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Inelastic Response of Prestressed Concrete Beams

Comportement inélastique de poutres précontraintes en béton

Unelastisches Verhalten vorgespannter Betonbalken

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Introduction

The purpose of the reported study was to develop an analytic technique to adequately describe the inelastic load-deflection behavior of prestressed concrete beams. The finite element method was used as the basis of solution. The stress distribution, and response to material nonlinearities such as the yielding of the reinforcement and the cracking and crushing of concrete are included. It was desired to produce a technique which would be efficient enough to be used as part of a larger nonlinear analysis problem, e.g., the overload analysis of beam-slab bridges. It will be seen that simplifying assumptions have been made in order to achieve the desired efficiency while continuing to allow a realistic treatment of beams subjected primarily to bending action. The emphasis of this paper will be on nonlinear analysis of prestressed concrete beams. Notation used herein will be defined when first encountered.

Basic Model

The basic model under consideration is a simply supported, essentially prismatic beam subjected to loading in a plane of symmetry. The formulation is general enough to allow for a wide range of materials and boundary conditions, but does not allow for the inclusion of local or lateral-torsional buckling of the beam. The nonlinear behavior of a beam is treated as a piecewise linear problem using an incremental loading path with iteration of each load step.

The beam is discretized as a series of beam type finite elements along its length. The elements are subdivided into layers. The node points at each end of the beam elements lie in an arbitrary plane of reference. Fig. 1 shows a beam element, coordinates and positive sign conventions. The plane sections assumed by the

Bernoulli beam theory is used to relate the strains in the layer to the displacements at the nodes. If a sufficient number of layers is used each layer may be assumed to be in a state of uniaxial tension or compression with the centroid of the layer assumed to be representative of the layer. These assumptions would become tenuous if high shearing stresses were present.

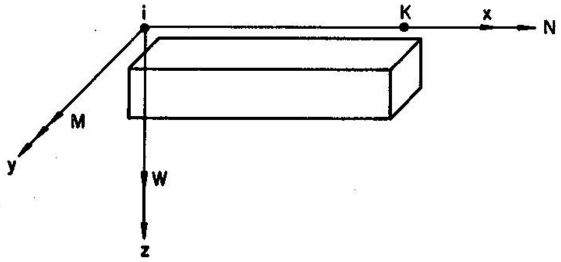

Fig. 1. Coordinate system and positive sign convention.

The effect of this simplified layered model on the economy of solution via the tangent stiffness approach is apparent from the following example. If 10 elements each having 15 layers is used with the proposed method there are 11 nodes each having 3 degrees of freedom. This results in 33 simultaneous equations. If, on the other hand, a continuum approach utilizing 300 triangular elements with 2 degrees of freeedom per node were used, there would be 352 simultaneous equations. Considering that incremental $\frac{I}{N}$ iterative approaches may require hundreds of solutions it is apparent that the savings in computational effort is enormous.

There are three degrees of freedom at each node of the beam element. They are the axial displacement, U, the lateral displacement, W, and the bending rotation, θ. The following polynomials are used to describe displacements within the element.

$$U = \alpha_1 + \alpha_2 X \tag{1}$$

$$W = \alpha_3 + \alpha_4 X + \alpha_5 X^2 + \alpha_6 X^3 \tag{2}$$

The α's are constants to be determined by using the boundary conditions at both ends of an element.

The generalized stresses are the normal force, N, and bending moment, M, at the plane of reference defined by $Z = 0.0$ in Fig. 1. The generalized strains are the axial strain and curvature at the plane of reference. The generalized stresses and strains are related by an elasticity matrix $[D]$. The assumption of plane sections makes it possible to generate the elasticity matrix using the usual equations of mechanics instead of the theory of elasticity. This is a trade-off of some accuracy and geometric generality for far greater computational efficiency. The result is:

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} \bar{A} & \bar{S} \\ \bar{S} & \bar{I} \end{bmatrix} \begin{Bmatrix} dU/dX \\ -d^2W/dX^2 \end{Bmatrix} \tag{3}$$

in which for n layers per element:

$$\bar{A} = \sum_{i=1}^{n} E_i A_i, \ \bar{S} = \sum_{i=1}^{n} E_i A_i Z_i, \ \bar{I} = \sum_{i=1}^{n} E_i A_i Z_i^2 \tag{4}$$

The element stiffness matrix can then be established using the well defined procedures of the finite element method, e.g., [16]. A detailed derivation can be found in [8].

Incremental Iterative Analysis Scheme

The load is applied in increments. The increments of forces, $\{F\}$, and displacements, $\{\delta\}$, are related by Eq. 5.

$$\{F\}=[K]\{\delta\} \tag{5}$$

Matrix $[K]$ is the assembled and reduced tangent stiffness matrix for the beam. The displacement increment is used to compute new trial increments of stress and strain. Each incremental displacement component is then checked against the corresponding component of the last trial. If all are within a relative tolerance of the last trial the iteration is stopped and the stress and displacement fields are incremented to include the new contributions from this load step. Each layer is then checked for tensile cracking or compressive crushing.

If no cracking or crushing has taken place, another load increment is added and the process is repeated with the generation of a new stiffness matrix which reflects the current state of stress.

If convergence of the current load step has not been attained the incremental stresses are temporarily added to the total stresses to find new tangent moduli using the layer stress-strain laws. A new stiffness matrix is generated and new incremental displacements are computed and compared with the last set to check convergence. This process is repeated until either convergence is attained in a limited number of trials or the maximum number of trials is reached at which time the load increment is reduced and the whole process is repeated.

Stress-Strain Curves

The following types of stress-strain curves can be used: (1) elastic-brittle, (2) elastic-plastic, not just elastic-perfectly plastic, (3) elastic-plastic with linear strain hardening, (4) elastic-plastic with tensile cracking, and (5) elastic-plastic with tensile cracking and compressive crushing.

The Ramberg-Osgood formulation, [12], has been chosen to provide generality in the shape of the stress-strain curve while maintaining a continuous mathematical expression, and to allow for a common base for all stress-strain curves.

$$\varepsilon=\frac{\sigma}{E}+\frac{1-m}{m}\left(\frac{\sigma_1}{E}\right)\left(\frac{\sigma}{\sigma_1}\right)^n \tag{6}$$

σ = Stress at some load.

ε = Strain at a stress equal to σ.

E = Initial modulus of elasticity.

σ_1 = Secant yield strength equal to the ordinate of intersection of the $\sigma\frac{I}{N}\varepsilon$ curve and a line of slope $(m)\cdot(E)$.

n = A constant.

m = A constant defining a line of slope $(m)\cdot(E)$ on a plot of stress and strain.

The following approach has led to stress-strain curves for concrete in uniaxial compression which compare well with similar curves in the literature:

1. Assume a value for Young's modulus from any acceptable equation or from laboratory tests.
2. Assume $\sigma_1 = f'_c$.
3. Assume that the stress-strain curve must pass through the point $(\bar{\varepsilon}, f'_c)$. This leads to the following equation for the coefficient m. $\bar{\varepsilon}$ is typically 0.002 for normal weight concrete.

$$m = \frac{f'_c}{E\bar{\varepsilon}}$$

4. Assume the Ramberg-Osgood curve stops at a strain of $\bar{\varepsilon}$.
5. Assume a horizontal straight line from a strain of $\bar{\varepsilon}$ to a strain given in Table 1 below as ε_1.
6. Assume a straight line sloping downward from ε_1 to a stress of zero. Suggested values for this slope, "E_{down}", are also found in Table 1. "E_{down}" will be used to compensate for compressive crushing.

Table 1

f'_c (psi)	E_{down} (ksi)	ε_1
5600	3000	0.0022
4750	1800	0.0022
3900	1250	0.0023
<3000	700	0.0024

7. From trial and error comparisons a value of $n = 9$ was found to give consistently good results for all strengths tried.

The results of this method of approximating the concrete compressive stress-strain curve are shown in Fig. 2. The approximate curves are quite close to the

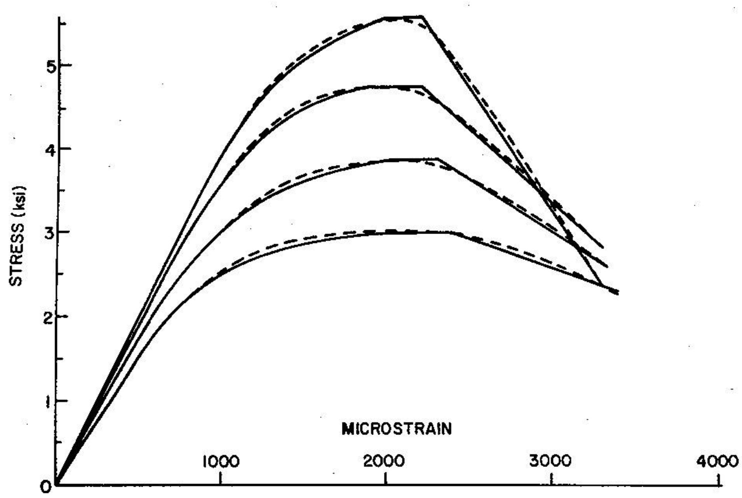

Fig. 2. Analytic and "actual" stress-strain curves.

typical smoothed concrete stress-strain curves as measured on the compressive side of flexural tests which are shown by the dotted lines. The overall shape is similar to that of Hognestad's stress-strain curve [5]. This curve has been compared with specific stress-strain curves for concrete [2 and 13]. The result is shown in Fig. 3.

The object of this analytic method is to produce the load-deflection curve of prestressed concrete beams. This implies that the strength and stiffness of concrete in tension cannot be neglected as is commonly done in ultimate strength type analyses. The inclusion of tensile concrete in this analysis procedure requires the employment of an appropriate tensile stress-strain curve for concrete. [3, 7 and 11] contain complete experimental tensile stress-strain curves for concrete. Fig. 4 represents the particular curves found in [3]. Unfortunately there is a small data base from comparative complete tensile and compressive stress-strain curves. This means that some assumptions about the tensile stress-strain curves are necessary. Fig. 5 shows the analytic tensile stress-strain curve actually used in the numerical examples to be presented as Curve *B*. The formulation has been left general enough to accept a curve as complex as Curve *A*, modeled after the curves in Fig. 4, so that the future developments in material behavior can be accomodated. The principle that concrete cracking does not create instantaneous unloading, as evidenced by the analytic tensile stress-strain curve used here, has also been employed by other investigators [1, 6, 10, 14]. The term tension stiffening stress-strain curve has also been used to describe similar phenomenon. Further information on the development of this tensile stress-strain curve can be found in [8].

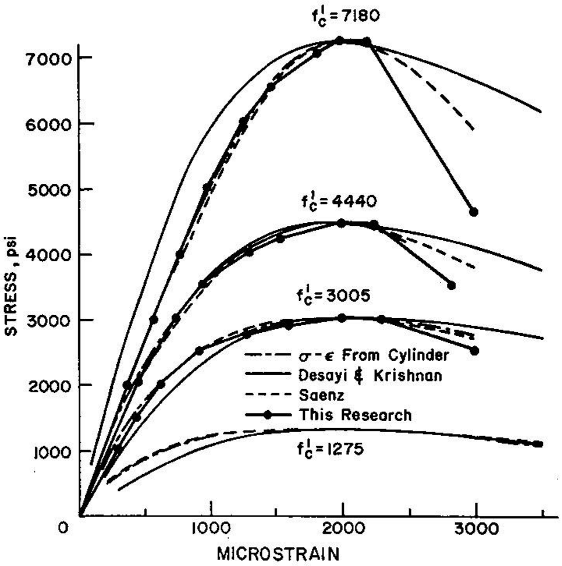

Fig. 3. Comparisons of concrete stress-strain curves.

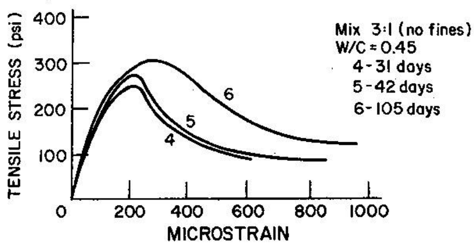

Fig. 4. Concrete tensile stress-strain curves.

The effect of various parameters used to define the tensile and compressive stress-strain curves on the resulting load-deflection behavior for under-reinforced prestressed concrete beams has been studied. This information as well as the effect of other geometric modeling parameters are contained in [9].

Application of Ramberg-Osgood formulation to reinforcing and prestressing steels is virtually exactly what it was intended for and deserves no more comment.

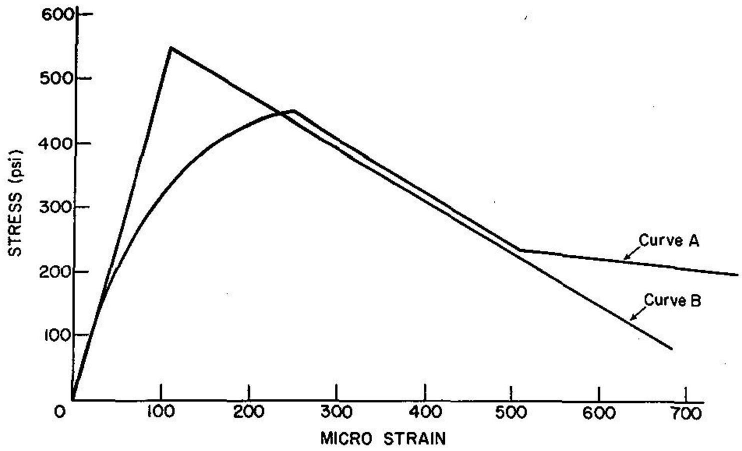

Fig. 5. Analytic tensile stress-strain curves.

A Technique for Cracking and Crushing Analysis

When the iterative procedure used to find the incremental displacements and stresses corresponding to a given load step has converged to an acceptable tolerance, the accumulated stresses and displacements are tentatively incremented. A prescanning and load reduction process is used to prevent large overstressing of the material for any load step. If no stresses exceed the compressive or tensile limit, another load step is taken.

If scanning reveals that the temporary accumulated stress is greater than the allowable tensile stress for any layer, then the layer is said to have cracked and steps are taken to set its modulus of elasticity to zero and redistribute the stresses in that layer.

The redistribution of stresses is accomplished by using the downward leg of the tensile stress-strain curves and a basic concept of the initial stiffness method. The amount of strain beyond that corresponding to cracking, or the incremental strain, whichever is appropriate, is multiplied by "E_{downt}" to produce a stress-like quantity called a fictitious stress. This is shown schematically in Fig. 6. This fictitious stress is applied to the layer which has cracked until the sum of the increments of fictitious stress and the accumulated tensile stress are zero. The redistribution to the rest of the beam is accomplished by using the layer area to convert stress into an eccentric force and thereby generating a fictitious load vector with axial force and corresponding moment terms. This is also shown in Fig. 6 in which element "i" is unloaded by the fictitious stress while the rest of the beam is being held in equilibrium.

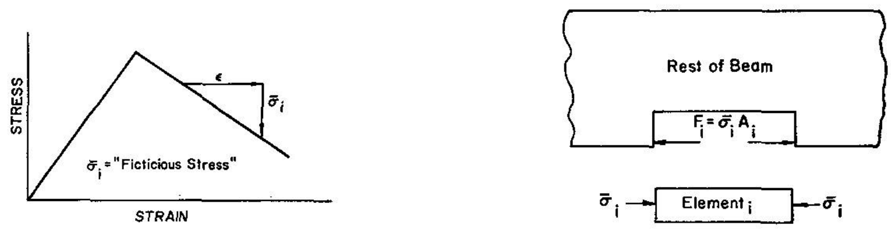

Fig. 6. Ficticious stresses and forces.

During the same scanning operation a test is also made to see if a given layer exceeds a crushing criteria. The crushing criteria for a layer is the attainment of the maximum compressive stress or a strain greater than ε_1 as given in Table 1. If it is ascertained that crushing has occurred, the first stage is to set Young's modulus equal to zero. If the strain is less than the value of ε_1 given in Table 1 no unloading or redistribution is considered. If the strain exceeds ε_1, the excess strain is converted to fictitious stresses and hence fictitious loads analogously to the tensile cracking analysis.

Once all layers have been scanned, the fictitious load vector is used to compute an auxiliary stress and displacement increment. Essentially the same iterative process is used to find convergence for the auxiliary displacement increment as that used for the actual load step. Once convergence has been obtained, the layers are rescanned to check if the redistribution of cracking and/or crushing stresses has caused any more layers to reach a cracking or crushing criteria. If any layers have reached these criteria the process of assembling a fictitious load vector and iterating to convergence is repeated. If no additional layers have reached cracking or crushing there may still be additional fictitious load vector components as a result of the additional strains computed from the increments of displacements. Therefore, the entire process is repeated until the fictitious loads are smaller than some tolerance. At that time the cracking-crushing analysis is terminated and the accumulated stress and displacement field are permanently updated for the effects of this load increment.

Prestressed Concrete Beams

The additional steps used with prestressed concrete beams follow from the physical actions involved in prestressing. An initial stress field is read in for each layer. This provides the initial steel tension. An eccentric prestressing force is applied using the nodal force vector. It is advisable to compensate this prestressing force for the elastic loss which well occur when it is applied. It should be apparent that the object of applying the nodal forces used in prestressing is to produce the same thrust and moment diagrams in the reference plane as would be generated by replacing the prestressing elements at each point along the beam by an eccentric force at that location. This concept is important in generalizing the process for cases other than straight strands or for conditions other than prestressed concrete.

Consider a simply supported prestressed concrete beam pretensioned with a draped strand such that the end eccentricity was e_1 and the eccentricity at a distance L_2 from an end was e_2 and the strand was straight line segments in between. e_1 and e_2 are measured from the reference plane. The prestressing forces would then be modeled as follows:

1. An axial force, P, is applied at each end of the beam.
2. End moments are applied to each end of the beam to (P) (e_1).
3. A concentrated load is applied to the point L_2 from an end such that $P(e_2 \frac{1}{N} e_1) = \bar{P}/L_2$.

In No. 3 $\bar{P}$ is the concentrated load, and L_2 is the distance from the end of the beam to drape point. If due consideration is given to algebraic sign this system of forces will be equivalent to draped strand prestressing. The inclined strand should be simulated by a series of horizontal line segments to approximate its contribution to the global stiffness matrix.

The beam deflects under the influence of the nodal force and moment used to apply the prestressing force. The prestress camber must be included when displacements are converted to total strains to test against strain based behavior criteria.

Comparisons with Test Beams

The experimental results with which comparisons will be made were available in the literature. No experimental work was done as a part of this study. Comparisons will be made with rectangular and I-shaped prestressed concrete beams. All the beams were pretensioned with six 7/16 inch seven wire strands. The rectangular beams had two layers of three strands each, the I-beams had three layers of strands; the top layer having one strand, the middle two and the bottom three.

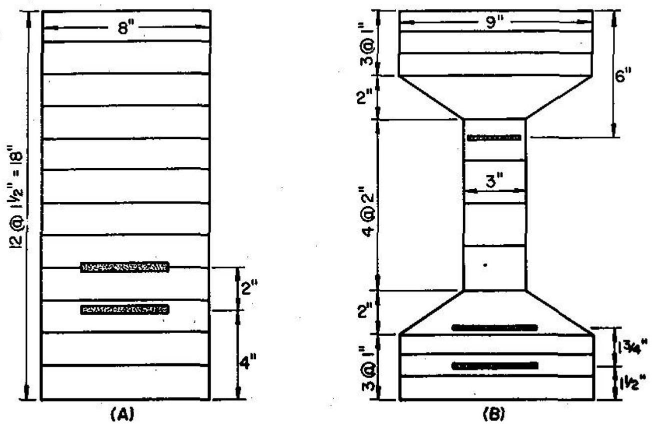

Fig. 7. Layering discretization.

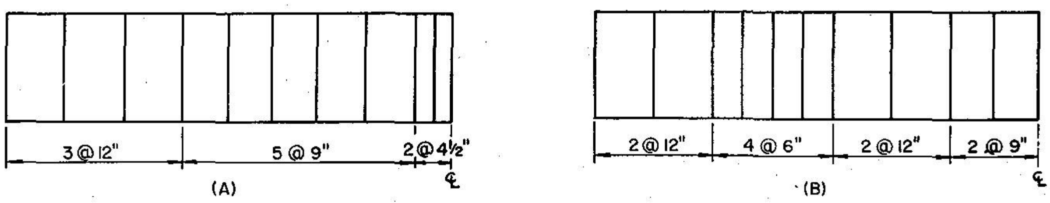

Fig. 8. Elemental discretization.

The prestressed concrete rectangular beams used for the comparative purposes in this study were tested by WALTHER and WARNER [15]. The cross-sectional layering and the elemental discritizations used to model this beam are shown in Figs. 7A and 8A. The cross-sectional dimensions and the half span lengths are also indicated in these figures. The rectangular beams were simply supported to produce a nine foot clear span and were subjected to third point loading within the span. Fig. 9 shows a comparison of the computed and measured load-deflection histories of the beams. Each curve in Fig. 9 (and 10) starts with zero load and zero deflection. The horizontal axes are therefore shown with a "typical" distance rather than an enumerated annotation. The beams had cylinder strength ranging from 6.14 ksi to 6.32 ksi, the prestressing forces varied from 85.73 kips to 93.85 kips, on the day of testing. A comparison of analytic and experimental ultimate strength is given in Table 2.

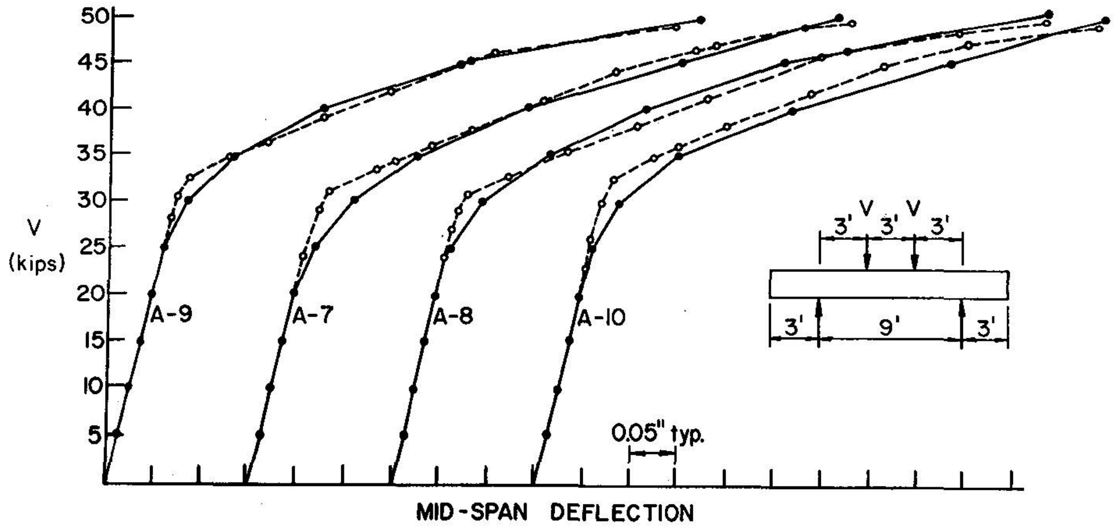

Fig. 9. Load-deflection-curves for prestressed concrete solid box beams.

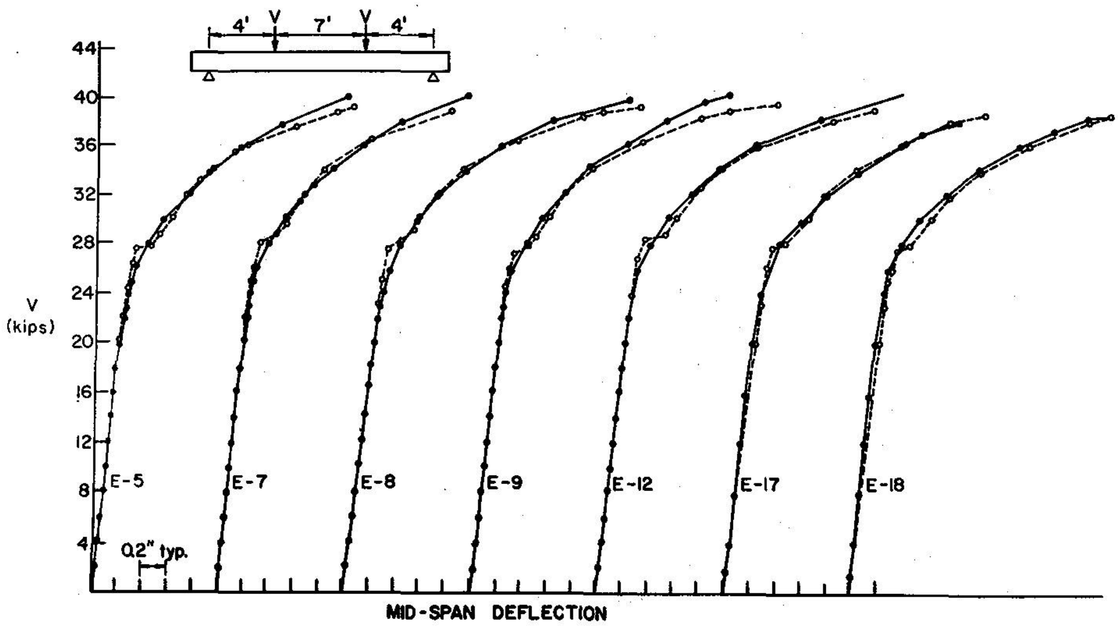

Fig. 10. Load-deflection curves for prestressed concrete I-beams.

The prestressed concrete I-beams were tested by Hanson and Hulsbos [4]. The discretization, layering and dimensions are shown in Figs. 7B and 8B. These beams were simply supported to produce a 15 ft. bending span and were loaded with a concentrated load 4 ft. from each end. Fig. 10 shows a comparison between the analytical and experimental load-deflection curves for these beams. For this series of examples the cylinder strength varied from 6.58 ksi to 7.23 ksi, the prestressing force at the time of test varied from 90.6 kips to 92.6 kips. A comparison of analytic and experimental ultimate strength is given in Table 2.

Table 2: Ultimate Loads

Beam No.	Test (kips)	Calculated (kips)	Deviation (%)
A7	49.9	49.0	1.8
A8	50.2	48.9	2.6
A9	49.8	48.7	2.2
A10	49.9	49.0	1.8
E5	42.0	39.2	6.7
E7	41.1	39.8	3.2
E8	41.2	39.3	4.6
E9	41.2	38.9	5.6
E12	41.2	39.0	5.3
E17	38.0	38.2	0.5
E18	38.7	38.2	1.3

Other comparisons with reinforced concrete beams having considerably lower cylinder strength, uniformly distributed loading and the fixed ended steel beam, as well as more discussions of the test comparisons included herein can be found in [8 and 9].

Implementation

The analysis schemes, as applied to prestressed concrete beams, are primarily aimed either at the elastic regime or the ultimate strength of the member. The application of the reported study allows the prediction of the full inelastic behavior of these beams. Through the predicted response, the possible occurrence of overstressing, cracking, etc., can be predetermined for various load levels; which in turn will define the serviceability and safety characteristics of the structure. Conversely, economy in dimensioning can be assured for the beams that were overdimensioned in order to enhance their serviceability and strength, if the predicted response happens to be too conservative.

The reported scheme can, and in some cases has been, extended to the study of beam-columns, beam-slab bridge superstructures, gridwork, stiffened floors and similar structural systems. Due to the generality of the formulation, it can be applied to structural systems with or without prestressing. The structures can be constructed of any material with known stress-strain curve, or composites consisting of these materials.

Conclusions

A simple, efficient but effective model has been developed for the inelastic flexural analysis of beams. This method is based on a layered beam type finite element. An incremental iterative tangent stiffness analysis technique is used to solve the nonlinear problem in a piecewise linear manner. The resulting load-deflection curves have been compared herein with experimental results obtained from tests of eleven prestressed concrete beams. Good correlation has been observed. This beam analysis technique has been developed for use in the analysis of bridge superstructures subjected to significant overloading.

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Summary

An analytical method is presented that describes the inelastic load deformation behaviour of prestressed concrete beams. Material nonlinearities such as cracking and crushing of concrete and yielding of steel are included. It is assumed that the response of the member is flexure dominated. Through simplifying assumptions maximum computational economy is achieved without any sacrifice in accuracy. The scheme is based on the finite element method.

Résumé

L'article présente une méthode analytique décrivant le comportement charge/déformation inélastique de poutres précontraintes en béton, tenant compte de non-linéarités, telles que fissuration et rupture du béton et écoulement de l'armature. Il est assumé que l'élément de construction est soumis essentiellement à la flexion. Des hypothèses simplificatrices permettent une économie de temps d'ordinateur sans diminution de la précision.

Zusammenfassung

Der Beitrag behandelt eine Methode zur Berechnung des unelastischen Last-Deformations-Verhaltens vorgespannter Betonbalken. Dabei sind Nichtlinearitäten, wie Rissebildung und Stauchung des Betons sowie das Fliessen der Bewehrung inbegriffen. Es wird angenommen, dass das Verhalten des Bauelementes vorwiegend auf Biegung zurückzuführen ist. Durch vereinfachende Annahmen wird eine grösstmögliche Einsparung an Rechenzeit ohne irgendwelche Beeinträchtigung der Genauigkeit erzielt. Das Verfahren stützt sich auf die Methode der finiten Elemente.