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Autor(en): **Soboyejo, A.B.O.**

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Stochastic Analysis for Time-Dependent Load Transfer in Reinforced Concrete Columns

Analyse aléatoire du transfert de la charge en fonction du temps dans les colonnes en acier-béton

Stochastische Analyse für zeitabhängige Lastübertragung in Stahlbetonstützen

A. B. O. SOBOYEJO
Faculty of Engineering, University of Lagos, Lagos, Nigeria

Introduction

A stochastic process model is developed for the phenomenon of time-dependent load transfer from concrete to steel in reinforced concrete columns subjected to a constant axially sustained loading. This phenomenon is caused mainly by the mechanisms of creep and shrinkage in concrete.

Having developed a mathematical model, a stochastic process model is then developed. The choice of the stochastic process model depends on the actual physical mechanisms of creep and shrinkage in reinforced concrete columns, and on the nature of the mathematical model developed. The stochastic process model is a probabilistic model which gives a mean value function which is functionally the same as the mathematical model; it also gives more useful information than the deterministic model, since it gives not only the mean value function, but also the variance and the covariance functions. The variance function gives a measure of the variation of the process from its mean value at any time $t \geqq 0$; while the covariance function describe the degree of correlation between the values of the process at any two times s and t, where $0 \leqq s \leqq t$.

The variance function enables statements to be made concerning the upper and lower bounds for the phenomenon of time-dependent load transfer in reinforced concrete columns. Moreover, the variance function enables statements to be made concerning the possible deviations from the mean value function, for the phenomenon of time-dependent load transfer in reinforced concrete columns.

The ability to predict not only the upper and lower bounds but also the possible deviations from the mean value, for this phenomenon should have useful application in practice.

The results of the stochastic process model developed are compared with results obtained in recent experimental work in this field; and very good agreements are obtained.

Deterministic Model

When a reinforced concrete column is subjected to a constant axially sustained loading, it can be shown that the instantaneous loads taken up by the concrete and the steel, at the initial time of loading, can be represented by (see Fig. 1)

$$P_c(0) = P\frac{A_c E_c}{A_c E_c + A_s E_s}, \tag{1}$$

$$P_s(0) = P\frac{A_s E_s}{A_c E_c + A_s E_s}, \tag{2}$$

where $P_c(0)$ = load on the concrete at the initial time of loading which in this case is considered as time $t = 0$;

$P_s(0)$ = load on the steel at the initial time of loading $t = 0$;

P = total load taken up by the column;

A_s = area of steel;

A_c = area of concrete;

E_s = modulus of elasticity of steel;

E_c = modulus of elasticity of concrete; this value is assumed to be constant.

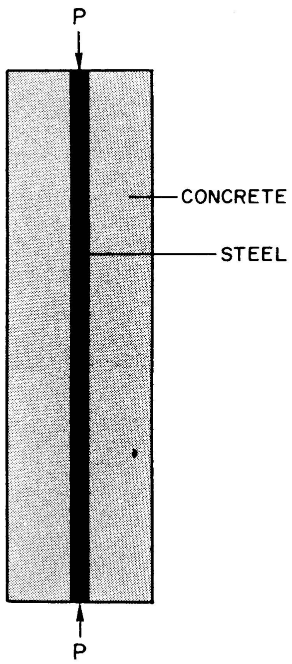

Fig. 1. Reinforced concrete column under constant axial loading.

It has been shown [1, 2, 3, 4] that due to the mechanisms of creep and shrinkage, there is a gradual transfer of load from concrete to steel in reinforced concrete columns, under constant axially sustained loadings. It is reasonable to assume that the following relationship [4, 5] holds at any small increment of time, *dt*.

Change in creep and shrinkage strains of concrete
= Change in Elastic Recovery Strain of steel,
+ Change in Elastic Recovery Strain of concrete;

that is,

$$\left(\frac{P_c(t)}{A_c}\right) dc(t) + ds(t) = \frac{dP_s(t)}{A_s E_s} - \frac{dP_c(t)}{A_c E_c}, \qquad t \geqq 0, \tag{3}$$

where $c(t)$ = specific creep strain for concrete at any time $t \geqq 0$, it has the dimensions $\frac{\text{strain}}{\text{stress}}$, since it represents creep strain per unit stress;
$s(t)$ = shrinkage strain at any time $t \geqq 0$ for the concrete.

Moreover, since the increase of load on the steel must be equal to the decrease of load on the concrete, in order to maintain equilibrium, therefore;

$$dP_s(t) + dP_c(t) = 0. \tag{4}$$

It is also reasonable to assume (5) that shrinkage strain and creep strain functions are related by

$$s(t) = k(s)\, c(t), \qquad t \geqq 0, \tag{5}$$

where $k(s)$ is approximately a constant which can be derived from experimental data by finding the ratio between the shrinkage strain and the creep strain at any time. The value of $k(s)$ will depend very much on factors (6) which can influence both shrinkage and creep.

Eqs. (3), (4) and (5) can be solved with the initial conditions $c(t) = s(t) = 0$ when $t = 0$, to give the following results, for the load on the concrete $P_c(t)$, and the load transferred from the concrete to the steel $\varDelta P(t)$, at any time $t \geqq 0$;

$$P_c(t) = P_c(0)\, e^{-\theta_c(t)} - S\,(1 - e^{-\theta_c(t)}), \qquad t \geqq 0, \tag{6}$$

$$\varDelta P(t) = (P_c(0) + S)\,(1 - e^{-\theta_c(t)}), \qquad t \geqq 0, \tag{7}$$

where $\theta = \dfrac{p\, E_s}{1 + (n-1)\, p}, \qquad S = k(s)\, A_c$

and p = steel ratio, $\qquad n = \dfrac{E_s}{E_c}.$

Moreover, the load on the steel $P_s(t)$ at any time $t \geqq 0$, can be expressed as:

$$P_s(t) = P_s(0) + \varDelta P(t), \qquad t \geqq 0. \tag{8}$$

The specific creep strain will be assumed to have a function of the type (3):

$$c(t) = \frac{t}{A + Bt}, \qquad t \geqq 0, \tag{9}$$

where A and B are constants for a given mix of concrete and age of application of load on the concrete.

In this development, it is assumed that the modulus of elasticity of concrete E_c is a constant. This is not strictly so, however, it is possible to consider the effect of the variation of E_c with time in this problem (5), this derivation will not be considered here in order to simplify the mathematical development.

Effect of Time of Loading

When the column is subjected to a sustained loading beginning at time $T \geqq 0$, then the load transferred from the concrete to the steel $\Delta P(t)$, which has been affected by the shrinkage of the concrete through, the total time $t \geqq 0$, and the creep of the concrete which only comes into consideration during the time period from T to t, can be expressed as:

$$\Delta P(t) = S(1 - e^{-\theta_c(t)}) + P_c(T)(1 - e^{-\theta_c(t-T)}), \qquad t \geqq T \geqq 0: \tag{10}$$

while the loads $P_c(t)$ and $P_s(t)$ on the concrete and steel are given respectively as:

$$P_c(t) = P_c(T) - \Delta P(t), \qquad t \geqq T \geqq 0, \tag{11a}$$

and

$$P_s(t) = P_s(T) + \Delta P(t), \qquad t \geqq T \geqq 0. \tag{11b}$$

Useful Simplification for the Mathematical Deterministic Model

It is good enough in most practical structural engineering applications to assume that the quantity,

$$\begin{aligned} \theta c(t-T) &= \theta \frac{(t-T)}{A + B(t-T)}, && t \geqq T \geqq 0, \\ &\cong \frac{\theta}{A}(t-T), && t \geqq T \geqq 0, \\ &\cong \mu (t-T) && t \geqq T \geqq 0, \end{aligned} \tag{12}$$

where

$$\mu = \frac{\theta}{A}. \tag{13}$$

The quantity μ can be regarded as a constant for any reinforced concrete column under known working conditions for the concrete. Known working conditions for the concrete may be such factors as the external temperature which is assumed constant here, moisture, or any other factors which can affect creep characteristics of the concrete and thereby affect the value of A for the concrete.

Eq. (12) is good enough for most normal practical structural engineering applications because experimental results for creep data normally show that (5):

$$\frac{B}{A}(t-T) \ll 1.00$$

particularly after a long time.

Using the results of Eqs. (12) and (13), the results given by Eqs. (10), (11a) and (11b) can now be expressed as:

$$\Delta P(t) = S(1-e^{-\mu t}) + P_c(T)(1-e^{-\mu(t-T)}), \qquad t \geqq T \geqq 0, \tag{14}$$

$$P_c(t) = P_c(T) - \Delta P(t), = P_c(T)e^{-\mu(t-T)} - S(1-e^{-\mu t}), \qquad t \geqq T \geqq 0, \tag{15}$$

$$P_s(t) = P_s(T) + \Delta P(t), = P_s(T) + S(1-e^{-\mu t}) + P_c(T)(1-e^{-\mu(t-T)}), \quad t \geqq T \geqq 0. \tag{16}$$

Similar modifications can be made to the results given by Eqs. (6) and (7).

The results given by Eqs. (14), (15) and (16) provide very useful bases in the formulation of a reasonable stochastic process model for the problem of time-dependent load transfer in reinforced concrete columns.

Formulation of Stochastic Process Model

The nature of the variations in the time-dependent load transfer due to the mechanisms of creep and shrinkage in reinforced concrete columns subjected to axially sustained loadings, suggest that a stochastic process model is appropriate to describe such a phenomenon (5). The results of the deterministic model and the physical nature of the phenomenon are very useful guides in the choice of the appropriate statistical model.

In this development it is assumed that the steel is inert and that it does not experience any appreciable time-dependent deformations at the environment in which the system is operating. Also it is assumed that the operating temperature of the column remains essentially constant.

Effect of Mechanism of Creep Alone

Creep strains of concrete can be assumed to be resisted by the presence of the inert steel, and that as a result of ths resistance, the steel becomes more highly stressed, while the stress in the concrete decreases with time. Since concrete creep can be assumed to be proportional to the applied stress on the concrete, or to some power of the applied stress (8), the rate of creep will be reduced more and more as the load continues to be transferred from the concrete to the steel. Even in short-period tests on reinforced concrete the creep rate decreases appreciably within a few minutes of loading, and it is thought that this effect is due partly to the transfer of load within the specimen. The creep rate is further reduced as the age of the concrete increases owing to the increase in the "resistance to flow" of the cement in the concrete (7).

The deterministic model shows that the load on the concrete $P_c(t)$, the load on the steel $P_s(t)$, and the load transferred from the concrete to the steel $\Delta P(t)$, are all functions of many variables including creep; and since creep itself can be assumed to be a random variable (9), therefore it is reasonable to assume that the quantities $P_c(t)$, $P_s(t)$, and $\Delta P(t)$ are all random variables (10). Furthermore, it can be shown from Eq. (15), that the rate of the function $P_c(t)$, when shrinkage is neglected, can be expressed as:

$$\frac{d\overline{P}_c(t)}{dt} = -\mu \overline{P}_c(T) e^{-\mu(t-T)} = -\mu \overline{P}_c(t), \qquad t \geqq T \geqq 0, \tag{17}$$

where
$$\overline{P}_c(t) = P_c(T) e^{-\mu(t-T)}, \qquad t \geqq T \geqq 0 \tag{18}$$

is the load on the concrete at any time, due to the effect of creep mechanism alone.

Eqs. (17) and (18) are the characteristic deterministic behavior of any "Markov homogeneous linear death process" [11], which can be used as the stochastic process model to describe the mechanism of creep in reinforced concrete columns subjected to sustained axial loading.

The statistical properties of "the Markov homogeneous linear death process" will be used to formulate the prediction equations for the phenomenon of creep mechanism in this study. Experimental and analytical results [2, 4, 5] confirm the validity of Eqs. (17) and (18). This means that the greater the initial load $P_c(T)$ on the concrete, the greater the subsequent load $\overline{P}_c(t)$ on the concrete at any time t, and that this load on the concrete depends on the internal mechanism of creep in the concrete.

Therefore, the following expression is proposed for the load on the concrete:

$$\overline{P}_c(t) = P_c(T) X(t-T), \qquad t \geqq T \geqq 0, \tag{19}$$

where the process $[x(t-T); (t-T) \geqq 0]$ can be regarded as "the Markov homogeneous linear death process", with unit initial value, and also represents the contribution of the internal creep mechanism to the whole process. It is assumed here that the initial load $P_c(T)$ on the concrete is a constant, the model developed here has been extended to account for the case when the initial load on the concrete is a random variable (5).

The moment functions of the process $\{\overline{P}_c(t), t \geqq T \geqq 0\}$ can now be expressed as (11):

$$\begin{aligned} M_1(t) &= E[\bar{P}_c(t)] = P_c(T), \\ &\quad E[X(t-T)] = P_c(T)\, e^{-\mu(t-T)}, \qquad t \geqq T \geqq 0, \end{aligned} \tag{20a}$$

$$\begin{aligned} \sigma_1^2(t,t) &= \operatorname{Var}[\bar{P}_c(t)] = P_c(T), \\ &\quad \operatorname{Var}[X(t-T)] = P_c(T)\, e^{-\mu(t-T)}\left(1-e^{-\mu(t-T)}\right), \qquad t \geqq T \geqq 0, \end{aligned} \tag{20b}$$

$$\begin{aligned} \sigma_1^2(t,s) &= \operatorname{Cov}[\bar{P}_c(t), \bar{P}_s(s)] = P_c(T), \\ &\quad \operatorname{Cov}[X(t-T), X(s-t)] = P_c(t)\, e^{-\mu(t-T)}\left(1-e^{-\mu(s-t)}\right), \\ &\qquad 0 \leqq (s-t) \leqq (t-T), \end{aligned} \tag{20c}$$

where $M_1(t)$ = the mean or expected value,

$\sigma_1^2(t,t)$ = the variance function,

and $\sigma_1^2(t,s)$ = the covariance function, of the load $\bar{P}_c(t)$ on the concrete due to creep mechanism alone.

A Markov process implies a process which does not depend on what has happened in the past trials of the process, but only on what is happening at the present time. This is the characteristic nature of any Markov process, and in this problem this behavior is quite evident (see Fig. 2).

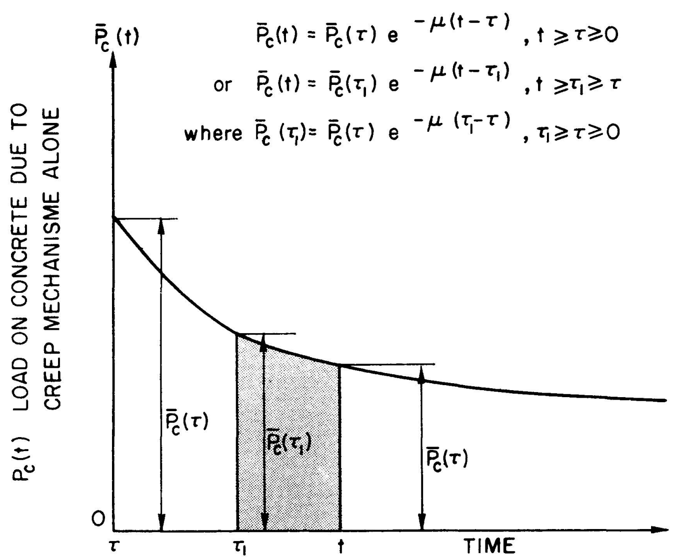

Fig. 2. Load variation on concrete due to creep alone.

The assumption that the Markov process is homogeneous is also a reasonable one for this problem, since the duration of the time interval, e.g. $(t-\tau_1)$ is very important in the evaluation of the load on the concrete at any time (see Fig. 2). In other words, the load $\bar{P}_c(t)$ on the concrete is a function of the time difference $(t-\tau_1)$ between the initial time τ_1 and the latter time t under consideration.

The linearity assumption for the Markov process is also a reasonable one for this problem because Eq. (17) shows that the rate function is a linear function of the load on the concrete. This is a characteristic feature of any Markov homogeneous linear process.

The assumption that the process is a "death process" follows directly from the fact that the load on the concrete is a decreasing function of time, as it is depicted by the physical nature of the problem under study.

Extension of Stochastic Process Model to Account for Combined Effects of Creep and Shrinkage Mechanisms

The results already obtained can benefit by extension of the stochastic process model to account for the effect of shrinkage mechanism. The time-dependent load which is transferred from the concrete to the steel due to the combined mechanisms of creep and shrinkage is greater than that due to creep alone [2, 4, 5, 6]. Eq. (14) shows this very clearly, that is:

$$P(t) = \underset{\text{(Creep Effect)}}{P_c(T)(1-e^{-\mu(t-T)})} + \underset{\text{(Shrinkage Effect)}}{S(1-e^{-\mu t})}, \qquad t \geqq T \geqq 0,$$

(see Eq. (14)).

Shrinkage forces can be considered as systems of internal forces in the concrete specimen.

The results of the deterministic solution (5) proposed in this work and actual observed results [1, 2, 4, 8], show that a functional relationship of the form:

$$\Delta P(t) = \underset{\text{(Creep Effect)}}{P_c(T)[1-X(t-T)]} + \underset{\text{(Shrinkage Effect)}}{S[1-X(t)]}, \qquad t \geqq T \geqq 0, \tag{21}$$

where $S = k(s)A_c$, can be proposed for the load transferred from the concrete to the steel. It is also assumed that the same process in the concrete accounts for the mechanisms of creep and shrinkage; the only difference is that while the creep contribution occurs at $t \geqq T \geqq 0$, the shrinkage contribution occurs at $t \geqq 0$. Moreover, the two processes $\{X(t-T); (t-T) \geqq 0\}$ and $\{X(t); t \geqq 0\}$ can also be considered as "homogeneous Markov linear death processes", with unit initial values.

The values of $S = k(s)A_c$ can usually be evaluated from actual data. As far as the total load transferred from concrete to steel is concerned; the quantity $P_c(T)$ which is the initial load on the concrete can be regarded as the maximum load which can be transferred from the concrete to the steel; while the quantity $S = k(s)A_c$ can be regarded as the maximum shrinkage load capacity for the concrete, which of course, is the maximum shrinkage load which can be transferred from the concrete to the steel.

Using the results of the deterministic analysis and the properties of "homogeneous Markov linear death process", the moment functions of the process $\{\Delta P(t); t \geqq T \geqq 0\}$ can be expressed simply as (11):

$$M(t) = E[\Delta P(t)] = P_c(T)(1-e^{-\mu(t-T)}) + S(1-e^{-\mu t}), \qquad t \geqq T \geqq 0, \tag{22a}$$

$$\sigma^2(t,t) = \mathrm{Var}[\Delta P(t)] = P_c(T)\, e^{-\mu(t-T)}(1-e^{-\mu(t-T)}) + S\, e^{-\mu t}(1-e^{-\mu t}), \qquad t \geqq T \geqq 0, \tag{22b}$$

$$\sigma^2(t,s) = \mathrm{Cov}[\Delta P(t), \Delta P(s)] = P_c(T)\, e^{-\mu(t-T)}(1-e^{-\mu(s-t)}) + S\, e^{-\mu t}(1-e^{-\mu s}), \qquad (t-T) \geqq (s-t) \geqq 0, \tag{22c}$$

where $M(t)$ = the mean or expected value;
$\sigma^2(t,t)$ = the variance function;
and $\sigma^2(t,s)$ = the covariance function;

for the total load transferred from the concrete to the steel.

The quantities $P_c(T)$ and $S = k(s) A_c$ are considered here as constants, which can be evaluated from experimental data.

The relationships given by Eqs. (15) and (16), which are:

$$P_c(t) = P_c(T) - \Delta P(t), \ t \geqq T \geqq 0 \ \text{ and } \ P_s(t) = P_s(T) + \Delta P(t), \ t \geqq T \geqq 0$$

and the results given by Eqs. (22a, b, and c), can now be used to obtain the moment functions for the process $\{P_c(t), t \geqq T \geqq 0\}$ for the load on the concrete, and the process $\{P_s(t); t \geqq T \geqq 0\}$ for the load on the steel.

Use of Stochastic Process Model in Predicting Upper and Lower Bounds for Loads

For instance, if a scatter of one standard deviation from the mean value is assumed, then the upper and lower bounds for $\Delta P(t)$ which is the load transferred from the concrete to the steel, can be expressed as, $M(t) + \sigma(t,t)$ and $M(t) - \sigma(t,t)$ respectively at any particular time, $t \geqq T \geqq 0$. Moreover, the upper and lower bounds for $P_c(t)$ which is the load on the concrete at any time, can be expressed as $E[P_c(t)] + \sigma(t,t)$ and $E[P_c(t)] - \sigma(t,t)$; while the upper and lower bounds for $P_s(t)$ which is the load on the steel at any time, can be expressed as $E[P_s(t)] + \sigma(t,t)$ and $E[P_s(t)] - \sigma(t,t)$.

The ability to make this type of conclusion should have great applications in many practical structural engineering situations.

Long-Time Correlation Between Creep and Shrinkage Loads

After a long time of loading, the ratio of the loads transferred from the concrete to the steel, due to the mechanisms of shrinkage and creep can be represented by:

$$h = \frac{S}{P_c(T)} = \frac{k(s) A_c}{f_c(T) A_c} = \frac{k(s)}{f_c(T)}, \tag{23}$$

where $P_c(T) = f_c(T)A_c$, $S = k(s)A_c$ and $f_c(T)$ is the initial stress on the concrete when it is loaded. The quantity h can be regarded as a constant for a given reinforced concrete column under given operating conditions, and under a given constant external loading. This is a very useful parameter in many practical structural engineering problems, since it is always important to know how to evaluate long time creep and shrinkage characteristics of reinforced concrete columns.

Application of Method Developed to the Prediction of Time-Dependent Stresses in Reinforced Lightweight Concrete Columns (4, 12)

The method developed here has been used by the author [12] to predict the time-dependent stresses in reinforced lightweight concrete columns. The recent data [4] obtained by HOLM and PISTRANG have been used by the author

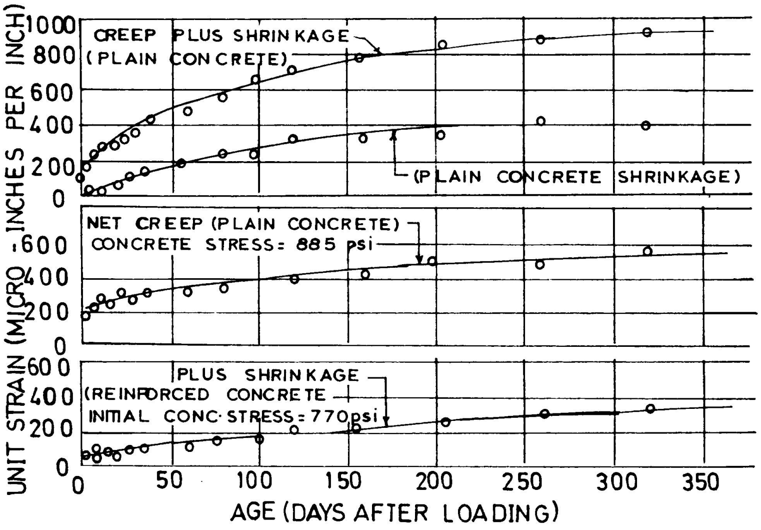

Fig. 3. Time-dependent shrinkage and creep (4).

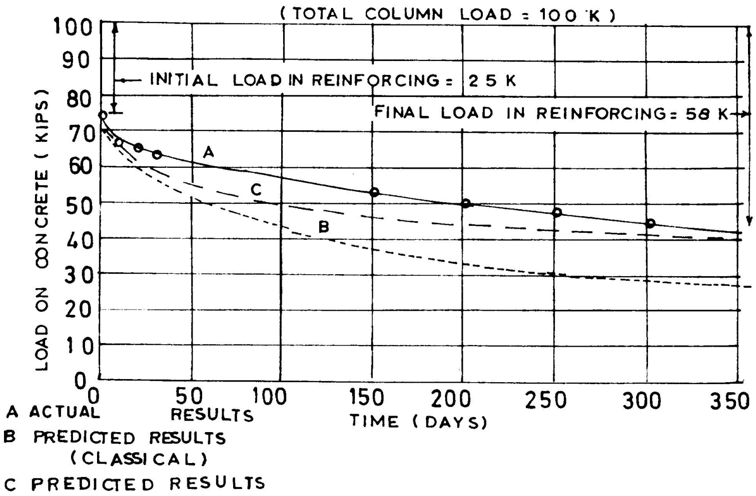

Fig. 4. Time-dependent load transfer (4).

[12]. The results obtained by HOLM and PISTRANG for time-dependent shrinkage and creep for the concrete used are given in Fig. 3; while the actual experimental time-dependent load transfer in the column is given by Fig. 4.

It is desired to use the present development to evaluate the stresses in the concrete and steel, and the predicted upper and lower values for these stresses, after one year; and then compare the results with actual experimental data [4].

The basic data [4] for the test results are as follows: Column cross section (gross);

10 in. × 10 in.	A	$= A_c + A_s = 100$ sq. in.
Steel cross-section: 4, No. 8 bars.	A_s	$= 3.14$ sq. in.
Steel ratio	p	$= 0.0314$
Column load	P	$= 100$ kips
Steel Elastic Modulus	E_s	$= 30.5 \times 10^6$ psi
Concrete Elastic Modulus	E_c	$= 2.91 \times 10^6$ psi
Modular Ratio, steel to concrete:	n	$= \frac{30.5}{2.91} = 10.5$
Load on 6 in. diameter test cylinders	P_{cyl}	$= 25$ kips
Time dependent cylinder strain (creep and shrinkage after 1 year)	$\rho + s$	$= 960 \times 10^{-6}$ in. per in.
Shrinkage – only cylinder strain (after 1 year)	s	$= 420 \times 10^{-6}$ in. per in.
Creep – only cylinder strain (after 1 year)	ρ	$= 540 \times 10^{-6}$ in. per in.

Initial Elastic Column Stresses

Concrete:
$$f_c(0) = \frac{P}{A_c} \frac{A_c E_c}{A_c E_c + A_s E_s} = 770 \text{ psi}.$$

Steel:
$$f_s(0) = \frac{P}{A_s} \frac{A_s E_s}{A_c E_c + A_s E_s} = 8080 \text{ psi}.$$

Concrete Cylinder Creep Stress

$$f_{cyl} = \frac{P_{cyl}}{A} = \frac{25{,}000}{\frac{\pi}{4} \times 6^2} = 885 \text{ psi}.$$

Specific Creep Strain for 6 in. Diameter Cylinder at One Year ($t = 365$ days)

$$c(t) = c(365) = \frac{\rho}{f_{cyl}} = \frac{540 \times 10^{-6}}{885} = 0.610 \times 10^{-6} \text{ in. per in. per psi}.$$

Time Dependent Creep and Shrinkage Strains Adjusted for a 10 In. × 10 In. Column Size

The authors [4] reported a size effect co-efficient of 0.8, and this was determined by comparing shrinkage specimens (10 in. × 10 in. square sections compared with 6 in. diameter cylinders), the same ratio is applied to creep calculations as well, because the inhibiting effect of size is approximately the same for both creep and shrinkage mechanisms. The value of 0.8 is comparable to that reported by other investigators [2, 13].

Adjusted value for specific creep strain at one year

$$c(365) = 0.8 \times 0.610 \times 10^{-6} = 0.488 \times 10^{-6} \text{ per in. per psi.}$$

Adjusted value for shrinkage strain at one year

$$s(365) = 0.8 \times 420 \times 10^{-6} = 336 \times 10^{-6} \text{ in. per in.},$$

$$\theta = \frac{p E_s}{1+(n-1)p} = \frac{10^6}{1.35} \text{psi},$$

$$k(s) = \frac{s(t)}{c(t)} = 292 \text{ psi (see Eq. (5))}.$$

The value of $k(s) = 292$ psi is the best fit to the actual data supplied by HOLM and PISTRANG [4].

When $t = 365$ days, $c(t) = 0.488 \times 10^{-6}$ in. per in. per psi, as shown above. Therefore, the following result follows (at $t = 365$ days):

$$\mu t = \theta_c(t) = (0.488 \times 10^{-6}) \left(\frac{10^6}{1.35}\right) = 0.36 \text{ (see Eq. (12))}.$$

Moreover, the value of $S = k(s) A_c$ is given by:

$$S = k(s) A_c = (292)(0.9686 \times 100) \text{ lb.} = 28200 \text{ lb.},$$

while the value of $$h = \frac{k(s)}{f_c(T)} \text{ (see Eq. (23))}$$

becomes, $$h = \frac{k(s)}{f_c(T)} = \frac{292}{770} = 0.38.$$

The following value can now be obtained for the load on the concrete, after one year:

$$\begin{aligned} E = [P_c(t)] &= P_c(0) e^{-\mu t} - S(1 - e^{-\mu t}) \\ &= (770 \times 96.86)(0.67) - (28200)(0.33) \cong 42{,}000 \text{ lb.} \end{aligned}$$

The mean stress on the concrete after one year now becomes

$$\frac{42{,}000}{96.86} = 440 \text{ psi}.$$

The mean load on the steel at $t = 365$ days and $T = 0$ is

$$\begin{aligned} E[P_s(t)] &= P_s(0) + P_c(0)(1 - e^{-\mu t}) + S(1 - e^{-\mu t}) \\ &= 8080 \times 3.14) + (770 \times 96.86)(0.33) + (28200)\ (0.33) \cong 58{,}000 \text{ lb.} \end{aligned}$$

The mean stress on the steel after one year now becomes

$$\frac{58{,}000}{3.14} = 18{,}500 \text{ psi.}$$

The expected total load transferred from concrete to steel during the one year period is

$$\Delta P(t) = P_c(0) - P_c(t) = 75{,}000 - 42{,}000 = 33{,}000 \text{ lb.}$$

The variance function for the stresses on the concrete can be expressed as [5, 12]:

$$\sigma_c^2(t,t) = f_c(T)\, e^{-\mu(t-T)}(1 - e^{-\mu(t-T)}) + k(s)\, e^{-\mu t}(1 - e^{-\mu t}), \qquad t \geqq T \geqq 0,$$

and when $t = 365$ days and $T = 0$.

$$\begin{aligned} \sigma_c^2(t,t) &= \{f_c(T) + k(s)\}\, e^{-\mu t}(1 - e^{-\mu t}) \\ &= (770 + 292)(0.33)(0.67) = 240\,(\text{psi})^2, \end{aligned}$$

hence $\sigma_c(t,t) \cong 15.5$ psi.

Therefore, assuming a scatter of one standard deviation from the mean value, the upper and lower values for the stresses in the concrete can be expressed as:

$440 + 15.5$ psi and $440 - 15.5$ psi or 456 psi and 425 psi, while the mean value is 440 psi. The mean stress of 440 psi on the concrete, corresponds to a load of 42,000 lb. on the concrete. This result compares very well with the actual experimental value of exactly 42,000 lb. obtained in the data of HOLM and PISTRANG [4].

The upper and lower values of 456 psi and 425 psi on the concrete, correspond to loads of 44,000 lb. and 41,000 lb. respectively on the concrete. These values mean that since the total external axial load acting on the concrete is 100,000 lb., then the upper and lower loads on the steel are given respectively as 59,000 lb. and 56,000 lb. These values mean upper and lower stress values of 19,000 psi and 18,000 psi respectively on the steel. The mean value of the load on the steel, which is approximately given as 58,000 lb. compares very well with the actual experimental value of exactly 58,000 lb. obtained in the data of HOLM and PISTRANG [4].

Furthermore, the predicted upper and lower values for loads and stresses on the concrete and steel, are useful additional information.

Conclusion

The following major results can be mentioned in the conclusion to this work:

1. It is possible to formulate a realistic, probabilistic, stochastic model for the phenomenon of time-dependent load transfer in reinforced concrete columns.

2. The stochastic model enables meaningful and realistic predictions to be made concerning the possible upper and lower bounds, in the values of resulting stresses occuring during the phenomenon of time dependent load transfer in reinforced concrete columns.
3. The ability of the stochastic model to predict not only the mean value, but also the possible bounds about the mean value, for the phenomenon under study should have useful applications in structural engineering practice.
4. The results obtained by the use of stochastic process model compare very well with actual experimental results.

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Summary

A stochastic process model is proposed for the phenomenon of time-dependent stress transfer from concrete to steel in reinforced concrete columns, subjected to axial loading. The phenomenon is caused by the mechanisms of creep and shrinkage. The importance of the stochastic model is brought out, in that it is now possible to have more meaningful information concerning the phenomenon under study. Furthermore, the importance of the model in practical problems involving the estimation of possible longtime stresses in structures is mentioned. The results of the stochastic model are compared with practical test results, and good agreements are obtained.

Résumé

On propose un modèle de procédé aléatoire pour le phénomène de transfert de la sollicitation en fonction du temps du béton à l'acier dans les colonnes en acier-béton soumises à des charges axiales. Le phénomène est causé par le fluage et le retrait. L'importance du modèle aléatoire est démontrée en tant qu'il permet de gagner une information plus importante du phénomène à étudier. En plus, l'importance du modèle pour les problèmes pratiques, comprenant l'estimation de sollicitations de longue durée dans les structures est mentionnée. Les résultats au modèle sont comparés à ceux obtenus en pratique et une bonne concordance a été constatée.

Zusammenfassung

Es wird ein stochastisches Modell zu einem Verfahren über das Phänomen der zeitbedingten Beanspruchungs-Übertragung von Beton auf Stahl in Stahlbetonstützen unter axialer Belastung vorgeschlagen. Das Phänomen wird durch Kriechen und Schrumpfen verursacht. Mittels des stochastischen Modells wird es jetzt möglich, wichtige Erkenntnisse über das genannte Phänomen zu gewinnen. Ferner wird die Bedeutung des stochastischen Modells bei praktischen Problemen unter Einschluss möglicher langzeitiger Beanspruchungen an Bauwerken hervorgehoben. Die Ergebnisse am Modell werden mit den praktischen Versuchsresultaten verglichen, wobei sich eine gute Übereinstimmung zeigte.

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