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Limit States Design of Flat Plates and Slabs

Calcul aux états limites de dalles plates

Bemessung von Flachdecken nach Grenzzuständen

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SUMMARY

In the limit states design method proposed in this paper, in addition to choosing thicknesses of slabs from code provisions, long term deflections are calculated and checked against accepted limits. To keep crack widths within desirable limits, the size of reinforcing bar is selected from a crack control criterion. The design of slabs for strength is carried out using the yield-line method and the conventional punching shear formula. A numerical example is included to illustrate this limit states design method.

RÉSUMÉ

Dans la méthode de calcul aux états limites proposée dans cet article, et partant du choix de l'épaisseur des dalles selon les normes de construction, on calcule les flèches à long terme et on les compare aux valeurs indiquées dans les normes. Le diamètre des barres d'armature est choisi selon un critère de contrôle des fissures, afin de limiter la grandeur de celles-ci aux valeurs admissibles. La résistance des dalles est calculée selon la méthode des lignes de rupture et au moyen de la formule conventionnelle de poinçonnement. Un exemple numérique illustre cette méthode de calcul aux états limites.

ZUSAMMENFASSUNG

Bei der hier vorgeschlagenen Bemessung nach Grenzzuständen werden die Grenzwerte der Langzeit-Durchbiegungen, wie sie sich aufgrund von nach Normvorschriften gewählter Plattendicke ergeben, mit als zulässig erachteten Werten verglichen. Um die Rissebildung in Grenzen zu halten, wird der Durchmesser der Bewehrungsstäbe nach einer Rissformel festgelegt. Die Bemessung auf Bruch erfolgt nach der Bruchlinienmethode und aufgrund der üblichen Durchstanzformel. Ein numerisches Beispiel zeigt den Ablauf des hier vorgeschlagenen Bemessungsvorganges.

1. INTRODUCTION

Serviceability and strength are two basic criteria to be satisfied in the structural design of flat plates and slabs. The term serviceability refers to control of deformations (in most cases deflections) and crack widths at working loads. Because deformations of concrete structures continue to increase with time due to creep and shrinkage of concrete, not only instantaneous but also long-term deformations of slabs require careful consideration. In addition, a slab should have adequate strength to carry the applied loads with a required margin of safety against failure in flexure, shear, or bond, etc.

In an earlier paper [9], the writer has presented a limit-states design method for slabs supported on edges. The aim of this paper is to develop a limit-states design method for flat plates and flat slabs. In this method, the long-term deflections are computed using an approximate method described elsewhere [10] so that these deflections may be given proper consideration in design. In order to keep crack widths at service loads within desired limits, the size of reinforcing bars is selected using a crack control criterion. For adequate strength in flexure, the reinforcement is designed for the bending moments given by the yield-line method. The thickness of the slab is checked against punching shear failure and if necessary, special shear reinforcement is added in the vicinity of the column.

2. SERVICEABILITY DESIGN

2.1 Deflections

In current design practice, not a great deal of attention is given to deflections of slabs. At best, the designer may choose span-to-thickness ratios which might supposedly keep deflections within certain maximum values [4,11]. These provisions are usually based on the assumption that creep and shrinkage deflections can be lumped together when computing long-term deflections. Field data accumulated so far have indicated that such an assumption may not be valid in the case of flat plates and slabs [5,7]. Moreover, ACI Committee 209 [1] has recommended that in the case of thin members such as flat plates and slabs it may be desirable to compute separately the long-term deflections due to creep and shrinkage. A study of long-term deflections of three flat slab floors reported by Heiman [7] confirms the validity of this recommendation.

There are other factors such as loading history, creep and shrinkage characteristics of concrete actually used, ambient temperature and humidity conditions, method of construction, etc., influencing long-term deflections. It will be a tedious task to attempt to formulate span-to-thickness ratios to account for all variables. Therefore, *in addition to* choosing thicknesses of slabs using the code provisions, it seems desirable to calculate long-term deflections using some simple methods. One such method, which shows good agreement with available test data, is described [10].

The following steps are suggested when designing for deflections:

- Make an estimate of the thickness of the slab using appropriate provisions given in the codes [4, 11].
- Compute the incremental parts of the long-term deflections which occur after various stages of loading using the method of deflection calculation described in Reference 10. Take necessary precautions to accommodate these deflections.

- Calculate the total deflections which might occur after a long period of time (say 5 years) and check whether these are within required limits. On the basis of a survey made in the United States, ACI Committee 435 has recommended deflections limitations for various general situations [2]. But only the Czech and Russian codes [5] contain limits on total deflections specifically applicable to flat slabs. These are, for span ≤ 7 m (23 ft) span/200; and for span > 7 m (23 ft), span/300.

2.2 Crack Widths

According to the codes [4,11], checking for crack widths is not required if the spacings of reinforcing bars do not exceed twice the thickness of slab. Research data collected in recent years have shown that merely relating spacings of bars to thickness of slab is not adequate for proper crack control in two-way action slabs [8]. By extensive research Nawy and Blair have shown that crack widths at service loads can be kept within desirable limits by properly choosing the spacing and diameter of reinforcing bars.

The crack control formula developed by Nawy and Blair [8] is recommended in the Commentary to ACI 318-71. Based on this equation, the writer derived a crack control criterion for two-way action slabs in an earlier paper [9]. If the diameters of reinforcing bars in two perpendicular directions are taken as equal, this criterion reduces to the following form:

$$d_b \leq \left[\frac{\lambda \; A_{s1} \; A_{s2}}{72 \; \pi \; d_c}\right]^{\frac{1}{3}} \qquad (1)$$

where the reinforcement index, $\lambda = (w_{max}/K\beta f_s)^2$, w_{max} = maximum crack width in inches, K = coefficient influenced by edge conditions of slabs, equal to 2.8 x 10^{-5} for most restrained slabs, β = ratio of distances to neutral axis from extreme tension fiber and from centroid of tension reinforcement, equal to 1.3 in most cases, f_s = tensile stress in reinforcing bar at service load in ksi, A_{s1}, A_{s2} = area of tension reinforcement in two directions, per foot width of slab, in square inches, d_c = average effective cover to tension reinforcement in inches and d_b = diameter of reinforcing bar in inches.

Available test data show that in the case of flat plates and slabs, regions close to the columns are those generally critical for checking crack widths. Therefore, it is usually sufficient to apply the crack control criterion, Eq.1, only to such regions.

3. STRENGTH DESIGN

3.1 Punching Shear Strength

Almost every flat slab system that has been tested in the laboratory has failed by punching of the column through the slab. The slab-column region should be designed against this type of failure. In a recent state-of-the-art report [3], the Joint ASCE-ACI Committee 426 has suggested that the punching shear strength V_u of slabs without shear reinforcement could be taken as

$$V_u = \phi(0.50 + 0.75\,\frac{c_s}{c_\ell})(4\sqrt{f'_c}\;\; b_o d) \leq \phi 4\sqrt{f'_c}\;\; b_o d \qquad (2)$$

in which c_s and c_ℓ are the short and long sides of the rectangular column, f'_c is the compressive strength of concrete in psi, b_o is the perimeter around a line d/2 from the column face, d is the effective depth of the slab, and ϕ=0.85.

If possible, the thickness of slab and drop panel (if any) is chosen so that the design ultimate shear force does not exceed the punching shear strength given by Eq. 2. Otherwise, shear reinforcement must be provided for the excess shear [3].

3.2 Flexural strength

According to the codes [4.11], the design of reinforcement for flexural strength is based on the elastic analysis of simplified models of slab systems. Such methods are generally tedious. For this reason, Yield-line method and Strip method based on collapse load theory are attractive alternatives.

In this paper, yield-line method will be used to design reinforcement for flexural strength. Because the yield-line method is based on the upper-bound theorem, it is necessary that all possible failure patterns should be examined and the design of reinforcement must be based on that pattern which gives the lowest possible value for the collapse load.

The yield-line method and how it can be applied to flat plates and slabs are described in several publications [6,12]. The design of slabs for adequate flexural strength may be carried out as follows:

- Calculate total panel moments using the yield-line method.
- Distribute these moments to column and middle strips using CEB Recommendations [6].
- Calculate areas of tension reinforcement using ultimate strength design procedure.

4. LIMIT STATES DESIGN

The following steps are recommended for limit-states design of flat plates and slabs:

- Choose thickness of slab from code provisions. Check whether this value is suitable to guard against punching shear failure (Eq.2).
- Determine areas of tension reinforcement required at column and middle strips using yield-line method [6].
- Use Eq.1 to select the diameter of the reinforcing bar. Calculate number of bars or spacings of bars to give the required areas of tension reinforcement.
- Calculate long-term deflections using the method described in Ref.10. For excessive deflections, correct total deflection by camber or by other construction techniques.

5. DESIGN EXAMPLE

The limit-states design procedure described in the previous section is illustrated by applying it to an interior panel of a flat plate floor. The dimensions of the panel are 24 ft (7.32 m) by 20 ft (6.10 m). Applied working load = 60 psf (2874 N/m^2); compressive strength of concrete, f'_c = 4,000 psi (27.6 MPa) and yield strength of steel, f_y = 40 ksi (276 MPa). Assume 18 in. (458 mm) square columns.

The solution is as follows:

- From ACI 318-71 [4], for an interior panel, thickness of slab, $h = \ell_n (800 + 0.005 f_y)/36000 = \ell_n/36 = (24 - 1.5) \times 12/36 = 7.5$ in. (190 mm). Therefore, self-weight ≃ 90 psf (4, 311 N/m^2). Assuming that the weight of partitions, etc. is 10 psf (479 N/m^2), total dead load is 100 psf (4,790 N/m^2).

Using the load factors given in ACI 318-71, ultimate load, w = (1.7 x 60) +(1.4 x 100) = 242 psf (11,600 N/m^2). Assuming an average effective cover of 1.25 in (32 mm), the effective depth of slab = 7.5 - 1.25 = 6.25 in. (158 mm).

For similar adjacent panels, from Eq. 2, nominal punching shear stress, $V_u/\phi b_o d$ = 242 (20 x 24 - 2^2)/(0.85 x 4 x 24.25 x 6.25) = 224 psi (1.55 MPa) which is less than the allowable value of $4\sqrt{f'_c}$, or 253 psi (1.75 MPa).

- If m and m' are positive and negative moments of resistance of reinforcing bars in the direction in which moments are being computed, then for m'/m = 1.5, the yield line method [6] gives $m = w\ell_n^2/20$ and $m' = w\ell_n^2/13.3$ where ℓ_n= length of clear span in the direction moments are being determined. Therefore, total moments are given by $M = m\ell_2$ and $M' = m'\ell_2$, where ℓ_2 is the length of span transverse to ℓ_n.

In this example, for long span-direction, total positive moment = $242(24-1.5)^2$ 20/(20 x 1000) = 122 kip-ft (167 kN m), and total negative moment = 242 $(24-1.5)^2$ 20/(13.3 x 1000) = 184 kip-ft (250 kN m). For short-span direction total positive moment = 242 $(20 - 1.5)^2$ 24/(20 x 1000) = 102 kip-ft (138 kN m), and total negative moment = 242 $(20 - 1.5)^2$ 24/(13.3 x 1000) = 153 kip-ft (208 kN m).

These moments are distributed between column and middle strips according to CEB Recommendations [6]. The results are given in Table 1, wherein the areas of tension reinforcement required to resist these computed moments are also given. For this layout of reinforcement, fan mode of failure around columns is found to be not critical.

TABLE 1 - DESIGN EXAMPLE

Moment of Resistance, kip-ft							
In Long-span Direction				In Short-span Direction			
Column strips, width = 2 x 5 ft.		Middle strip width = 10 ft		Column strips, width = 2 x 5 ft.		Middle strip, width = 14 ft.	
+ve	-ve	+ve	-ve	+ve	-ve	+ve	-ve
67	138	55	46	56	115	46	38
0.38	0.79	0.30	0.25	0.30	0.66	0.18	0.18

- Assume w_{max} = 0.012 in. (0.3 mm), K = 2.8 x 10^{-5}, f_s = 0.6 x 40 ksi and β = 1.3 and, therefore, λ = 188. Near columns, from Table 1, A_{s1} = 0.79 sq.in. (510 mm^2) and A_{s2} = 0.66 sq.in. (425 mm^2). Therefore, d_c = 1.25 in.(32 mm), from Eq.1, for crack control, $d_b \leq 0.70$ in. (18 mm). If No.4 bars (approx. 12.7 mm in diameter) are selected, the calculated crack width will be about 0.010 in. (0.25 mm).
- For the example slab, long-term deflections after five years at the mid-point of the panel are calculated using the method described in Ref.10. The creep coefficient, C_t and the shrinkage curvature, ϕ_{sh} required in the analysis were computed from the expressions proposed by ACI Committee 209 [1]. The

following data were assumed in the calculations: moist cured concrete; relative humidity = 60%, minimum thickness of member = 7.5 in. (190 mm); age when props removed = 14 days; age when non-structural elements attached = 90 days; age when a part of live load, if any, applied as a sustained load = 365 days; modulus of elasticity of concrete, E = 3.6×10^6 psi (2.5×10^4 MPa); and no compression reinforcement.

The results obtained from the deflection analysis are summarized in Table 2. The extent of live load that should be considered as part of sustained load would depend on the function of the slab. For example, if the slab is part of a car-park, the sustained load will mostly be dead load only. On the other hand, if the slab is the floor of a records room of an office building, most of the live load should be considered as part of the sustained load. In Table 2, computed deflections are given for three values of the percentage of live load as part of sustained load.(See Appendix for sample calculations).

TABLE 2 - RESULTS OF DEFLECTION ANALYSIS FOR EXAMPLE SLAB

Portion of Total Deflection.	Computed Deflection after five years, in inches. Percentage of Live Load Included in the Sustained Load			Limiting Deflection, in inches.
	Zero	20	80	
Incremental part which occurs after attachment of non-structural elements.	0.63	0.68	0.83	0.30*, 0.67** (Ref.2) 0.60 (Ref.4) 0.58 (Ref.11)
Total Deflection	1.64	1.69	1.84	0.96 (Czech Code) 1.20 (Ref.2)

Note: 1 inch = 25.4 mm. * masonry walls. ** plaster ceiling.

Two types of deflection require attention: (1) the incremental part of total deflection which occurs after the attachment of non-structural elements should be limited for proper functioning of such elements; and (2) the total deflection should be accommodated in order to ensure serviceability of the slab. The last column of Table 2 gives the limiting deflections recommended by various authorities. With respect to incremental deflection, the example slab reasonably satisfies the limits, except when the non-structural element is a masonry wall and when 80 percent of the live load is part of the sustained load. On the other hand, the total deflection is in excess of the limiting value in every case and, therefore, need to be corrected by camber.

6. CONCLUSIONS

Serviceability design of flat plates and slabs requires more attention than it is being given in current practice. In the limit-states design method outlined in the paper, it is proposed that, *in addition to* choosing span-to-thickness ratios from code provisions, long-term deflections are computed using a simple method described in Ref.10 so that they can be given proper consideration in design.

The design method uses Eq.1 for proper crack control at regions close to the columns.

To satisfy the limit-state for strength, yield-line method and the punching shear formula (Eq.2) are used. If the aim is to simplify the strength design of flat slabs, then the yield-line method seems to be an attractive alternative to the elastic methods described in the codes. The total panel moments calculated by the yield-line method may be distributed to the column and middle strips using CEB Recommendations [6].

The proposed limit-states design method is illustrated by a numerical example.

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APPENDIX

The results of the deflection analyses for the example slab given in Table 2 will be illustrated by the following sample calculation:

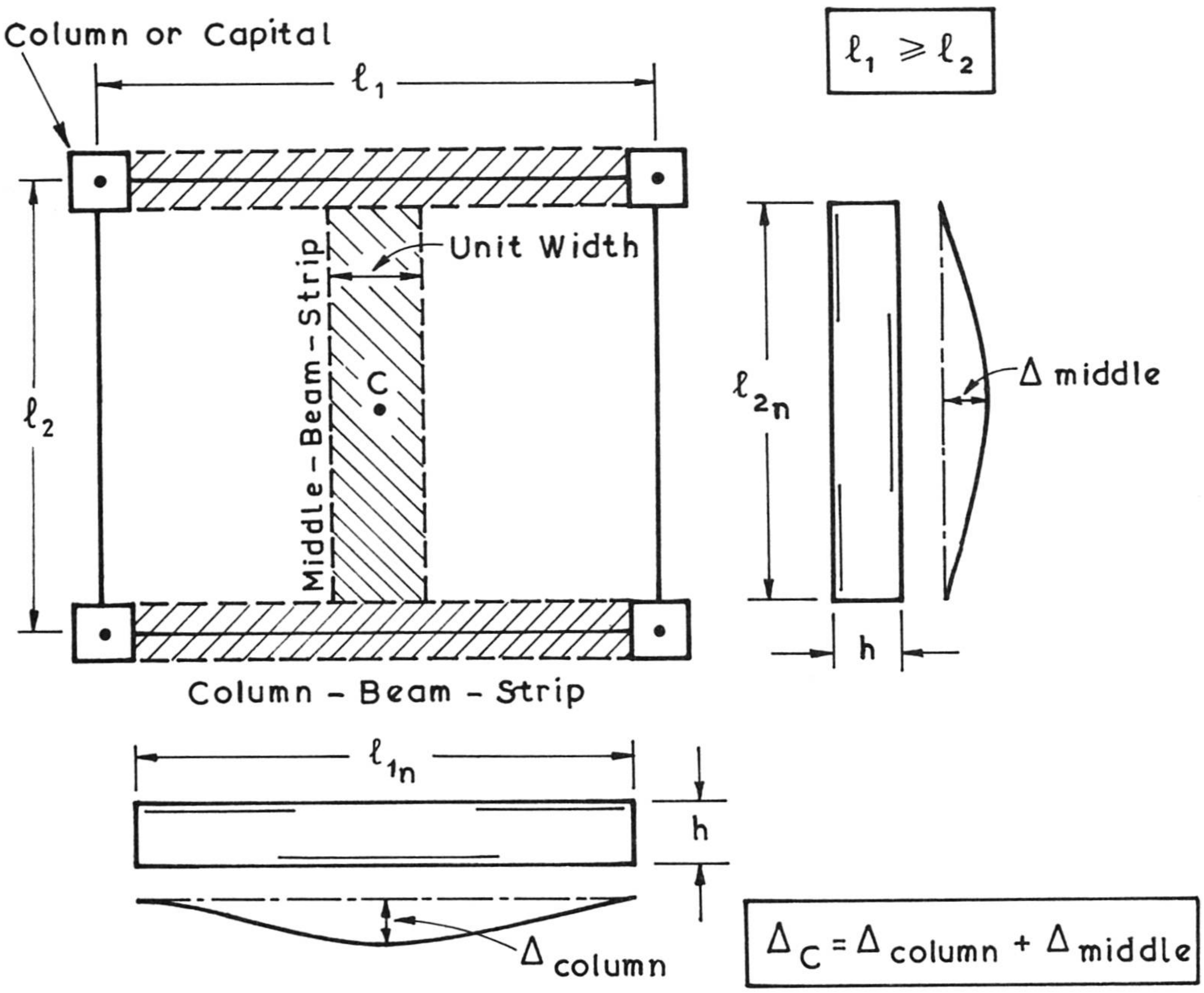

Fig. 1 Model for Calculation of Long-Term Deflections of Flat Plates

The long-term deflection calculation method described in Ref.10 is based on the model shown in Figures 1 and 2. In this method, the column-beam-strip is assumed to be *fully* cracked, that is, second moment of area, $I = I_{cr}$ and the middle-beam-strip is considered to be only partially cracked, i.e.

$$I = 0.5\ (I_g + I_{cr})$$

where I_g is the second moment of area of gross concrete section. Also, the average value of I is taken as [4,11]

$$I_{avg} = I_{mid}\left[1 - \left(\frac{M_e}{M_o}\right)^2\right] + I_{end}\left(\frac{M_e}{M_o}\right)^2 \qquad (3)$$

where M_e is the average of end moments and M_o is the total static moment equal to the sum of M_e and the midspan moment, M_m. The value of M_e/M_m is obtained from the design for strength, i.e. $(M_e/M_m) = (\rho_e/\rho_m) \leq 2$, where ρ_e and ρ_m are respectively, the average tensile steel ratios at the end spans and the midspan of the beam-strips shown in Figures 1 and 2.

For the example slab, using the data given in Table 1, for the column-beam-strip in the long direction (Figure 1), $I_{mid} = 82$ in^4/ft.width, $I_{end} = 141$ in^4/ft. width.Also, $M_e/M_m = (0.79/0.38) \leq 2$, or, equal to 2 and therefore $M_e/M_o = 2/3$.

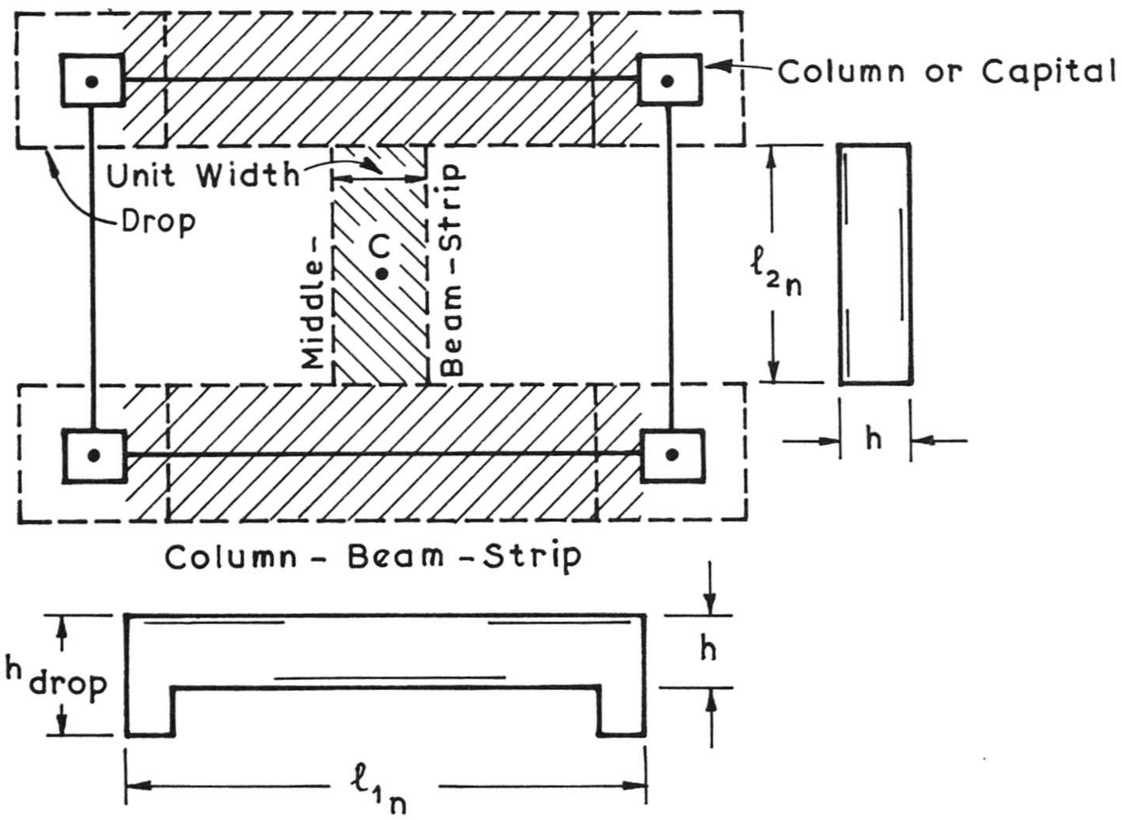

Fig. 2 Model for Calculation of Long-Term Deflections of Flat Slabs

Then, from Eq.3, $I_{avg} = 82\,[1 - (\frac{2}{3})^2] + 141\,(\frac{2}{3})^2 = 108\ in^4/ft$. For the middle beam-strip in the short direction, $I_g = 422$ and $I_{cr} = 44$, and therefore $I_{avg} = 0.5\,(422 + 44) = 233\ in^4/ft$.

The long-term deflections will be calculated for the case when the sustained load includes no part of the live load (Table 2).

Let us first calculate the *total* deflection Δ_{total} at the age of 90 days when non-structural elements will be installed. Using the data given by ACI Committee 209 [1], the creep coefficient C_t and the shrinkage strain ε_{sh} are computed as $C_t = 1.10$ and $\varepsilon_{sh} = 438 \times 10^{-6}$.

In order to calculate Δ_{column} (Figure 1), the elastic deflection is given by

$$\Delta_e = \frac{5}{384}\,\frac{W\ell^3}{EI}\left[\frac{1 - 0.2\,(M_e/M_m)}{1 + (M_e/M_m)}\right] \qquad (4)$$

$$= \frac{5}{384}\,\frac{(90 \times 1.5 \times 24)\,(22.5 \times 12)^3}{(3.6 \times 10^6)\,(108 \times 1.5)}\left[\frac{1 - (0.2 \times 2)}{1 + 2}\right]$$

$$= 0.28\ \text{in.}$$

The creep deflection Δ_{cp} is given by

$$\Delta_{cp} = k_r \, C_t \, \Delta_e \tag{5}$$

in which

$$k_r = [0.85 - 0.45\ (\rho'/\rho)] \geq 0.4 \tag{6}$$

and ρ' is the compression steel ratio [1]. Therefore,

$$\Delta_{cp} = 0.85 \times 1.10 \times 0.28 = 0.26 \text{ in.}$$

The shrinkage deflection Δ_{sh} is given by

$$\Delta_{sh} = \alpha \, \phi_{sh} \, \ell^2 \tag{7}$$

in which

$$\phi_{sh} = 0.7 \ \frac{\varepsilon_{sh}}{h} \ (\rho-\rho')^{1/3} \ \left(\frac{\rho-\rho'}{\rho}\right)^{\frac{1}{2}} \tag{8}$$

$\alpha = 1/16$ for continuous spans, and h is the thickness of slab [1].
Therefore,

$$\Delta_{sh} = \frac{1}{16} \times 0.7 \times \frac{438 \times 10^{-6}}{7.5} \ (0.51)^{1/3} \ (1)^{\frac{1}{2}} \ (22.5 \times 12)^2$$

$$= 0.15 \text{ in.}$$

Then,

$$\Delta_{column} = \Delta_e + \Delta_{cp} + \Delta_{sh} \tag{9}$$

$$= 0.28 + 0.26 + 0.15 = 0.69 \text{ in.}$$

By similar calculations, we obtain

$\Delta_{middle} = 0.12 + 0.11 + 0.08 = 0.31$ in. and hence

Δ_{total} at the age of 90 days is given by $(\Delta_{total})_{90} = 0.69 + 0.31 = 1.00$ in.

Performing similar computations, we obtain Δ_{total} at the age of 1825 days (5 years) as $(\Delta_{total})_{1825} = 1.63$ in. Note that this total deflection includes the instantaneous deflection due to live load applied at the age of 365 days (one year).

The incremental part of total deflection which occurs after the attachment of non-structural elements, Δ_{incre} is therefore given by

$$\Delta_{incre} = (\Delta_{total})_{1825} - (\Delta_{total})_{90}$$

$$= 1.63 - 1.00 = 0.63 \text{ in.}$$

The other results given in Table 2 are obtained by similar calculations.