Flat plates supported on walls

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Flat Plates Supported on Walls

Dalles plates supportées par des parois

Durch Wandscheiben gestütze Flachdecken

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Abstract

In this paper values of the bending stiffness of slabs in cross wall structures subjected to lateral loads are presented. The cross sectional dimensions of the walls and the dimensions of the slab are fully taken into account. The case investigated is an interior bay of regular cross wall building, having a large number of bays in the longitudinal direction and one transverse bay. The walls are subjected to a simultaneous rotation in the same direction about their centre lines in the longitudinal direction. From considerations of symmetry only a quarter of the bay is analysed.

The stiffness values are obtained from elastic analyses of the slab using meshes of compatible quadrilateral plate bending finite elements. With these elements quite coarse meshes may be used. In regions of rapidly changing curvatures, such as near the wall, fine mesh is used. The transition from fine to coarse mesh is facilitated using a mesh grading technique. This technique allows two or more small elements to abut the side of a larger element without, however, violating interelement compatibility. In this manner the monotonic convergence characteristics of compatible elements are maintained in an unorthodox arrangement.

The results are presented as a series of graphs giving the variation of the non-dimensional slab stiffness with the aspect ratio of the slab, the span and the thickness of the wall. Curves giving the variation of the effective width of an "equivalent beam" are also given. In addition some results showing the distribution of the moments in the slab also presented.

1. Introduction

The widespread use of shear walls in multistorey buildings to resist lateral loads has led to a search for more efficient methods of analysis of such structures. Of particular interest in this paper is the lateral response of a cross wall structure which is a structural system consisting of plane walls connected solely by the slab as shown in Fig. 1. The existing methods of analysis of shear walls connected by beams under lateral loads [1-5] have been applied to cross wall structures by analysing its equivalent frame. The system of the equivalent frame is made up of the plane walls joined by strips of floor slabs considered as connecting beams. The ACI Committee 442 report on the Response of Buildings to Lateral Forces [6] indicated that little research works has been done to ascertain the width of the slab strip to be considered effective as a connecting beam. In fact, available information gives values of less than full width [7], equal to the full width [8] and greater than the full width [9], which were shown to be valid under different circumstances. In the study made by QADEER and STAFFORD

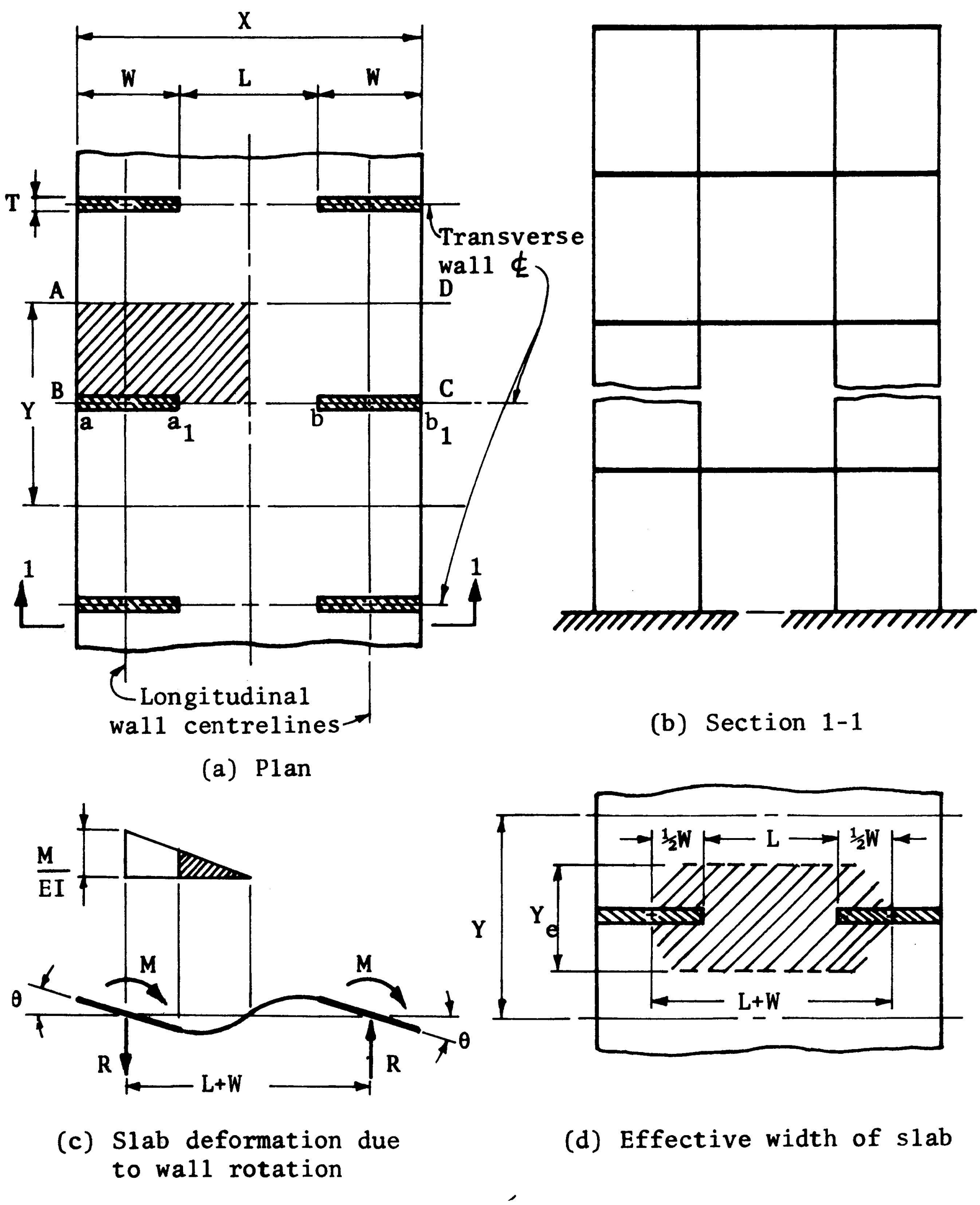

Fig. 1. Slab and Cross Wall Structure and Wall Deformations.

SMITH [7], the method of finite differences was employed to determine bending stiffness values for slabs connecting plane shear walls. Values for the effective width of the slab were also given. The results showed that the bending stiffness and effective width are affected significantly by variations in the slab proportions, the lengths of the walls and the clear span. However the thickness of the walls was not taken into account, thereby neglecting its effect. The ACI Committee 442 [6] and MICHAEL [10] have recognised that the slab stiffness is a function of the relative dimensions of the cross-section of the supporting member with respect to the dimensions of the slab panel. This has also been shown, to a limited extent, by PULMANO, et. al [11].

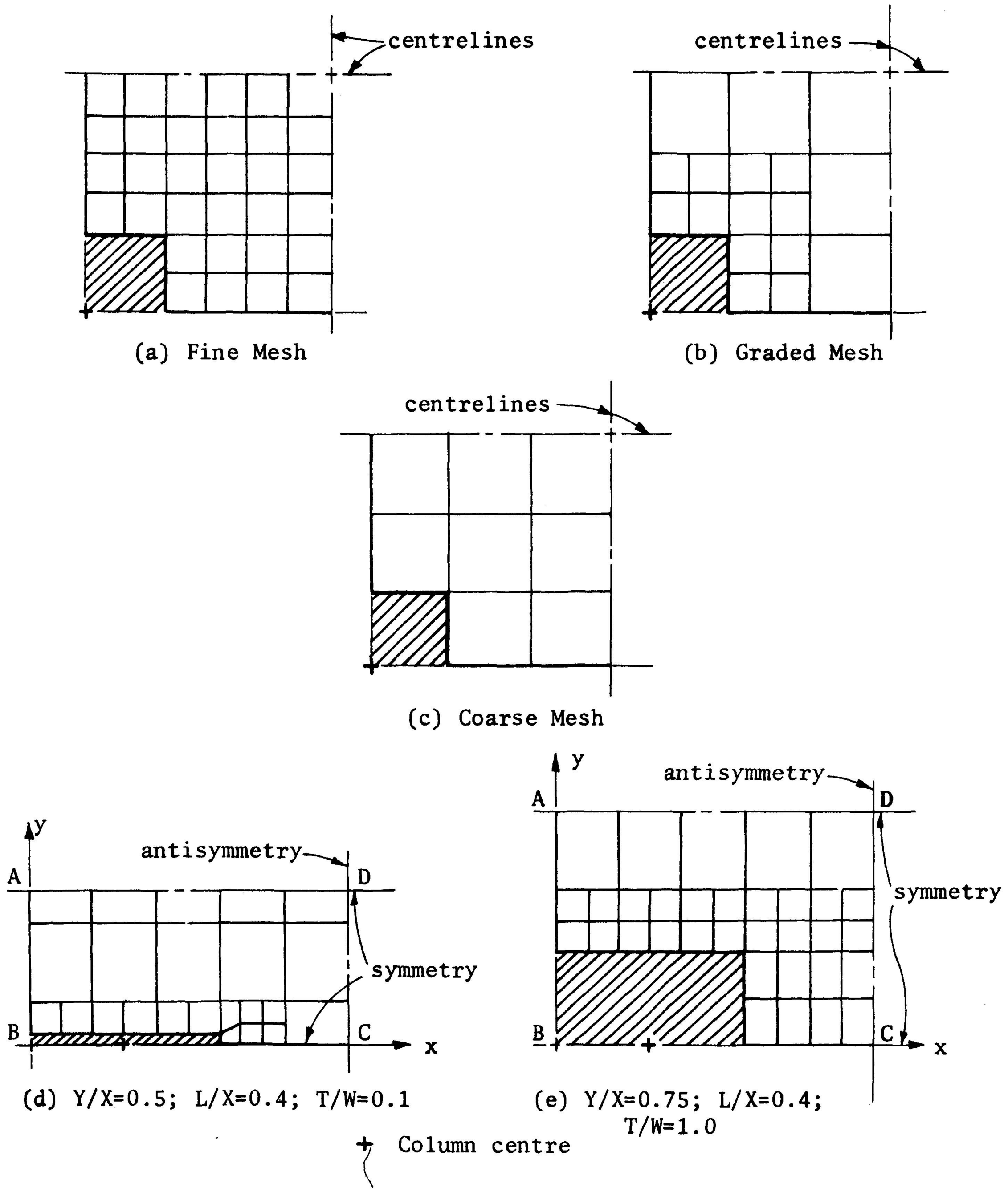

Fig. 2. Typical Finite Element Meshes.

In this investigation, a finite element approach is used to determine the variation of the stiffness of the slab and its effective width with the parameters involving the dimensions of the wall section and of the slab panel. In particular, the effects of the thickness of the shear walls have been fully taken into account. The investigation is limited to the study of an interior bay of a cross wall structure, assumed to have a large number of bays in the longitudinal direction and one bay in the transverse direction.

The floor slabs are represented by finite element meshes consisting of compatible quadrilateral plate bending elements [12, 13]. For brevity of presentation the formulation of this element is omitted. The use of these elements enables quite

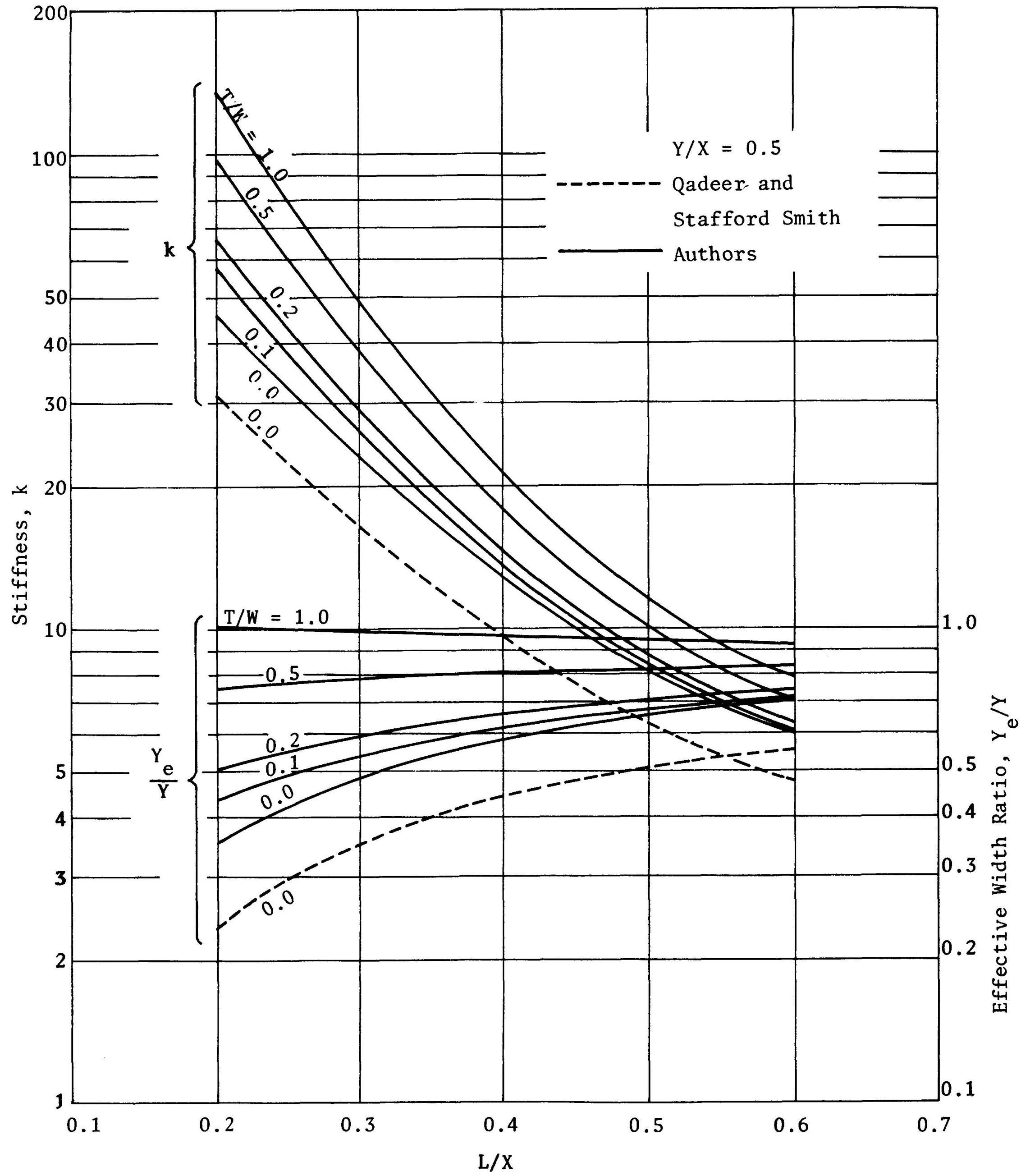

Fig. 3. Stiffness, k, and Effective Width, $Y_e Y$, of Slab.

coarse meshes to be used in the analyses. However, in regions of rapidly changing curvatures such as around the wall, finer mesh should be used. The transition from the fine to the coarser mesh is achieved with the aid of a mesh grading technique [14]. This enables abrupt changes from fine to coarse mesh to take place while the aspect ratios of the elements may be kept within reasonable limits and the need to use triangular elements is eliminated. By this technique two or more elements may abut the side of a larger one without violating interelement compatibility. Other characteristics of compatible elements such as monotonic convergence of resultant displacements or forces and the yielding of a lower bound for the load potential are retained. The technique has been successfully applied

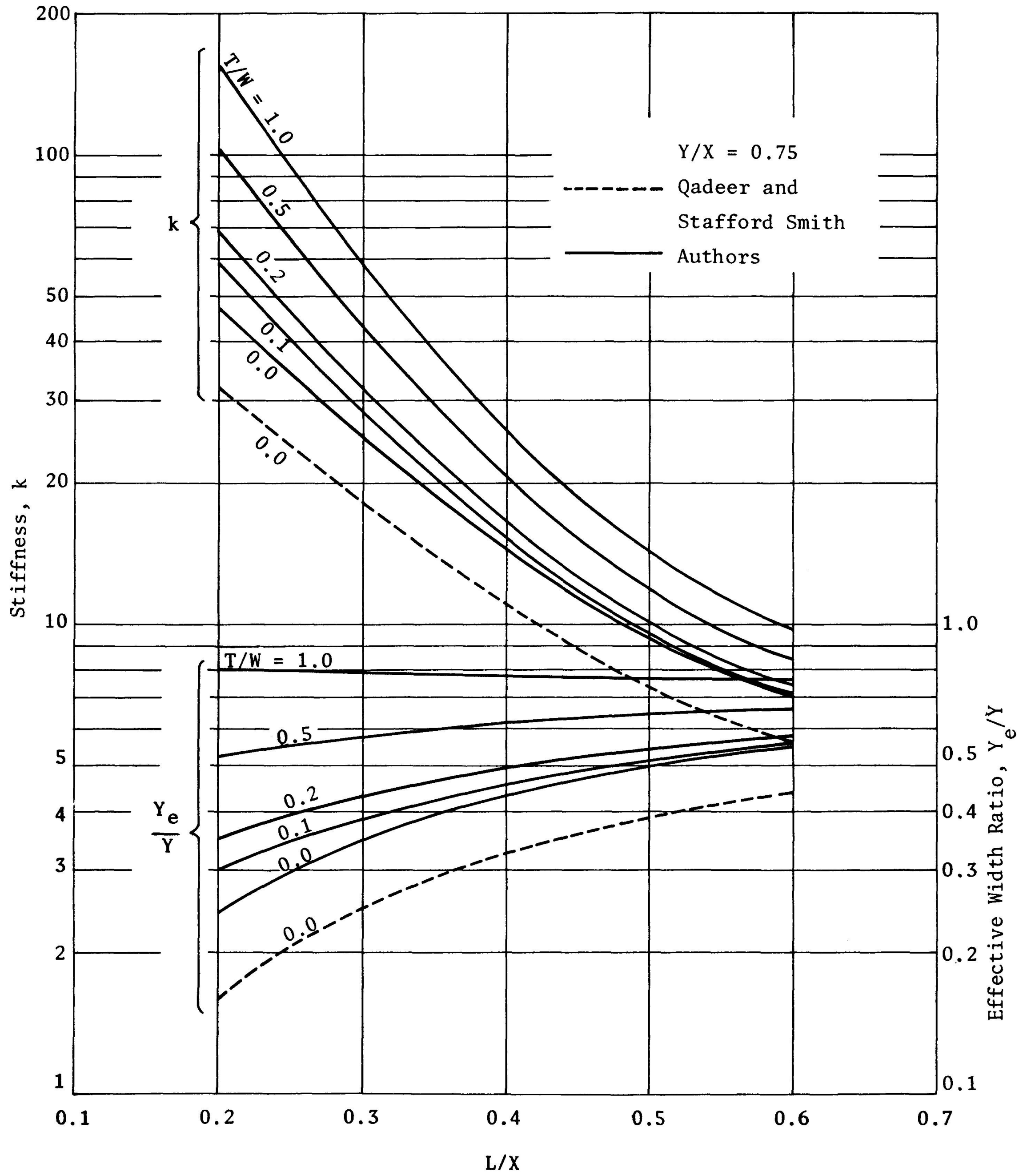

Fig. 4. Stiffness, k, and Effective Width, Y_e/Y, of Slab.

to several different plate bending elements of the displacement and the equilibrium type [17]. It has been exhaustively tested in several applications [14, 16, 17] by comparing the results obtained from graded meshes with the results from finer and coarser ungraded meshes. Examples of a fine, a graded and a coarse mesh are shown in Fig. 2a, b and c. In each case tested the results obtained for the graded mesh represented a significant improvement over the results for the coarser mesh. The results for the graded mesh were practically identical with the results for the finer mesh, though the number of degrees of freedom was reduced considerably. Some examples of the meshes used in this investigation are shown in Fig. 2d and e.

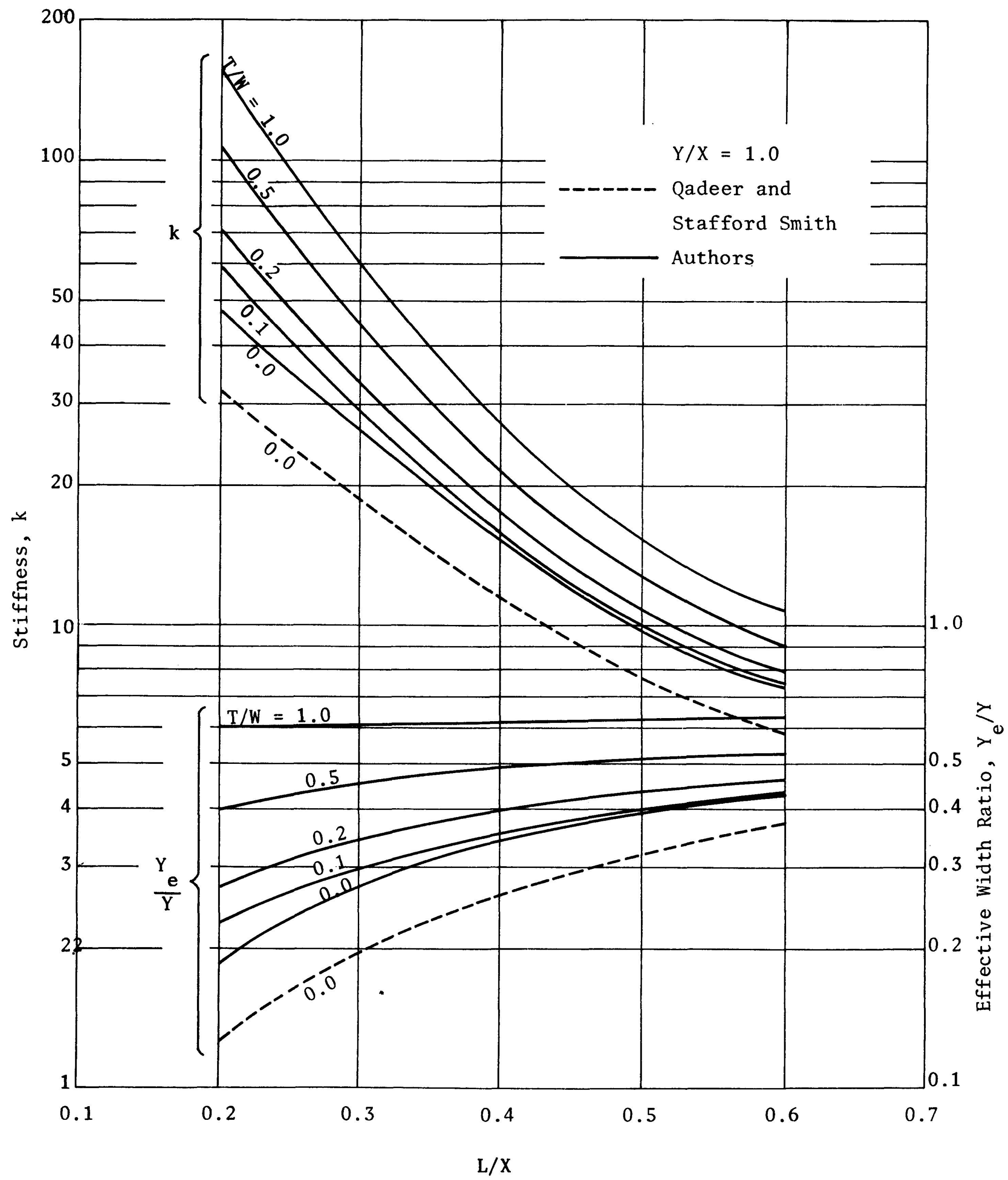

Fig. 5. Stiffness, k, and Effective Width, Y_e/Y, of Slab.

2. Stiffness of Slab and Effective Width

To conduct this investigation an interior bay of the idealised cross wall structure, shown in Fig. 1a and b, is considered. The structure is assumed to consist of a large number of regularly spaced pairs of walls. Each wall has the cross section shown in Fig. 1a. The walls are connected solely by the floor slab. The stiffness evaluated is for the simultaneous rotation of all the walls in the same direction as shown in Fig. 1c. Such deformation will occur when a cross wall structure is subjected to lateral loading or when differential vertical movement of the walls occurs. The deformation in which the walls rotate in opposite directions would occur if the slab was subjected to a uniform vertical load. This case will not be investigated here. It is assumed that as the walls rotate, planar cross-sections remain plane.

The stiffness is evaluated for a typical one half interior bay *ABCD*, shown in Fig. 1a, subjected to a simultaneous rotation, in the same direction, of the shear walls $a - a_1$ and $b - b_1$ about their centre lines. In view of the assumptions made above, the wall and slab panel centre lines in the transverse direction are lines of symmetry while the longitudinal centre line of the bay is a line of anti-symmetry. Thus, only one half of portion *ABCD* need be analysed, as indicated by the shaded area in Fig. 1a. The stiffness of the slab is obtained from the finite element solution when a unit rotation of the two shear walls is specified. It has been suggested [7] that the slab stiffness may be expressed in terms of a nondimensional parameter, k, as follows:

$$k = \frac{M}{D\theta}$$

where θ = applied rotation about centre of wall.
M = bending moment conjugate to applied rotation.
$D = \dfrac{Et^3}{12(1-\mu^2)}$ = slab stiffness per unit width.
E = Young's modulus.
u = Poisson's ratio.
t = thickness of slab.

Further, it has also been shown [7] that the ratio of the effective width to the full bay width of the slab, Y_e/Y, may be expressed as follows:

$$\frac{Y_e}{Y} = \frac{kL^2}{6(1-\mu^2)(L+W)^2} \tag{2}$$

where Y_e is defined in Fig. 1d and Y, L and W are defined in Fig. 1a. The expression given in Eq. (2) relates to an "equivalent beam" of span $(L+W)$ where the length $(W/2)$ at each end is considered to be rigid, as indicated in Fig. 1d.

The graphs of k and Y_e/Y versus L/X for various values of T/W are plotted for three values of Y/X in Figs. 3, 4 and 5 for Poisson's ratio = 0.2. The results are also given in Table 2.

3. Discussion of Results

To determine suitable patterns for finite element meshes and to find how many degrees of freedom the meshes should have, two convergence tests were conducted. The *first convergence test* was for $T/W = 0.0$, $L/X = 0.4$ and $Y/X = 0.5$. In this case the walls had an elongated cross section. The configurations of the wall and slab provided a severe test of the ability of the finite elements to model the deflected shape of the slab. Three different mesh patterns, with fine mesh around the wall, were used. The coarsest and finest meshes had 98 and 289 degrees of freedom (*DOF*), respectively. The results of the test, as given in Table 1a, indicate that the percentage difference between the values of k and Y_e/Y obtained using the coarsest and finest meshes was about 4%.

Table 1: Convergence Tests

(a) $T/W = 0.0$; $L/X = 0.4$; $Y/X = 0.5$

Mesh Type	Y_e/Y	k
Coarse 15 Elements 98 DOF	0.586	13.00
Graded 36 Elements 199 DOF	0.564	12.52
Fine 50 Elements 298 DOF	0.564	12.52

(b) $T/W = 1.0$; $L/X = 0.2$; $Y/X = 0.5$

Mesh Type	Y_e/Y	k
7 Elements 46 DOF	1.018	132.0
18 Elements 105 DOF	1.005	130.5
33 Elements 184 DOF	1.005	130.5

The second convergence test was conducted for the case where $T/W = 1.0$, $L/X = 0.2$ and $Y/X = 0.5$. This case had the stockiest walls of all the cases to be analysed. Three different mesh patterns were used. The coarsest mesh had 46 *DOF* while the finest mesh had 184 *DOF*. The resultant values of k and Y_e/Y are given in Table 1b. The percentage difference between the results obtained for the coarsest and finest meshes was 1%.

Table 2: Ratio of Effective Width to Total Width of Slab, Y_e/Y, and Slab Stiffness, k.

		Y/X								
	$\frac{T}{W}$	0.5			0.75			1.0		
		L/X								
		0.2	0.4	0.6	0.2	0.4	0.6	0.2	0.4	0.6
$\frac{Y_e}{Y}$	0.0	0.352	0.586	0.702	0.243	0.436	0.547	0.184	0.346	0.432
	0.1	0.436	0.614	0.714	0.302	0.456	0.554	0.228	0.354	0.439
	0.2	0.504	0.664	0.744	0.350	0.494	0.580	0.272	0.398	0.464
	0.5	0.750	0.806	0.828	0.526	0.620	0.660	0.400	0.488	0.524
	1.0	1.018	0.970	0.924	0.806	0.774	0.768	0.606	0.620	0.636
k	0.0	45.6	13.0	6.0	47.2	14.4	7.0	47.8	15.3	7.4
	0.1	56.6	13.6	6.1	58.8	15.2	7.1	59.3	15.7	7.5
	0.2	65.6	14.6	6.3	68.2	16.4	7.4	70.4	17.6	7.9
	0.5	97.0	17.8	7.1	102.4	20.6	8.4	103.8	21.6	9.0
	1.0	132.0	21.4	7.9	152.2	25.7	9.8	157.0	27.3	10.8

Analyses for various combinations of T/W, L/X and Y/X were made using meshes having *DOF*'s in the range from 46 to 205. Meshes with a large number of *DOF*'s were used in cases similar to the first convergence test. This was to ensure that the mesh was fine enough to adequately model the deflected shape of the slab in these cases. The coarse meshes were used for wall-slab configurations similar to the second convergence test. In view of the good convergence characteristics of the test results, the error in the slab stiffness values, calculated by using finite elements is deemed to be less than 4%. It is considered that this accuracy is quite adequate for practical purposes.

The results for the stiffness and effective width of the slab joining the supporting walls are shown in Figs. 3, 4 and 5. They are also summarized in Table 2. The results obtained by QADEER and STAFFORD SMITH [7], using finite differences, for the case where $T/W = 0.0$ are also plotted in Figs. 3, 4 and 5. It can be observed

Table 3: Distribution of Moments
($T/W = 0.1$; $L/X = 0.4$; $Y/X = 0.5$)

Mx at Section AA		My at Section BB	
x/X	$Mx\ t/D$	y/Y	$My\ t/D$
0.3	−0.334	0.03	0.022
0.33	−0.221	0.13	0.003
0.365	−0.118	0.27	−0.006
0.4	−0.078	0.45	−0.009
0.5	−0.003		

that the differences between the finite element and the finite difference solutions for k and Y_e/Y range up to about 33%. The differences may be attributed to the methods of analysis used. In the light of the known convergence characteristics of compatible finite elements and the results of the convergence tests the finite element solution was considered to give the correct results.

Figure 6a shows a typical distribution of the bending moment, M_x, along the wall centre line in the transverse direction. Fig. 6b shows the plot of the bending moment, M_y, along a section passing through the nose of the wall in the longitudinal

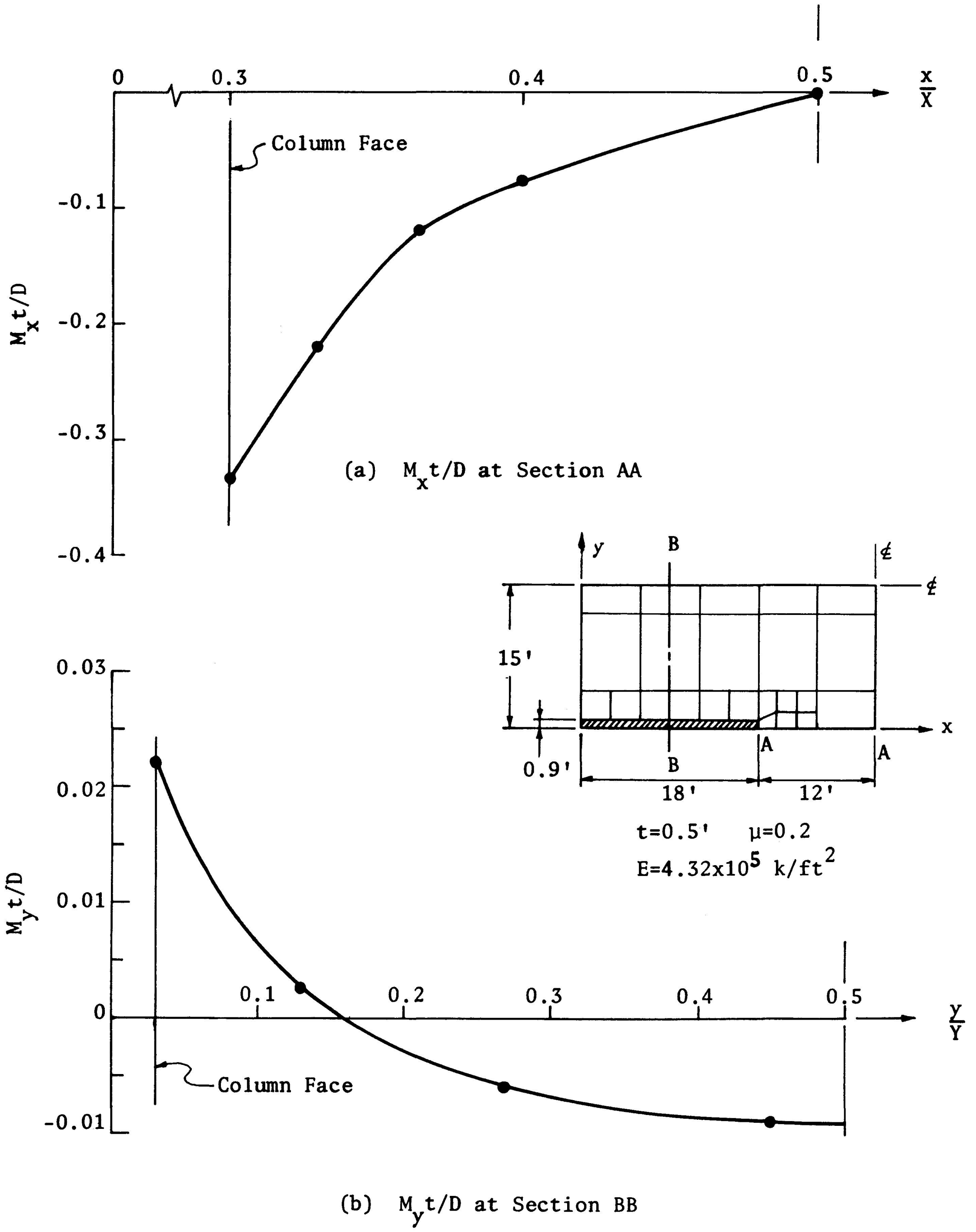

Fig. 6. Distribution of Bending Moments ($Y/X = 0.5$, $L/X = 0.4$, $T/W = 0.1$).

direction. The results for M_x and M_y are also given in Table 3. From these bending moment diagrams it can be observed that there is a high stress concentration at the nose of the wall. This must be considered in the design of the slab at these regions to prevent local failure, such as excessive cracking.

The curves, as presented in Figs. 3, 4 and 5, may be used in conjunction with some existing design curves [15] to predict stresses and deflections of coupled plane shear walls. However, if the stiffness method of matrix analysis is employed to analyse an interior bay of a cross wall structure, then the values of the slab stiffness, k, may be used directly. The curves for effective width are no longer required since there is no need to develop the stiffness of an "equivalent beam". By using these curves to evaluate the slab bending stiffness and, if necessary, the effective width, the reliability of the analysis of cross wall structures will be improved. This will enable the design to be more economical without impairing its safety.

The stiffness of exterior bay slabs, though not treated in this study, may be determined in a similar manner to interior bay slabs by considering a cross wall structure having two transverse bays. Previous experience in the analysis of flat plate structures shows that the effect of the rotation of a supporting member does not significantly go beyond one bay. QADEER and STAFFORD SMITH [7] suggested that the stiffness of the end bay slab would be 42% of the interior bay slab, however no numerical results were presented to support this approximation.

4. Safety and Economy

The practical significance of design aids, such as those presented in this paper, is that their use will enable engineers to design safer and more economical structures.

Approximate methods of analysis either overestimate, or underestimate the stresses in a structure. If the stresses are overestimated, then the structure tend to be uneconomical. On the other hand, if the stresses are underestimated, then the structure may be unsafe.

The method presented in this paper is more accurate than other simple methods suitable for practical design applications. For this reason, it will contribute to both the economy and the safety of the types of structures under consideration.

The design aids given in this paper will enable engineers to design flat plates supported by walls with the same ease as that of the conventional stiffness analysis of framed structures. The accuracy of the method is similar to the accuracy which could be attained by modelling the whole structure by a large number of finite elements. Thus, for a given accuracy, the application of the method presented herein would result in substantial economies of design costs.

5. Conclusions

The curves showing the stiffness and the effective width of the interior bay slabs, against simultaneous rotation of the plane shear walls may be used to analyse an interior bay of a flat plate floor supported on cross walls. The stiffness, and the effective width, of the slab is clearly a function of the parameters involving the dimensions of the slab panel and the wall section, including its thickness. It would be desirable to extend this investigation to cross wall structures with shear walls of different cross-section and with more than one bay in the transverse direction.

6. Notation

D	flexural stiffness of plate per unit width.
E	Young's modulus.
k	non-dimensional slab stiffness.
L	clear span between shear walls.
M	moment about centre of shear wall.
M_x	bending moment resultant in x direction.
M_y	bending moment resultant in y direction.
t	thickness of plate.
T	smaller cross sectional dimension of shear wall.
W	larger cross section dimension of shear wall.
X	distance between outer edges of shear walls.
Y	slab width in typical bay.
Y_e	effective width of slab.
θ	rotation of shear wall and slab connection.
μ	Poisson's ratio.

7. Acknowledgment

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Summary

Graphs for bending stiffness of slabs in cross wall structures, subjected to lateral loads, are presented. These graphs can be used as a design aid, and they lead to an analysis which is very similar to the ordinary frame analysis. A feature of this paper is that the cross sectional dimensions of the walls and the dimensions of the slab are fully taken into account.

Résumé

L'article donne une représentation graphique de la rigidité de flexion de dalles dans des structures combinées de dalles et parois soumises à des charges latérales. Cette représentation peut servir d'aide au projet et mène à une analyse très similaire à l'analyse des cadres habituels. Une des caractéristiques de cette étude est que les sections des parois et des dalles sont intégralement prises en considération.

Zusammenfassung

Die Arbeit stellt Diagramme für die Biegesteifigkeit von Flachdecken zwischen Querwänden unter seitlicher Belastung zur Verfügung. Die Diagramme sind eine Hilfe bei der Berechnung und führen zu einem, der üblichen Rahmenberechnung sehr ähnlichen, Berechnungsgang. Die Querschnittswerte der Wände und Platten können dabei in der Berechnung voll berücksichtigt werden.

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