

# Effect of variation of interface connection modulus in elastically connected elements of composite beams

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Objektyp: **Article**

Zeitschrift: **IABSE publications = Mémoires AIPC = IVBH Abhandlungen**

Band (Jahr): **35 (1975)**

PDF erstellt am: **09.07.2024**

Persistenter Link: <https://doi.org/10.5169/seals-26942>

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**Effect of Variation of Interface Connection Modulus  
in Elastically connected Elements of Composite Beams**

*Influence de la variation de la déformabilité des connecteurs  
dans les poutres mixtes élastiques*

*Einfluss der Änderung des Verformungsmoduls  
der Verbundmittel bei elastischen Verbundträgern*

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**Abstract**

Various forms of shear connectors are the medium through which interaction in composite beams is ensured. The connectors most commonly in use nowadays are the flexible type either in the form of studs, spiral shear connectors or cleats connected to the beam element and embedded in the concrete. In the present analysis an attempt is made to study the effect of distribution of shear connectors in a span where they are needed for ensuring interaction. Implicit in the analysis is the assumption that the profile for the shear connection modulus is analogous to that defining the juxtaposition of the shear connectors. This is of course only partially true since other effects like the flexural rigidity of the connectors, totally embedded within an "elastic" medium which itself undergoes deformation due to bending, and the presence of the connectors come into play. The mechanics by which flexible connectors ensure interaction is too complex, that only a simplified and idealised model like that used in this work can provide some insight into connector interaction behaviour.

The analysis shows quite clearly that for a given number of connectors to be employed for the transfer of interacting axial forces from one element to the other of a composite section the best arrangement is not the evenly spaced type, nor is the arrangement whereby connectors are concentrated near the point of maximum moment to be recommended. The best arrangement is the one that ensures that the greater number of connectors to be used are concentrated in the regions of zero external moments.

## Introduction

The design of flexible shear connectors in composite beams is usually based on conditions at ultimate loads or collapse. The design objective is to ensure that at collapse there will be enough connectors, based on some predetermined connector design capacity obtained from standard push-out tests, to transfer the total axial force developed as a result of interaction from one to the other of the interacting steel beam and concrete slab. This design capacity is chosen on the basis of a limiting slip at failure as the design criterion. CHAPMAN and YAM [1] suggested that the juxtaposition of the connectors should be determined by spacing evenly over the distance between two adjacent points of maximum and zero or minimum moments the shear connectors needed to resist the maximum interacting axial force developed at ultimate load conditions between such points. They presented results for inelastic behaviour of continuous composite beams employing a predictor-corrector method of numerical integration.

Although it is an accepted design practice, and indeed the vogue, to select structural elements on the basis of conditions at collapse, safe structures however generally remain "elastic", or at any rate are far from collapse, at working loads. The present paper is therefore an attempt to examine the influence of shear connectors and their distributions on interaction under so called elastic conditions. It is believed that the insight gained from this work will lead to a better understanding of interaction at working loads and may result in more rational design basis or economy or both.

Previous interaction analyses of composite beams have assumed constant interface connection modulus along the length of the beams irrespective of the type of imposed loading or whether the beams are simply supported or continuous. In the present analysis a variation of the interface connection modulus is assumed in the form of an exponential function. Harmonic representation can also be used to study this variation. This representation enables a number of variations which idealise possible distributions of shear connectors to be studied as well as the limiting cases of partial interaction with constant interface connection modulus, the complete interaction and no interaction. Because in the design of a bridge deck system one is normally considering moment envelopes, the present analysis has been limited to simply supported beams of symmetrical loading.

## Formulation of the problem

We will consider a system of an infinite number of steel beams supported over finite and equal spans equally spaced transversely, interacting with a concrete slab extending over the entire beam system, with each steel beam identically loaded. We choose as our origin the middle plane of the concrete slab at a point midway between two adjacent beams for the  $y$  coordinate in the transverse direction and midspan for the  $x$  coordinate in the spanwise direction.

The assumed representation for the interface connection modulus is of the form  $Ae^{r \frac{|x|}{a}}$  for all cases of symmetrical loading over a continuous or a simply supported span. This will enable the effect of any one of the four spanwise variations of the connection modulus shown in Fig. 1 to be studied.

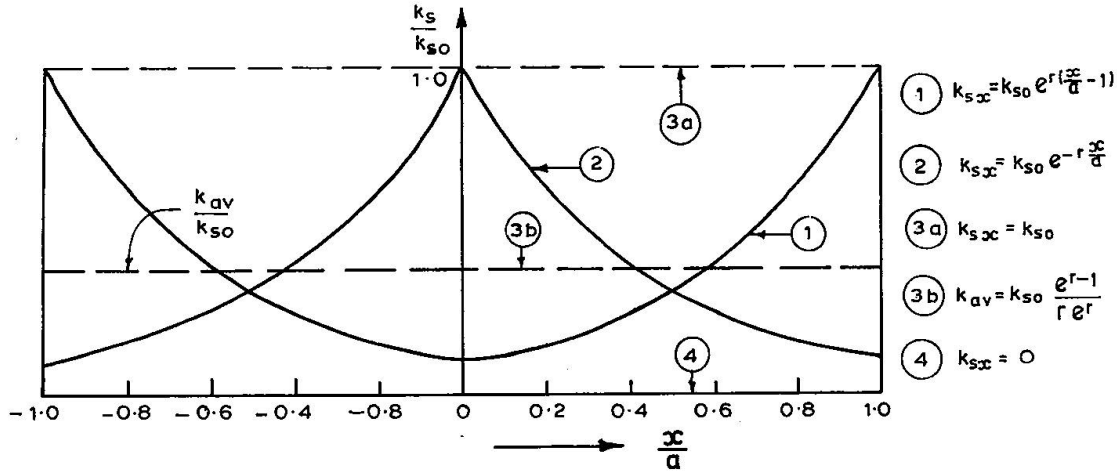


Fig. 1. Assumed variations of foundation modulus over the span of simply supported beam.

In general

$$k_{sx} = \begin{cases} \frac{k_{so}}{p} e^{\frac{|x|}{a} \ln p} \\ k_{so} e^{-\frac{|x|}{a} \ln p} \end{cases} \quad (1)$$

according as whether the profile of 1 or 2 is assumed. Both representations yield the average value (3) when the connectors are assumed evenly spread but with the same overall resistance to slip over the span as in the expressions (1) and (2), and comply with the limiting case (4) of no interaction when  $p$  is  $-\infty$  for representation (2), whilst complete interaction condition may be simulated by setting  $p$  to  $+\infty$  for (1) and  $-\infty$  for (2).

The equilibrium and compatibility conditions to be satisfied at the cross-section of each longitudinal steel beam, assumed prismatic, are given by [3]

$$\begin{aligned} -E_s I_s w_{,1111} + \frac{d}{2} \cdot F_{,11} - Q_{c,1} + p_n &= 0, \\ [E_s R_s]^{-1} \cdot F + \frac{(d+t)}{2} \cdot w_{,11} - u_{x,1} &= (k_{sx}^{-1} \cdot F_{,1})_{,1} \end{aligned} \quad (2)$$

where  $w$  is the transverse deflexion,  $u_x$  the longitudinal in-plane slab displacement,  $E_s I_s$ ,  $R_s$  and  $d$  refer to the elastic modulus, the second moment of area, cross-sectional area and overall depth of each of the steel beams;  $t$  the slab thickness;  $k_{sx}$  the interaction modulus at  $x$ ,  $F$  the interacting axial forces and  $p_n$  the component of a Fourier representation for the loading.

$$Q_{c,1} = -2[D_y \cdot w_{,222} + H \cdot w_{,112}] \quad (4)$$

A comma, followed by 1 or 2 implies differentiation with respect to  $x$  or  $y$  respectively.

The displacement and membrane stress relations satisfying equilibrium conditions in the absence of body forces for a special case of isotropy are as follows:

$$\left. \begin{aligned} u_x &= \Psi_{,1} - 4(1 + \nu)^{-1} \phi_1 \\ \alpha u_y &= \alpha \Psi_{,2} - 4(1 + \nu)^{-1} \phi_2 \\ c_{11}^{-1} \sigma_{xx} &= \Psi_{,11} + \alpha^2 \nu \Psi_{,22} - 4(1 + \nu)^{-1} (\phi_{1,1} + \alpha \nu \phi_{2,2}) \\ \alpha^{-2} c_{11}^{-1} \sigma_{yy} &= \nu \Psi_{,11} + \alpha^2 \Psi_{,22} - 4(1 + \nu)^{-1} (\nu \phi_{1,1} + \alpha \phi_{2,2}) \\ \alpha^{-2} c_{11}^{-1} \sigma_{xy} &= (1 - \nu) [\Psi_{,12} - 2(1 + \nu)^{-1} (\phi_{1,2} + \alpha \phi_{2,1})] \end{aligned} \right\} \quad (5)$$

where

$$\Psi = \phi_0 + x \phi_1 + Y \phi_2, \quad Y = \alpha^{-1} y, \quad (6)$$

$$\phi_0 = \sum_{n=1}^{\infty} [A_{n1} \cos h(K_n Y) \cos(K_n x) + B_{n1} \cos h(L_n X) \cos(L_n y)], \quad (7)$$

$$\phi_1 = \sum_{n=1}^{\infty} B_{n2} \sin h(L_n X) \cos(L_n y), \quad (8)$$

$$\phi_2 = \sum_{n=1}^{\infty} A_{n2} \sin h(K_n Y) \cos(K_n x), \quad (9)$$

Similarly we write the relation giving the deflexion surface satisfying equilibrium conditions for a plate having properties that result in a special case of isotropy as follows:

$$w(x, y) = \sum_{n=1}^{\infty} [K_n^{-4} q_n + \bar{A}_{n1} \cos h(K_n \bar{Y}) + \bar{A}_{n2} \bar{Y} \sin h(K_n \bar{Y})] \cos(K_n x), \quad (10)$$

where

$$q = D_x \sum_{n=1}^{\infty} q_n \cos(K_n x), \quad K_n = (2n - 1) \frac{\pi}{2a}, \quad L_n = \frac{2a}{b} K_n, \quad X = \alpha x, \quad \bar{Y} = \lambda^{-1} y, \quad (11)$$

By our choice of the eigen-values we have ensured the satisfaction of the condition for simply supported edges, namely  $w = M_x = 0$  at  $x = \pm a$ . The conditions that  $\sigma_{xx} = \sigma_{xy} = 0$  at  $x = \pm a$  are satisfied provided

$$B_{n1} = L_n^{-1} (\alpha^2 c_{11} - c_{12})^{-1} [2(1 - \nu)(1 + \nu)^{-1} c_{11} - (\alpha^2 c_{11} - c_{12}) L_n a \tan h(L_n \alpha a)] \cdot B_{n2} \quad (12)$$

$$B_r = L_r \left[ \frac{(1 - \nu)(\alpha^2 c_{11} + c_{12})}{(1 + \nu)(\alpha^2 c_{11} - c_{12})} \sin h(L_r \alpha a) + L_r \alpha a / \cos h(L_r \alpha a) \right] \cdot B_{r2}, \quad (13)$$

$$B_r = 4b^{-1} \sum_{m=1}^{\infty} \frac{(-1)^m L_r K_m \alpha^{-1} \sin h(K_m b \alpha^{-1})}{\alpha^2 K_m^2 + L_r^2} \left[ (1 + \nu)^{-1} - \frac{K_m^2}{K_m^2 + \alpha^2 L_r^2} \right] \cdot A_{m2} \quad (14)$$

The other conditions to be satisfied from a consideration of the symmetry of the problem are

$$u_y = 0 \text{ at } y = 0 \text{ and } y = \pm b \text{ for } ||x| < a$$

$$w_{,2} = 0 \text{ at } y = 0 \text{ and } y = \pm b \text{ for } |x| < a$$

These are automatically satisfied if

$$A_{n1} = K_n^{-1} [(3 - \nu)(1 + \nu)^{-1} - K_n b \alpha^{-1} \cot h(K_n b \alpha^{-1})] \cdot A_{n2} \quad (15)$$

$$\bar{A}_{n1} = -K_n^{-1} [1 + K_n b \lambda^{-1} \cot h(K_n b \lambda^{-1})] \cdot \bar{A}_{n2} \quad (16)$$

Using the result in (15) above we obtain the interacting axial force as

$$F = \tau_0 t \sum_{n=1}^{\infty} A_{n2} \sin h(K_n b \alpha^{-1}) \cos(K_n x) \quad (17)$$

In satisfying the compatibility condition (3) using the new representation for  $k_s$  as  $k_{s0} e^{r \frac{|x|}{a}}$ , that is case 1 of Fig. 1, and employing orthogonality relations for taking out from under the summation signs the extractable superposition coefficients we encounter the integrals

$$\int_{-a}^a e^{-r \frac{|x|}{a}} \cdot \sin(K_n x) \cos(K_m x) dx \quad (18)$$

$$\int_{-a}^a e^{-r \frac{|x|}{a}} \cdot \cos(K_n x) \cos(K_m x) dx \quad (19)$$

which upon solution reduces to zero for (18) and for (19) to

$$g_{mn} = \frac{r}{a} \left[ \frac{(-1)^{n-m+1} e^{-r} + 1}{r^2 + (K_n - K_m)^2} + \frac{(-1)^{n+m} e^{-r} + 1}{r^2 + (K_n + K_m)^2} \right] \quad (20)$$

the case when the variation of  $k_s$  is of the type 2 of Fig. 1, namely  $k_{s0} e^{-r \frac{|x|}{a}}$  is obtained by replacing  $r$  in (20) by  $-r$ .

The equilibrium and compatibility equations (2) and (3) now reduce respectively to

$$\bar{A}_{m2} J_m = E_s I_s q_m - p_m + \tau_0 t d K_m^2 \sin h(K_m b \alpha^{-1}) \cdot A_{m2} \quad (21)$$

$$A_{m2} + \tau_0 t V_m k_{s0}^{-1} \sum_{p=1}^{\infty} K_p^2 g_{mp} \sin h(K_p b \alpha^{-1}) \cdot A_{p2} + V_m \sum_{p=1}^{\infty} \sum_{n=1}^{\infty} R_{mp} S_{np} A_{n2} =$$

$$\frac{a(d+t)}{2} J_m^{-1} V_m [c_{1m} p_m + 4q_m D_y \lambda^{-3} \sin h(K_m b \lambda^{-1})] \quad (22)$$

$$J_m = K_m^2 [E_s I_s C_{1m} + 4D_y \lambda^{-3} \sin h(K_m b \lambda^{-1})] \quad (23)$$

$$V_m^{-1} = a \left\{ \tau_0 t \sin h(K_m b \alpha^{-1}) \left[ (E_s R_s)^{-1} + \frac{d(d+t)}{4} C_{1m} J_m^{-1} K_m^2 \right] + C_{2m} \right\} \quad (24)$$

For a constant  $k_s$

$$V_m^{-1} = a \left\{ \tau_0 t \sin h(K_m b \alpha^{-1}) \left[ (E_s R_s)^{-1} + \frac{d(d+t)}{4} C_{1m} J_m^{-1} K_m^2 + K_s^{-1} K_m^2 \right] + C_{2m} \right\} \quad (25)$$

since  $g_{mp} = a$  when  $p = m$  and zero  
when  $p \neq m$

where

$$C_{1m} = K_m (\cos h(K_m b \lambda^{-1}) + K_m b \lambda^{-1} / \sin h(K_m b \lambda^{-1})) \quad (26)$$

$$C_{2m} = K_m ((3 - \nu)(1 + \nu)^{-1} \cos h(K_m b \alpha^{-1}) - K_m b \alpha^{-1} / \sin h(K_m b \alpha^{-1})) \quad (27)$$

$$R_{mp} = \frac{4\alpha L_p K_m (-1)^{m+1} \cos h(L_p \alpha a)}{K_m^2 + \alpha^2 L_p^2} \left[ \frac{(1 - \nu)c_{12}}{(1 + \nu)(\alpha^2 c_{11} - c_{12})} - \frac{\alpha^2 L_p^2}{K_m^2 + \alpha^2 L_p^2} \right] \quad (28)$$

$$S_{np} = \frac{4S_p^{-1} (-1)^n \alpha L_p K_n \sin h(K_n b \alpha^{-1})}{K_n^2 + \alpha^2 L_p^2} \left[ (1 - \nu)^{-1} - \frac{K_n^2}{K_n^2 + \alpha^2 L_p^2} \right] \quad (29)$$

$$S_p = L_p b \left[ \frac{(1 - \nu)(\alpha^2 c_{11} + c_{12})}{(1 + \nu)(\alpha^2 c_{11} - c_{12})} \sin h(L_p \alpha a) + L_p \alpha a / \cos h(L_p \alpha a) \right] \quad (30)$$

The method of analysing two-span continuous composite beams as well as the method of obtaining effective widths based on a comparison of the moment-curvature relationship of the composite assembly with that of the steel beams alone supporting the imposed loading have been discussed elsewhere [5].

Complete interaction is attained for type 1 variation of the  $k_s$  as  $r$  tends to infinity in equation (20) since  $g_{mm}$  will tend to zero correspondingly resulting in an earlier established result [4]. The case of no interaction is obtained by letting  $r$  tend to  $-\infty$ , thus leading to a zero  $k_s$ . Under this condition, equation (22) resulting from the interface compatibility condition becomes indeterminate since the coefficients of  $A_{r2}$  become infinite and the  $A_{r2}^s$  consequently take zero values. The surviving condition of equilibrium (21), also leads thereby to another earlier established result for no interaction case [4].

In the results that follow, degree of interaction, effective widths, steel bottom flange stress reduction, slip and deflexion profiles are all studied with the variation of  $r$  or  $p$ .

For the special case of isotropy considered in this analysis

$$\alpha = \lambda = (c_{22} c_{11}^{-1})^{1/4} = (D_y D_x^{-1})^{1/4}$$

and both become unity for the case of ordinary isotropy.

For the purpose of computations symmetrical and prismatic steel I sections are assumed to be interacting with a 152 mm thick concrete slab assumed isotropic. The dimensions of each steel beam are as follows:

Overall depth	625 mm
Flange width	254 mm

$$\begin{aligned} \text{Flange thickness} & 24 \text{ mm} \\ \text{Web thickness} & 13 \text{ mm} \\ I_s & = 1.309579 \times 10^{-3} \text{ m}^4 \\ R_s & = 1.969300 \times 10^{-2} \text{ m}^2 \end{aligned}$$

The total load applied either as midspan point load or uniformly distributed over a span of 4.0 metre is 250 kN.

### Concluding Remarks

The analysis demonstrates clearly the influence of the variation of shear connection modulus on interaction. It also once again confirms that considerable reduction in steel bottom flange stress and deflexions can be achieved at relatively low degree of interaction (see Tables 1 and 2  $\frac{k_s}{E_s}(1.5 - 2.5\%)$ ). It is assumed that the total number of connectors used partly determines the interface shear connection modulus of the composite element. Quite clearly the advantages to be gained by employing a large number of shear connectors to provide a shear connection modulus much above the range of values given above are minimal. What would seem to be paramount from the analysis is the physical arrangements of the connectors between adjacent points of minimum or zero moments.

If it is assumed that the physical arrangements of shear connectors bears a direct relation to the profile of shear connection modulus developed during interaction, then from the present analysis the more efficient distribution of connectors will be that which for a given number of shear connectors to be employed, varies from a minimum at the point of maximum positive or negative moment to a maximum at the point of zero moment. The system of evenly distributed shear connectors based on the above assumption would appear to be less efficient. However it must be pointed out that the method by which shear connectors develop interaction shear connection modulus is complex and least understood, and it could very well be that even the evenly distributed system of shear connectors does not offer a constant shear connection modulus in service.

Table 1

$100 \frac{k_s}{E_s}$	Maximum Deflexion Span Ratio $\times 10^{-4}$ for $\frac{b}{a} = 0.4, p = 10$		
	Average Constant $k_s$	$\frac{k_{sx}}{k_{so}} = \frac{e^{-x/a}}{p} 1n p$	$\frac{k_{sx}}{k_{so}} = e^{-x/a} 1n p$
0	22.6	22.6	22.6
0.125	18.5	14.0	17.2
0.250	16.4	12.5	15.0
0.500	14.3	11.5	13.1
1.000	12.6	11.0	11.7
1.500	11.9	10.6	11.2
2.000	11.5	10.5	10.9
2.500	11.2	10.4	10.7
$\infty$	10.0	10.0	10.0



Table 2

Bottom Flange Stress Factor for $\frac{b}{a} = 0.4, p = 10$			
$100 \frac{k_s}{E_s}$	Average Constant $k_s$	$\frac{k_{sx}}{k_{so}} = \frac{e^{-\frac{1 \times 1}{a}}}{p} 1np$	$\frac{k_{sx}}{k_{so}} = e^{-\frac{1 \times 1}{a}} 1np$
$p = 10$			
0	0.934	0.934	0.934
0.125	0.894	0.852	0.880
0.250	0.874	0.836	0.857
0.500	0.853	0.824	0.837
1.000	0.835	0.815	0.822
1.500	0.827	0.811	0.816
2.000	0.822	0.808	0.812
2.500	0.818	0.806	0.811
$\infty$	0.800	0.800	0.800

Fig. 2 demonstrates the performance of the various distributions of shear connection moduli assumed, in terms of bottom flange stress reduction and central deflexion reduction as compared with bottom flange stress and deflexion of the steel beams acting alone. The practical range of the diagram is from  $p = 2$  to 10. Below this range one encounters infinite values of  $k_s$  and it will not be meaningful to draw conclusions from these.

Type 1 distribution as shown in fig. 1 is clearly the most efficient. Fig. 3 further demonstrates this fact in the degree of interaction achieved as defined by the

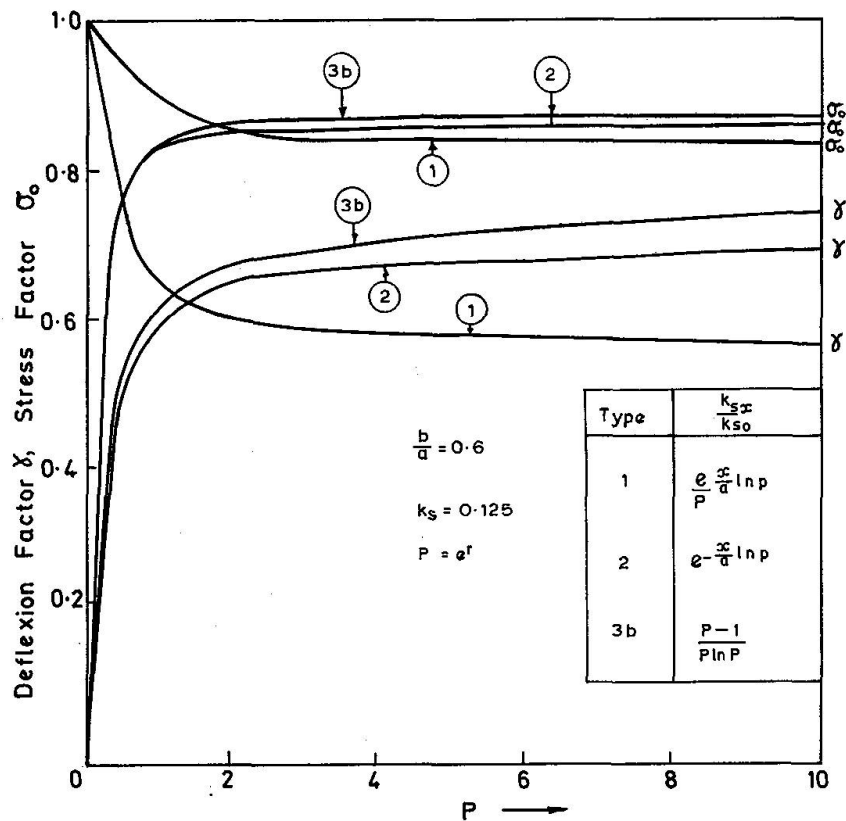


Fig. 2. The effect of foundation modulus reduction parameter  $p$  on deflexions and stresses.

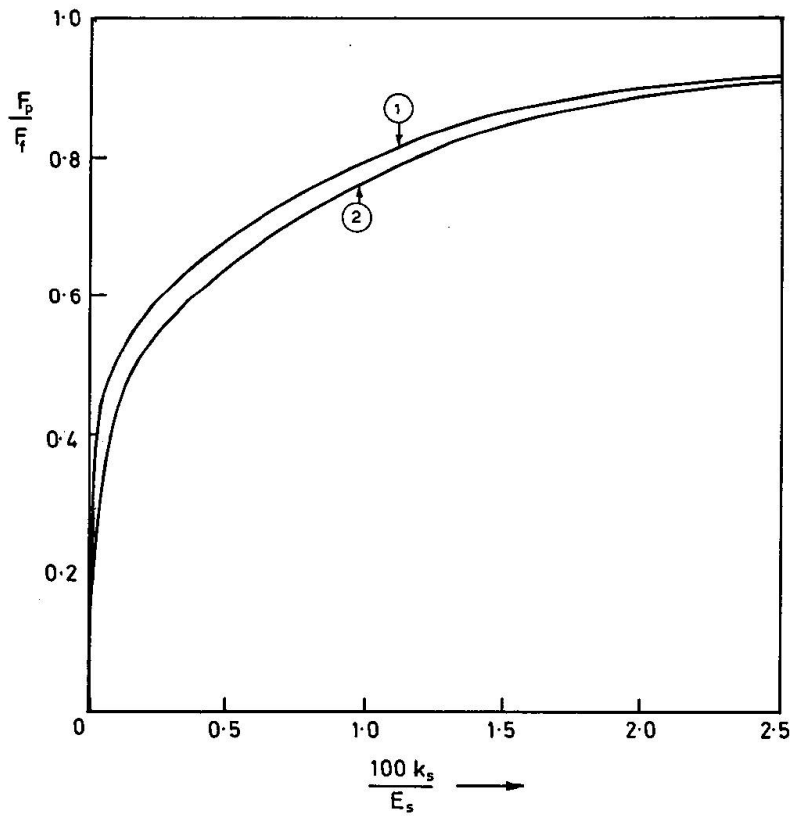


Fig. 3. Degree of interaction against foundation modulus.

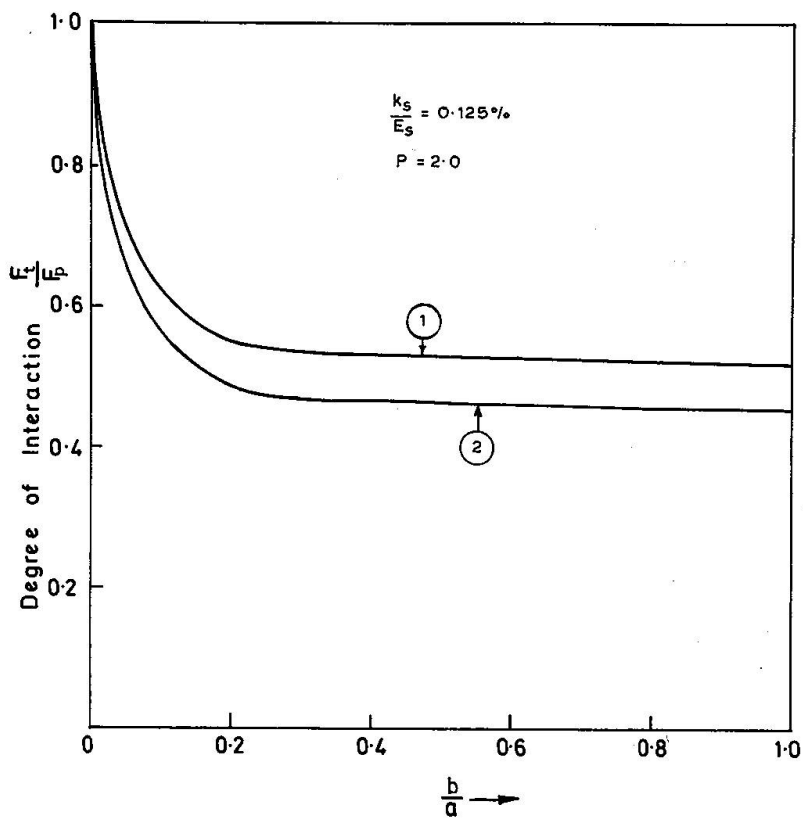


Fig. 4. Degree of interaction against aspect ratio.

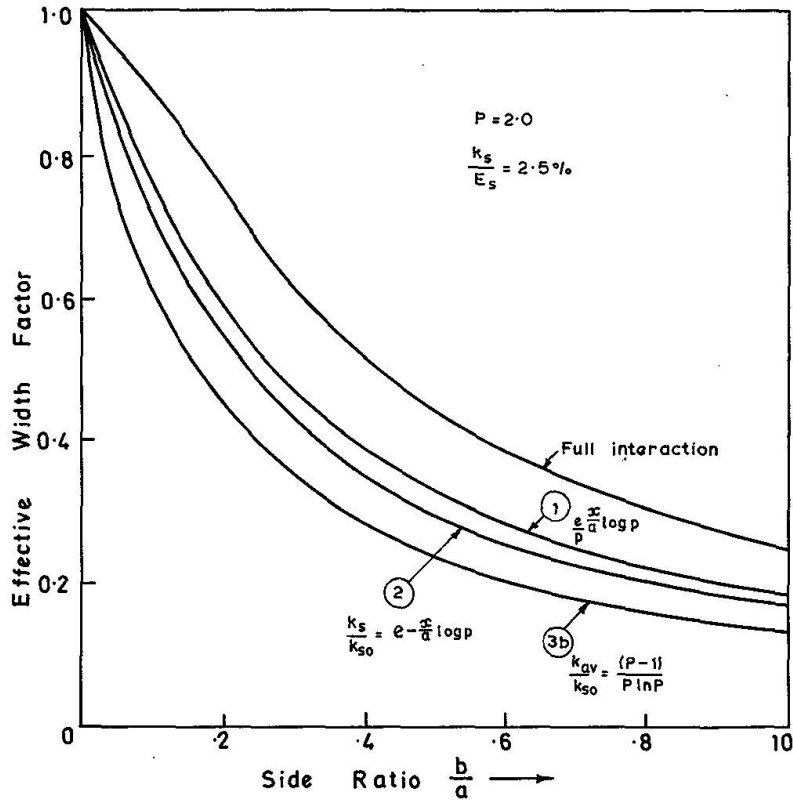


Fig. 5. Variation of Effective width with aspect ratio.

interacting axial force developed. Fig. 4 and 5 show respectively the variation of degree of interaction with aspect ratio ( $\frac{b}{a}$ ) and the variation of effective width with aspect ratio for various distributions of shear connection modulus including full interaction. In all these the superiority of Type 1 distribution of Fig. 1 is manifested.

Finally mention must be made of the rather interesting slip profiles resulting from the various representations of shear connection moduli, Fig. 6a and 6b. Slip profile for the Type 2 representation of shear connection modulus of Fig. 1 has not been reported from any experimental work known for a beam simply supported. Slip profile for the Type 3b (Fig. 1) representation of modulus is however fairly well known. The slip profile similar to that for Type 1 representation of shear connection modulus of Fig. 1 was first observed in experiments performed by Balakrishnan [2] on simply supported composite beams subjected to symmetrical loading.

In conclusion the analysis has shown, first, that distribution of shear connectors is an important factor in achieving a high degree of interaction in composite beams under static working loads, and secondly, that the existence of small amount of interface slips does not negate the attainment of a high degree of interaction. Greater economy can therefore be achieved by seeking not to provide enough connectors to prevent slip totally but by providing sufficient number of connectors, distributed in accordance with the earlier observations in these concluding remarks, to develop an interface shear connection modulus not greater than 2.5% of the modulus of elasticity of the steel beam. The modulus offered by different types of connectors will still need to be determined from standard push-out tests.

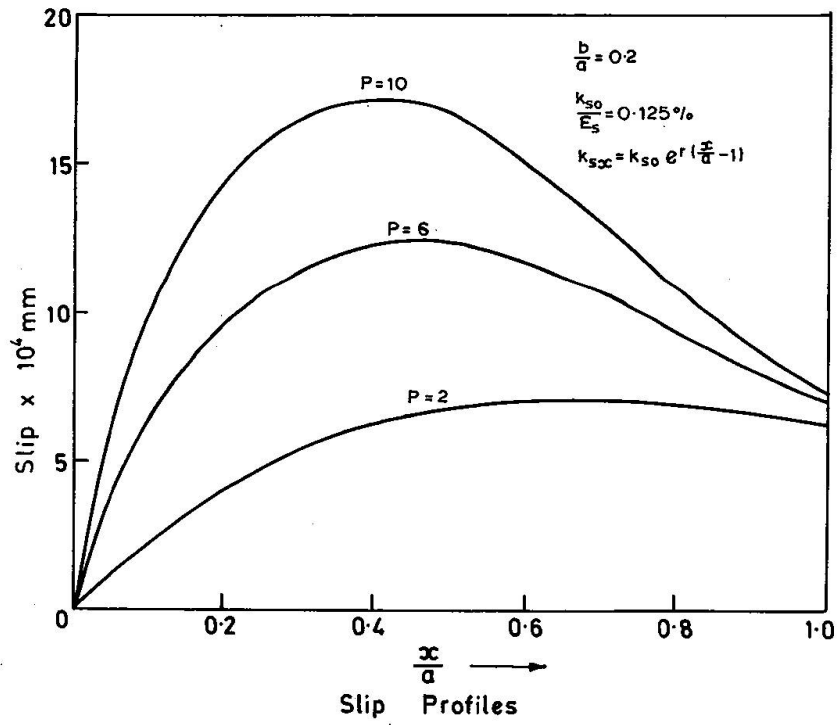


Fig. 6a. Slip Characteristics for Type 1 Spanwise variation of foundation modulus.

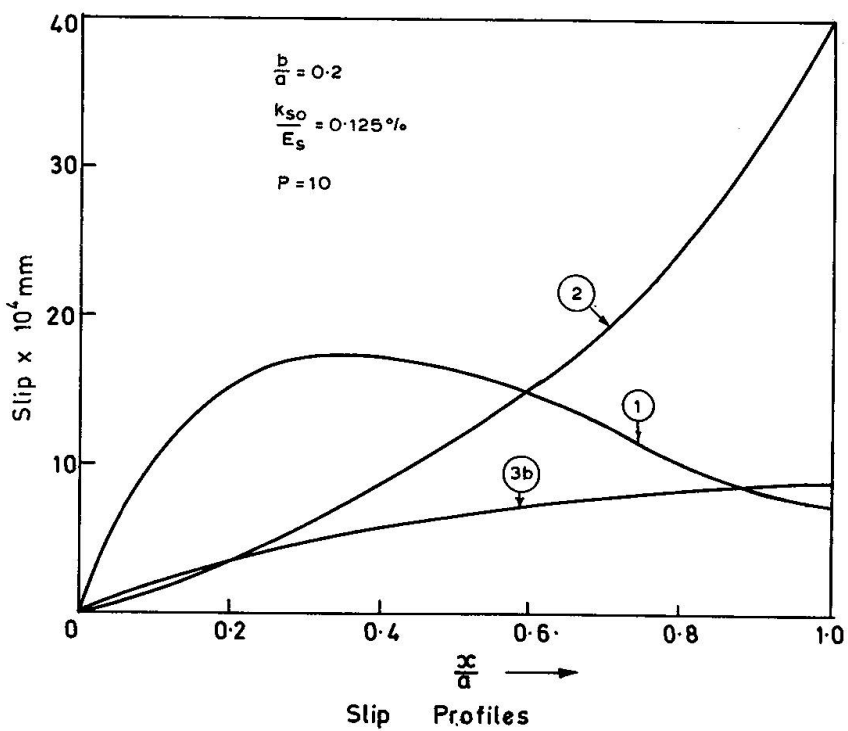


Fig. 6b. Slip Characteristics for different span-wise variation of foundation modulus.

### Safety and Economy

Composite construction has been accepted in practice as an economic method of design. By exploiting the presence of the concrete slab when it is in compression and its steel reinforcement when the concrete is in tension, some reduction in sizes of the principal moment resisting elements (e.g. steel beams or concrete ribs) is effected. However the attainment of this reduction depends almost exclusively on the efficiency and reliability of the connecting mechanism — in this case shear connectors of various forms. Design techniques must therefore have as their goals not only reduction in beam sizes, but also the provision and arrangement within a beam of a number of shear connectors needed to ensure good performance and safety under working loads (i.e. not permitting excessive interface slips).

Distribution of shear connectors in a given beam could most probably influence the pattern or mode of failure of the connectors in the beam. It is reasonable to suppose that the worst strained connectors will be those located in the region of greatest slips within a beam. Such regions become the location for commencement of connector failures. The mathematical representations assumed in the present analysis for the variation of connection modulus along the span (and hence the spacing of connectors) are very idealised and such idealised distributions of modulus variation are not readily attainable in practice. It can however be inferred from the study that for reasons of safety, the best arrangement of shear connectors is one in which connector spacing is small in the regions of maximum shears or of maximum slips e.g. ends of a simply supported beam.

Current design practice is either based on connection modulus that will permit little or no interface slips or the attainment of ultimate carrying capacities by connectors at collapse of the composite beams. The objective is to eliminate or limit severely interface slip that can occur in a composite beam under working loads using the results of standard push-out tests. Consequently the value of working load connection modulus aimed at in practice by current design methods is rather high. The conclusion of the analysis that a high degree of interaction is attainable if sufficient number of connectors to develop an interface shear connection modulus not greater than 2.5% of the modulus of elasticity of the steel beam is employed should lead to some savings on the number of connectors.

### Notations

$a$	half span.
$b$	distance between steel beams.
$c_{11}, c_{22}, c_{12}, c_{66}$	plate material elastic constants.
$D_x, D_y$	plate flexural stiffness in the $x$ and $y$ directions.
$d$	overall depth of steel beams.
$E_s$	steel modulus of elasticity.
$F$	interacting axial force.
$I_s$	steel second moment of area.
$k_{so}$	maximum shear connection modulus.
$p, r$	non-dimensional parameters for the variation of $k_s$ .

$P_n$	harmonic component of loading.
$q$	superimposed loading.
$q_n$	harmonic component of superimposed loading.
$R_s$	cross-sectional area of steel beam.
$u, v$	plate displacements in the $x, y$ directions.
$w$	deflexion.
$\alpha$	non-dimensional parameter for plate plane stress.
$\lambda$	non-dimensional parameter for plate bending.
$\tau_0$	$8 c_{66} \alpha^{-1} (1 + \nu)^{-1}$ .
$\nu$	Poisson's ratio.
$\sigma_0$	<u>bottom flange stress of composite assembly</u>
	<u>bottom flange stress of steel beam alone</u>
	<u>deflexion of composite beam</u>
	<u>deflexion of steel beam alone</u>

### References

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### Summary

Various forms of shear connectors, in the form of studs, spirals or cleats provide the means for ensuring composite action between beams and slabs. The present analysis attempts to study the influence, if any, of the span-wise variation in the shear connection modulus. This variation can be directly related to the physical arrangement of the connectors along the span of the beam. The results lead to the obvious conclusion that the concentration of shear connectors should be more in the region of large shears and that an even distribution of these connectors is not the most effective for ensuring composite action.

### Résumé

Différents genres de connecteurs de cisaillement sous forme de goujons, spirales et profilés fournissent les moyens propres à assurer la transmission des efforts rasants entre poutres et dalles. L'auteur étudie l'influence éventuelle de la variation

le long de la portée de la déformabilité des connecteurs sur le comportement des poutres mixtes. Cette variation peut être directement mise en rapport avec la distribution des connecteurs. Les résultats obtenus mènent à la conclusion évidente que les connecteurs doivent être concentrés dans les régions à cisaillements élevés et qu'une distribution uniforme ne constitue pas le moyen le plus efficace à assurer la liaison.

### **Zusammenfassung**

Zur Gewährleistung der Verbundwirkung zwischen Träger und Platte werden verschiedene Arten von Verbundmitteln in Form von Bolzen, Wendeln oder Knaggen verwendet. Der Einfluss der Änderung des Verformungsmoduls der Verbundmittel Diese Änderung kann direkt mit der Verteilung der Verbundmittel längs der Spannweite eines Trägers in Beziehung gebracht werden. Die Ergebnisse führen zum offensichtlichen Schluss, dass die Verbundmittel mehr im Bereich grosser Schubbeanspruchungen liegen sollten und eine gleichmässige Verteilung nicht das wirksamste Mittel zur Gewährleistung der Verbundwirkung darstellt.