Zeitschrift: IABSE publications = Mémoires AIPC = IVBH Abhandlungen

Band: 32 (1972)

Artikel: Finite element analysis of skew vault bridges

Autor: Sisodiya, R.G. / Cheung, Y.K. / Ghali, A.

DOI: https://doi.org/10.5169/seals-24959

Finite Element Analysis of Skew Vault Bridges

Analyse de ponts en arc biais à l'aide des éléments finis

Berechnung schiefer Bogenbrücken mittels der finiten Elemente

R. G. SISODIYA
Ph. D., MEIC, Assistant Prfessor of Civil Engineering,
McGill University, Montreal, Quebec, Canada

Y. K. CHEUNG
Ph. D., MICE, Professor of Civil Engineering, The University of Calgary

A. GHALI
Ph. D., MASCE, Professor of Civil Engineering, The University of Calgary

Introduction

In the past vault bridges such as in Fig. 1 have been analyzed approximately by treating them as arch beams. The approximation involved may not be acceptable when the bridge is skew or when the bridge width is of the same order as the span. In such cases a spatial structural analysis is necessary and the finite element method is used here for this purpose.

SABIR and ASHWELL [1] analyzed the rectilinear bridge in Fig. 1 by finite element method using rectangular plate bending element for the slab and cylindrical shell element for the vault. Their analysis assumed that the slab rests on the crown of the vault, and the interaction between the deck and the vault is of only vertical forces along the crown. This assumption is valid only for special cases.

A parallelogram shell element developed in References 2 and 3 is used in the present analysis. The stiffness of this element is a combination of the bending stiffness developed by DAWE [4] and in-plane stiffness derived [2], [3] specially for bridge analysis. The latter refers to a high order in-plane element which includes in-plane rotations as nodal parameters and is used in the present analysis because of its superior accuracy over the standard bilinear parallelogram element.

The examples analyzed in the present paper are for rectilinear and skew

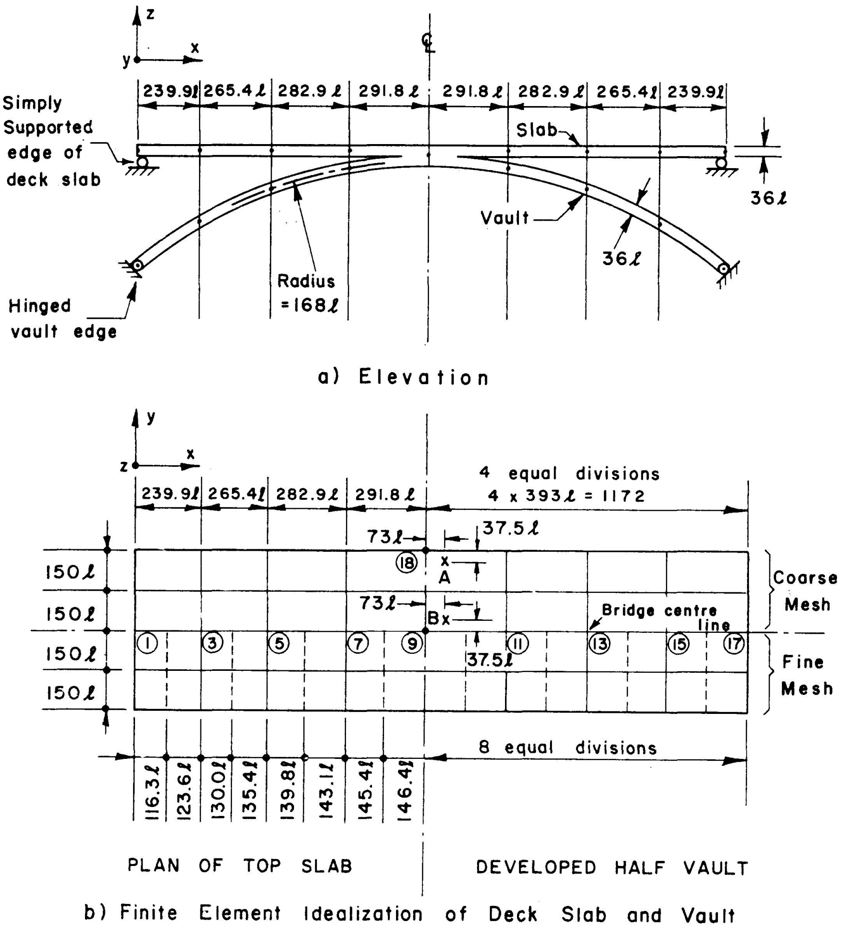


Fig. 1. Bridge No. 1 Analyzed in Example 1 (Sabir and Ashwell's Bridge).

bridges made up of a vault, a slab deck and interconnecting walls or columns with monolithic joints (Fig. 2). The computer program used is also capable of analyzing bridges for which the vault or the deck are of box girder or slab-beam construction.

Method of Analysis

The paralelogram element used has the following nodal parameters at its corners (Fig. 3): w, θ_x and θ_y (for bending) and u, v, $\theta_z = (\partial v / \partial x)$ (for in-plane displacements). The displacement function used to derive the in-plane stiffness are given below for easy reference:

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \sum_{i=1}^{4} \begin{bmatrix} f_1 & f_4 & f_5 \\ 0 & f_2 & f_3 \end{bmatrix}_i \begin{Bmatrix} u \\ v \\ \theta_z \end{Bmatrix}_i ,$$

where

$$f_{1i} = \tfrac{1}{4}(1 + \xi\,\xi_i)(1 + \eta\,\eta_i),$$

$$f_{2i} = \tfrac{1}{2}(2 + \xi\,\xi_i - \xi^2) f_{1i},$$

$$f_{3i} = -\tfrac{1}{4} a\,\xi_i (1 - \xi^2) f_{1i},$$

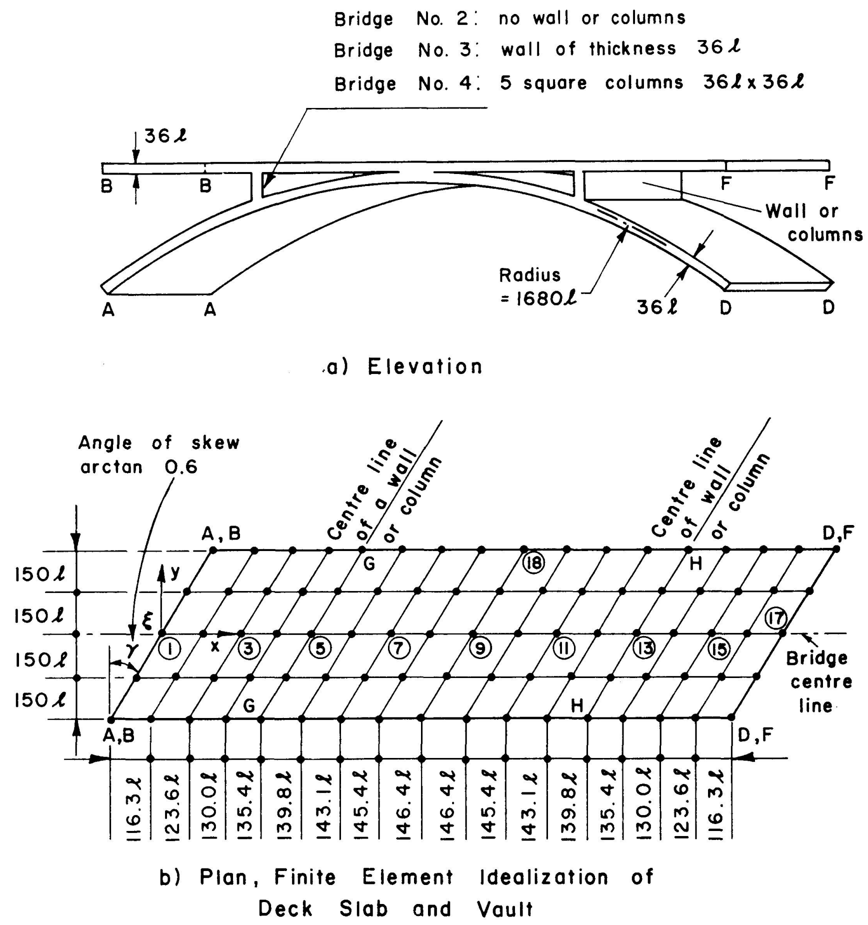


Fig. 2. Skew Bridges Nos. 2, 3 and 4 Analyzed in Example 2.

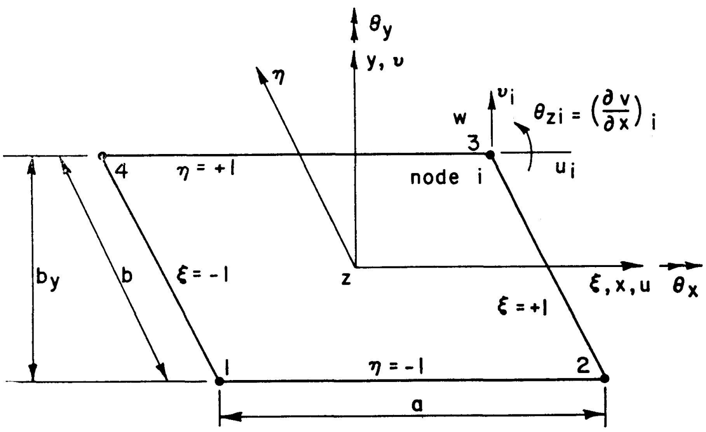


Fig. 3. An In-Plane Stiffness Parallelogram Element.

$$f_{4i} = \frac{3}{8a} b_y \xi_i \eta_i (1-\xi^2)(1-\eta^2),$$

$$f_{5i} = -\frac{1}{16} b_y \eta_i (1-\eta^2)(1-2\xi\xi_i - 3\xi^2).$$

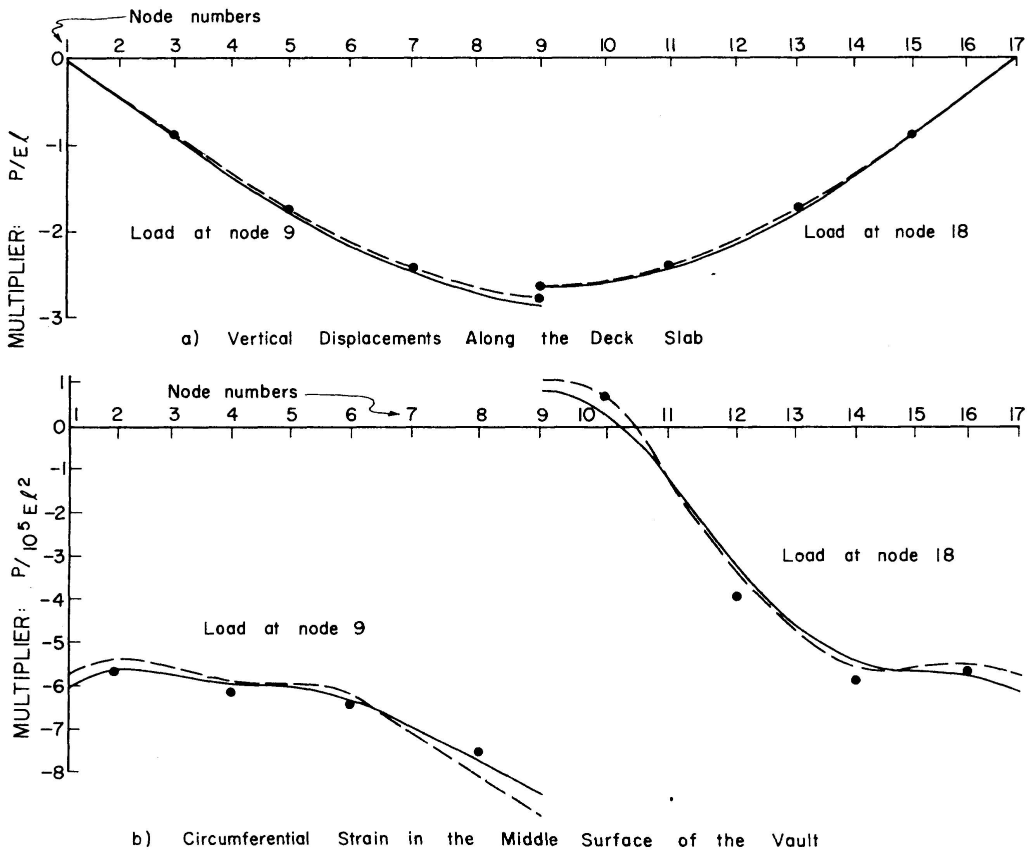
The quantities a and b_y and the co-ordinates x, y, ξ and η are defined in Fig. 3.

For beams or columns (if any) a standard beam element [5] is used with three translations and three rotations at each end. For beams monolithic with the deck slab the 12 displacements are taken at "eccentric" nodes on the centroid of the beams, and these are then transformed to nodes in the middle surface of the slab (assuming rigid connection). In the examples discussed below, Poisson's ratio is considered equal to 0.2.

Example 1: Rectilinear Bridge

The bridge analyzed by Sabir and Ashwell (Bridge No. 1, Fig. 1) is symmetrical in structure and loading, and thus their assumption that only vertical forces exist between the slab and the vault at the crown is correct, even if the two are monolithically connected. Sabir and Ashwell carried out the analysis for a vertical concentrated load P at points 9 and 18, Fig. 1b, using a 12×12 mesh for the deck and an 8×8 mesh for the vault. Their results are used for checking the present analyses, in which both a coarse mesh and a fine mesh idealization (Fig. 1b) are used.

Since the displacement functions of the plane stress element are not balanced in the x and y directions, a study was made to determine the effect of this by running two identical coarse mesh solutions with the elements' axes interchanged. The results shown in Fig. 4 indicate that the general behaviour



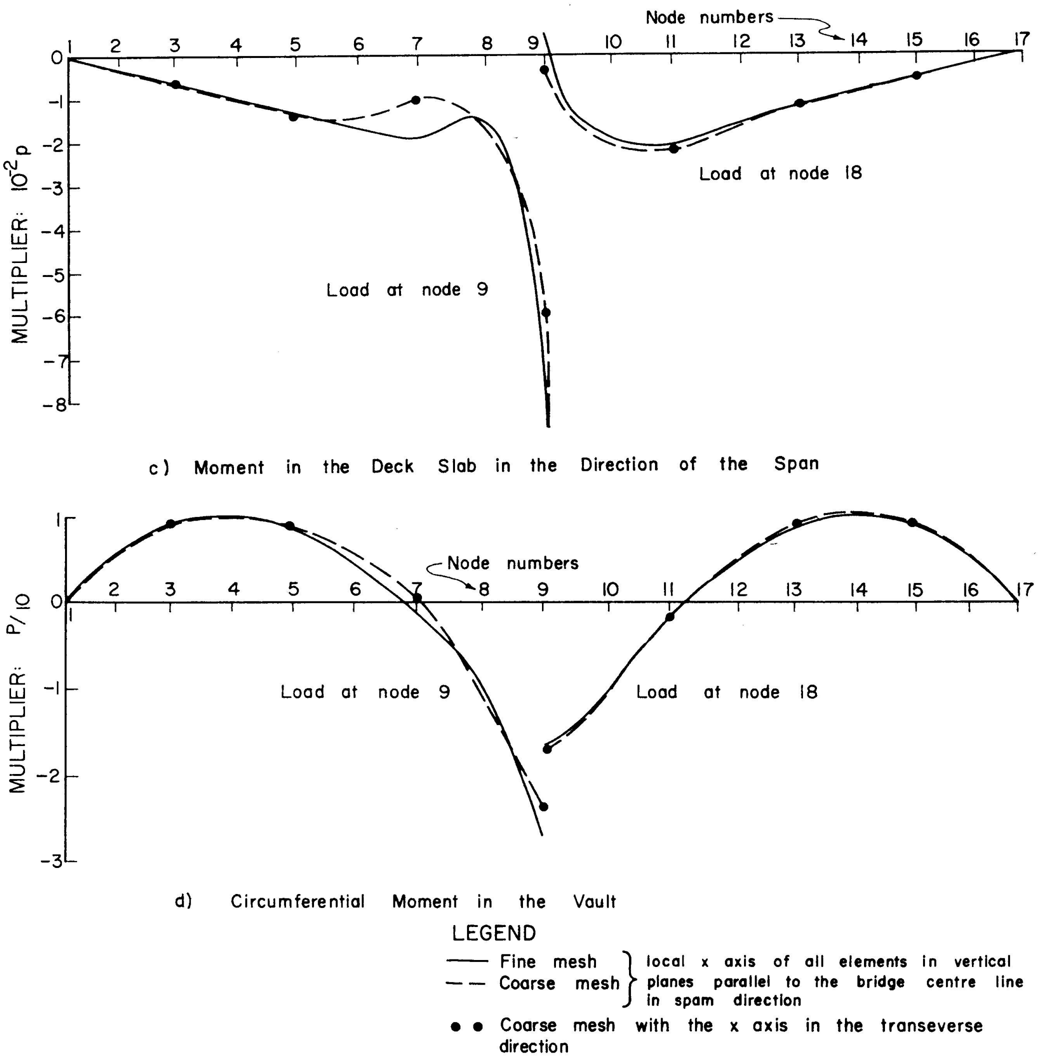


Fig. 4. Variation of Vertical Displacement, Strain and Moment Along the Centre Line of Bridge in Fig. 1.

Table 1. Comparison of Results of Analysis of the Bridge No. 1 in Fig. 1

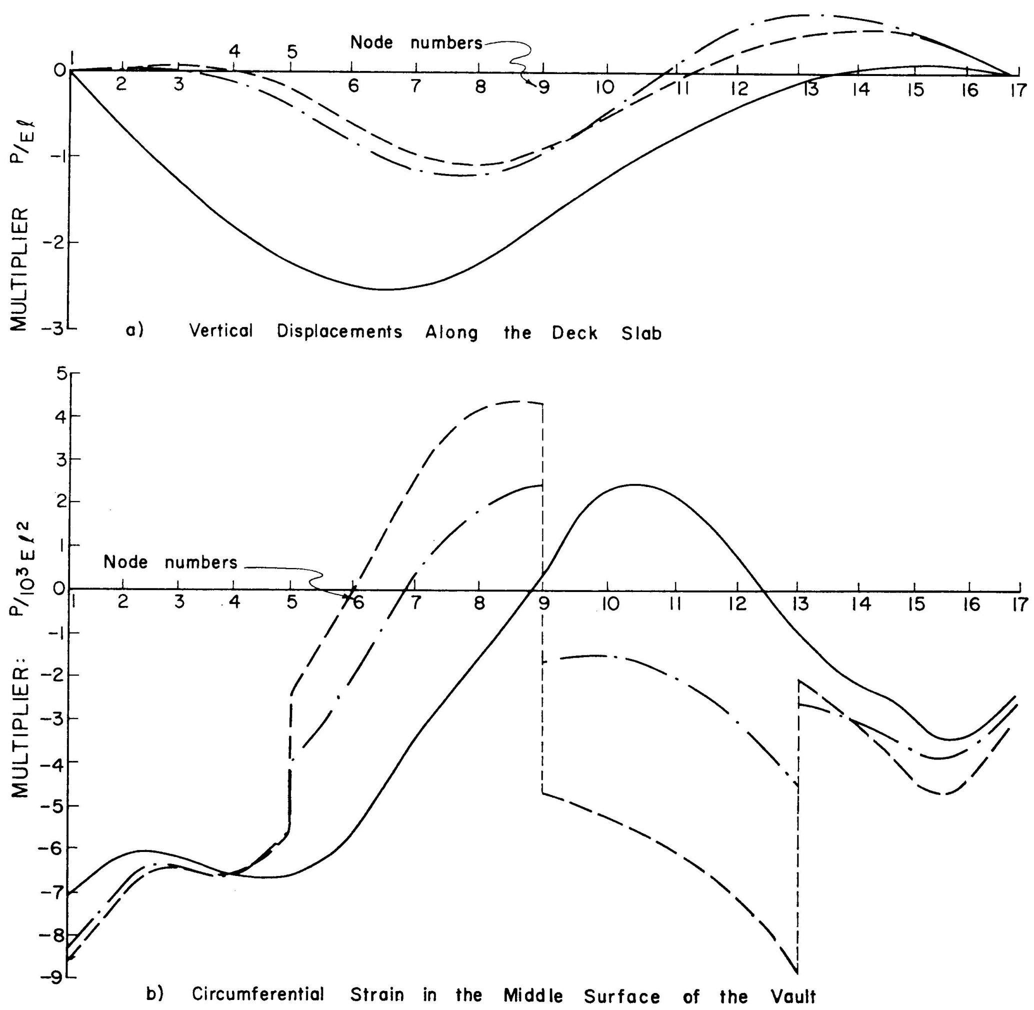
	Multiplier	Point	Load at Node 18			Load at Node 9		
			Ref. [1]	Present Analysis		Ref. [1]	Present Analysis	
				Coarse Mesh	Fine Mesh		Coarse Mesh	Fine Mesh
Deflections	$-\frac{P}{10^2 El}$	1	531	506	525	—	253	263
		2	—	253	263	290	275	285
Circumferential Forces	$-\frac{P}{10^4\, l}$	A	100	84	91	—	8	6
		B	—	4	6	37	27	31
Circumferential Bending Moment	$-\frac{P}{10^3}$	A	281	257	262	—	145	148
		B	—	158	152	176	157	166

of the bridge is unaltered. For this reason, in all the subsequent analyses the local x-axis of the elements will always be in a vertical plane parallel to the direction of span.

The results given in Reference [1] are in agreement with the present analysis (Table 1). The moments and stresses in the present analysis are calculated at the nodes and the centre of the element sides respectively, these are then plotted in graphs from which the results in Table 1 are deduced. The values of the moments and forces quoted from Reference [1] are the "maximum" mid-element values; these are assumed here to represent Sabir and Ashwell's results at points A and B.

Example 2: Skew Bridge

Three skew bridges are analyzed; they have the same plan and elevation shown in Fig. 2. The Bridge No. 2 has no connection between the deck slab and vault along lines GG and HH; in Bridge No. 3 vertical walls are provided along these lines and in Bridge No. 4 each wall is replaced by 5 vertical square columns ($36\,l \times 36\,l$). The conditions of support at the ends of the deck slab



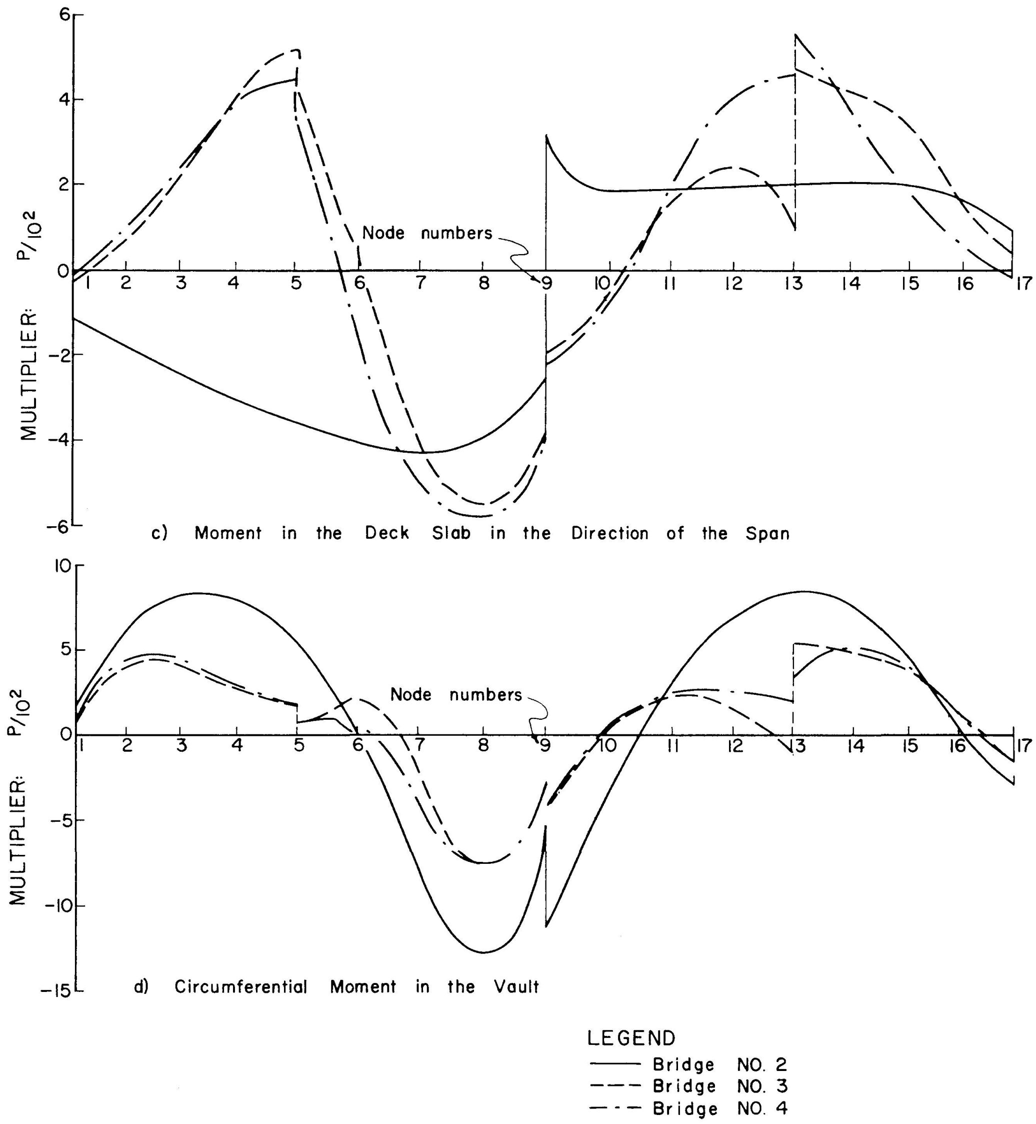


Fig. 5. Variation of Vertical Displacement, Strain and Moment Along the Centre Line of Bridge in Fig. 2.

and the vault are the same as in Example 1. The three skew bridges are analyzed for a concentrated load P at node 18. The finite element idealization of the deck slab is indicated in Fig. 2b; this figure also represents the projection of the mesh division in the vault which divides the vaults into equal elements. Each of the two walls in Bridge No. 3 is divided into 4 elements by vertical lines joining the nodes in the deck slab and the vault.

Fig. 5 shows the variation of vertical displacements, moments and strains along the centre line of the deck slab and vaults of the three bridges. Comparison of the results of Bridges No. 1 and No. 2, which are identical except for the skew angle, for the same case of loading shows totally different values. Thus an analysis in which the skew effect is properly accounted for is essential for this type of bridges.

The effect of shew is further studied for the dead loading. Bridge No. 3

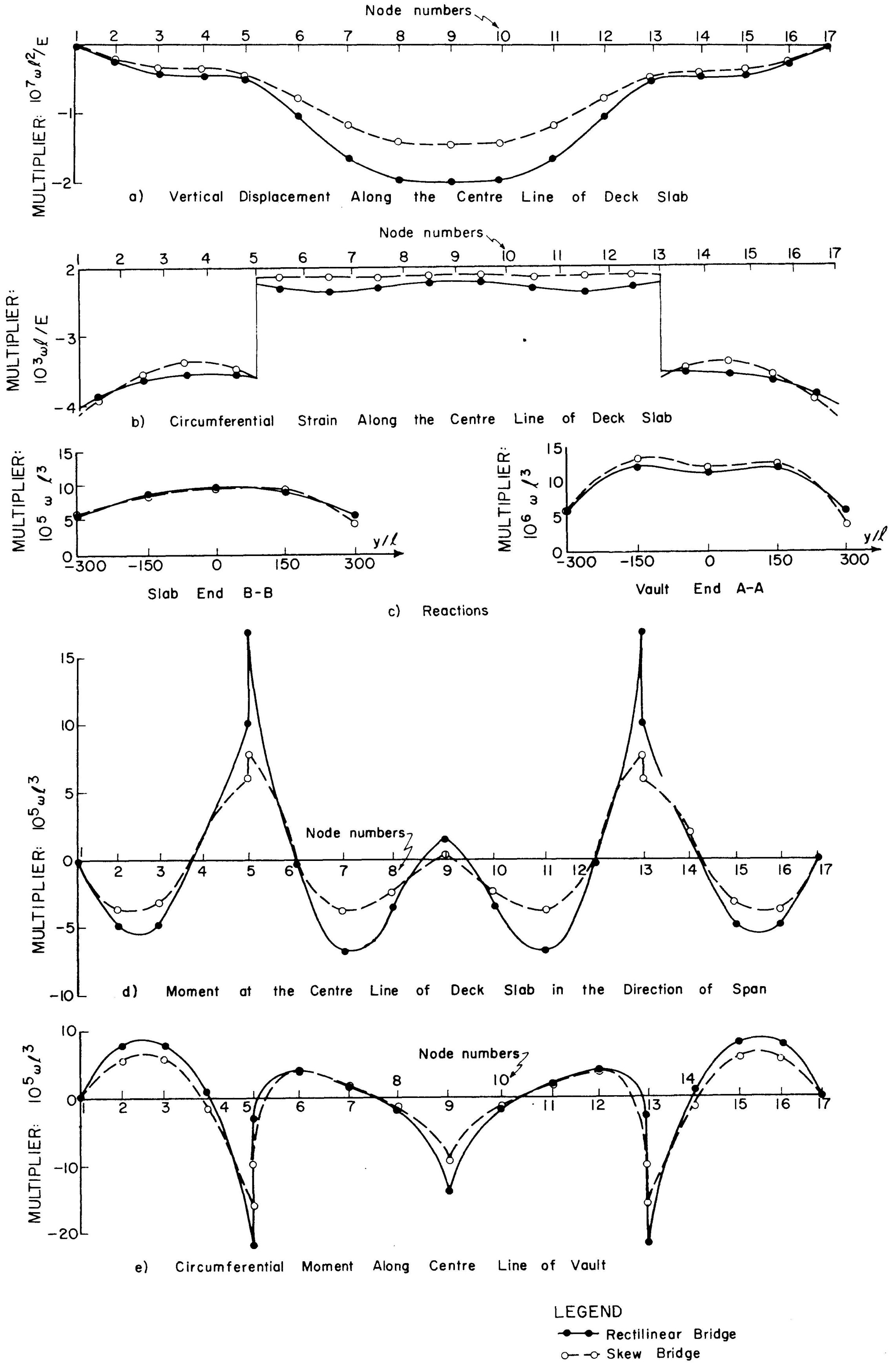


Fig. 6. Dead Load Effects on a Rectilinear and Skew Bridge (See Fig. 2).

and an identical bridge but with the skew angle $\gamma = 0$ (Bridge No. 5) are analyzed for the dead loading, which is represented for each element by 4 equal vertical forces at its corners. The results are shown in Fig. 6, which includes the reactions at the supports in addition to the vertical displacements, strains and moments considered in Fig. 5.

Compilation and execution time for complete analyses for one loading case CDC/6400 computer using KRONOS operating system compiler FTN with optimization level 1 is 90 and 110 seconds, respectively. Execution time is increased by about 5 seconds for each additional load case.

Conclusions

The finite element analysis described can be used economically for skew vault bridges. The accuracy of the method is verified for a rectilinear bridge by comparison with published results. The results of examples analyzed show the drastic effect of the angle of skew, thus demonstrating the necessity of an appropriate spatial structural analysis.

Notations

$a, b. b_y$	dimensions of a parallelogram element (see Fig. 3).
E	Young's modulus.
i	integer.
l	length parameter.
P	concentrated vertical point load.
u, v, w	translations in x, y and z directions respectively.
x, y, z	axes.
γ	skew angle defined in Fig. 5b.
θ	rotation which can be represented by a vector along any of the axes x, y, and z.
ξ, η	dimensionless co-ordinates in a parallelogram element (see Fig. 3).
ω	specific weight.

Acknowledgement

Financial support of this research provided by the National Research Council of Canada is gratefully acknowledged.

References

1. A. B. Sabir and D. G. Ashwell: Finite Element Analysis of a Broad Arch Bridge with a Slab Deck. Development in Bridge Design and Construction, Cardiff, Great Britain, 29 March – 2 April, 1971.

2. R. G. SISODIYA, Y. K. CHEUNG, and A. GHALI: New Finite Elements with Application to Box Girder Bridges. Proceedings I.C.E., London Paper No. 7479.
3. R. G. SISODIYA: Finite Element Analysis of Box Girder Bridges. Ph. D. thesis, The University of Calgary, Calgary, Alberta, Canada, 1971.
4. D. J. DAWE: Parallelogramic Elements in the Solution of Rhombic Cantilever Plate Problems. Journal of Strain Analysis, Vol. 1, No. 3, 1966.
5. A. GHALI and A. M. NEVILLE: Structural Analysis – A Unified Classical and Matrix Approach. International Textbook Co., Scranton, Pennsylvania, 1972, p. 79.

Summary

The finite element method is used to analyse skew bridges composed of a vault and a horizontal deck connected by walls or columns. A parallelogram element combining in-plane and plate bending stiffnesses is used. The in-plane element stiffness is derived (in a earlier publication) particularly for use in bridge design and is proved to lead to accurate results with small number of elements. The analysis is applied to rectilinear and skew bridges and the effect of the skew angle is discussed.

Résumé

La méthode des éléments finis est utilisée pour analyser des ponts biais composés d'un arc et d'une dalle de chaussée horizontale, liés entre eux par des parois ou des piles. On travaille avec un élément ayant la forme d'un parallèlogramme qui combine la rigidité dans le plan et la rigidité de flexion. La rigidité dans le plan des éléments a été traitée spécialement (voir la publication antérieure) pour l'application dans la construction de ponts. On a démontré qu'elle mêne, même avec un petit nombre d'éléments, à de très bons résultats. L'analyse est appliquée à des ponts droits et biais et l'effet de l'inclinaison est discutée.

Zusammenfassung

Die Methode der endlichen Elemente wird verwendet, um schiefe Brücken, deren Bögen durch Wände oder Stützen mit der Fahrbahn verbunden sind, zu berechnen. Es wird ein parallelogrammförmiges Element, das Scheiben- und Plattensteifigkeit kombiniert, verwendet. Die Scheibenelementsteifigkeit wurde speziell für die Anwendung im Brückenbau (in einer früheren Publikation) entwickelt; es wurde bewiesen, dass sie bereits bei einer kleinen Anzahl von Elementen zu genauen Resultaten führt. Die Berechnung wurde an geraden und schiefen Brücken durchgeführt, und der Einfluss der Schiefe wird diskutiert.