Dynamics of asymmetric multistory structures

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Dynamique de structures asymétriques à plusieurs étages

Dynamik asymmetrischer mehrstöckiger Bauten

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Introduction

In multistory building design impulsive lateral loads and earthquake ground motion lead to a dynamic analysis of such structures. A rigorous solution of the problem involves a large amount of numerical calculations which may be carried out with aid of digital computers only. It is well known that severe earthquakes will produce nonlinear dynamic response while in the case of a moderate earthquake ground motion the response of the structure may be assumed linear. Herein is described a method for linear dynamic response of multistory structures.

Previous reports treat extensively linear and nonlinear dynamic analysis of multistory buildings by the discrete method, but most of them involve symmetric structures which leads to two-dimensional analytical models [4], [5]. A three-dimensional discrete model for linear dynamic analysis of multistory buildings was presented by WEAVER [13]. The structure is laid out in a rectangular grid pattern and consists of shear walls and columns with principal axes parallel to those of the system. A threedimensional model for statical analysis was presented by WINOKUR [14].

Another common approach to lateral load analysis of multistory structures is based on the continuous-medium concept, in which the discrete structure is represented by a continuous one, by replacing the horizontal connecting beams and frame by an equivalent continuous medium. The forces borne in this medium are assumed to be proportional to the second derivative of the displacement. The statical analysis of two-dimensional models was presented by

CHITTY [3], BECK [2], ROSMAN [11], DESPEYROUX [6] and others, and the dynamics of this model were presented by OSAWA [9] and others [10], [8]. A three dimensional model for statical analysis was presented in a previous paper [7], and the dynamics by BARUCH [1]. This model neglects the displacements due to normal strains in columns and shear walls. As mentioned by previous investigators the effect of normal strains in shear walls is of considerable magnitude and is not to be neglected in dynamic analysis of multistory structures.

The object of this paper is to present a more accurate approach to the linear dynamic analysis of asymmetric multistory structures, using a three-dimensional continuous model, including the effects due to normal strains in columns and shear walls. The lay-out of the stiffening elements in the plane of the structure is arbitrary, without restrictions as to shape of the shear wall cross section, direction of its principal axes and orientation of the frames. Vlasov's thin-walled bar theory is used for establishing the torsional properties of shear walls with non-prismatic section, in which the centroid and shear center do not coincide. The equations of motion for lateral and normal displacement functions are formulated in matrix form. The evaluation of the normal modes leads to an eigenvalue problem of order $2n+12$, independent of the number of stories, n being the total number of coupled shear walls. With the normal modes known the dynamic response to impulsive lateral loads and earthquake ground motion may be evaluated.

Fig. 1 shows a horizontal section of an asymmetric structure, where the circled numbers refer to the shear walls and those enclosed in squares refer to

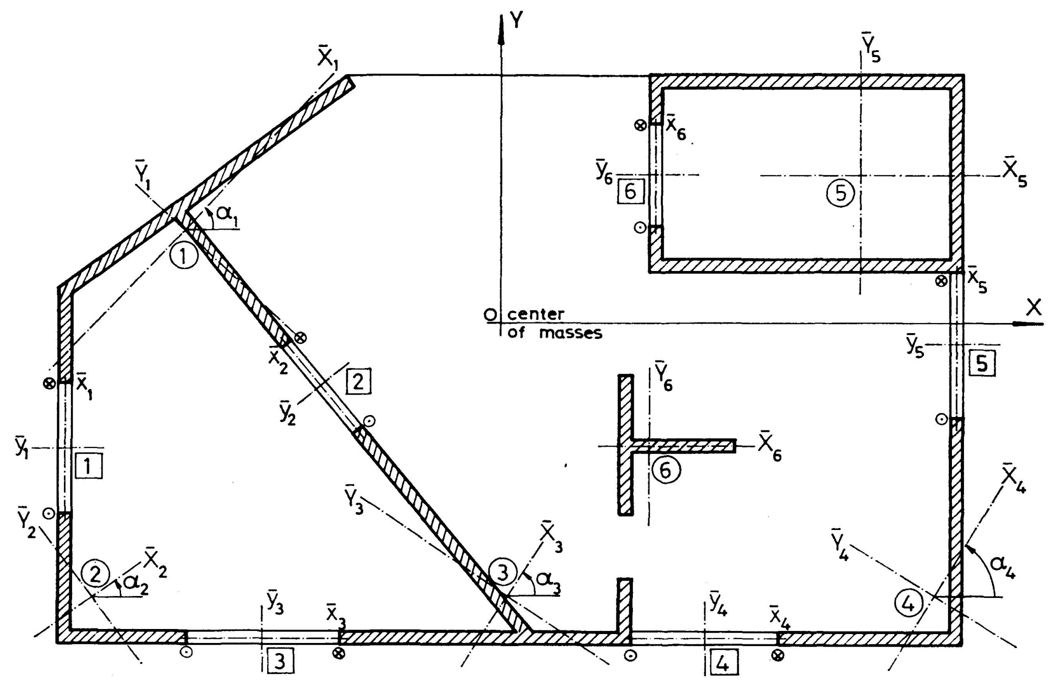


Fig. 1. Floor plane of asymmetric structure.

connecting beams transformed into a continuous medium. The local system of axes $(\overline{X}_i, \overline{Y}_i)$ coincides with the principal axes of shear wall cross section. System axes are denoted with X, Y, Z.

Assumptions and Limitations

1. Floors have infinite plane rigidity and negligible rigidity perpendicular to their plane.
2. Columns, beams, shear walls etc. have uniform geometric properties with regard to height.
3. Shear deformations in shear walls are negligible.

Geometrie and Elastic Properties of Shear Walls

The relation between actions and displacements, expressed in matrix form, for local axes, including warping effect, is given by:

$$\overline{\boldsymbol{p}}_i(z) = \overline{\boldsymbol{K}}_i \boldsymbol{D}_i(z)^{\mathrm{IV}} - \overline{\boldsymbol{K}}_{Ti} \overline{\boldsymbol{D}}_i(z)'', \tag{1}$$

in which

$$\overline{\boldsymbol{p}}_i(z) = \begin{Bmatrix} \overline{p}_{xi}(z) \\ \overline{p}_{yi}(z) \\ \overline{p}_i(z) \end{Bmatrix}. \tag{2}$$

Terms $\overline{p}_{xi}(z)$ and $\overline{p}_{yi}(z)$ = the respective lateral load in $\overline{X}_i$ and $\overline{Y}_i$ directions and $\overline{p}_i(z)$ the torque in shear wall i.

The stiffness matrices of shear wall i are:

$$\overline{\boldsymbol{K}}_i = \begin{vmatrix} E\bar{J}_{xi} & 0 & 0 \\ 0 & E\bar{J}_{yi} & 0 \\ 0 & 0 & E\bar{J}_{wi} \end{vmatrix}, \tag{3}$$

$$\overline{\boldsymbol{K}}_{Ti} = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & G\bar{J}_{Ti} \end{vmatrix}, \tag{4}$$

in which $\bar{J}_{xi}$ and $\bar{J}_{yi}$ = the respective moments of inertia about the $\overline{X}_i$ and $\overline{Y}_i$ axes; $\bar{J}_{wi}$ = the sectorial moment of inertia of shear wall i; and $\bar{J}_{Ti}$ = the Saint-Venant torsion coefficient of shear wall i.

The displacement vector of shear center of wall i in local axes is:

$$\overline{\boldsymbol{D}}_i(z) = \begin{Bmatrix} \overline{u}_i(z) \\ \overline{v}_i(z) \\ \theta_i(z) \end{Bmatrix}, \tag{5}$$

in which $\overline{u}_i(z)$, $\overline{v}_i(z)$ = the respective displacement functions in $\overline{X}_i$ and $\overline{Y}_i$ directions; and $\theta_i(z)$ = the rotation function about $\overline{Z}_i$ axis.

Equilibrium equations of the whole structure refer to the (X, Y, Z) system axes. Transformation of the geometric and physical properties from local into system axes is obtained by combined rotation and translation [12].

Transformation of the displacement vector is given by:

$$\bar{\boldsymbol{D}}_i(z) = \boldsymbol{T}_i \boldsymbol{R}_i \boldsymbol{D}(z), \tag{6}$$

in which the rotation matrix is given by:

$$\boldsymbol{R}_i = \begin{vmatrix} \cos\alpha_i & \sin\alpha_i & 0 \\ -\sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 1 \end{vmatrix}, \tag{7}$$

the translation matrix is given by:

$$\boldsymbol{T}_i = \begin{vmatrix} 1 & 0 & -Y_i \\ 0 & 1 & X_i \\ 0 & 0 & 1 \end{vmatrix} \tag{8}$$

and the system displacement vector is:

$$\boldsymbol{D}(z) = \begin{Bmatrix} u(z) \\ v(z) \\ \theta(z) \end{Bmatrix}. \tag{9}$$

Terms X_i, Y_i = the coordinates of shear center of wall in system axes; $u(z)$, $v(z)$ = the respective displacement functions in X and Y directions of the system axes; and $\theta(z)$ = the rotation function about Z axis.

Stiffness matrices $\boldsymbol{K}_i$ for system axes are obtained by congruent transformation (12).

$$\boldsymbol{K}_i = \boldsymbol{T}_i^T \boldsymbol{R}_i^T \bar{\boldsymbol{K}}_i \boldsymbol{R}_i \boldsymbol{T}_i, \tag{10}$$

in which $\boldsymbol{R}_i^T$ and $\boldsymbol{T}_i^T$ = the transpose of $\boldsymbol{R}_i$ and $\boldsymbol{T}_i$ respectively.

Stiffness matrix $\boldsymbol{K}_{Ti}$ does not change with transformation.

$$\boldsymbol{K}_{Ti} = \bar{\boldsymbol{K}}_{Ti}. \tag{11}$$

Transformation of the action vector into system axes is given by (12)

$$\boldsymbol{p}_i(z) = \boldsymbol{T}_i^T \boldsymbol{R}_i^T \bar{\boldsymbol{p}}_i(z). \tag{12}$$

Substituting Eqs. (1) and (6) in Eq. (12) yields:

$$\boldsymbol{p}_i(z) = \boldsymbol{K}_i \boldsymbol{D}(z)^{\mathrm{IV}} - K_{Ti} D(z)''. \tag{13}$$

Geometric and Elastic Properties of Connecting Beams

Since the stiffness of the shear walls is much more higher than that of the connecting beams, the walls may be assumed rigid, undeformable compared with the latter. In this case the mid point of the connecting beam is a contraflexure point with zero bending moment. In a cut at this section as shown in Fig. 2a only a shear force will suffice to provide continuity. These concentrated shear forces are transformed into an equivalent distributed shear force (by dividing it by story height) acting on a continuous media with shear stiffness equal to that of the cut connecting beam divided by story height as shown in Fig. 2b.

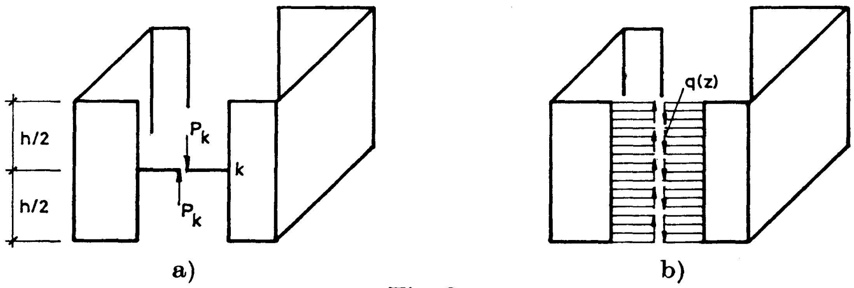


Fig. 2.

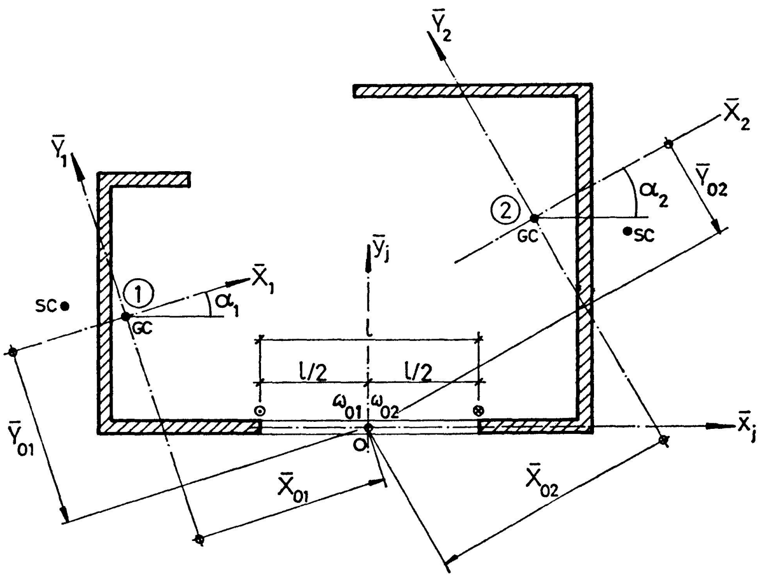


Fig. 3. Cross section of coupled nonprismatic shear wall.

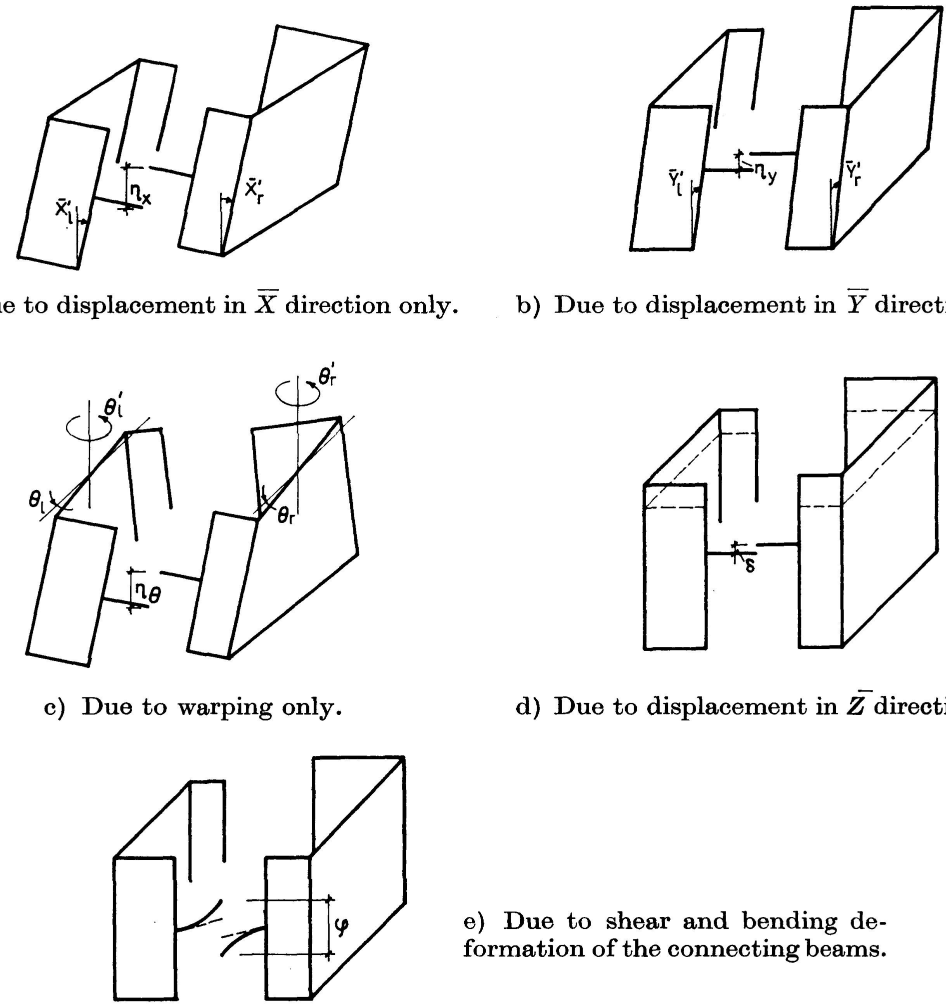
A cross section of a coupled shear wall with notations used in the text is shown in Fig. 3. Lateral deflections and rotation of the shear wall at the cut ends will result in a gap η (see Fig. 4a, b, c) which expressed in a matrix form is:

$$\eta_j = \eta_{jx} + \eta_{jy} + \eta_{j\theta} = -\boldsymbol{e}_l^T \bar{\boldsymbol{D}}_l(z)' + \boldsymbol{e}_r^T \bar{\boldsymbol{D}}_r(z)', \tag{14}$$

in which

$$\boldsymbol{e}_l = \begin{Bmatrix} \bar{X}_{l0} \\ \bar{Y}_{l0} \\ \bar{w}_{l0} \end{Bmatrix}, \tag{15}$$

$$\boldsymbol{e}_r = \begin{Bmatrix} \bar{X}_{r0} \\ \bar{Y}_{r0} \\ \bar{w}_{r0} \end{Bmatrix}. \tag{16}$$



a) Due to displacement in $\bar{X}$ direction only.

b) Due to displacement in $\bar{Y}$ direction only.

c) Due to warping only.

d) Due to displacement in $\bar{Z}$ direction only.

e) Due to shear and bending deformation of the connecting beams.

Fig. 4. Gaps in cut ends due to displacements in $\bar{X}$, $\bar{Y}$ and $\bar{Z}$ directions and rotation.

In Eqs. (15) and (16), $\bar{X}_{l0}$, $\bar{Y}_{l0}$ and $\bar{X}_{r0}$, $\bar{Y}_{r0}$ = coordinates of mid point 0 in left element local axes respective right; and $\bar{w}_l$ and $\bar{w}_r$ the sectorial coordinates, respectively left and right element shear centers.

Substituting Eq. (6) into Eq. (14) yields:

$$\eta_j(z) = -(\boldsymbol{e}_l^T \boldsymbol{R}_l \boldsymbol{T}_l - \boldsymbol{e}_r^T \boldsymbol{R}_r \boldsymbol{T}_r)\,\boldsymbol{D}(z)', \tag{17}$$

denoting

$$\boldsymbol{e}_J = \boldsymbol{e}_l^T \boldsymbol{R}_l \boldsymbol{T}_l - \boldsymbol{e}_r^T \boldsymbol{R}_r \boldsymbol{T}_r. \tag{18}$$

Eq. (17) may be written as

$$\eta_j(z) = -\boldsymbol{e}_j\,\boldsymbol{D}(z)'. \tag{19}$$

The gap at cut ends due to axial displacements of the shear walls may be written as

$$\delta_j(z) = \boldsymbol{U}_j^T \boldsymbol{W}, \tag{20}$$

in which

$$\boldsymbol{W}(z) = \begin{Bmatrix} W_1(z) \\ W_2(z) \\ \vdots \\ W_n(z) \end{Bmatrix}. \tag{21}$$

In Eq. (21) $\boldsymbol{W}$ is the vector of axial displacements of the shear walls; $n =$ the total number of coupled shear walls of the structure.

It is assumed that $q_j(z)$ is positive when producing tension in the left shear wall (denoted with $\odot$) and compression in the right one (denoted with $\otimes$). In Eq. (20) $\boldsymbol{U}_j$ = a Boolean vector related to positive action of $q_j(z)$. If tension is produced in the shear walls connected to continuous media j, a plus one appears in vector $\boldsymbol{U}_j$, a minus one if compression, and a zero for all other coupled shear walls. For the example building given in Fig. 1 the matrix U containing all vectors U_j is

$$\boldsymbol{U} = (\boldsymbol{U}_1, \boldsymbol{U}_2, \boldsymbol{U}_3 \ldots \boldsymbol{U}_6), \tag{22}$$

in which

$$\boldsymbol{U} = \begin{vmatrix} -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{vmatrix}. \tag{23}$$

Due to shear and bending of the connecting beam, the gap which results at cut ends is given by

$$\varphi_j = f_j q_j(z), \tag{24}$$

in which

$$f_j = \left(\frac{L_j^3}{12\,E\,I_j} + \frac{L_j}{G\bar{A}_j}\right) h. \tag{25}$$

In Eq. (25), $\bar{A}_j$ = the effective area of the connecting beam cross section; and I_j its moment of inertia.

Compatibility Equations

The compatibility equations at the cut end of the connecting beams at row j may be expressed as follows:

Substituting Eqs. (19), (20), and (24) into Eq. (26) results

$$-\boldsymbol{e}_j \boldsymbol{D}(z)' + f_1 q_j(z) + \boldsymbol{U}_j^T \boldsymbol{W}(z) = 0. \tag{27}$$

Denoting

$$\boldsymbol{f} = \begin{vmatrix} f_1 & 0 \ldots 0 \\ 0 & f_2 \ldots 0 \\ \vdots & \\ 0 & f_s \end{vmatrix}, \tag{28}$$

$$\boldsymbol{e} = \begin{Bmatrix} e_1 \\ e_2 \\ \vdots \\ e_s \end{Bmatrix}, \tag{29}$$

in which s = the total number of connecting beams.

The system of differential equations expressing compatibility at cut ends of connecting beams may be expressed in matrix form as:

$$-\boldsymbol{e}\,\boldsymbol{D}(z)' + \boldsymbol{f}\,\boldsymbol{q}(z) + \boldsymbol{U}^T\,\boldsymbol{W}(z) = \boldsymbol{0}, \tag{30}$$

in which

$$\boldsymbol{q} = \begin{Bmatrix} q_1 \\ q_2 \\ \vdots \\ q_s \end{Bmatrix}. \tag{31}$$

Equations of Motion

Denoting

$$\boldsymbol{e}_j^T = \boldsymbol{T}_l^T\,\boldsymbol{R}_l^T\,\boldsymbol{e}_l - \boldsymbol{T}_r^T\,\boldsymbol{R}_r^T\,\boldsymbol{e}_r \tag{32}$$

and

$$\boldsymbol{e}^T = (\boldsymbol{e}_1^T\,\boldsymbol{e}_2^T \ldots \boldsymbol{e}_s^T). \tag{33}$$

The equations of motion for given dynamic loads may be written:

$$\boldsymbol{K}\,\boldsymbol{D}(z,t)^{\mathrm{IV}} - \boldsymbol{K}_T\,\boldsymbol{D}(z,t)'' - \boldsymbol{e}^T\,\boldsymbol{q}(z,t)' + \boldsymbol{M}\,\ddot{\boldsymbol{D}}(z,t) = \boldsymbol{p}(z,t), \tag{34}$$

$$\boldsymbol{F}^{-1}\,\boldsymbol{W}(z,t)'' + \boldsymbol{U}\,\boldsymbol{q}(z,t) + \boldsymbol{M}^*\,\ddot{\boldsymbol{W}}(z,t) = \boldsymbol{P}(z,t), \tag{35}$$

in which

$$\boldsymbol{K} = \sum_{i=1}^{N} \boldsymbol{K}_i, \tag{36}$$

$$\boldsymbol{K}_T = \sum_{i=1}^{N} \boldsymbol{K}_{Ti}, \tag{37}$$

$$\boldsymbol{M} = \begin{vmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & J_p \end{vmatrix}, \tag{38}$$

$$\boldsymbol{M}^* = \begin{vmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ \vdots & & \\ 0 & & m_n \end{vmatrix}, \tag{39}$$

$$\boldsymbol{p}(z,t) = \begin{Bmatrix} p_x(z,t) \\ p_y(z,t) \\ p_\theta(z,t) \end{Bmatrix}, \tag{40}$$

$$\boldsymbol{P}(z,t) = \begin{Bmatrix} P_1(z,t) \\ P_2(z,t) \\ \vdots \\ P_n(z,t) \end{Bmatrix}, \tag{41}$$

$$\boldsymbol{F} = \begin{vmatrix} \frac{1}{E\,A_1} & 0 & 0 \\ 0 & \frac{1}{E\,A_2} & 0 \\ \vdots & & \\ 0 & & \frac{1}{E\,A_n} \end{vmatrix}. \tag{42}$$

In Eq. (36), N = the total number of shear walls (coupled or not); in Eq. (38), m = the average mass per unit height; in Eq. (39), $m_1, m_2, \ldots m_n$ = the masses of the shear walls, per unit height; J_p = the polar moment of inertia per unit height:

$$J_p = \int \rho^2 \, dm, \tag{43}$$

in which ρ = the radius vector in horizontal plane.

By solving the vector of the shear force functions from Eq. (30) results

$$\boldsymbol{q} = \boldsymbol{f}^{-1} \boldsymbol{e} \boldsymbol{D}' - \boldsymbol{f}^{-1} \boldsymbol{U}^T \boldsymbol{W}. \tag{44}$$

The substitution of Eq. (44) ind Eqs. (34) and (35) yields the following systems of equations:

$$\boldsymbol{K} \boldsymbol{D}^{\mathrm{IV}} - (\boldsymbol{K}_T + \boldsymbol{e}^T \boldsymbol{f}^{-1} \boldsymbol{e}) \boldsymbol{D}'' + \boldsymbol{e}^T \boldsymbol{f}^{-1} \boldsymbol{U}^T \boldsymbol{W}' + \boldsymbol{M} \ddot{\boldsymbol{D}} = \boldsymbol{p}, \tag{45}$$

$$\boldsymbol{F}^{-1} \boldsymbol{W}'' - \boldsymbol{U} \boldsymbol{f}^{-1} \boldsymbol{U}^T \boldsymbol{W} + \boldsymbol{U} \boldsymbol{f}^{-1} \boldsymbol{e} \boldsymbol{D}' + \boldsymbol{M}^* \ddot{\boldsymbol{W}} = \boldsymbol{P}. \tag{46}$$

A generalized vector of displacements $\boldsymbol{X}(z,t)$ is defined by

$$\boldsymbol{X}(z,t) = \begin{Bmatrix} \boldsymbol{D}(z,t) \\ \boldsymbol{W}(z,t) \end{Bmatrix}. \tag{47}$$

Eqs. (45) and (46) may then be written as:

$$\boldsymbol{a} \boldsymbol{X}^{\mathrm{IV}}(z,t) + \boldsymbol{b} \boldsymbol{X}''(z,t) + \boldsymbol{c} \boldsymbol{X}'(z,t) + \boldsymbol{d} \boldsymbol{X}(z,t) + \boldsymbol{g} \ddot{\boldsymbol{X}} = \boldsymbol{L}, \tag{48}$$

in which

$$\boldsymbol{a} = \begin{vmatrix} \boldsymbol{K} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{vmatrix}, \tag{49}$$

$$\boldsymbol{b} = \begin{vmatrix} -(\boldsymbol{K}_T + \boldsymbol{e}^T \boldsymbol{f}^{-1} \boldsymbol{e}) & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{F}^{-1} \end{vmatrix}, \tag{50}$$

$$\boldsymbol{c} = \begin{vmatrix} \boldsymbol{0} & \boldsymbol{e}^T \boldsymbol{f}^{-1} \boldsymbol{U}^T \\ \boldsymbol{U} \boldsymbol{f}^{-1} \boldsymbol{e} & \boldsymbol{0} \end{vmatrix}, \tag{51}$$

$$\boldsymbol{d} = \begin{vmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & -U f^{-1} U^T \end{vmatrix}, \tag{52}$$

$$\boldsymbol{g} = \begin{vmatrix} \boldsymbol{M} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{M}^* \end{vmatrix}, \tag{53}$$

$$\boldsymbol{L} = \begin{Bmatrix} \boldsymbol{p} \\ \boldsymbol{P} \end{Bmatrix}. \tag{54}$$

Eqs. (48) represent a system of $n+3$ differential equations with the same number of unknown functions (3 lateral displacement functions $u(z,t)$, $v(z,t)$, $\theta(z,t)$ and n axial displacement functions $w_1, w_2, \ldots w_n(z,t)$.

Homogeneous Solution of the System of Differential Equations

The proposed solution is:

$$\boldsymbol{X}_h(z,t) = \boldsymbol{A}\, e^{rz} \sin \omega t = \boldsymbol{X}_h(z) \sin \omega t, \tag{55}$$

in which r = the characteristic value; ω = the circular frequency and $\boldsymbol{A}$ = the characteristic vector

$$\boldsymbol{A} = \begin{Bmatrix} A_1 \\ A_2 \\ \vdots \\ A_{n+3} \end{Bmatrix}. \tag{56}$$

Substitution of Eq. (55) into the homogeneous part of Eq. (48) yields

$$(r^4 \boldsymbol{a} + r^2 \boldsymbol{b} + r \boldsymbol{c} + \boldsymbol{d} - \omega^2 \boldsymbol{g}) \boldsymbol{A}\, e^{rz} \sin \omega t = \boldsymbol{0}. \tag{57}$$

The condition for existence of nonzero $\boldsymbol{A}$ in Eq. (57) is vanishing of the determinant of the coefficients. The characteristic equation is therefore

$$\det (r^4 \boldsymbol{a} + r^2 \boldsymbol{b} + r \boldsymbol{c} + \boldsymbol{d} - \omega^2 \boldsymbol{g}) = 0. \tag{58}$$

In Eq. (58) there are two unknowns, r and ω, the solution of which requires a second equation. This equation will be obtained from the boundary conditions. For a given ω^2, $2n+12$ eigenvalues r are obtained, and for each one a corresponding eigenvector $\boldsymbol{A}$ will be calculated.

The normal mode $\boldsymbol{X}_n(z,t)$ for a given ω_n^2 is

$$\boldsymbol{X}_{hn}(z) = \sum_{i=1}^{2n+12} \boldsymbol{A}_i C_i e^{r_i z} = \begin{Bmatrix} \boldsymbol{D}_{hn}(z) \\ \boldsymbol{W}_{hn}(z) \end{Bmatrix}. \tag{59}$$

The solution obtained contains $2n+12$ constants which are calculated from the boundary equations, expressed by a system of $2n+12$ homogeneous algebraic equations. The solution of this system calls for vanishing of the coefficients determinant.

Boundary Conditions

At $z=0$ with lateral and normal deflections = 0

$$\boldsymbol{D}_h(0) = \boldsymbol{0}, \tag{60}$$

$$\boldsymbol{W}_h(0) = \boldsymbol{0}. \tag{61}$$

At $z=0$ with full restraint at support

$$\boldsymbol{D}_h'(0) = \boldsymbol{0}. \tag{62}$$

At $z = H$ with moments in shear walls = 0

$$\boldsymbol{D}_h''(0) = \boldsymbol{0}. \tag{63}$$

At $z = H$ with shear forces at the top of the structure = 0

$$\boldsymbol{K}\,\boldsymbol{D}_h'''(H) - (\boldsymbol{K}_T + \boldsymbol{e}^T \boldsymbol{f}^{-1} \boldsymbol{e})\,\boldsymbol{D}_h'(H) + \boldsymbol{e}^T \boldsymbol{f}^{-1}\,\boldsymbol{U}^T\,\boldsymbol{W}_h(H) = \boldsymbol{0}. \tag{64}$$

At $z = H$ with normal strains at the top of the shear walls = 0

$$\boldsymbol{W}_h'(H) = \boldsymbol{0}. \tag{65}$$

In case of structures with foundations on elastic subgrade similar boundary conditions may be expressed.

General Solution of the System of Differential Equations

Assuming that the dynamic load may be expressed in the form

$$\boldsymbol{L}(z,t) = \boldsymbol{L}(z)\,\boldsymbol{\Phi}(t), \tag{66}$$

in which $L(z)$ = a vector showing the distribution of the load with height of the structure

$$\boldsymbol{L}(z) = \begin{Bmatrix} \boldsymbol{p}(z) \\ \boldsymbol{P}(z) \end{Bmatrix} = \begin{Bmatrix} p_x(z) \\ p_y(z) \\ p_\theta(z) \\ P_1(z) \\ P_2(z) \\ \vdots \\ P_n(z) \end{Bmatrix} \tag{67}$$

and $\Phi(t)$ = a time function, Eq. (48) may be written:

$$\boldsymbol{a}\,\boldsymbol{X}^{\mathrm{IV}}(z,t) + \boldsymbol{b}\,\boldsymbol{X}''(z,t) + \boldsymbol{c}\,\boldsymbol{X}'(z,t) + \boldsymbol{d}\,\boldsymbol{X}(z,t) + \boldsymbol{g}\,\ddot{\boldsymbol{X}}(z,t) = \boldsymbol{L}(z)\,\boldsymbol{\Phi}(t). \tag{68}$$

The solution proposed for Eq. (68) will be the sum of the normal modes $\boldsymbol{X}_{hi}(z)$ multiplied by a corresponding time function $B_i(t)$, i.e.

$$\boldsymbol{X}(z,t) = \sum_{i=1}^{\infty} \boldsymbol{X}_{hi}(z)\,B_i(t). \tag{69}$$

Premultiplying Eq. (68) with $\boldsymbol{X}_{hn}(z)^T$ and integrating results

$$\begin{aligned} \int_0^H X_{hn}(z)^T\,[\boldsymbol{a}\,\boldsymbol{X}^{\mathrm{IV}}(z,t) + \boldsymbol{b}\,\boldsymbol{X}''(z,t) + \boldsymbol{c}\,\boldsymbol{X}'(z,t) + \boldsymbol{d}\,\boldsymbol{X}(z,t) + \boldsymbol{g}\,\ddot{\boldsymbol{X}}(z,t)]\,dz = \\ \int_0^H \boldsymbol{X}_{hn}^T(z)\,\boldsymbol{L}(z)\,\boldsymbol{\Phi}(t)\,dz. \end{aligned} \tag{70}$$

Substituting Eq. (69) into Eq. (70) yields

$$
\begin{aligned}
\sum_{i=1}^{\infty} \int_0^H [\boldsymbol{D}_{hn}^T (\boldsymbol{a}_{11} \boldsymbol{D}_{hi}^{\mathrm{IV}} + \boldsymbol{b}_{11} \boldsymbol{D}_{hi}'' + \boldsymbol{c}_{12} \boldsymbol{W}_{hi}') \\
+ \boldsymbol{W}_{hn}^T (\boldsymbol{b}_{22} \boldsymbol{W}_{hi}'' + \boldsymbol{d}_{22} \boldsymbol{W}_{hi} + \boldsymbol{c}_{21} \boldsymbol{D}_{hi}')]\, dz\, B_i(t) \\
+ \int_0^H (\boldsymbol{D}_{hn}^T \boldsymbol{M} \boldsymbol{D}_{hi} + \boldsymbol{W}_{hn}^T \boldsymbol{M}^* \boldsymbol{W}_{hi})\, dz\, \ddot{B}_i(t) = \\
\int_0^H (\boldsymbol{D}_{hn}^T \boldsymbol{p} + \boldsymbol{W}_{hn}^T \boldsymbol{P})\, dz\, \Phi(t).
\end{aligned} \tag{71}
$$

Due to orthogonality of the normal modes all crossed products of two normal modes are equal to zero, leading to the decoupling of the normal modes.

Denoting the generalized mass with

$$
M_n = \int_0^H [\boldsymbol{D}_{hn}^T \boldsymbol{M} \boldsymbol{D}_{hn} + \boldsymbol{W}_{hn}^T \boldsymbol{M}^* \boldsymbol{W}_{hn}]\, dz, \tag{72}
$$

the generalized exciting force with

$$
L_n \Phi(t) = \Phi(t) \int_0^H [\boldsymbol{D}_{hn}^T \boldsymbol{p} + \boldsymbol{W}_{hn}^T \boldsymbol{P}]\, dz \tag{73}
$$

and the generalized stiffness with

$$
\begin{aligned}
K_n = \int_0^H [\boldsymbol{D}_{hn}^T (\boldsymbol{a}_{11} \boldsymbol{D}_{hn}^{\mathrm{IV}} + \boldsymbol{b}_{11} \boldsymbol{D}_{hn}'' + \boldsymbol{c}_{12} \boldsymbol{W}_{hn}') \\
+ \boldsymbol{W}_{hn}^T (\boldsymbol{b}_{22} \boldsymbol{W}_{hn}'' + \boldsymbol{d}_{22} \boldsymbol{W}_{hn} + \boldsymbol{c}_{21} \boldsymbol{D}_{hn}')]\, dz,
\end{aligned} \tag{74}
$$

the uncoupled equation of motion for the n-th mode will be:

$$
K_n B_n(t) + M_n \ddot{B}_n(t) = L_n \Phi(t). \tag{75}
$$

The solution of Eq. (75) may be written

$$
B_n = \frac{\gamma_n}{\Omega_n^2} \int_0^t \Phi(\tau) \sin \Omega_n (t-\tau)\, d\tau, \tag{76}
$$

in which

$$
\Omega_n^2 = \frac{K_n}{M_n} \tag{77}
$$

and

$$
\gamma_n = \frac{L_n}{M_n}. \tag{78}
$$

In case of earthquake motion the dynamic load vector may be expressed in the form

$$
\boldsymbol{L}(z) \Phi(t) = - \begin{Bmatrix} \boldsymbol{M} \ddot{D}_0(t) \\ \boldsymbol{M}^* \ddot{W}_0(t) \end{Bmatrix}. \tag{79}
$$

The components of the earth acceleration vector may be independent and variable according to $n+3$ different time functions

$$\ddot{\boldsymbol{D}}_0(t) = \begin{Bmatrix} \ddot{u}_0(t) \\ \ddot{v}_0(t) \\ \ddot{\theta}_0(t) \end{Bmatrix} \tag{80}$$

and
$$\ddot{\boldsymbol{W}}_0(t) = \begin{Bmatrix} \ddot{W}_{10}(t) \\ \ddot{W}_{20}(t) \\ \vdots \\ \ddot{W}_{n0}(t) \end{Bmatrix}. \tag{81}$$

Denoting with
$$\gamma_{B_n} = -\frac{\int_0^H \boldsymbol{D}_{hn}^T(z)\,\boldsymbol{M}\,dz}{M_n} \tag{82}$$

and
$$\gamma_{N_n} = -\frac{\int_0^H \boldsymbol{W}_{hn}^T(z)\,\boldsymbol{M}^*\,dz}{M_n}. \tag{83}$$

The solution of Eq. (75) may be expressed

$$B_n(t) = \frac{1}{\Omega_n^2}\int_0^t [\gamma_{B_n}\ddot{\boldsymbol{D}}_0(\tau) + \gamma_{N_n}\ddot{\boldsymbol{W}}_0(\tau)]\sin\Omega\,(t-\tau)\,d\tau. \tag{84}$$

Interior Forces and Displacements

Knowing the displacement vector $\boldsymbol{D}(z,t)$ for system axes, that for local axes $\bar{\boldsymbol{D}}_i(z,t)$ for shear wall i will be obtained by Eq. (6).

The shear force vector $\boldsymbol{q}(z,t)$ is calculated from Eq. (44).

The shear force in the connecting beam belonging to row j and storey k with ordinate z_k is given by

$$P_{Kj} = \int_{Z_K-h/2}^{Z_K+h/2} q_j(z,t)\,dz. \tag{85}$$

The shear forces and torque in shear wall i are

$$\bar{\boldsymbol{Q}}_i(z,t) = \begin{Bmatrix} Q_{xi}(z,t) \\ Q_{yi}(z,t) \\ Q_{\theta i}(z,t) \end{Bmatrix} = -\bar{\boldsymbol{K}}_i \sum_{i=1}^{\infty} \bar{\boldsymbol{D}}_i(z)'''\,B_i(t)$$
$$+\boldsymbol{K}_T \sum_{i+1}^{\infty} \bar{\boldsymbol{D}}_i(z)'\,B_i(t) + \sum_l \boldsymbol{e}_l q_l(z,t) - \sum_r \boldsymbol{e}_r q_r(z,t). \tag{86}$$

In Eq. (86) the sum on l refers to connecting beams joining the shear wall from left and r that from right.

The bending moments and bimoment in shear wall are given by

$$\bar{\boldsymbol{M}}_i(z,t) = \begin{Bmatrix} M_{xi}(z,t) \\ M_{yi}(z,t) \\ B_i(z,t) \end{Bmatrix} = -\bar{K}_i \sum_{i=1}^{\infty} \bar{\boldsymbol{D}}_i(z)''\,B_i(t). \tag{87}$$

Conclusions

A general method for dynamic analysis of multistory structures is presented. On the basis of this method a computer program may be prepared for dynamic analysis of multistory structures with arbitrary layout of shear walls. With adequate boundary conditions the method may be applied for cases with sudden changes in height of the geometric properties and loads. For g number of changes the evaluation of normal modes leads to an eigenvalue problem of order $(2n+12)(g+1)$. The influence of normal strains in the shear wall is included. Vertical dynamic loads can be taken into account.

List of Symbols and Notations

Upper bar	magnitudes in local systems of axes.
i	index for shear wall.
j	index for connecting beam or continuous media.
$\boldsymbol{A}$	characteristic vector.
A	cross section area.
B	bimoment.
$B(t)$	displacement time function.
C_i	constants.
$\boldsymbol{D}(z)$	lateral displacement vector.
$\boldsymbol{D}_0(t)$	earth acceleration vector.
E	modulus of elasticity.
$\boldsymbol{F}$	flexibility matrix to normal strains.
G	shear modulus.
H	height of the structure.
J_{xi}, J_{yi}, J_{wi}	moment of inertia in X respective Y direction and sectorial moment of inertia.
J_T	Saint-Venant torsion coefficient.
$\boldsymbol{K}, \boldsymbol{K}_T$	stiffness matrices.
L	clear span of connecting beam.
$\boldsymbol{L}$	load vector.
$\boldsymbol{M}, \boldsymbol{M}^*$	mass matrices.
$\boldsymbol{M}_i$	3×1 vector consisting bending moment in X and Y direction and bimoment with respect to local axes.
M_n	generalized mass.
M_x, M_y	moments.
N	total number of shear walls (coupled or not).
$\boldsymbol{P}(z,t), \boldsymbol{p}(z,t)$	dynamic load vectors.
P_k	shear force in connecting beam at story level k.
P_n	generalized exciting force.

$\boldsymbol{R}$	rotation matrix.
$\boldsymbol{T}$	translation matrix.
$\boldsymbol{Q}$	3×1 vector consisting shear forces in X and Y direction and torsion moment with respect to origin of the system axes.
$\boldsymbol{U}$	matrix dfined in the text.
X, Y, Z	coordinates.
$\boldsymbol{X}$	generalized displacement vector.
$\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}, \boldsymbol{g}$	matrices defined in the text.
$\boldsymbol{e}_l, e_r$	vector defined in the text.
$\boldsymbol{f}$	flexibility matrix of lamella system.
g	number of changes in characteristics of stiffening elements.
h	story height.
i, j, k, l, r	indexes.
n	number of shear walls having joints with connecting beams.
$q(z)$	shear force function in lamella system.
r	characteristic values.
x, y, z	local system of axes for connecting beams.
s	number of lamella systems.
α	angle between main axis $\bar{X}$ and system axis X.
φ, δ, η	gap at cut end due to bending and shear of connecting beam, respectively strains in shear walls and bending and warping of the shear walls.
ω	circular frequency.
$\boldsymbol{\Phi}(t)$	load time functions.
Ω_n	circular frequency of the nth decoupled equations.
w	sectorial coordinate.
$u(z), v(z), \theta(z)$	lateral displacement functions in X and Y directions and rotation with respect to origin of the system axes.
$W_1(t), -W_n(t)$	axial displacement functions.

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Summary

A general method for three dimensional dynamic analysis of asymmetric multistorey structures based on the continuum method approach is presented. There are no restrictions referring to types of stiffening elements and layout in floor plane. The dynamic loads may be vertical and lateral at arbitrary locations. The lateral and axial displacements of the shear walls are selected as independent variables. Normal strains and vertical dynamic loads and earth accelerations can also be taken into account.

Résumé

On présente une méthode générale pour l'analyse tridimensionnelle de structures asymétriques à plusieurs étages basée sur la méthode approchée du continuum. Il n'existe pas des restrictions quant au types des éléments raidisseurs et au layout dans le plan des étages. Les charges dynamiques peuvent agir verticalement ou latéralement à des endroits arbitraires. Les déplacements latéraux et axiaux des parois de cisaillement sont sélectionnés comme variables indépendants. Des sollicitations normaux et des charges dynamiques verticaux ainsi que des accélérations terrestres peuvent également être prises en considération.

Zusammenfassung

Es wird eine allgemeine Methode für die dreidimensionale Analyse von asymmetrischen mehrstöckigen Bauten auf Grund der Kontinuum-Näherungsmethode dargelegt. Es bestehen keine Einschränkungen hinsichtlich der Typen von Versteifungselementen und des Entwurfs in der Stockwerkebene. Die dynamischen Belastungen können vertikal und seitlich an beliebigen Stellen wirken. Die seitlichen und axialen Verschiebungen der Schubwände werden als unabhängige Variable gewählt. Normale Beanspruchungen und vertikale dynamische Belastungen sowie Erdbeschleunigungen können ebenfalls berücksichtigt werden.