Minimum wall thickness of circular concrete tanks

Minimum Wall Thickness of Circular Concrete Tanks

Epaisseur minimale de réservoirs en béton armé de section circulaire

Mindestwandstärke kreisförmiger Stahlbetontanks

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Introduction

"Primary tensile cracking", cracks transversing the entire thickness [2] in the walls of reinforced concrete, liquid carrying tanks, creates an undesirable situation [4]. Appreciable circumferential tensile stresses develop in the concrete of the walls of circular tanks due to (*a*) the shrinkage tendency of concrete, (*b*) drop in ambient temperature and temperature gradient in the concrete, and (*c*) the ring tension induced by the hydrostatic pressure. "Primary tensile cracks" form, vertically, when the average of these stresses exceeds the tensile capacity of concrete, necessitating expensive repairs.

This study is an attempt to develop a formula to determine the minimum wall thickness of circular reinforced concrete tanks, sufficient to prevent "primary tensile cracking" of concrete. It takes into account the time-dependent nature of shrinkage, relaxation of the stresses due to tensile creep of concrete, frictional restraint at the base of the tank, and thermal effects. Thermal coefficients of steel and concrete are assumed to be equal and effects of any temperature gradient along the height of the tank are ignored. Furthermore, instantaneous modulus of elasticity of concrete under tension is considered to be independent of time, since it tends to approach a constant value, much faster than the modulus of elasticity of concrete under compression, following a relatively short curing period [9].

Shrinkage and Tensile Creep Strains in Concrete

The average time-dependent shrinkage strain of unrestrained concrete, $(\varepsilon_{sh})_t$, may be expressed as [7]

$$(\varepsilon_{sh})_t = (\varepsilon_{sh})_\infty \; (1 - e^{-\xi(t - t_o)}) \tag{1}$$

where t is the age of concrete at the time of strain measurement, t_o is the age of concrete at the start of shrinkage, $(\varepsilon_{sh})_\infty = (\varepsilon_{sh})_{t=\infty}$, and ξ is the coefficient determining the change of slope of the shrinkage curve [6].

The total, initial and time-dependent linear strain of concrete per unit of tensile stress, $1/(E_c)_t$, may be expressed as [1, 10]

$$1/(E_c)_t = 1/E_{ci} + \kappa(\varepsilon_\infty + \eta/t_i)(1 - e^{-\zeta(t - t_i)}) \tag{2}$$

where t_i is the age of concrete at the loading time, E_{ci} is the instantaneous tensile modulus of elasticity of concrete, ε_∞ is the maximum strain in concrete loaded at a very old age, η is the coefficient determining the relation between maximum creep strain and ε_∞, ζ is the coefficient determining the change of slope of the creep curve, and κ is a coefficient introducing the influences of the climatic conditions, geometric dimensions of the member, composition of the concrete, etc., on the creep of concrete [3].

Shrinkage Stresses in Concrete Restrained by Reinforcement

Using relations (1) and (2) stated above, it is shown in Reference [11] that the average concrete stress in the sections of concentrically reinforced concrete bars under pure shrinkage for any specific age of concrete, t_1, can be expressed as $-\rho\chi(t_1)$. Here, ρ is the percentage of steel and

$$\chi(t_1) = (\xi(\varepsilon_{sh})_\infty/\Phi)\int_{t_o}^{t_1} [(\xi - \zeta)e^{\xi t_o}\int_{t_o}^{\bar{t}} t^{\Lambda} e^{t(\Omega - \xi)}dt - t_o^{\Lambda} e^{\Omega t_o}]\bar{t}^{-\Lambda} e^{-\Omega\bar{t}}d\bar{t} \tag{3}$$

where $\Phi = (\rho/E_{ci}) + (1/E_s)$, E_s is the modulus of elasticity of steel, $\Lambda = \zeta\rho\kappa\eta/\Phi$, and $\Omega = (\zeta\rho\kappa\varepsilon_\infty/\Phi) + \zeta$.

The value of $\chi(t_1)$ as given by Eq. (3) can easily be computed with the help of a digital computer for any ρ, t_o, t_1 combination in terms of the material constants E_s, E_{ci}, $(\varepsilon_{sh})_\infty$, ξ, κ, η, ε_∞, and ζ, which can all be determined from test data. χ values for a particular set of these constants are given in Fig. 1 for various ages of concrete, t_1, various percentages of reinforcement, ρ, and for various maximum unrestrained shrinkage strains of concrete, $(\varepsilon_{sh})_\infty$. Effort has been made to choose realistic values for constants in the preparation of Fig. 1; the following were assumed: $t_o = 7$ days, $E_s = 29 \times 10^6$ psi (2.04×10^6 kg per sq cm), $E_{ci} = 5 \times 10^6$ psi (0.352×10^6 kg per sq cm), $\xi = 0.037$, $\kappa = 1.0$. Also, based on tensile creep data of Rose Dam concrete [9], constants η, ε_∞, and ζ were taken as 3.2×10^{-6}/psi (45.4×10^{-6}/kg per sq cm), 0.04×10^{-6}/psi (0.568×10^{-6}/kg per sq cm), and 0.06 respectively.

Stresses in Concrete Wall Due to Base Restraint

Uniform circumferential tensile stresses develop in the concrete of tank walls, due to environmental temperature drop and to shrinkage of concrete, whenever free contraction of the tank is restrained by the frictional resistance of its subbase. These stresses quickly vanish with height [8]. Deflected wall shape for such a tank is shown in Fig. 2. A free tank contracts from the center of its base and, unless the tank dimensions are unusually large, the frictional force developed can

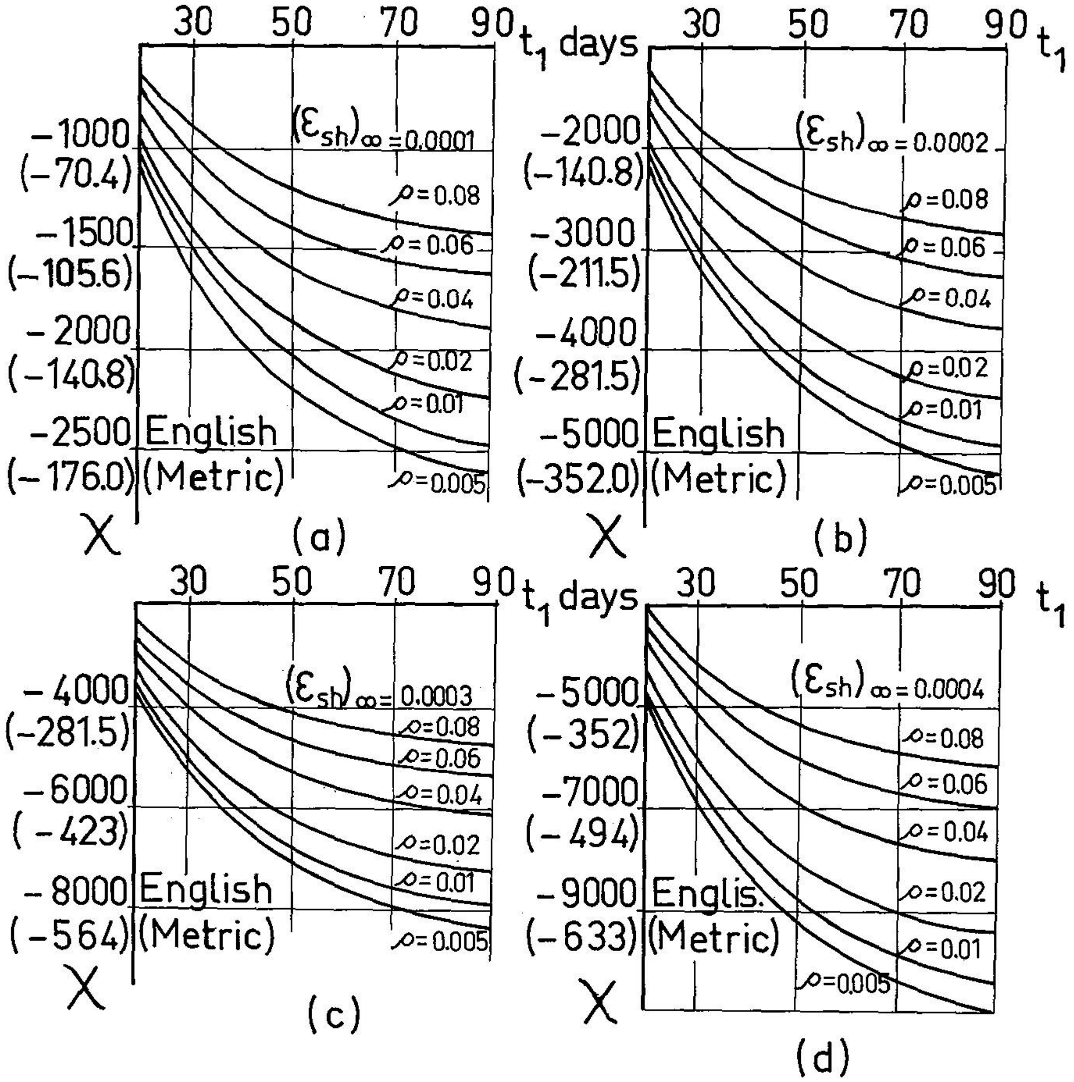

Fig. 1. $\chi - t_1$ relationship.

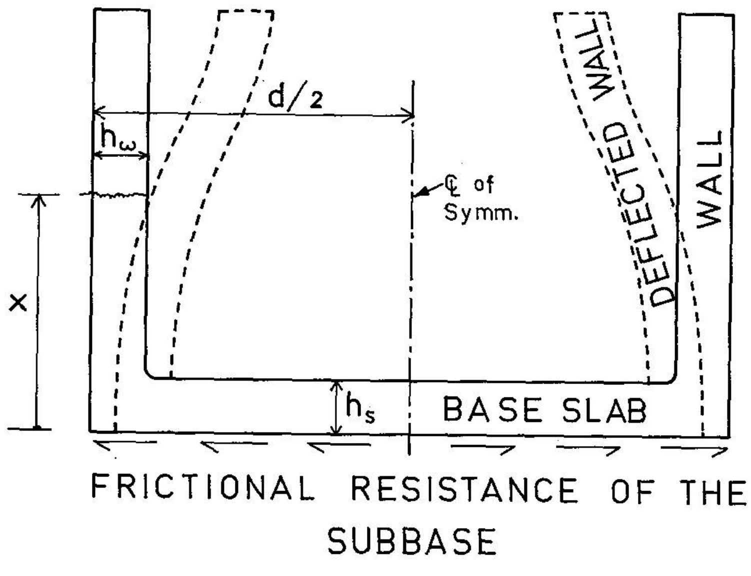

Fig. 2. Cross section of the tank.

nowhere reach a magnitude high enough to arrest the movement of the tank's base completely [5]. If the subgrade resistance is assumed to be a linear function of the tank diameter, d, and if the average friction coefficient, μ, between the tank base and the ground is taken as constant, then, these stresses can be expressed as $\delta Z\mu/2dh_s$ [5]. Here, Z is the total ground reaction and is equal to the weight of the tank and the enclosed liquid, h_s is the thickness of the base slab of the tank, and δ is restraint reduction factor, introducing the height effect. Assuming rotational fixity at the base and uniform wall thickness, δ varies with the distance from the wall base, x, the wall thickness, h_w, and the tank diameter as shown in Fig. 3 [8]. δ should be taken as equal to zero for elevated tanks.

The average friction coefficient varies with the displacement of the base slab and, in the absence of accurate pertinent data, it can be determined with the help of Fig. 4 [5], where α is the thermal coefficient of expansion of concrete and T is the maximum expected drop in ambient temperature.

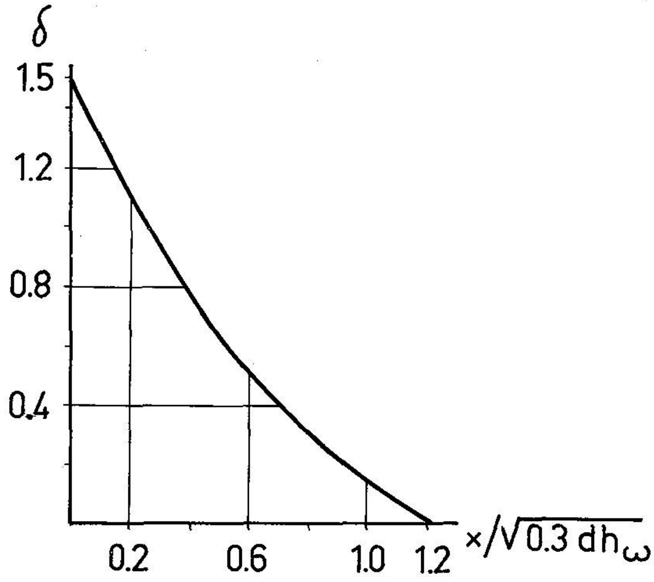

Fig. 3. $\delta - x/\sqrt{0.3\,dh_\omega}$ relationship.

Stresses in Concrete Wall Due to Temperature Gradient

Circumferential thermal stresses develop in the concrete of the tank walls under a temperature gradient in radial direction. It can be shown that (8) when linear variation of temperature through the wall thickness is assumed and the effect of Poisson's ratio is ignored, these stresses vary uniformly and reach $\pm 0.8E_{ci}\alpha|T_1 - T_2|$ values at the exterior and interior surfaces. Here, T_1 and T_2 are the temperatures of the tank wall at the interior and exterior surfaces, respectively. Although these stresses do not alter the average concrete stresses, they may force flexural type of cracking from one face, reducing the cross-sectional area of concrete resisting tensile cracking.

A temperature gradient along the height of the tank will increase the average circumferential concrete stresses (8). However, this effect is assumed to be relatively unimportant for normal tank conditions and is ignored.

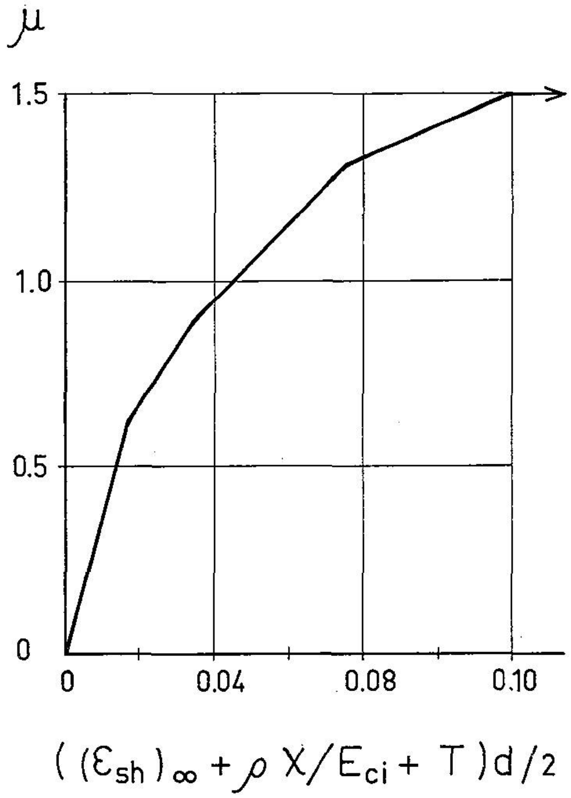

Fig. 4. μ – maximum displacement relationship (5).

Minimum Wall Thickness

The average circumferential tensile stress in the concrete of the tank walls for any age t_1 can be found by summing up the stresses developed due to the shrinkage tendency of concrete, environmental temperature drop, and ring tension, F, caused by hydrostatic pressure as

$$f_{ct} = \{-\rho\chi(t_1) + \delta Z\mu/(2dh_s) + F/[A_c(1+\rho E_s/E_{ci})]\} \qquad (4)$$

Here, A_c is the cross-sectional area of concrete. If h_w is the wall thickness in inches, and the tensile force, F, is computed for a ring depth of 12 in., then, $A_c = 12\ h_w$. On the other hand, the usual procedure in tank design is to provide sufficient circumferential steel reinforcement to carry all the ring tension, at a certain allowable stress, f_s, as though designing for a cracked section (4). Accordingly, $\rho = F/(A_c f_s)$. Substituting these values of A_c and ρ into Eq. (4) and introducing the tensile stresses caused by temperature gradient, and assuming a linear interaction between tension and flexural types of cracking, one obtains:

$$\{F[-\chi/(12h_w f_s) + (f_s E_{ci})/(12h_w f_s E_{ci} + F E_s)] + (\delta Z\mu)/(2dh_s)\}\gamma/f_t + \{0.8E_{ci}\alpha|T_1 - T_2|\}\gamma/f_r = 1 \qquad (5)$$

Here, f_t is the average tensile strength of concrete per unit area, f_r is the modulus of rupture of concrete, and γ is the appropriate safety factor against "primary tensile cracking" of concrete in tank walls.

χ values, given by Eq. (3), decrease in time as can be seen in Fig. 1 and approach an asymptotic value for all intensities of shrinkage and percentages of reinforcement. In designing for wall thickness, the minimum χ value should be used with Eq. (5) in order to cover all the significant effects of the shrinkage of concrete. For practical purposes, it would be accurate enough to take minimum χ value $= \chi$ $(t_1 = 90$ days). Such minimum values of χ, based on the same set of material constants used in the preparation of Fig. 1, are given in Fig. 5 for various maximum unrestrained shrinkage strains of concrete and for varying percentages of reinforcement.

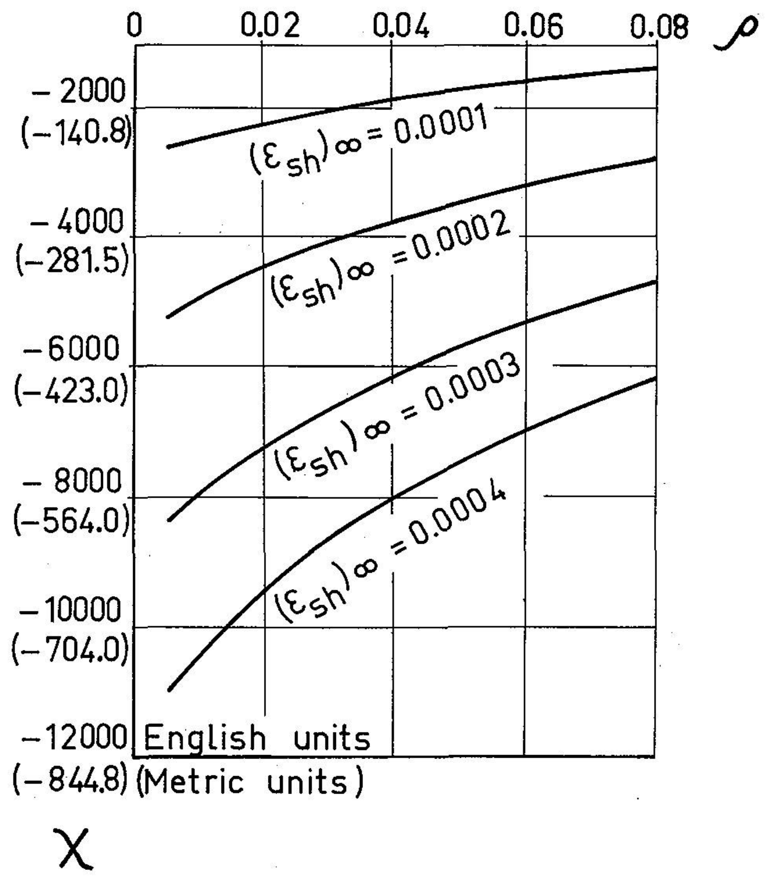

Fig. 5. $\chi(t_1 = 90$ days$) - \rho$ relationship.

Fig. 6 shows the variation in the minimum χ values given in Fig. 5 with change in E_{ci} and κ. The minimum χ values of Fig. 5 can be adjusted for use with different E_{ci} and κ values when multiplied by the corresponding adjustment factors β_1 and β_2 given in Fig. 6, respectively.

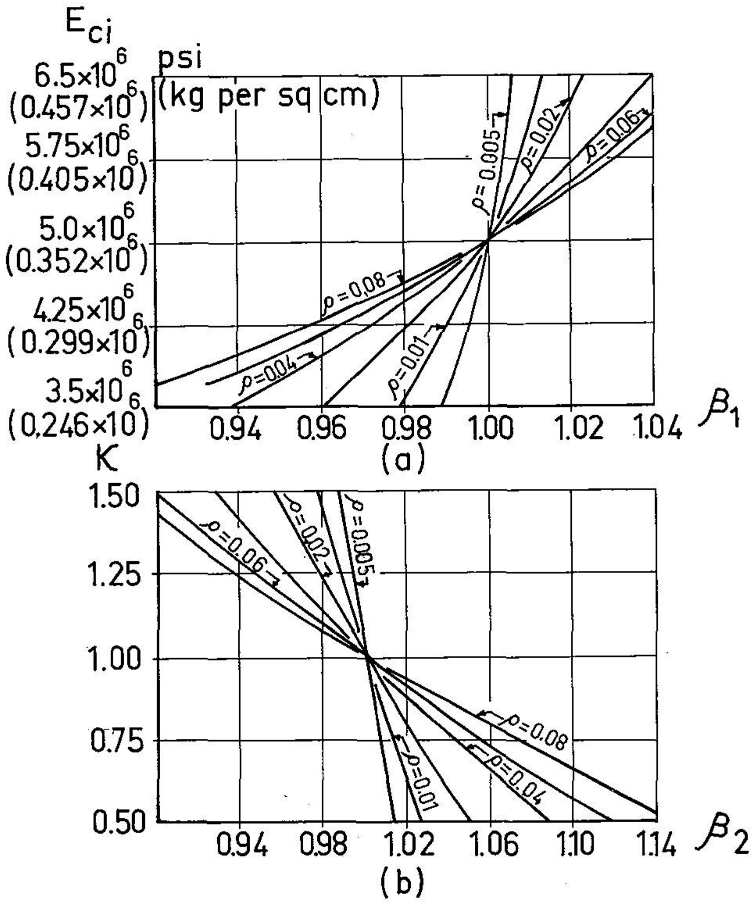

Fig. 6. Adjustment factors, β_1 and β_2 for corresponding E_{ci} and κ values, respectively.

Example

Determine the minimum wall thickness, h_w, for the given circular water tank, sufficient to prevent "primary tensile cracking" at the specified depth. Given: $d = 300$ in., $\chi = 24$ in., $F = 10{,}000$ lb per ft of wall depth, $f_t = 280$ psi, $f_r = 560$ psi, $f_s = 20{,}000$ psi, $E_{ci} = 4.0 \times 10^6$ psi, $E_s = 29 \times 10^6$ psi, $(\varepsilon_{sh})_\infty = 0.0003$, $\kappa = 1.25$, $Z = 0.7 \times 10^6$ lb, $h_s = 12$ in., $T = 30$ deg F, $|T_1 - T_2| = 6$ deg F, $\alpha = 6.0 \times 10^{-6}$ per deg F, and $\gamma = 1.4$.

Area of circumferential reinforcement, $A_s = F/f_s = 10{,}000/20{,}000 = 0.5$ sq. in. Assume $h_w = 14$ in. For the given d, x, and assumed h_w, Fig. 3 gives $\delta = 0.43$. Assume $\rho = 0.005$. For the given $(\varepsilon_{sh})_\infty$ and assumed ρ, Fig. 5 gives $\chi(t_1 = 90 \text{ days}) = -8333$. For the given E_{ci} and κ, Fig. 6 gives $\beta_1 = 0.994$ and $\beta_2 = 0.994$. Therefore, adjusted $\chi(t_1 = 90) = \beta_1 \beta_2 \chi = 0.994 \times 0.994\ (-8333) = -8230$. Then, $((\varepsilon_{sh})_\infty + \rho\chi/E_{ci} + \alpha T)d/2 = 0.0705$ and Fig. 4 gives $\mu = 1.18$. Substituting the values given and found above into Eq. (5) and solving it for h_w one finds $h_w = 12.4$ in. Therefore, use 12.5 in. thickness.

Actual $\rho = A_s/A_c = 0.5/12 \times 12.5 = 0.0033$, less than assumed ρ, therefore, O.K.

In the above example, about 25% of the concrete strength is used up by the frictional restraint at the base of the tank and 29% by the temperature gradient.

For the given tank, effect of base restraint vanishes 36 in. above ground, Fig. 3. Disregarding the base restraint and the temperature gradient, and then using the rest of the previously given data, minimum wall thickness is found to be, $h_w = 5.7$ in. from Eq. (5). This value is only 1.7 in. larger than the wall thickness found by feeding the same data into the thickness formula given in Reference (4).

Conclusion

Minimum wall thickness of circular reinforced concrete tanks, sufficient to prevent "primary tensile racking", can directly be determined from Eq. (5). The solution takes into consideration the effects of hydrostatic pressure, shrinkage and tensile creep of concrete, ground restraint, thermal stresses and the interaction between the tensile and flexural type of cracking forces in concrete. For usual design purposes, values of δ, μ, and χ used in Eq. (5) can readily be obtained from Fig. 3, 4, and 5 and 6 respectively.

Notation

The following symbols are used in this paper:

A_c	cross-sectional area of concrete.
A_s	area of circumferential reinforcement.
d	diameter of the tank.
E_{ci}	instantaneous tensile modulus of elasticity of concrete.
$(E_c)_t$	time-dependent tensile strain modulus of concrete.
E_s	modulus of elasticity of steel.
F	ring tension per unit depth of tank wall due to hydrostatic pressure.
f_{ct}	average circumferential tensile stress in concrete.
f_t	average tensile strength of concrete per unit area.
f_r	modulus of rupture of concrete.
f_s	allowable stress in steel.
h_s	thickness of the base slab of the tank.
h_w	thickness of the tank wall.
T_1 and T_2	temperature of the tank wall at the interior and exterior surfaces, respectively.
T	maximum expected drop in ambient temperature.
t	age of concrete at the time of strain measurement.
t_o	age of concrete at the start of shrinkage.
t_1	a specific age for concrete.
t_i	age of concrete at the time of loading.
x	distance from ground to the tank wall slice under consideration.
Z	total ground reaction under the tank.
α	thermal coefficient of expansion of concrete.
β_1 and β_2	adjustment factors.
γ	appropriate factor of safety against "primary tensile cracking of concrete" in tank walls.

δ	restraint reduction factor.
ε_∞	maximum strain of concrete loaded at a very old age.
$(\varepsilon_{sh})_t$	average time-dependent shrinkage strain of unrestrained concrete.
$(\varepsilon_{sh})_\infty$	$(\varepsilon_{sh})_{(t=\infty)}$.
ζ	coefficient determining the change of slope of the creep curve.
η	coefficient determining the relation between maximum creep strain loaded at a very young age and ε_∞.
κ	a coefficient introducing the influences of the climatic conditions, geometric dimensions of the member, composition of the concrete, etc., on the creep of concrete.
Λ	$\zeta\rho\kappa\eta/\Phi$.
μ	average friction coefficient between the tank base and the ground.
ξ	coefficient determining the change of slope of the shrinkage curve.
ρ	percentage of reinforcement.
Φ	$(\rho/E_{ci})+(1/E_s)$.
$\chi(t_1)$	a function given by Eq. (3).
Ω	$(\zeta\rho\kappa\varepsilon_\infty/\Phi)+\zeta$.

Practical Consequences

The environmental conditions around the building site, the method of construction, the properties of materials used in the construction, the existing foundation conditions, the time of initial loading, and utilization greatly vary from one reinforced concrete tank to the other. Different minimum tank wall thicknesses are needed to prevent "primary tensile cracking" in different tanks because the above stated factors significantly influence the ultimate tensile strength, the shrinkage, and the tensile creep properties of concrete, the amount of base friction restraining the displacement tendencies of the tank, and the amount of the maximum temperature gradient which may develop in the tank walls. Eq. (5), which accounts separately for all these effects, enables the designer to determine the required minimum wall thickness for a circular reinforced concrete tank under any given set of conditions. Use of tank wall thicknesses greater than those thus found not only leads to waste in material and labor, but, in extreme cases, may force cracking because of the adverse effect of wall thickness on the base restraint of the tank. Therefore, the minimum wall thickness found with the help of Eq. (5) is the most economical solution to the problem ensuring safety against cracking under all conditions.

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Summary

A formula is developed for determining the minimum wall thickness of circular reinforced concrete tanks, sufficient to prevent "primary tensile cracking" of concrete. It accounts for the effects of hydrostatic pressure, time-dependent shrinkage and tensile creep of concrete, ground restraint, thermal stresses and the interaction between the tensile and flexural type of cracking forces in concrete and easily lends itself to solution with the help of accompanying charts.

Résumé

On développe une formule pour la détermination de l'épaisseur minimale de réservoirs en béton armé, à section circulaire, suffisante à prévenir la rupture intégrale du béton. Elle s'explique par l'effet de la pression hydrostatique, par le retrait dépendant du temps et l'effet du fluage du béton ainsi que par le serrage au fond du réservoir; en plus par les contraintes thermiques et l'interaction entre l'effet de dilatation et de flexion des forces de rupture dans le béton. On arrive facilement à la solution du problème à la main des diagrammes accompagnants.

Zusammenfassung

Es wird eine Formel zur Bestimmung der Mindestwandstärke kreisförmiger Stahlbetontanks entwickelt, die das durchgehende Reissen des Betons verhindert. Dieses erklärt sich aus der Wirkung des hydrostatischen Druckes, des zeitabhängigen Schrumpfens und Kriechens des Beton, der Einspannung am Boden des Tanks, aus Wärmebeanspruchungen und der Wechselwirkung zwischen der durch Dehnung und Biegung veranlassten Risskräfte im Beton. Die angegebene Formel verhilft unschwer zur Lösung mit Hilfe der beigefügten Diagramme.

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