia
$\boldsymbol{K}$	stiffness matrix
L	clear span of connecting beam
$\boldsymbol{M}$	3×1 vector consisting bending moment in X and Y direction and bimoment with respect to the origin of the system axes
N	normal force
P_k	shear force in connecting beam at story level k
$\boldsymbol{R}$	rotation matrix
$\boldsymbol{T}$	translation matrix
$\boldsymbol{Q}$	3×1 vector consisting shear forces in X and Y direction and torsion moment with respect to origin of the system axes
$\boldsymbol{U}, \boldsymbol{V}$	vector defined in the text
X, Y, Z	coordinates
$\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$	matrices defined in the text
d	ordinate of characteristic change level

$\boldsymbol{e}$, $\boldsymbol{e}_l$, $\boldsymbol{e}_r$	vectors defined in the text
$\boldsymbol{f}$	flexibility matrix of lamella system
g	number of changes in characteristics of stiffening elements
h	story height
m	number of lamella systems
n	number of shear walls
$\boldsymbol{p}$	vertical load vector
$\boldsymbol{q}(z)$	shear force function in lamella system
$\psi(z)$	function defined in the text
x, y, z	particular system of axes for connecting beams
α	angle between main axis $\bar{X}$ and system axes X
$\boldsymbol{\varrho}_i$	vector defined in the text
η, δ, φ	gap at cut end due to bending and shear of connecting beams, respectively strains in shear walls and bending and warping of the shear walls
λ_i	eigenvalue
θ	torsion angle
w	sectorial coordinate

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Summary

A method for three dimensional analysis of multistorey structures based on the continuum approach is presented. There are no restrictions referring to types of stiffening elements and layout in floor plane. The number of stiffening elements which may be included in analysis is practically not limited.

The loads may be vertical and lateral at arbitrary locations. Elastic working of thin-walled members is included in analysis, as well as axial deformations in vertical elements.

Résumé

On présente une méthode pour l'analyse tridimensionnelle des structures à plusieurs étages, basée sur l'approchement du continuum. Il n'existe pas de restrictions quant aux types des éléments raidisseurs et du layout dans le plan de l'étage. Le nombre des éléments raidisseurs qui peuvent être compris dans l'analyse n'est pratiquement pas limité. Les charges peuvent agir verticalement ou latéralement à des endroits arbitraires. Le travail élastique de membres à parois minces est compris dans l'analyse, aussi bien que les déformations axiales dans les éléments verticaux.

Zusammenfassung

Es wird eine Methode zur dreidimensionalen Analyse mehrstöckiger Bauten auf Grund der Kontinuum-Näherung vorgelegt. Es bestehen keine Einschränkungen hinsichtlich der Typen der Versteifungselemente und des Entwurfes in der Stockwerksebene. Die Zahl der Versteifungselemente, die in die Analyse einbezogen werden können, ist praktisch nicht begrenzt. Die Lasten können vertikal und seitlich an beliebigen Stellen wirken. Das elastische Arbeiten dünnwandiger Glieder ist in der Analyse inbegriffen, ebenso axiale Deformationen in Vertikalelementen.