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Critical Loads of Building Frames

Charges critiques de constructions en cadres

Kritische Lasten von Rahmentragwerken

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Introduction

The theoretical solution of the problem of finding the critical load of orthogonal building structures within the elastic range is already known. In practice, however, the determination of that load – excepting very simple cases – is difficult, for it involves the solution of a great number of transcendental equations. It is sufficient for the Structural Engineer the knowledge of the approximate value of that load, if it leads to an acceptable safety factor. Therefore, if a simple method is known that yields a sufficiently approximate answer, the exact solution may be waved. The conventional simplistic procedure consisted in finding the buckling load of the columns of the building, one by one, and assuming that the effect of the link with the remainder of the structure could be represented by a single buckling length. In this article a method of solution [1] is suggested which lies between the simplistic and the theoretically exact solution. "Theoretically" exact since even the most complicated solution mentioned above, with the transcendental equations, does not avoid certain facts that lead to discrepancies in relation to the actual structure, due to the lack of knowledge of certain values and the way of considering certain factors, such as: the value of the modulus of elasticity E, the effects of the beam-column connections and of the rigidity of the slabs, the uncertainty of the load distribution, the heterogeneity of the material, in particular if it is reinforced concrete.

Hypothesis

Consider a plane structure with several vertical columns fixed at the same level at the lower end, and m stories with horizontal uninterrupted beams [2]. The cross section of the columns remains constant within each story, but it may vary from one

[1] The method originated from another more elementar one which considers only one of the various systems of stacked columns of the building, with only one parameter to be determined; it has been suggested by the Author several years ago [1].

[2] If it happens that some of the columns, and attached girders, do not extend all the way to the top of the structure, the method may still be applied if we assume that they do exist, but with $I = 0$.

story to another; there are no restrictions to the beam sizes, and the loads – assumed vertical – are applied at the joints of the structure without any intensity relations among them[1].

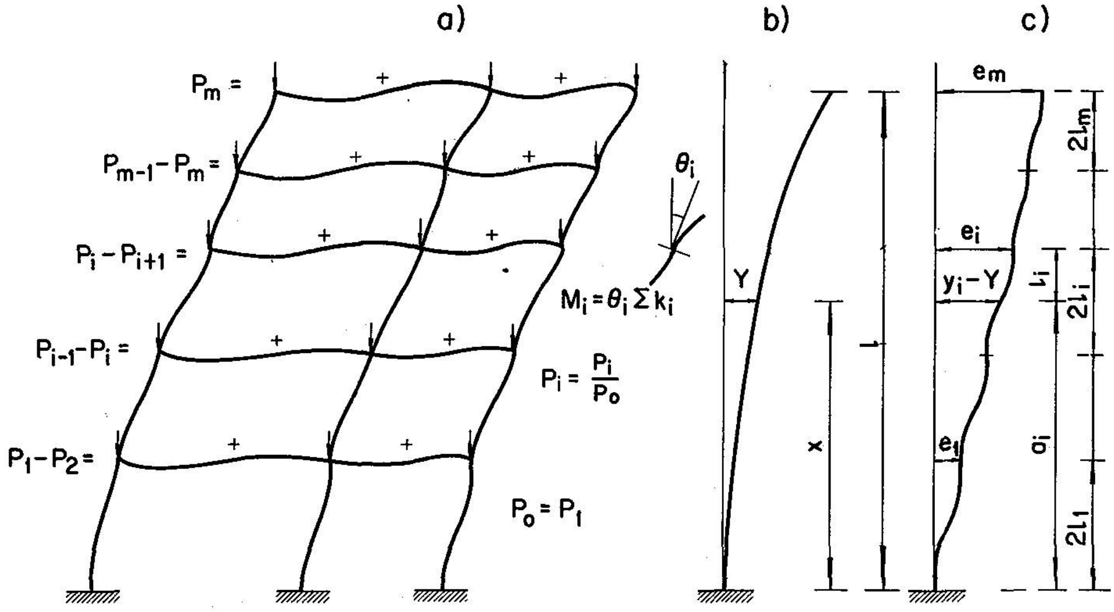


Fig. 1.

The simplifying hypothesis consists in assuming for the elastic curves of the deformed columns, which remain within elastic range until buckling, a combination of two curves (Fig. 1a): one corresponding to the deformation resulting by disregarding the floor girders (Fig. 1b), the other by assuming girders of infinite rigidity (Fig. 1c). The relative magnitude of the respective displacements is evaluated in such a manner as to result minimum buckling load.

Since the method is based on the assumption of a certain shape of the deflection curve of the structure, with a few parameters to be determined by comparing energies – it will yield a larger value than the correct one. In order to stay on the safe side a reduction coefficient for the deformation energy is used, thus resulting, for the majority of the less favorable cases, a strength which is nearly the correct one, but in other cases might yield safety factors in excess of 20% when the assumed curve resembles the correct one (as in the case of the buckling of isolated columns in two adjacent floors, mentioned ahead, where Euler's formula would be applicable).

[1] The loads applied on the girders are transferred to the columns, without significant influence on the structural instability. A closer look will be taken regarding this aspect in a later publication, including the action of horizontal loads as well as the deformability at the supports.

Deflection Curve

The deflection curve of the column shall be the one defined along each portion between floors by

$$y_i = e_{i-1} + c_i \left[2 + 3 \frac{x - a_i}{l_i} - \left(\frac{x - a_i}{l_i} \right)^3 \right] l_o + Y, \tag{4}$$

the three terms of the second member representing, respectively:
- the displacement of the starting joint in the portion of the structure with rigid girders (e_{i-1}),
- the complement of this displacement along that portion (the equation of the cubic parabola was used because – as it is known – it adjusts itself very well to the corresponding sine wave) including the factor c_i, to be determined,
- the displacement (Y) of the column (without girders) assumed fixed at the lower end and free at the top.

For the latter, any curve that satisfies the boundary conditions can be used; however, for simplicity, a cubic parabola will again be used:

$$Y = \frac{x^2}{l_o^2} (Kl - x) \tag{5}$$

This equation assumes fixity at the bottom; the condition at the top is caracterized by the parameter K, which varies from $K = 1{,}5$ (infinitely rigid beam at the top) to $K = 3$ (rigidity zero of the beam at the top). A better value of K could be obtained assuming it being a new parameter to be determined along with c_i; however, in the case of buildings, as long as the beams at the top are not stronger than the ones below, which is usual, one may take $K = 3$, as has been done in the present article. For any other value of K it is sufficient to substitute λ by $K\lambda/3$ in the formulas (9)[1].

Since the assumed line is not the real one, which is the most unfavorable, the proposed equation will lead to a larger critical load; in order to compensate for this error it is suitable to multiply the coefficient c_i, which appears in the expression of the deformation energy of the column, by $\sim 0{,}9$. This explains, in the equations for B_i and C_i presented further, the coefficient 1,8 (instead of 2), and 0,8 ($\sim 0{,}9^2$), respectively.

Buckling Load

It is a known fact that the buckling load of a member may be obtained by establishing the equivalence between the developed energy and the summation of energies from the reaction forces and the deformation of the member; i.e. (the summations extend between $i = 1$ and $i = m$):

$$\tfrac{1}{2} \sum P_i I_i^* = \tfrac{E}{2} \sum I_i I_i^{**} + \tfrac{1}{2} \sum M_i \theta_i \tag{6}$$

[1] Only λ; the λ_i remain the same.

If the assumed deflection curve is correct, this equation would yield the buckling load (by substituting P_i by $p_i P_o$, I_i by $j_i I_o$, M_i by $\theta_i \sum k_i$, θ_i by $Y'_{a_i+l_i}$):

$$P_o = \frac{\sum l_o^2 j_i I_i^{**} + \sum (l_o^2 \sum k_i / EI_o)\, Y'^2_{a_i+l_i}}{\sum p_i I_i^*} \frac{EI_o}{l_o^2} \tag{7}$$

or:

$$P_o = \frac{A + B + C}{A' + B' + C'} \frac{EI_o}{l_o^2} = G \frac{EI_o}{l_o^2} \tag{8}$$

with A, B, C, A', B' and C' as defined in (1) and (2), and also:

$$A_i = \lambda_i j_i \left[3(\lambda - \alpha_i)^2 + \lambda_i^2\right] + \tfrac{3}{8}\, \kappa_i (\alpha_i + \lambda_i)^2 (2\lambda - \lambda_i - \alpha_i)^2$$

$$B_i = 1{,}8 j_i, \qquad C_i = 0{,}8 \frac{j_i}{\lambda_i^3}$$

$$A'_i = p_i \lambda_i \left[\tfrac{3}{4}(\alpha_i^4 + 2\alpha_i^2 \lambda_i^2 + 0{,}2\lambda_i^4) - 3\lambda\alpha_i (\alpha_i^2 + \lambda_i^2) + \lambda^2 (3\alpha_i^2 + \lambda_i^2)\right] \tag{9}$$

$$B'_i = p_i (2\lambda\alpha_i - \alpha_i^2 - 0{,}2\lambda_i^2), \qquad C'_i = 0{,}4 \frac{p_i}{\lambda_i}.$$

The problem will have been solved when the expression (8) reaches its minimum value, which can be accomplished by assigning adequate values to the coefficients c_i, obtained from the following equations:

$$\frac{B_i + 2C_i c_i}{B'_i + 2C'_i c_i} = G. \tag{10}$$

For the solution of this system of equations, assign values G_o for G (neither larger than A/A' nor larger than the smaller of the quotients C_i/C'_i) and determine the corresponding c_i values [1]:

$$c_i = \tfrac{1}{2} \frac{B'_i G_o - B_i}{C_i - C'_i G_o}; \tag{11}$$

the substitution of these c_i values in (3) will lead to the corresponding G values. If they do not match with the assumed G_o values, the process should be repeated by assigning another value to G_o (equal, for instance, to the preceding value of G, but never greater than any of the mentioned limiting values A/A' and C_i/C'_i), until $G_o = G$. Usually a few trials will suffice (the plotting of curves $G = f(G_o)$ and their intersection with the line $G = G_o$ will aid in reducing the number of trials).

[1] The limit A/A' results from all c_i values equal to zero, which would be the solution of the problem if the beams had no effect on the buckling load. The limit C_i/C'_i, which is a minimum for a certain value $i = i_o$, corresponds to $c_{i_o} = \infty$ meaning isolated buckling of the columns of the floor i_o. In this case the buckling load should be given by Euler's formula; however the value will be smaller, since the idea is to remain on the safe side by using the reduction coefficient 0,9 while calculating the deformation energy (refer to the last observation of item *Deflection Curve*).

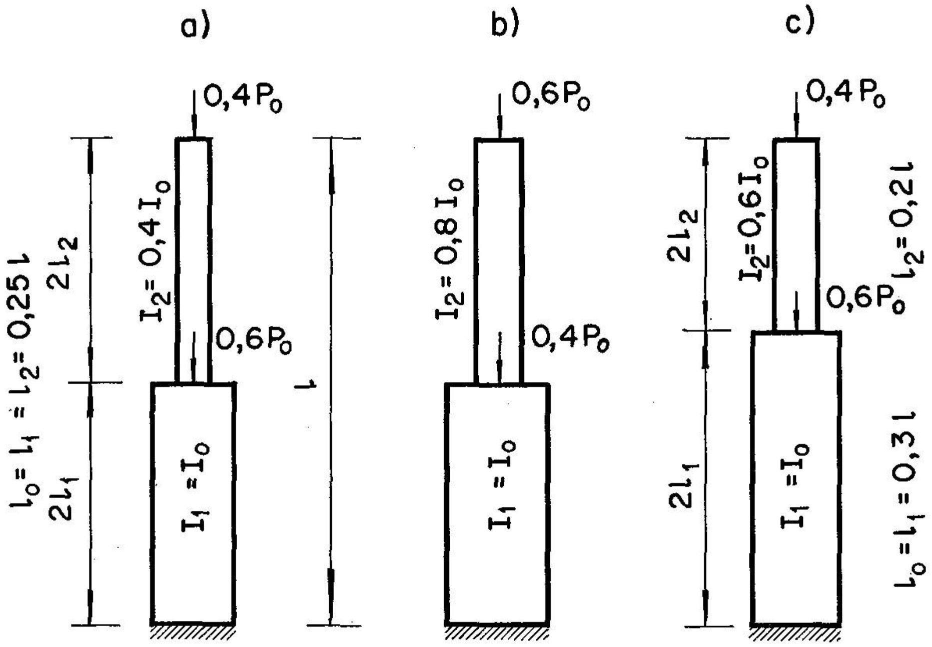
The problem will have been solved when the value of G has been found, since the critical load is equal to:

$$P_o = G \frac{EJ_o}{l_o^2} \tag{12}$$

The precision will be shown by comparing results already known with the ones obtained using the method being presented.

1st Example

The extreme case of a structure with beams with zero rigidity will be presented in this example. Therefore the column is isolated and subjected to a concentrated load at the top end of each of the two portions, each with its own cross section. Three cases with different dimensions will be considered (Fig. 2).



	a)	b)	c)
$\kappa_1 = \kappa_2 =$	0	0	0
$\lambda =$	4	4	10/3
$\lambda_1 = \alpha_1 = p_1 = j_1 =$	1	1	1
$\lambda_2 =$	1	1	2/3
$\alpha_2 =$	3	3	8/3
$p_2 =$	0,4	0,6	0,4
$j_2 =$	0,4	0,8	0,6

Fig. 2.

In the first case, $A/A' = 0{,}2757$, $C_1/C_1' = 1$, $C_2/C_2' = 1$. Assigning the smaller of these values (0,275) to G_o, from (11), $c_1 = 0{,}0507$ and $c_2 = 1{,}645$, resulting $G = 0{,}269$ if we replace these values in (3). Repeating the process, starting with $G_o = 0{,}269$, will lead to $c_1 = 0{,}081$, $c_2 = 1{,}575$, $G = 0{,}269$. This value replaced in (11) yields the final answer:

a) $P_o = 0{,}269 \dfrac{EJ_o}{l_o^2}$

The exact answer [5] is $G = 0{,}253$, i.e., 6% smaller. For the other two cases the discrepancy is much smaller, the results being obtained in a similar manner:

b) $G = 0{,}222$ (instead of [5]: 0,221)
c) $G = 0{,}367$ (instead of [5]: 0,366).

2nd Example

Three different story heights and two columns, as shown in Fig. 3. $A/A' = 46{,}13$ and the smallest value C_i/C_i', for $i = 1$, equals 18,75. Assuming G_o equal to 18 and then 17, $G = G_o$ for the latter, thus the answer is $G = 17$. Other authors found, for the same problems, the values 16,84 [3] and 16,93 [4]. The latter one shows also the buckling mode, i.e., the relative floor displacements under the buckling load: 1, 0,924 and 0,621, respectively at the top, 3rd and 2nd floor level.

The buckling mode can also be determined by using formulas presented herein. With $G = 17$, $c_1 = 45{,}43$, $c_2 = 6{,}45$ and $c_3 = 3{,}65$; correspondingly:

$$\frac{e_1}{l_o} = 4c_1 = 181{,}7. \qquad \frac{e_2}{l_o} = \frac{e_1}{l_o} + 4c_2 = 207{,}5$$

$$\frac{e_3}{l_o} - \frac{e_2}{l_o} + 4c_3 = 222{,}1$$

and also the respective Y values (5):

$$\frac{Y_1}{l_o} = \frac{240^2}{150^3}(3 \times 570 - 240) = 25{,}1$$

$$\frac{Y_2}{l_o} = \frac{420^2}{150^3}(3 \times 570 - 420) = 67{,}4$$

$$\frac{Y_3}{l_o} = \frac{570^2}{150^3}(3 \times 570 - 570) = 109{,}7$$

with $(Y_3 + e_3)/l_o = y_3^*/l_o = 331{,}8$, and the relative values:

$$\frac{y_3^*}{y_3^*} = 1 \qquad \frac{y_2^*}{y_3^*} = 0{,}829 \qquad \frac{y_1^*}{y_3^*} = 0{,}623.$$

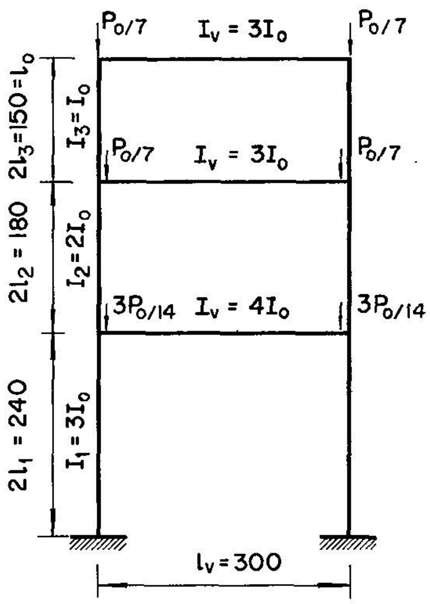
3rd Example

Five stories equally high, $h = 2l_o$ each, one single load applied at the top, girders on both sides, and $(l_v I_o)/(l_o I_v) = 3$:

$\lambda_i = 1$, $\alpha_1 = 2i - 1$, $\lambda = 10$, $p_i = 1$, $j_i = 1$
$c_1 = 13{,}7$ $c_2 = 40{,}3$ $c_3 = 60{,}2$ $c_4 = 73{,}5$ $c_5 = 80{,}1$.

Therefore:

$P_o = 0{,}80\ EJ_o/l_o^2$, since $G = 0{,}798$ (exact value [2]: 0,77).



$\lambda = 3{,}8$
$\lambda_1 = 0{,}8$
$\lambda_2 = 0{,}6$
$\lambda_3 = 0{,}5$

$\alpha_1 = 0{,}8$
$\alpha_2 = 2{,}2$
$\alpha_3 = 3{,}3$

$p_1 = 1$
$p_2 = 4/7$
$p_3 = 2/7$

$j_1 = 6$
$j_2 = 4$
$j_3 = 2$

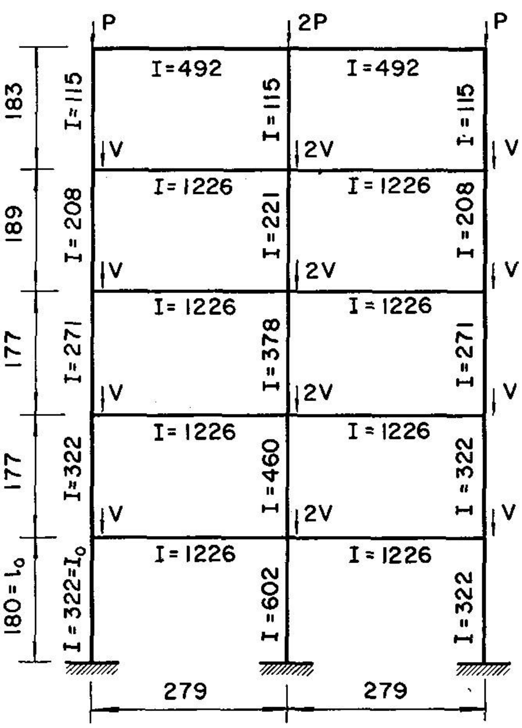
$\kappa_1 = 24$
$\kappa_2 = 18$
$\kappa_3 = 18$

Fig. 3.

4th Example

Five stories and three different columns, see Fig. 4. Proceeding as in the previous examples:

$c_1 = 186{,}7$, $c_2 = 39{,}6$, $c_3 = 29{,}1$, $c_4 = 36{,}6$, $c_5 = 11{,}0$,
$G =$ 30,5 (as compared to 28,7 in [6], 30,3 in [3] and 31,4 in [4]).



$\lambda = 5{,}033$	$V/P = 1{,}962$
$\lambda_1 = 0{,}500$	$p_1 = 1{,}000$
$\lambda_2 = 0{,}492$	$p_2 = 0{,}778$
$\lambda_3 = 0{,}492$	$p_3 = 0{,}557$
$\lambda_4 = 0{,}525$	$p_4 = 0{,}335$
$\lambda_5 = 0{,}508$	$p_5 = 0{,}113$
$\alpha_1 = 0{,}500$	$j_1 = 3{,}870$
$\alpha_2 = 1{,}492$	$j_2 = 3{,}429$
$\alpha_3 = 2{,}475$	$j_3 = 2{,}857$
$\alpha_4 = 3{,}492$	$j_4 = 1{,}978$
$\alpha_5 = 4{,}525$	$j_5 = 1{,}071$

$\kappa_1 = \kappa_2 = \kappa_3 = \kappa_4 = 14{,}74 \times 4$
$\kappa_5 = 5{,}915 \times 4$

Fig. 4.

5th Example

This example considers the less common case of the upper beam being more rigid than the others; therefore, assume $K = 1{,}5$ in (5), which corresponds to substituting λ by $K\lambda/3 = 0{,}5\lambda$ in (9). Using again the example shown in Fig. 3, and changing the value of I_v of the upper beam from $3I_o$ to $20I_o$, among the values shown on Fig. 3 only κ_3 changes from 18 to 120 (besides the substitution of $\lambda = 3{,}8$ by $0{,}5\lambda = 1{,}9$ in the formulas).

The result thus obtained $G = 15{,}1$ is 10,3% smaller than the exact one.

Notations

The stories are numbered 1 through m, starting from below; therefore the properties related to the portion of the column between the foundation and the first floor shall be identified by the subscript 1, and so forth. The properties related to the joints (column-girder intersection) shall have the same subscript of the adjacent column below. The notation is as follows (Fig. 1):

$$A = \sum A_i, \qquad B = \sum B_i c_i, \qquad C = \sum C_i c_i^2 \tag{1}$$

$$A' = \sum A_i', \qquad B' = \sum B_i' c_i, \qquad C' = \sum C_i' c_i^2 \tag{2}$$

$A_i, B_i, C_i, A_i', B_i', C_i'$ = values defined in (9).

a_i distance from the lower end to the center of segment i of the column.

c_i coefficient, to be determined, representing the relative sidesway magnitude at the various floors.

E modulus of elasticity of the material.

e_i displacement of joint i in the structure with rigid girders.

G $(A + B + C)/(A' + B' + C')$ (3)

$$I_i^* = \int_{a_i - l_i}^{a_i + l_i} y'^2 \, dx, \qquad I_i^{**} = \int_{a_i - l_i}^{a_i + l_i} y_i''^2 \, dx.$$

i subscript referring to the generic column portion between two floors.

I_v moment of inertia of beam cross section.

I_i summation of the moments of inertia of the cross sections of the portions i of the columns.

I_o moment of inertia of a section used as reference.

j_i I_i/I_o.

K parameter.

k_i coefficient characterizing the rigidity of the joints $i (k_i = M_i/\theta_i)$[1].

$\sum k_i$ summation of all k_i with the same subscript i.

[1] For instance, at a joint with only one girder (end columns) of length l_v and constant cross section with moment of inertia I_v, $k_i = 6EI_v/l_v$; should two girders be present, the summation of the corresponding $6EI_v/l_v$ yields the value of k_i.

l	total column length.
l_i	length of the portion i of the column, divided by 2.
l_o	reference length.
l_v	length of girder.
M_i	moments at the supports of the girders, at the joints i.
P_i	summation of the loads applied above the joints i (including the ones applied at these joints).
P_o	total buckling load ($= P_1$), to be determined.
p_i	P_i/P.
x	distance of a section from the lower end of a column.
Y	portion of the horizontal displacement of the column (function of x), for the case of total absence of girders (Fig. 1b).
Y_i	value of Y at the joint i.
y_i	horizontal displacement of the column at x, in the segment $i\,(a_i - l_i < x < a_i + l_i)$.
y_i^*	horizontal displacement of joint i.
α_i	a_i/l_o.
θ_i	$Y'_{a_i+l_i}$ = angular displacement at joint i.
κ_i	$(\sum k_i) l_o / E I_o$.
λ	l/l_o.
λ_i	l_i/l_o.

The derivatives in respect to x are indicated by an accent (y', y'', Y', Y''). The values of these derivatives for particular values of x are identified by using these as subscripts (for instance, Y' for $x = a_1$ will be represented by Y'_{a_1}).

Remark concerning Economy

The method proposed in the present article for calculating the buckling load of the structure permits to obtain it with a sufficient approximation and in a much simplier and economical manner than by the general method.

Therefore, without spending a great deal of his time and of the computer, the proposed method will yield the Structural Engineer a fairly accurate solution of the problem.

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Summary

The Author proposes a simplified method for the determination of the critical load of orthogonal building structures, assuming plane buckling. The method presented, although much easier to apply than most of the exacting ones — which require the use of large computers — is sufficiently accurate.

Résumé

L'auteur propose une méthode simplifiée pour la détermination de la charge critique aux structures orthogonales, en assumant le voilement en plaine. La méthode présentée, bien que plus facilement applicable que la plupart des méthodes exactes qui demandent l'application de grands ordinateurs, s'avère suffisamment exacte.

Zusammenfassung

Der Verfasser schlägt eine vereinfachte Methode zur Bestimmung der kritischen Last an rechteckigen Tragwerken unter Annahme der Beulung in einer Ebene vor. Die vorgelegte Methode, obschon weit leichter anwendbar als die meisten genauen Verfahren, welche die Verwendung grosser Computer erfordern, erweist sich als hinreichend genau.