Rigidity of structures against aerodynamic forces

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Rigidité des constructions et forces aérodynamiques

Steifigkeit von Baukonstruktionen gegenüber aerodynamischen Kräften

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1. Preface

In the different building codes, the wind forces are based on turbulent force fluctuations. The strength requirements which follow are independent of the rigidity, mass and form of the construction.

This, however, is not always sufficient. The wind forces can be greater when a self-excited oscillation is possible and is always destructive when flutter (coupled bending-torsional oscillation) can arise even in a turbulent wind. These severe wind forces can be avoided by choosing the critical wind speed of action sufficiently far above the apparent maximum wind speed.

This means that, besides a strength requirement in view of turbulent forces, a stiffness requirement is necessary in a structure to avoid large forces caused by its own motion. A simple stiffness requirement is given in this article.

2. Aerodynamic forces

From a numerical computation of the flow field with aid of the flow equations of EULER [1], it follows that a complete description of this field is possible (fig. 1), as, for instance, the shedding of the von Karman vortices and unstable, irregular motion at a sufficiently high wind speed. When the unity of time in the calculation is taken small enough, the turbulent motion is also resolvable from the Euler equations. The flow pictures are entirely provided by the principal properties of the flow and by the constraints.

2.1 Approximate solution

When the Reynolds number, $R = V \cdot d/v$ (V = wind speed, d = width of the object, fig. 1, v = viscosity of air) is high, a term in the Euler equation is negligible and the equation of the main stream is similar to that of irrotational potential flow.

This equation can be expressed in terms of mean flow velocities and instantaneous deviations from the mean velocities (turbulence). The interaction of the turbulent and the quasi-stationary resistance, also specifies the force on the structure [2]. The influence of turbulence is small when the scale of turbulence is very different from the width, *d*. This can be the case at hurricane wind velocities. This influence can also be small for structures oscillating violently at the eigenfrequency, when by coupling, the influence of the Karman vortices predominates [3], [4].

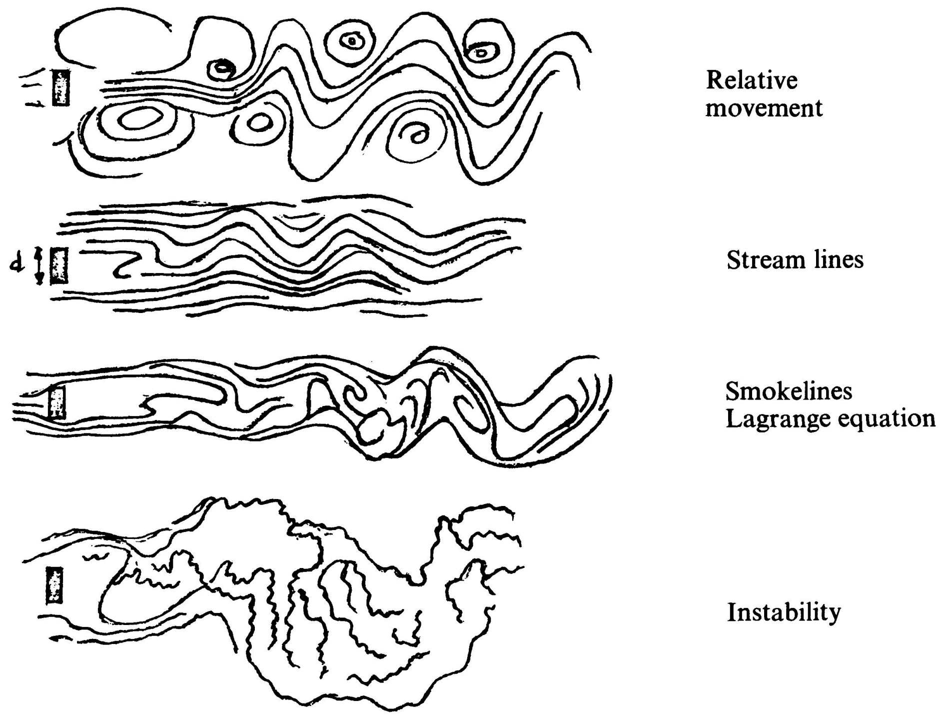


Fig. 1

2.2 Wind oscillations of structures

Two kinds of unstable oscillations are commonly distinguished: resonance and flutter.

2.2.1 Resonance

When air flows around an object, vortices arise periodically, loosening at some definite frequency. These vortices cause a periodic lift force on the object causing it to oscillate. The object is thus considered as a vortex with opposite rotation to the stream vortices. Under sufficiently strong oscillation there will be reversed, from equilibrium considerations, vortex shedding at the frequency of the moving object. This gives rise to feed-back, causing self excited oscillation. From this, it can be proved that large amplitudes will arise (depending on the form) when the wind force is already causing amplitudes *a* of order of the width *d* of the object, regardless of feed-back (single degree of freedom flutter) [4]. Also, the region of instability by PARKINSON [5], NOVAK [6] and SCRUTON [5], is explicable from this feed-back [4].

2.2.2 *Flutter*

The particular form of an object makes it possible for the amplitude of motion to be magnified.

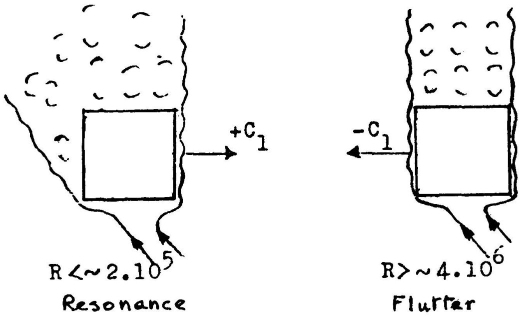


Fig. 2.

The equations of motion of an element of a structure are:

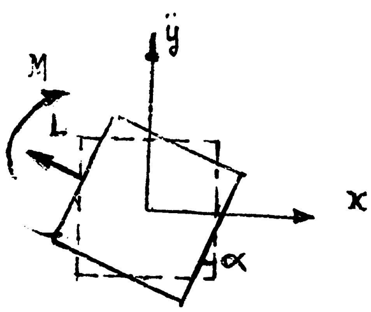


Fig. 3.

$$m \cdot \frac{d^2 y}{dt^2} + m\,\omega_y^2 \cdot y + m\,\frac{\omega_y}{\pi}\,\theta_y\,\frac{dy}{dt} - L = 0$$

$$I \cdot \frac{d^2 \alpha}{dt^2} + I\,\omega_\alpha^2 \cdot \alpha + I\,\frac{\omega_\alpha}{\pi}\,\theta_\alpha\,\frac{d\alpha}{dt} - M = 0$$

where m = mass of the body, ω_y and ω_α the uncoupled eigenfrequencies of translation and rotation respectivelly θ_y and θ_α are logarithmic decrements of (respectively) translational and rotational oscillations, L and M are the aerodynamic lift force and moment calculated for a flat sheet with the aid of quasipotential theory [7] where L contains terms which are linear functions of α and $\frac{d\alpha}{dt}$ and M has terms in y and $\frac{dy}{dt}$. The translational and rotational oscillations are coupled by L and M. The system of equations consists also of two homogeneous linear equations in the extreme amplitudes y_0 and α_0. The motion becomes unstable when the denominator determinant becomes zero. In spite of damping, the amplitudes become infinite. When damping and rigidity are small, flutter in a single degree of freedom system is also possible (galloping instability) [8]. This is a particular solution of the flutter equations ($\alpha_0 = 0$, L neutralizes the damping). The only possibility of avoiding flutter is to ensure that the critical wind speed of this instability is sufficiently far above the maximum possible wind speed. Flutter must be avoided otherwise the structure will collapse.

2.3 Influence of turbulence

2.3.1 Influence on resonance

Outside the main vortex system of von Karman, there are also secondary vortices behind a body and also to a lesser extent a turbulent boundary layer that can be considered as vorticity with many frequencies and intensities. These types generate a spectrum of forces; usually, therefore, the turbulent force is dominant for the behaviour of the structure. Relatively rigid, sufficiently damped constructions will consequently have a broad, not peaked, force spectrum and will move randomly. This can normally be measured (DAVENPORT [6], VAN KOTEN). In extreme conditions, however, for example during a hurricane, when turbulent vortices are relatively small and also the influence on the main vortex system is small as a result of the feed-back of the movement of the structure on the vortex shedding, there are clear peaks in the spectrum at the Stouhal frequencies (WOOTON [6], NAKAGAWA [5], [9]) and resonance and flutter are possible (hula-hoop movement of steel stacks [3]). These peaks can be explained by equating the magnification, starting from the turbulent lift force on a rigid object, obtained by determining the feed-back from the Karman vorticity [4].

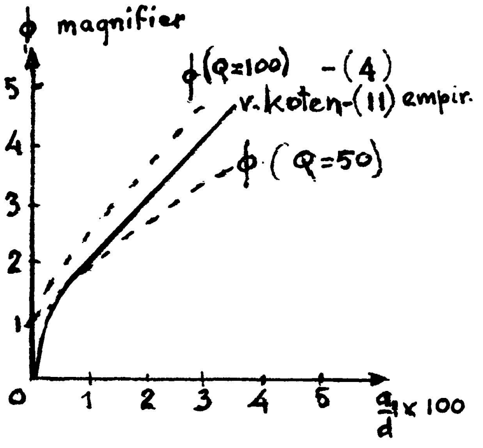


Fig. 4.

2.3.2 Influence on flutter

Turbulence lowers slightly the critical wind speed of flutter [10]. This can be omitted from consideration.

3. Stability criterion (rigidity requirement)

3.1 Towers, slender structures or components

To avoid single degree of freedom flutter it is necessary to satisfy the condition: $\frac{a_1}{d} \ll \sim 0.7$ where a_1 = amplitude at resonance without feed-back.

d = width of the object perpendicular to the wind direction. To avoid inconvenient resonance oscillations of blunt bodies the condition [4] becomes.

$\frac{a_1}{d} < \sim 0.03.$ The critical wind speed is $V_e = \frac{\omega_e \cdot d}{0{,}2\,\pi}$ or a multiple of this and

$\frac{a_1}{d} = \frac{Q}{k} \cdot C_L \cdot \frac{\rho}{2} V_c^2$ where.

ω_e = resonance frequency in rad./sec. $= \sqrt{k/m}$.

C_l = Lift factor.

k = rigidity of the structure, $k = m\omega_c^2$, where m = mass of the oscillating part.

ρ = Mass density of air.

Q = magnification factor at resonance $Q = \pi/\theta$ = logarithmic decrement.

3.2 Bridges, roofs, plate construction

The frequency of resonance is $\omega = 0.2\pi\, V/d = \omega_e$. The rigidity requirement against inconvenient resonance oscillation is also $a_l/d < \sim 0.03$.

The flutter speed can be estimated, according to fig. 5 [10]. At high damping ($\theta_b = \theta_t = 0.20$) to be expected close to the state of fracture, it follows from fig. 5 that the critical wind speed, V_c, of flutter is:

$$V_c = \eta\left(1 + \left(\frac{\omega_c}{\omega_b} - 0.5\right)\sqrt{(r/b), 0.72\mu}\right)\omega_b, b \text{ with } \frac{\omega_c}{\omega_b} > \sim 1.2 \text{ and}$$

$\mu = m/\pi\rho b^2$ = mass ratio, construction to air.

$\rho = 1/8$ kgf. sec^2/m^4 = mass density of air.

b = half width of the plate.

m = mass density of the plate.

ω_t and ω_b are respectively resonance frequencies of torsion and bending of the plate in rad./sec.

$r/b = (1/b)\sqrt{I/m}$ is the ratio of inertia radius to half width of the plate.

I = Inertia moment per unit length.

η = An empirical form factor giving the difference of the critical wind speed of a certain profile with reference to the theoretical value [10] of a flat plate such that

$\eta = \sim 0.1$ for profiles with flat plates the ends.
~ 0.3 for blunt profiles.
~ 0.5 for slender profiles.
~ 0.7 for streamlined profiles (fig. 6).

3.3 Examples (fig. 7)

1. For the beam bridge $\eta \approx 0.3$ and $V_c = 0.3(1 + 0.8\sqrt{0.7, 0.72, 27})\ 2\pi,\ 0.5,\ 25/2 =$ 46 m/sec. The rigidity of the bridge has to be increased.

2. For the suspension bridge $\eta \approx 0.7$ and $V_c = 0.7\ (1 + 3.0\sqrt{0.6, 0.72, 30})\ 2\pi,\ 0.3,$ $3{,}15/2 = 97$ m/sec. Because of the great torsional rigidity and favourable aerodynamic form, there will be no flutter.

3. For the arch bridge $\eta \approx 0.1$ and $V_c = 0.1\ (1 + \sqrt{0.7, 0.72, 70})\ 2\pi,\ 1.9,\ 5 = 41$ m/sec. A more favourable aerodynamic form has to be chosen. This is also the case for the suspension roof with $\eta = 0.1$ and $V_c = (1 + \sqrt{0.6, 0.72, 20})\,2\pi, 2.2, 3.5 = 19$ m/sec. Besides, it is desirable to raise the rigidity of the mast and with that the frequency of 2.2 Hz. Resonance is not important for these types of construction because $a/d < 0.05$. The wind force is to be taken from the Code in this case.

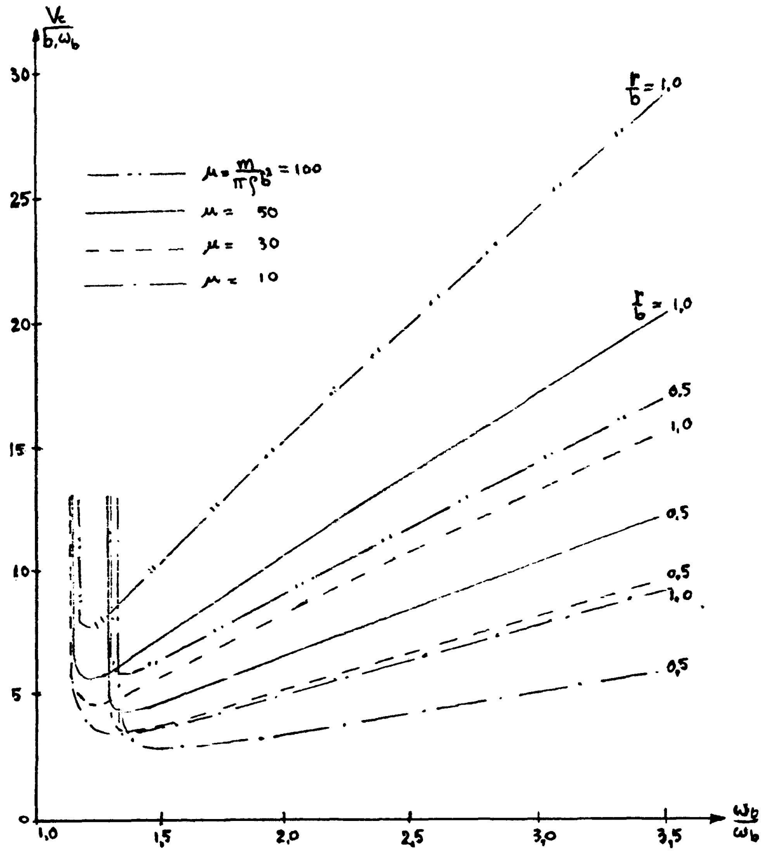
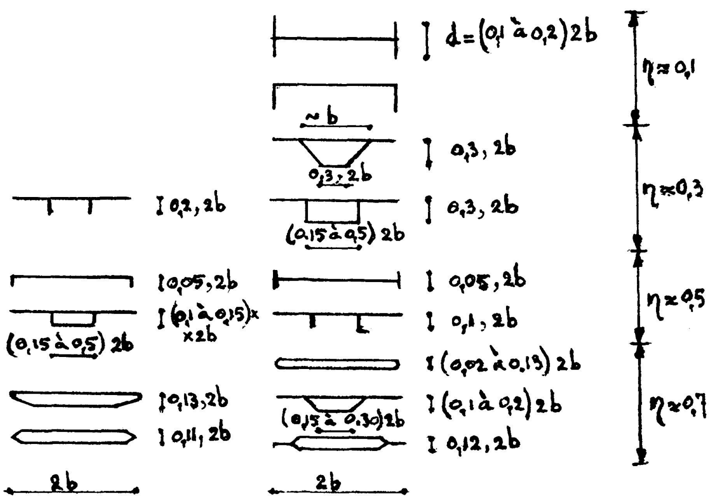


Fig. 5. Critical Wind Speed for $\theta_b = \theta_t = 0{,}20$ [10].

Fig. 6.



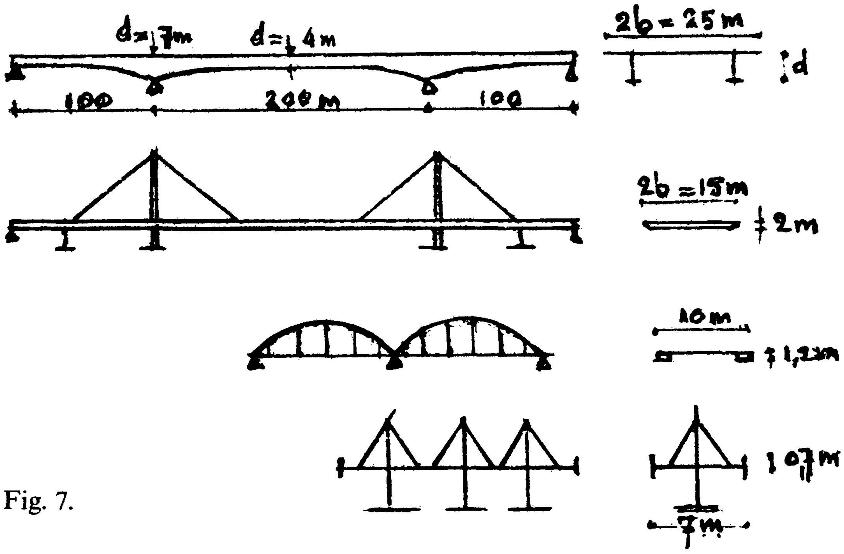


Fig. 7.

3.4 Conclusion

A rigidity requirement against unstable wind oscillations is given by:

a) The amplitude of the object, a/d, must satisfy $a/d \ll 0.7$.

b) The critical wind speed, V_c, of flutter of a plate profile must lie well above the wind speeds obtained by chosing the correct rigidity (sufficiently high ω_r and ω_c) where $V_c = \eta\left(1+\left(\frac{\omega_t}{\omega_b}-0.5\right)\sqrt{\left(\frac{0.72\sqrt{m, I}}{\pi\rho b^2}\right)}\right)\omega_b, b$ with $\frac{\omega_t}{\omega_b} > 1{,}2$.

c) As a first approximation, to check if an exact analysis is necessary, it is possible to start with resonance at the highest possible wind speed, because many resonance frequencies are usually possible. In that case:

$\frac{a_1}{d} = \frac{W}{dk} \ll 0.7$ (requirement against collapse $Q = 10$ or 15) [3].

$\frac{a_1}{d} < 0.03$ (to avoid inconvenient oscillations)

where a = amplitude in m, a_1 = amplitude without feed-back in m, W = wind force from the Code in kgf, K = rigidity of construction in kgf/cm, d = width of the construction perpendicular to the wind direction in cm.

For flutter: $V_c \approx \eta\,(1 + 0.8\sqrt{0.578, 0.72, m/(\pi\rho b^2)})\,\omega_b, b$ or

$V_c = \eta\,(1 + \sqrt{0.3\ G/B})\ 1.5\ B/\sqrt{a_{ct}} > \sim 60$ m/sec.

where η = form factor (see 3.2) $B = 2b$ = width of the plate in m., a_{st} = max. static bending displacement as a result of self-weight in m, G = weight of the plate in kgf/m^2.

d) Safety to assess the safety, an indication of the chance of exceeding a windspeed V is: $V \approx 37$ m/sec. with a 50% chance once in 25 years. The chance of $V \approx 60$ m/sec. is nearly zero (once in 2000 years).

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Summary

Codes only give strength requirements for wind forces. This is not always sufficient to avoid destructive forces (flutter). For this, a rigidity requirement is necessary.

Zusammenfassung

Die Baunormen geben nur die nötige Festigkeit gegenüber Windstärken an. Dies genügt jedoch nicht immer, um zerstörende Kräfte (Flatterwirkung) zu vermeiden. Deshalb ist ein Steifigkeitserfordernis notwendig.

Résumé

Les règlements de construction ne parlent que des valeurs de résistance aux forces du vent. C'est insuffisant car il existe des forces destructives dues aux vibrations. On en tient compte au moyen de la rigidité.