Torsional displacements in multi-storey structures

Torsional Displacements in Multi-Storey Structures

Déplacements torsionnels dans les structures à plusieurs étages

Verdrehungen in mehrstöckigen Tragwerken

J. H. WYNHOVEN
Dr. Ing., John Connell & Associates, Consulting Engineers, Melbourne, Australia

P. F. ADAMS
Professor, The University of Alberta, Department of Civil Engineering, Edmonton, Alberta, Canada

Introduction

In designing a structure such as that shown in Fig. 1 for lateral load, it is first necessary to determine the distribution of the load to the various stiffening elements. In addition, the lateral drift of the structure must be limited [1] to prevent discomfort to the occupants or damage to the non-structural components.

Although elastic methods of analysis are available for space frames [2], they are expensive to apply for large structures. Instead, the loads are generally distributed on the basis of either the tributary area or the relative stiffnesses of the elements. The additional overturning moments (P-Δ moments) resulting from the vertical load acting through the respective storey displacements are not considered.

First order elastic analyses [3, 4, 5] of asymmetric structures have shown that the members on one side of the structure will undergo larger displacements than those near the center of the structure. These increased displacements are caused by the rotation of the floor diaphragms. Second-order elastic-plastic analyses of planar bents [6, 7] indicate that the P-Δ moments considerably reduce the ultimate load carrying capacity. Reports [8, 9] on the collapse of multi-storey structures during earthquakes, have mentioned the influence of the torsional displacements in causing catastrophic failures.

This paper considers the influence of torsional displacements throughout the complete load-displacement response of a structure. The discussion is supported by results obtained from a behavior study [11] performed on a series of ten and twenty-four storey structures. The structures are composed of two series of orthogonal bents, consisting of frame elements or of frame elements coupled to reinforced concrete shear walls. The approximate analysis considers the second-order (P-Δ) effect of the vertical load and the elastic-plastic response of the members.

Method of Analysis

The analytical model and the method of analysis have been previously described [12] and only an outline will be presented here.

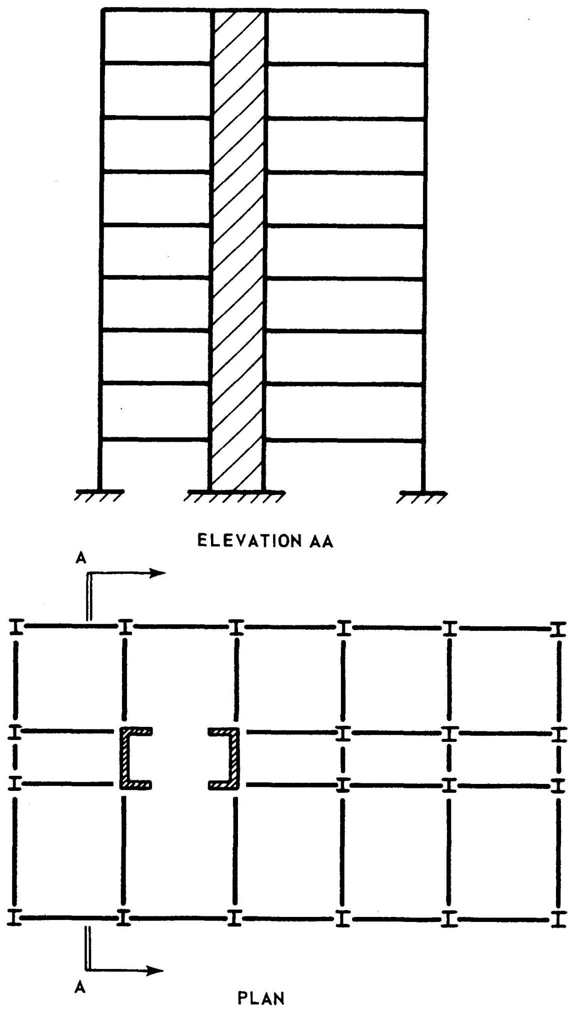

Fig. 1. Typical three dimensional building.

For the structure shown in Fig. 1, the floor diaphragms are assumed to be infinitely stiff in their own planes, while the torsional stiffnesses of the beams are neglected. This enables the structure to be represented as two series of intersecting planar bents. The displacement of each bent, at a particular floor, may be expressed in terms of the rigid body displacement of the floor diaphragm (two translations and a rotation about the vertical axis). The model is further simplified by representing each planar bent by an equivalent lumped bent [10, 13]. The members of the lumped bent have the same stiffness and strength as the sum of the stiffnesses and strengths of the individual members in the original unlumped bent. The response of individual members is idealized as elastic-perfectly plastic. A rectangular interaction relationship is adopted to determine the ultimate strength of the vertical stiffening elements under biaxial bending moments. The plastic moment capacities of the wide flange steel columns are reduced to account for the presence of axial loads [19].

Slope-deflection equations are used to formulate equations of equilibrium for the simplified model. The overturning moments (P-Δ) of the vertical load are expressed as additional horizontal shears to be resisted in each storey of every bent. The equations of equilibrium must be rewritten each time the plastic hinge configuration changes because the stiffness coefficients depend on the plastic hinge configuration of the members.

The equations are arranged in matrix form and solved for the unknown displacements by using a modified Gauss Elimination technique. The complete load displacement relationship of the model is predicted up to, and beyond, the ultimate load carrying capacity. The ultimate load is the load at which the stiffness of a structure changes from positive to negative. A negative stiffness means that the lateral displacement increases under a decreasing lateral load (unloading).

Influence in the Elastic Range

When the structural layout and/or the applied lateral or vertical load are asymmetric, a structure must twist in order to satisfy equilibrium. The rotations increase the displacements on one side of the structure, which means that the members in these bents are subjected to increased forces. Consequently, the lateral load may no longer be distributed on the basis of the relative stiffnesses of the bents; the arrangement of the stiffening elements must also be taken into consideration. A structural layout in which the center of stiffness does not coincide with the point of load application will result in torsional displacements. These may be limited by increasing the torsional stiffness of the structure.

The quantitative influence of the torsional displacement is illustrated by the results of a behavior study using the ten storey structure shown in Fig. 2. The lateral load consists of a uniform wind pressure of 16 p.s.f., resulting in the floor loads shown on the elevation in Fig. 2. The vertical load was assumed to be equal to zero. Elastic analyses were carried out with $X = 0$, 90, 180, 270, and 360 inches, where X defines the location of the shear wall bent (bent 2 in Fig. 2) relative to the center of the structure.

The lateral displacements of bents 1 to 4 at the top floor are plotted, in Fig. 3, against the eccentricity, X, of the shear wall bent (bent 2 in Fig. 2).

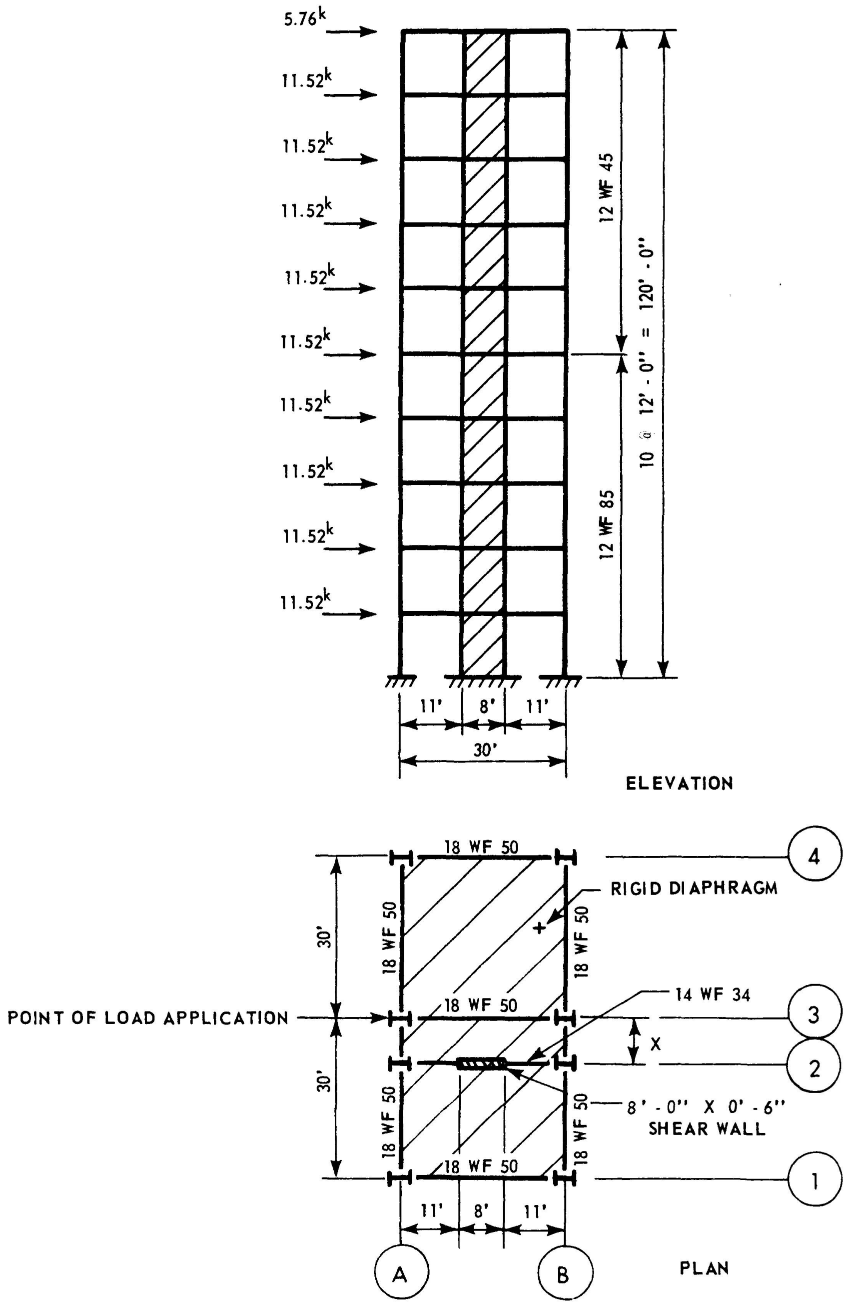

Fig. 2. Ten story structure *M*.

The influence of the torsional displacement with increasing eccentricity X, is clearly shown by the rapid increase in the displacement of bent 4. The redistribution of the lateral load is indicated in Table 1 which lists the shears for the wall in bent 2, and the columns in bent 4 (Fig. 2), for the various eccentricities of the shear wall bent. In the bottom storey, the shear that must be resisted by the columns of bent 4 increases from 5 to 37 percent, while the shear in the wall decreases from 78 to 37 percent, of the total lateral load, as X increases from 0 to 360 inches.

For $X = 0$ the drift at the top floor is 1.9 inches (Fig. 3); at $X = 360$ inches

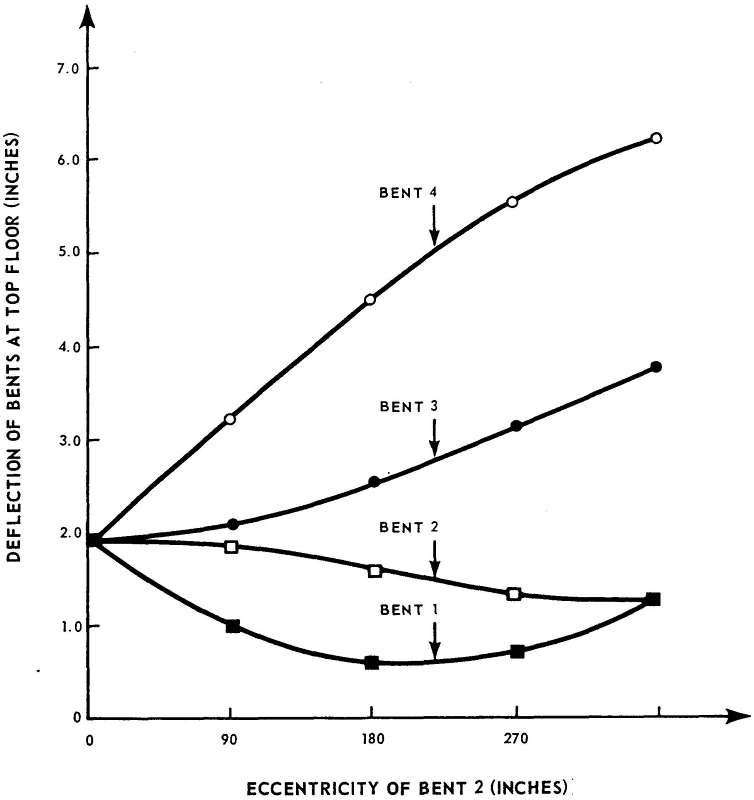

Fig. 3. Bent deflections at top floor.

Table 1. Shear tabulation for bent 4 and shear-wall

STORY NUMBER	EFFECT OF INCREASING ASYMMETRY									
	Story Shear in Bent 4 Kips					Story Shear in Shear Wall Kips				
X	0	90	180	270	360	0	90	180	270	360
1	5.53	16.68	27.35	35.31	40.22	85.21	79.88	67.01	52.94	40.94
2	8.55	16.31	24.12	30.35	34.54	59.01	56.09	48.74	39.98	31.93
3	9.16	15.43	21.62	26.76	30.28	44.33	42.40	37.29	31.08	25.17
4	8.65	13.80	18.97	23.24	26.21	35.06	33.59	29.74	24.97	20.36
5	8.59	12.30	16.05	19.35	21.84	23.49	22.89	20.98	18.23	15.27
6	5.22	9.76	14.04	17.34	19.32	27.45	25.83	21.84	17.45	13.67
7	5.01	8.05	10.99	13.28	14.87	16.85	16.02	13.92	11.42	9.13
8	3.76	5.86	7.94	9.56	10.60	11.17	10.60	9.25	7.62	6.11
9	2.63	3.78	4.84	5.74	6.33	4.93	4.76	4.21	3.55	2.91
10	2.37	2.12	1.92	1.89	1.97	-5.61	-5.10	-3.96	-2.80	-1.94

the drift at the top floor of bent 4 is 6.2 inches or 1/230 of the height of the structure. A normally acceptable limit for the lateral drift is 1/500 of the height of the structure, at the working load level [1].

The following approximate method may be used to estimate the influence of the P-Δ moments. For the $(n+1)$ storey from the top of the structure shown in Fig. 1, the vertical load P, the P-Δ shear V, and the lateral load H, are expressed as follows:

$$P = b\,d\,w\,(n+0.5), \tag{1}$$

$$V = P\left(\frac{\Delta}{h}\right), \tag{2}$$

$$H = b\,h\,p\,(n+0.5), \tag{3}$$

where b, d and h are the width, depth and storey height of the structure, respectively, w and p are the vertical and lateral load per unit area, while (Δ/h) is the storey sway at the working load level. It being assumed in Eqs. (1) and (3) that the load on the top floor is equal to one half of the load on the other floors, and that the storey height is uniform. The P-Δ shear V, may be expressed relative to the applied lateral load H, as follows:

$$\frac{V}{H} = \frac{d\,w}{h\,p}\left(\frac{\Delta}{h}\right). \tag{4}$$

By substituting for the ten storey structure in Fig. 2 for a drift limitation of 1/600 the ratio would be,

$$\frac{V}{H} = \frac{30 \cdot 100}{12 \cdot 16} \cdot \frac{1}{600} = \frac{2.6}{100}. \tag{5}$$

A second order analysis of the structures described above with a floor load of 100 p.s.f., indicated that the increase in top floor displacements were of the order of 2 to 3 percent. Consequently, the P-Δ effect may be ignored for these structures at the working load level. The P-Δ effect only becomes significant for more heavily loaded or slender structures.

Influence in the Elastic-Plastic Range

As loading increases above the working load level, the members on the side of the structure undergoing the larger displacements will be first to yield. This yielding not only reduces the lateral and torsional stiffness of the structure but also increases the effective torsional load. The increased torsional load is caused by the redistribution of the stiffness which moves the center of stiffness further away from the point of application of the lateral load. Corresponding to the shift in the center of stiffness the center of rotation also moves further from the point of application of the load. This causes a net increase in the P-Δ moments. The center of rotation is that point on a particular floor which does not experience a change in the lateral displacement due to the torsional displacement of that floor.

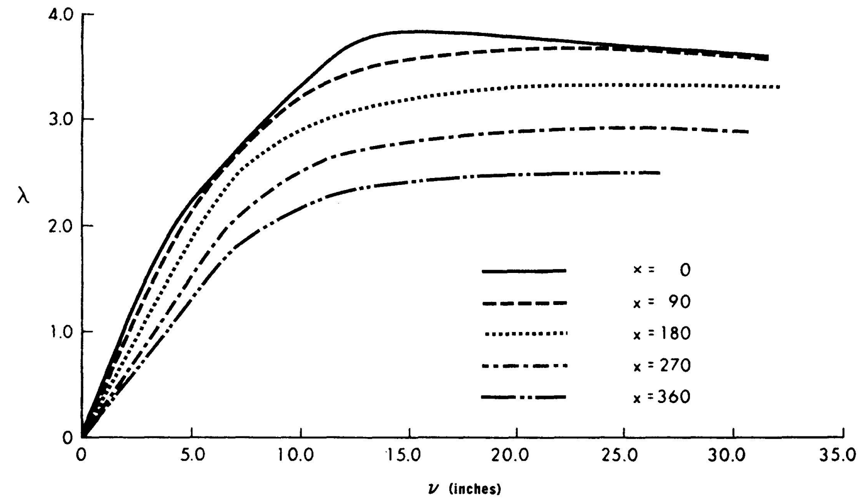

Fig. 4. Load-displacement response structures M 1, M 6–M 9.

Once an asymmetric structure enters the elastic-plastic range the asymmetry increases as yielding progresses. The ability of the structure to carry higher lateral loads decreases rapidly, as for each increase in displacement an additional part of the load carrying capacity is required to resist the P-Δ moments.

The influence of the torsional displacement in the elastic-plastic range is illustrated by the load-displacement curves obtained for structures M 6 to M 9 and M 1, and shown in Fig. 4. The structures are identical to those discussed in the previous section but with a uniformly distributed load of 100 p.s.f. applied to each floor. In Fig. 4 the load factor, λ, is plotted against the displacement, v, of the center of the top floor. A load factor λ, of 1.0 corresponds to the lateral loads shown on the elevation of Fig. 2. The curves differ in the

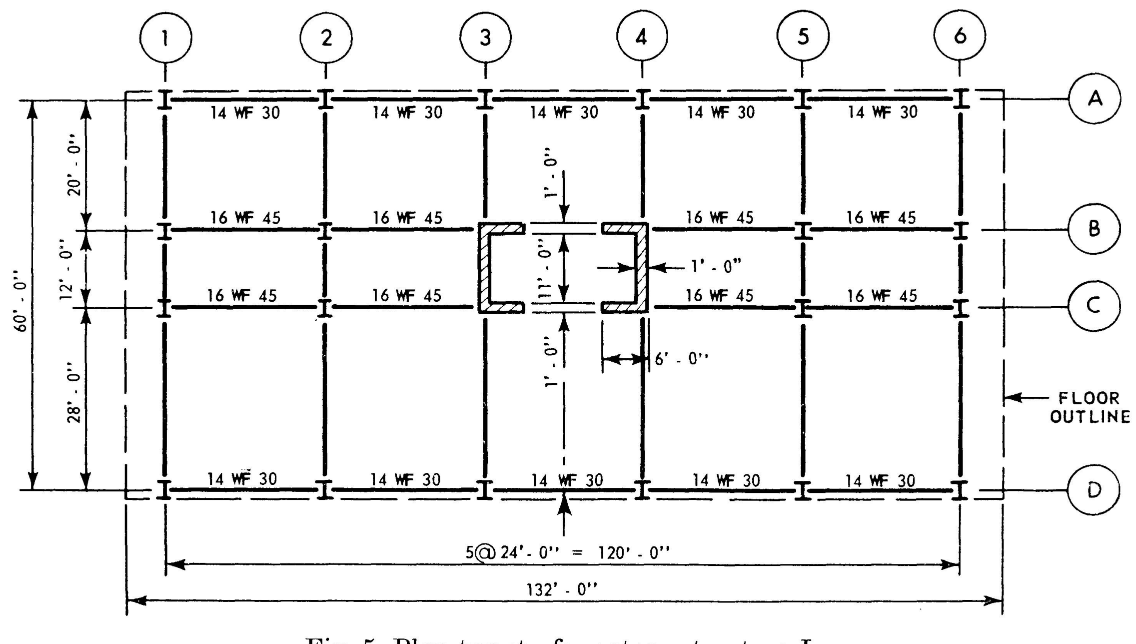

Fig. 5. Plan twenty-four story structure L.

elastic range (Fig. 4) because of the asymmetry of the structural layout as defined by the eccentricity X, of the shear wall bent (Fig. 2). As yielding commences there is a more rapid decrease in stiffness for the structures with higher asymmetry. The reductions in the ultimate load carrying capacity are 3, 12, 23 and 34 percent (relative to structure M 1) for structures M 6, M 7, M 8, and M 9, respectively.

The increased effect of the P-Δ moments for a heavily loaded structure is illustrated by the load-displacement, and load-floor rotation curves obtained for the 24 storey structure shown in Figs. 5 and 6. The structure was analyzed for eccentricities, e, of the lateral load of 0, 72, 144, and 288 inches (Fig. 5). The vertical load on each of the floors is 280 p. s. f. The load-displacement curves in Fig. 7 show the load factor, λ, plotted against the lateral displacement, v, of the center of the top floor. A load factor, λ, of 1.0 corresponds to a wind load of 20 p. s. f. applied to the wide face of the structure. The load-

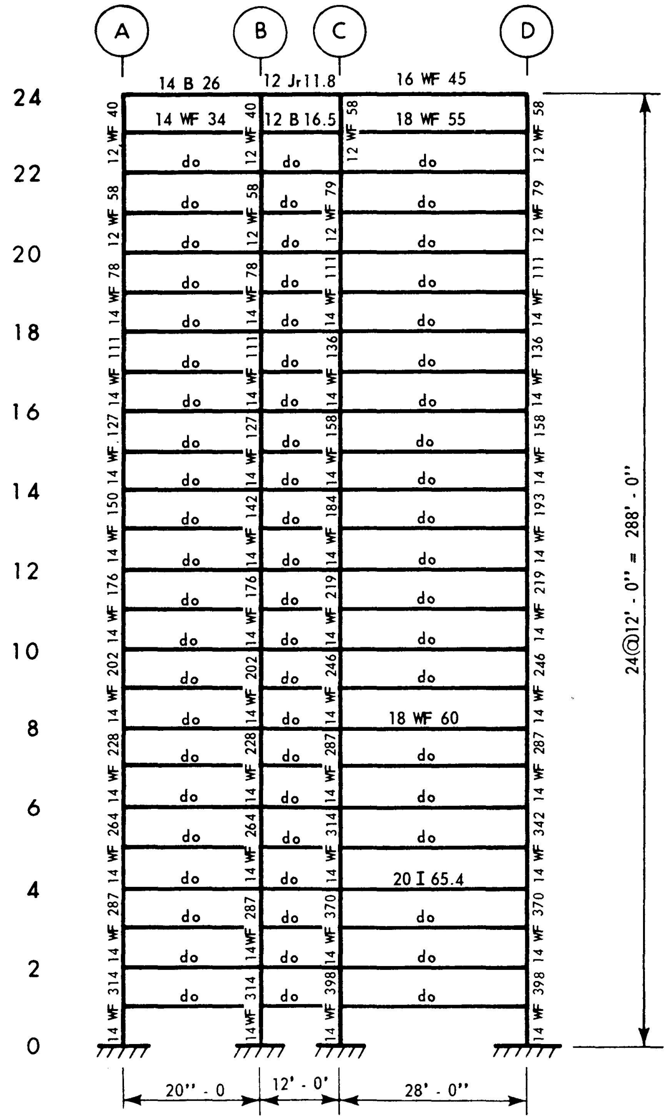

Fig. 6. Elevation twenty-four story structure L.

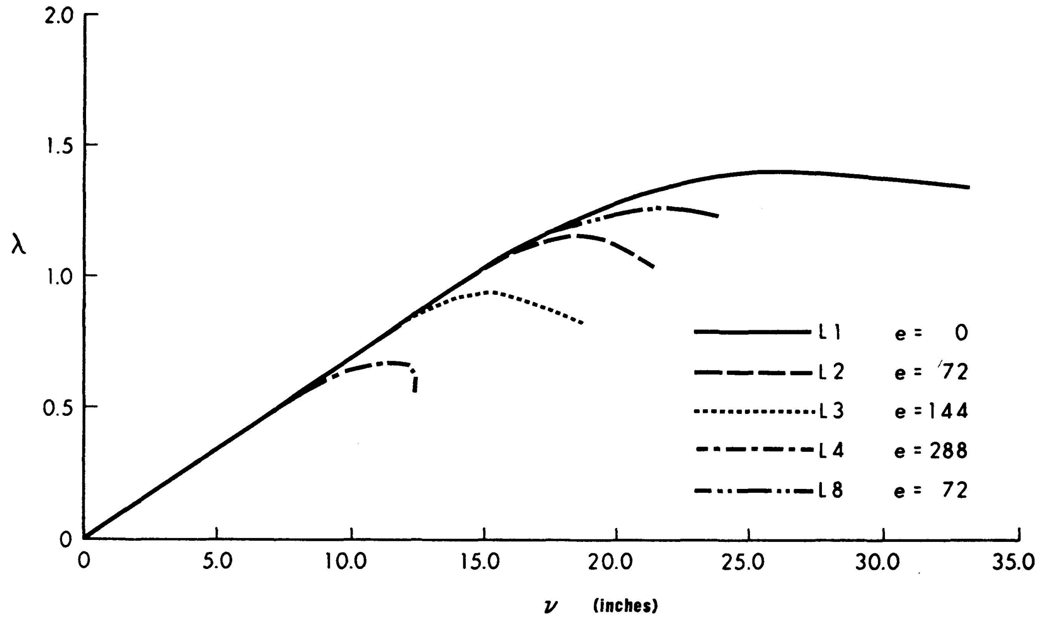

Fig. 7. Load-displacement response structures L 1–L 4, L 8.

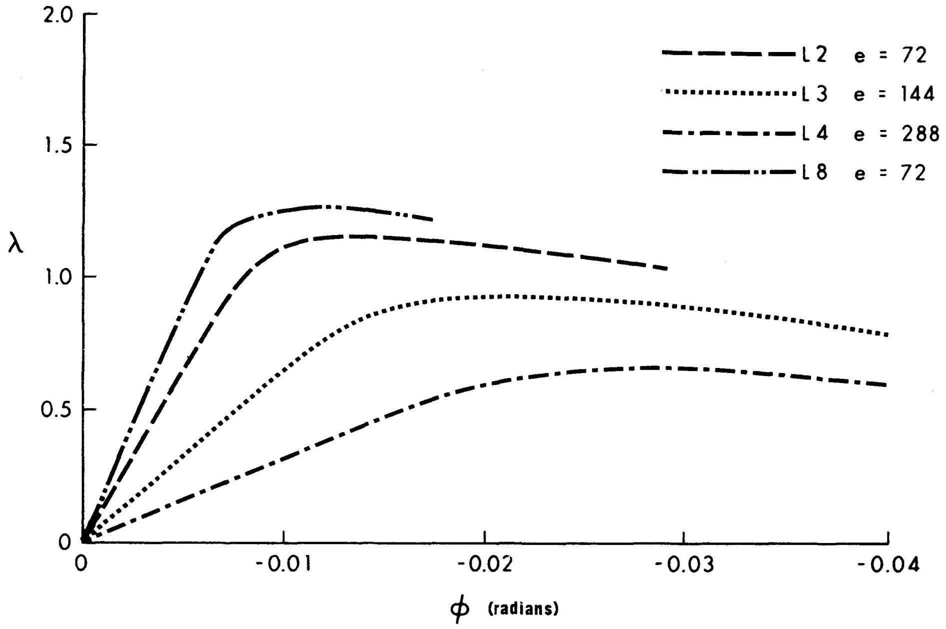

Fig. 8. Load-floor rotation response structures L 2–L 4, L 8.

floor rotation curves are shown in Fig. 8, where ϕ is the floor rotation of the top floor.

The curves in Fig. 7 coincide in the elastic range because, until yielding starts in the members, the center of rotations are at the center of the structure. The reductions in the ultimate load carrying capacity are 17, 33, and 53 percent (relative to structure L 1) for structures L 2, L 3, and L 4, respectively. The curves for structure L 8 (Figures 7 and 8) show the effect of increasing the St. Venant torsional stiffness (GK_T) of the structure. The GK_T value for structure L 8 was increased to 10 times the value for structures L 1 to L 4. The reduction in ultimate load carrying capacity for structure L 8 ($e = 72$ inches) is only 9 percent as opposed to 17 percent for structure L 2. In the analysis (11) it is assumed that the St. Venant torsional stiffness does not change throughout the loading history.

References

1. Response of Buildings to Lateral Forces. Report by Committee 442, Journal of ACI, February 1971.
2. ICES STRUDL II, Engineering User's Manual, Vol. 1, Frame Analysis, Report R 68-91, Massachusetts Institute of Technology, Department of Civil Engineering, Cambridge, Massachusetts, November 1968.
3. WEAVER, W. and NELSON, M. F.: Three-Dimensional Analysis of Tier Buildings. Journal of the Structural Division, ASCE, Vol. 92, ST 6, 1966.
4. WINOKUR, A. and GLUCK, J.: Lateral Loads in Asymmetric Multi-Storey Structures. Journal of the Structural Division, ASCE, Vol. 94, ST 3, 1968.
5. GLUCK, J.: Lateral Load Analysis of Asymmetric Multi-Storey Structures. Journal of the Structural Division, ASCE, Vol. 96, ST 2, 1970.
6. KORN, A. and GALAMBOS, T. V.: Behavior of Elastic-Plastic Frames. Journal of the Structural Division, ASCE, Vol. 94, ST 5, 1968.
7. CLARK, W. J., MACGREGOR, J. G., and ADAMS, P. F.: Inelastic Behavior of Reinforced Concrete Shear Wall Frame Structures. Proceedings of the Eighth Congress International Association for Bridge and Structural Engineering, September 1968.
8. BERG, G. V. and STRATTA, J. L.: Anchorage and the Alaska Earthquake of March 12, 1964. American Iron and Steel Institute, New York, 1964.
9. HANSON, R. D. and DEGENKOLB, H. J.: The Venezuela Earthquake, July 29, 1967. American Iron and Steel Institute, New York, 1968.
10. KHAN, F. R. and SBAROUNIS, J. A.: Interaction of Shear Walls with Frames in Concrete Structures under Lateral Loads. Journal of the Structural Division, ASCE, Vol. 90, ST 3, 1964.
11. WYNHOVEN, J. H. and ADAMS, P. F.: Elastic-Plastic Analysis of Three Dimensional Structures. Structural Engineering Report No. 29, University of Alberta, September 1970.
12. WYNHOVEN, J. H. and ADAMS, P. F.: Analysis of Three Dimensional Structures. Journal of the Structural Division, ASCE, Vol. 98, ST 1, January 1972.
13. MAJUMDAR, S. N. G., NIKHED, R. P., MACGREGOR, J. G., and ADAMS, P. F.: Approximate Analysis of Frame-Shear Wall Structures. Structural Engineering Report No. 14, University of Alberta, May 1968.
14. GALAMBOS, T. V.: Structural Members and Frames. Prentice Hall Inc., Englewood Cliffs, N. J., 1968.

Summary

Structures which have either an asymmetric layout or load distribution (lateral or vertical) will undergo torsional displacements. The influence of the torsional displacement on the structural behavior depends directly on the eccentricity between the center of stiffness and the point of application of the load.

In the elastic range, the twisting of the structure causes a redistribution of the horizontal shear, resulting in increased stresses in the exterior bents. In the elastic-plastic range the torsional displacements reduce the ultimate load carrying capacity because of the increased severity of P-Δ moments.

Résumé

Les structures de forme ou de charge asymétrique (latérale ou verticale) sont soumises à des déplacements torsionnels. L'influence de la torsion sur le comportement structurel dépend directement de l'excentricité entre le centre de raidissement et le point d'application de la charge.

Dans le domaine élastique la torsion de la structure entraîne une nouvelle distribution du cisaillement horizontal dont une augmentation de la résultante des parties extérieures. Dans le domaine élasto-plastique la torsion réduit la charge ultime ce qui est dû à l'influence croissante des moments P-Δ.

Zusammenfassung

Tragwerke von asymmetrischem Aufbau oder asymmetrischer Lastverteilung (seitlich oder vertikal) sind einer Verdrehung ausgesetzt. Der Einfluss der Verdrehung auf das Tragverhalten hängt direkt vom Abstand zwischen dem Steifigkeitszentrum und dem Angriffspunkt der Last ab.

Im elastischen Bereich bewirkt die Verdrehung des Tragwerkes eine Neuverteilung des Horizontalschubes und daher der daraus resultierenden höheren Beanspruchung der aussenliegenden Teile. Im elastisch-plastischen Bereich vermindert die Verdrehung die Traglast infolge des zunehmenden Einflusses der P-Δ-Momente.

Leere Seite
Blank page
Page vide